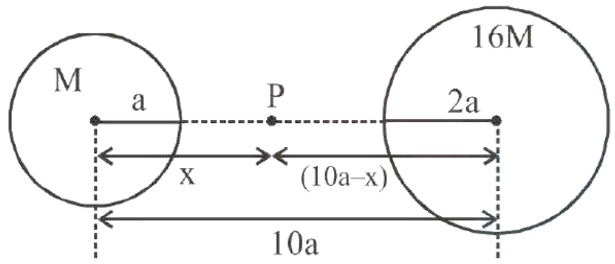
1. (b)



$$\frac{GM}{x^2}=\frac{G(16M)}{(10a-x)^2}$$

$$\frac{1}{x}=\frac{4}{(10a-x)} \Rightarrow 4x=10a-x$$

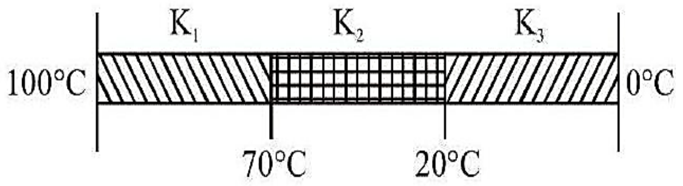
x = 2a

COME

$$-\frac{GMm}{8a}-\frac{G(16M)m}{2a}+KE$$
$$=-\frac{GMm}{2a}-\frac{G(16M)m}{8a}$$
$$KE=GMm\left[\frac{1}{8a}+\frac{16}{2a}-\frac{1}{2a}-\frac{16}{8a}\right]$$

$$\mathrm{KE}=\mathrm{GMm}\left[\frac{1+64-4-16}{8\mathrm{a}}\right]$$
$$\frac{1}{2}\mathrm{mv}^2=\mathrm{GMm}\left[\frac{45}{8\mathrm{a}}\right]$$
$$\mathrm{V}=\sqrt{\frac{90\mathrm{GM}}{8\mathrm{a}}}$$
$$\mathrm{V}=\frac{3}{2}\sqrt{\frac{5\mathrm{GM}}{\mathrm{a}}}$$

2. (a)



Rods are identical have same length (ℓ) and area of cross-section (A)

Combination are in series, so heat current is same for all Rods

$$\left(\frac{\Delta Q}{\Delta t}\right)_{AB}=\left(\frac{\Delta Q}{\Delta t}\right)_{BC}=\left(\frac{\Delta Q}{\Delta t}\right)_{CD}=\text{Heat current}$$

$$\frac{(100-70)K_1\,A}{\ell}=\frac{(70-20)K_2\,A}{\ell}=\frac{(20-0)K_3\,A}{\ell}$$

$30\ K_1 = 50\ K_2 = 20\ K_3$

$3\ K_1 = 2\ K_3$

$\frac{K_1}{K_3} = \frac{2}{3} = 2:3$

$5\ K_2 = 2\ K_3$

$\frac{K_2}{K_3} = \frac{2}{5} = 2:5$

3. (b)

4. (d)

$R_1 = R_2 = R$

Power (P)

Refractive index is assume (μ_ℓ)

$P = \frac{1}{f} = (\mu_\ell - 1)\left(\frac{2}{R}\right)$

$R'/$

$P' = \frac{1}{f'} = (\mu_\ell - 1)\left(\frac{1}{R'}\right)$

$P' = \frac{3}{2}P$

$(\mu_\ell - 1)\left(\frac{1}{R'}\right) = \mu\frac{3}{2}(\mu_\ell - 1)\left(\frac{2}{R}\right)$

$\therefore\ R' = \frac{R}{3}$

5. (a)

6. (c) $\frac{dv_x}{dt} = \frac{k}{m}v_y$

$\frac{dv_y}{dt} = \frac{k}{m}v_x$

$\frac{dv_y}{dv_x} = \frac{v_x}{v_y} \Rightarrow \int v_y dv_y = \int v_x dv_x$

$v_y^2 = v_x^2 + C$

$v_y^2 - v_x^2 =$ constant

Option (c)

$\vec{v} \times \vec{a} = (v_x\hat{\imath} + v_y\hat{\jmath}) \times \frac{k}{m}(v_y\hat{\imath} + v_x\hat{\jmath})$

$= (v_x^2\hat{k} - v_y^2\hat{k})\frac{k}{m}$

$= (v_x^2 - v_y^2)\frac{k}{m}\hat{k}$

= Constant

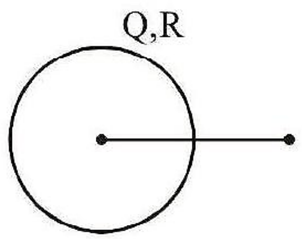
7. (a) Inside the shell

E = 0

hence F = 0

Oustside the sheell

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$



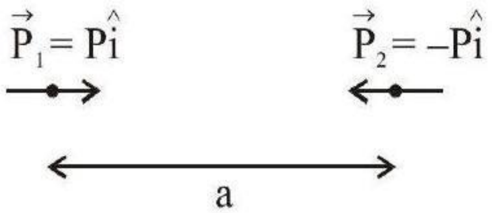
hence $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ for r > R

8. (a) Only in case-I, $M_{LHS} > M_{RHS}$ i.e.

total mass on reactant side is greater then that on the product side. Hence it will only be allowed.

9. (c) Using energy conservation:

$KE_i + PE_i = KE_f + PE_f$

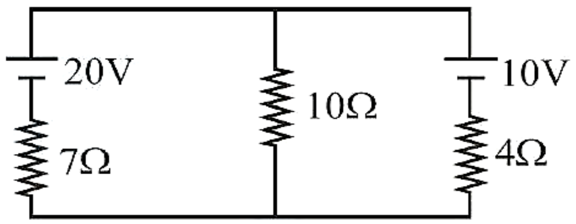


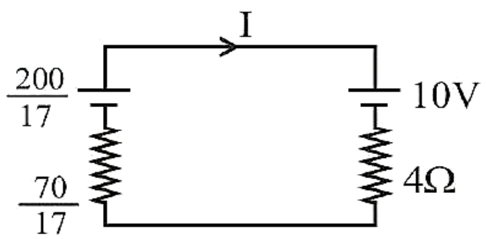
$$0 + \frac{2KP}{a^3} \times P = \frac{1}{2} mv^2 \times 2 + 0$$

$$V = \sqrt{\frac{2P^2}{4\pi\varepsilon_0 a^3\ m}} = \frac{P}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 am}}$$

10. (c) $E_{eq} = \frac{20\times10}{17} = \frac{200}{17}$

and $R_{eq} = \frac{7\times10}{17} = \frac{70}{17}$

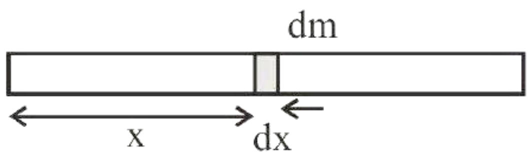




$$\therefore\ I = \frac{\frac{20}{17} - 10}{4 + \frac{70}{17}} = 0.21\ A$$

from $+ve$ to $-ve$ terminal

11. (d)



$$I = \int r^2 dm = \int x^2 \lambda dx$$

$$I = \int_0^L x^2 \lambda_0 \left(1 + \frac{x}{L}\right) dx$$

$$I = \lambda_0 \int_0^L \left(x^2 + \frac{x^3}{L}\right) dx$$

$$I = \lambda\left[\frac{L^3}{3} + \frac{L^3}{4}\right]$$

$$I = \frac{7\ L^3 \lambda_0}{12}$$

$M = \int_0^L \lambda dx = \int_0^L \lambda_0 \left(1 + \frac{x}{L}\right) dx$

$M = \lambda_0 \left(L + \frac{L}{2}\right) = \lambda_0 \frac{3\,L}{2}$

$\frac{2}{3} M = (\lambda_0\ L)$

From (i) & (ii)

$I = \frac{7}{12}\left(\frac{2}{3}M\right) L^2 = \frac{7ML^2}{18}$

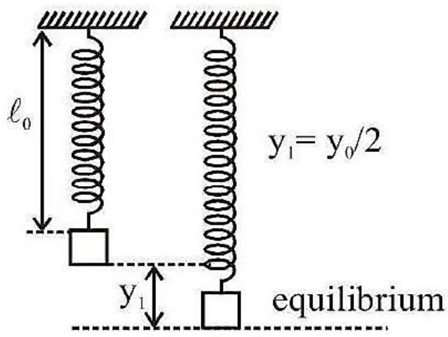
12. (c) Use significant figures. Answer must be upto three significant figures.

13. (b) $y = y_0 \sin^2 \omega t$

$y = \frac{y_0}{2}(1 - \cos 2\omega t)$

$y - \frac{y_0}{2} = -\frac{y_0}{2} \cos 2\omega t$

Amplitude : $\frac{y_0}{2}$



$\frac{y_0}{2} = \frac{mg}{K}$

$2\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{2\,g}{y_0}}$

$\omega = \sqrt{\frac{g}{2y_0}}$

14. (c) $V_{avg} \propto \sqrt{T}$

t_0: mean time

λ: mean free path

$t_0 = \frac{\lambda}{v_{avg}} \propto \frac{1}{\sqrt{T}}$

15. (b) Applying Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P + \frac{1}{2}\rho v^2 = \frac{P}{2} + \frac{1}{2}\rho V^2$$

$$\frac{2P}{2\rho} + \frac{1}{2}\frac{\rho v^2}{\rho} \times 2 = V^2$$

$$\sqrt{\frac{P}{\rho} + v^2} = V$$

16. (b)

$$v_{rms} = \sqrt{\frac{3KT}{m}}$$

m → mass of one molecule (in kg) = $\frac{\text{molar mass}}{\text{NA}}$

de-Broglie wavelenth,

$$\lambda = \frac{h}{mv}$$

given, $v = v_{rms}$

$$\lambda = \frac{h}{m\sqrt{\frac{3KT}{m}}}$$

$$\lambda = \frac{h}{\sqrt{3KTm}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.38 \times 10^{-23} \times 400 \times \left(\frac{28 \times 10^{-3}}{6.023 \times 10^{-23}}\right)}}$$

$$\lambda = \frac{6.63 \times 10^{-11}}{2.77} = 2.39 \times 10^{-11} \text{ m}$$

$$\lambda = 0.24\text{Å}$$

17. (d) $\vec{v}_{01} = (\sqrt{3}\hat{\imath} + \hat{\jmath})\text{m/s}$

$\vec{v}_{02} = \vec{0}$

$m_1 = 2\, m_2$

After collision, $\vec{v}_1 = (\hat{\imath} + \sqrt{3}\hat{\jmath})\text{m/s}$

$\vec{v}_2 = ?$

Applying conservation of linear momentum,

$$m_1\vec{v}_{01} + m_2\vec{v}_{02} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$2\ m_2(\sqrt{3}\hat{\imath} + j) + 0 = 2\ m_2(\hat{\imath} + \sqrt{3}\hat{\jmath}) + m_2\vec{v}_2$$

$$\vec{v}_2 = 2(\sqrt{3}\hat{\imath} + \hat{\jmath}) - 2(\hat{\imath} + \sqrt{3}\hat{\jmath})$$

$$= 2(\sqrt{3}\hat{\imath} - j) + 2(\hat{\imath} - \sqrt{3}\hat{\jmath})$$

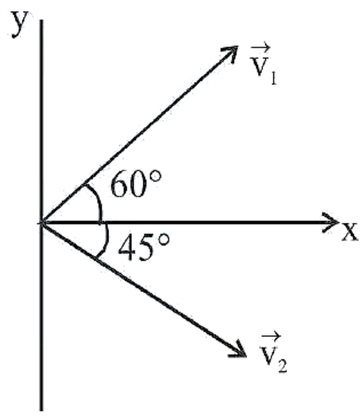
$$\vec{v}_2 = 2(\sqrt{3} - 1)(\hat{\imath} - \hat{\jmath})$$

for angle between $\vec{v}_1 \& \vec{v}_2$,

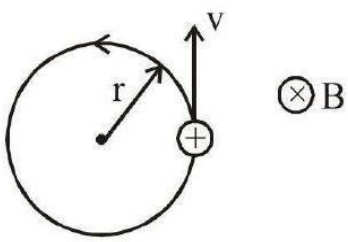
$$\cos\theta = \frac{\vec{v}_1, \vec{v}_2}{\vec{v}_1\vec{v}_2} = \frac{2(\sqrt{3} - 1)(1 - \sqrt{3})}{2 \times 2\sqrt{2}(\sqrt{3} - 1)}$$

$$\cos\theta = \frac{1 - \sqrt{3}}{2\sqrt{2}} \Rightarrow \theta = 105^\circ$$

or



18. (d)



Magnetic moment

$$M = iA$$

$$M = \left(\frac{q}{T}\right) \times \pi r^2 = \frac{q\pi r^2}{\left(\frac{2\pi r}{v}\right)} = \frac{qvr}{2}$$

$$M = \frac{qv}{2} \times \frac{vm}{qB}$$

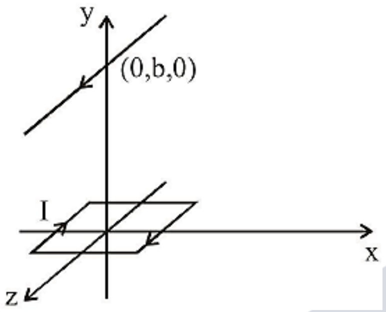
$M = \frac{mv}{}v^{2\ B}$

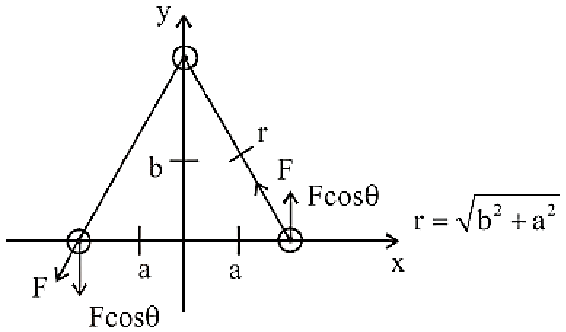
As we can see from the figure, direction of magnetic moment (M) is opposite to magnetic field.

$$\vec{M} = -\frac{mv^2}{2\ B}\hat{B}$$

$$= -\frac{mv^2}{2B^2}\vec{B}$$

19. (a)





$$F = BI2a = \frac{\mu_0 I}{2\pi r} I \times 2a$$

$$F = \frac{\mu_0 I^2 a}{\pi\sqrt{b^2 + a^2}}$$

$$\tau = F\cos\theta \times 2a$$

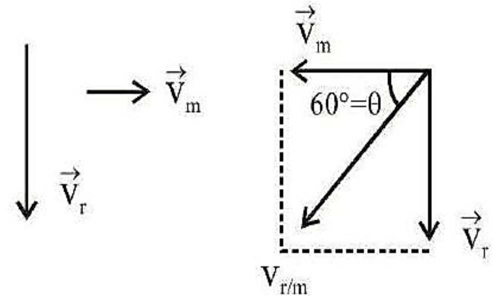
$$= \frac{\mu_0 I^2 a}{\pi\sqrt{b^2 + a^2}} \times \frac{b}{\sqrt{b^2 + a^2}} \times 2a$$

$$\tau = \frac{2\mu_0 I^2 a^2 b}{\pi(a^2 + b^2)}$$

If $b > a$ then $\tau = \frac{2\mu_0 I^2 a^2}{\pi b}$

20. (d) Rain is falling vertically downwards.

$$\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m$$



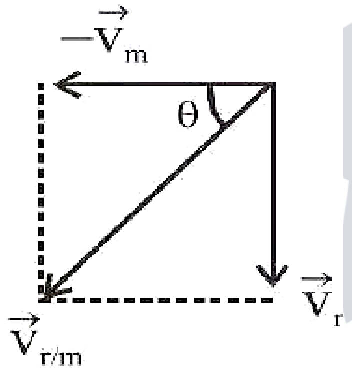
$\tan 60° = \frac{v_r}{v_m} = \sqrt{3}$

$v_r = v_m\sqrt{3} = v\sqrt{3}$

Now, $v_m = (1 + B)v$

and $\theta = 45°$

$\tan 45 = \frac{v_r}{v_m} = 1$



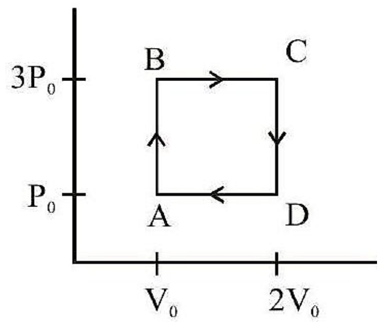
$v_r = v_m$

$v\sqrt{3} = (1 + \beta)v$

$\sqrt{3} = 1 + \beta$

$\Rightarrow \beta = \sqrt{3} - 1 = 0.73$

21. (19.00)



$W_{ABCDA} = 2P_0\, V_0$

$Q_{in} = Q_{AB} + Q_{BC}$

$Q_{AB} = nC(T_B - T_A)$

$= \frac{n3R}{2}(T_B - T_A)$

$= \frac{3}{2}(P_BV_B - P_AV_A)$

$= \frac{3}{2}(3P_BV_0 = P_0V_0) = 3P_0V_0$

$Q_{BC} = nC_P(T_C - T_B)$

$= \frac{n5R}{2}(T_C - T_B)$

$= \frac{5}{2}(P_CV_C - P_BV_B)$

$= \frac{5}{2}(6P_0V_0 - 3P_0V_0) = \frac{15}{2}P_0V_0$

$\eta = \frac{W}{Q_{in}} \times 100 = \frac{2P_0V_0}{3P_0V_0 + \frac{15}{2}P_0V_0} \times 100$

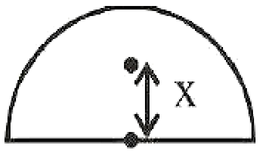
$\eta = \frac{400}{21} = 19.04 \approx 19$

$\eta = 19$

22. (3)

$x = \frac{3R}{8} = 3\text{ cm}$

$x = 3$



23. (150)

$\Delta I_B = (30 - 20) = 10\mu A$

$\Delta I_C = (4.5 - 3)mA = 1.5\text{ mA}$

$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{1.5\text{ mA}}{10\mu A} = 150$

$\beta_{ac} = 150$

24. (400)
25. (9.00)

$I_{max} = k$

$I_1 = I_2 = K/4$

$\Delta x = \lambda/6 \Rightarrow \Delta\phi = \pi/3$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$I = \frac{K}{4} + \frac{K}{4} + 2 \times \frac{K}{4}\frac{1}{2}$

$= \frac{K}{2} + \frac{K}{4} = \frac{3K}{4} = \frac{9K}{12}$

$n = 9$

26. (b) $N_2 + 3H_2 \rightleftharpoons 2NH_3 \to K_C = 64$

$2NH_3 \rightleftharpoons N_2 + 3H_2 \to K_C = \frac{1}{64}$

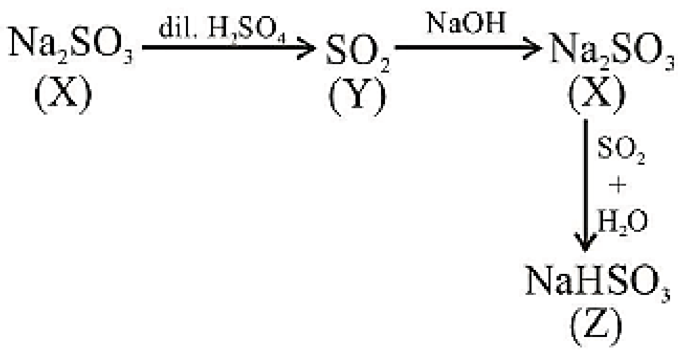
$NH_3 \rightleftharpoons \frac{1}{2} N_2 + \frac{3}{2} H_2 \to K_C = \left(\frac{1}{64}\right)^{1/2} = \frac{1}{8}$

27. (b) Impure zinc is refined by distillation method.

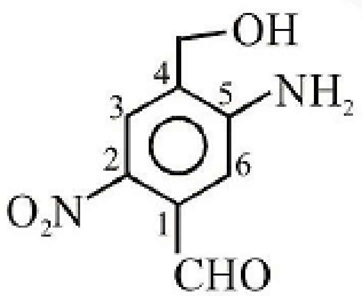
28. (b)

29. (a) Alloys of lanthanides with Fe are called Misch metal, which consists of a lanthanoid metal ($\sim$ 95%) and iron ($\sim$ 5%) and traces of S, C, Ca and Al.

30. (b)



31. (d)



32. (a) High purity ($> 99.95\%$) dihydrogen is obtained by electrolysing warm aqueous barium hydroxide solution between nickel electrodes.

33. (a)

34. (d) $2NO + N_2O_4 \xrightarrow{250\ K} 2\ N_2O_3$ blue solid

35. (d) $\Delta G = -nFE_{cell}$

ΔG is negative, if E_{cell} is positive

Anode : $Cu(s) \longrightarrow Cu^{+2}(C_1) + 2e^- : E^\circ$

Cathode: $Cu^{+2}(C_2) + 2e^- \longrightarrow Cu(S) : -E^\circ$

Cell reaction : $Cu^{+2}(C_2) \longrightarrow Cu^{+2}(C_1) E^\circ_{scl} = 0$

$$E_{cell} = E^\circ_{cell} - \frac{2.303RT}{nF} \log Q$$

$$E_{cell} = 0 - \frac{2.303RT}{nF} \log\left(\frac{C_1}{C_2}\right)$$

$E_{cell} > 0 : \text{if } \frac{C_1}{C_2} < 1 \Rightarrow C_1 < C_2$

36. (a) O^{-2} ions form ccp. $\underset{(-8 \text{ charge})}{\underset{\downarrow}{O_4}}$

$M_1 = 50\%$ of O.V. $\Rightarrow \frac{50}{100} \times 4 = 2 : (M_1)_2$

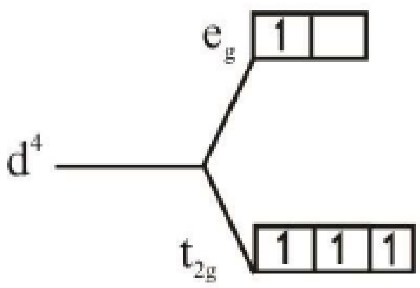
$M_2 = 12.5\%$ of T.V. $\Rightarrow \frac{12.5}{100} \times 8 = 1 : (M_2)_1$

So formula is : $(M_1)_2(M_2)_1O_4$

This must be neutral. Both metals must have +8 charge in total.

From given options: $\begin{Bmatrix} \text{O.N. of } M_1 = +2 \\ M_2 = +4 \end{Bmatrix}$

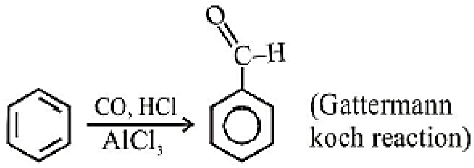
37. (c)

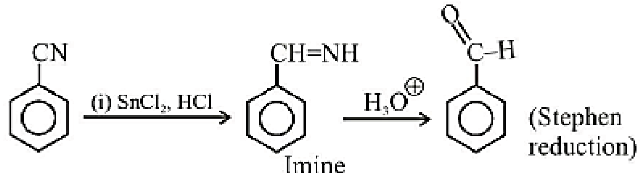


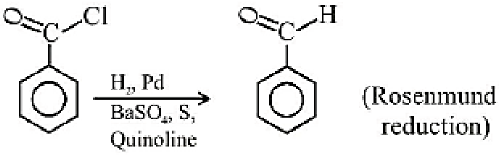
back pairing is not possible because pairing energy $> \Delta_O$.

38. (a)

39. (a)

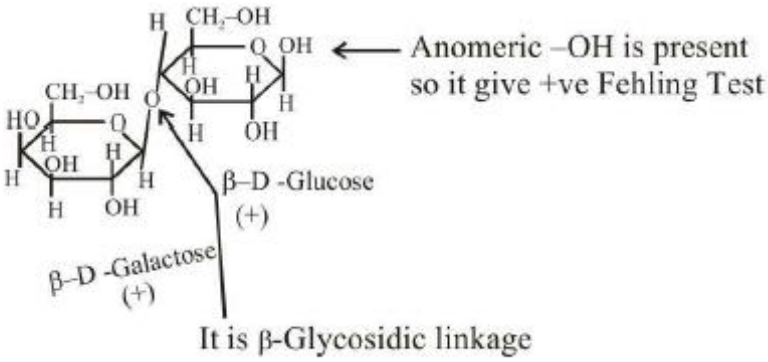
Sol.(i) 

(ii) 

(iii) 

40. (d)

41. (a)



Structure of Lactose

structure of lactose

42. (a) Relative lowering of V.P. $= \frac{\Delta P}{P^0} = x_{solute}$

$$\left(\frac{\Delta P}{P^0}\right)_A = \frac{\frac{10}{100}}{\frac{10}{100}+\frac{180}{18}} : \left(\frac{\Delta P}{P^0}\right)_B = \frac{\frac{10}{200}}{\frac{10}{200}+\frac{180}{18}}$$

$$\left(\frac{\Delta P}{P^0}\right)_C = \frac{\frac{10}{10{,}000}}{\frac{10}{10{,}000}+\frac{180}{18}} : \left(\frac{\Delta P}{P^0}\right)_A > \left(\frac{\Delta P}{P^0}\right)_B > \left(\frac{\Delta P}{P^0}\right)_C$$

43. (b)

44. (d) (I) $Ca(OH)_2$ is used in white wash

(II) NaCl is used in preparation of washing soda

$$2NH_3 + H_2O + CO_2 \longrightarrow (NH_4)_2CO_3$$
$$(NH_4)_2CO_3 + H_2O + CO_2 \longrightarrow 2NH_4HCO_3$$
$$NH_4HCO_3 + NaCl \longrightarrow NH_4Cl + NaHCO_3(s)$$
$$2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + CO_2 + H_2O$$

(III) $CaSO_4 \cdot \frac{1}{2}H_2O$ (Plaster of Paris) is used for making casts of statues

(IV) $CaCO_3$ is used as an antacid

45. (b)

46. (48) $\frac{x}{m} = KP^{1/n}$

$$\log\left(\frac{x}{m}\right) = \frac{1}{n}\log P + \log K$$

$$\text{slope } = \frac{1}{n} = 2$$

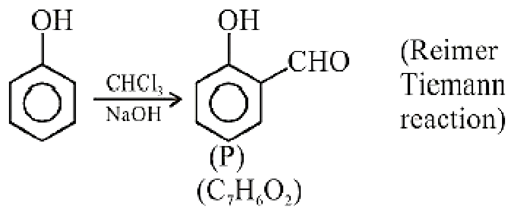
$$\text{intercept} = \log K = 0.4771$$

$$K = 3$$

mass of gas adsorbed per gm of adsorbent $= \frac{x}{m}$

$$\frac{x}{m} = 3 \times (0.04)^2 = 48 \times 10^{-4}$$

47. (69.00)



Molecular weight of $C_7H_6O_2 = 122$

$$\%C = \frac{12 \times 7 \times 100}{122} = 68.85 \approx 69$$

48. (2.00)

49. (100.00)

$$nn\left(\frac{K_{T_2}}{K_{T_1}}\right) = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$T_1 = 303$ K; $T_2 = 313$ K

$$\frac{K_{T_2}}{K_{T_1}} = 3.555$$

$$\ln(3.555) = \frac{E_a}{8.314}\left[\frac{1}{303} - \frac{1}{313}\right]$$

$E_a = 99980.715$

$E_a = 99.98 \frac{kJ}{mole}$

50. (101.00) Unnilunium $\Rightarrow 101$

51. (d) $f(x) = (1 - \cos^2 x)(\lambda + \sin x)$

$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$f(x) = \lambda \sin^2 x + \sin^3 x$

$f'(x) = 2\lambda \sin x \cos x + 3\sin^2 x \cos x$

$f'(x) = \sin x \cos x (2\lambda + 3\sin x)$

$\sin x = 0, \frac{-2\lambda}{3}, (\lambda \neq 0)$

for exactly one maxima & minima

$\frac{-2\lambda}{3} \in (-1,1) \Rightarrow \lambda \in \left(\frac{-3}{2}, \frac{3}{2}\right)$

$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

52. (a)

$f(0) = f(a) = f'(0) = 0$

Apply Rolles theorem on $y = f(x)$ in $x \in [0,1]$

$f(0) = f(a) = 0$

$\Rightarrow f'(\alpha) = 0$ where $\alpha \in (0,1)$

Now apply Rolles theorem on $y = f'(x)$

in $x \in [0, \alpha]$

$f'(0) = f'(\alpha) = 0$ and $f'(x)$ is continuous and differentiable

$\Rightarrow f''(\beta) = 0$ for some,$\beta \in (0, \alpha) \in (0,1)$

$\Rightarrow f''(x) = 0$ for some $x \in (0,1)$

53. (c) $f(x) = x \log_e x$

$f'(x)|_{(c,f(c))} = \frac{e - 0}{e - 1}$

$f'(x) = 1 + \log_e x$

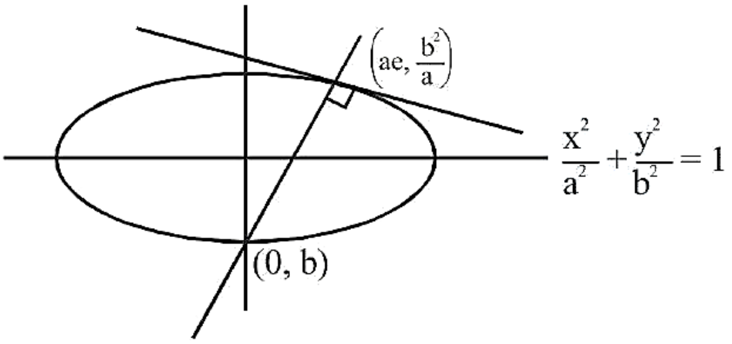
$f'(x)|_{(c,f(c))} = 1 + \log_e c = \frac{e}{e - 1}$

$\log_e c = \frac{e - (e - 1)}{e - 1} = \frac{1}{e - 1} \Rightarrow c = e^{\frac{1}{e-1}}$

54. (b) Contrapositive of $(p \to q)$ is $\sim q \to \sim p$

For an integer n, if n is even then $(n^3 - 1)$ is odd

55. (d)



$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2e^2$$
$$\frac{a^2x}{ae} - \frac{b^2y}{b^2} \cdot a = a^2e^2$$
$$\frac{ax}{e} - ay = a^2e^2 \Rightarrow \frac{x}{e} - y = ae^2$$

passes through $(0, b)$

$-\mathrm{b} = \mathrm{ae}^2 \Rightarrow \mathrm{b}^2 = \mathrm{a}^2\mathrm{e}^4$

$\mathrm{a}^2(1 - \mathrm{e}^2) = \mathrm{a}^2\mathrm{e}^4 \Rightarrow \mathrm{e}^4 + \mathrm{e}^2 = 1$

56. (b) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$A \equiv (a, 0, 0), B \equiv (0, b, 0), C \equiv (0, 0, c)$

Centroid $\equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1,1,2)$

$a = 3, b = 3, c = 6$

Plane : $\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

$2x + 2y + z = 6$

line $\perp$ to the plane (DR of line $= 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$)

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$$

57. (c) α and β are the roots of the equation $4x^2 + 2x - 1 = 0$

$$4\alpha^2 + 2\alpha = 1 \Rightarrow \frac{1}{2} = 2\alpha^2 + \alpha$$

$$\beta = \frac{-1}{2} - \alpha$$

using equation (a)

$\beta = -(2\alpha^2 + \alpha) - \alpha$

$\beta = -2\alpha^2 - 2\alpha$

$\beta = -2\alpha(\alpha + 1)$

58. (c) $z = x + iy$

$z^2 = i|z|^2$

$(x + iy)^2 = i(x^2 + y^2)$

$(x^2 - y^2) - i(x^2 + y^2 - 2xy) = 0$

$(x - y)(x + y) - i(x - y)^2 = 0$

$(x - y)((x + y) - i(x - y)) = 0$

$\Rightarrow x = y$

z lies on $y = x$

59. (b) $a_1, a_2, \ldots, a_n \rightarrow (CD = d)$

$b_1,\ b_2, \ldots, b_m \rightarrow (CD = d + 2)$

$a_{40} = a + 39d = -159$

$a_{100} = a + 99d = -399$
Subtract : $60\ d = -240 \Rightarrow d = -4$

using equation (a)

$a + 39(-4) = -159$

$a = 156 - 159 = -3$

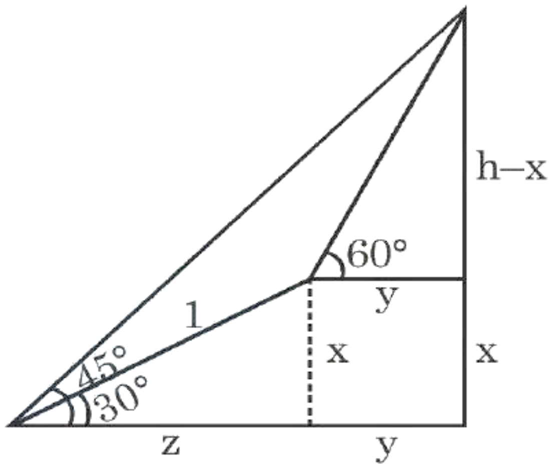
$a_{70} = a + 69d = -3 + 69(-4) = -279 = b_{100}$

$b_{100} = -279$

$b_1 + 99(\ d + 2) = -279$

$b_1 - 198 = -279 \Rightarrow b_1 = -81$

60. (a)



$\sin 30^\circ = x \Rightarrow x = \frac{1}{2}$

$\cos 30^\circ = z \Rightarrow z = \frac{\sqrt{3}}{2}$

$\tan 45^\circ = \frac{h}{y+z} \Rightarrow h = y + z$

$\tan 60^\circ = \frac{h-x}{y} \Rightarrow \tan 60^\circ = \frac{h-x}{h-z}$

$\sqrt{3}(h-z) = h - x$

$(\sqrt{3}-1)h = \sqrt{3}z - x$

$\Rightarrow (\sqrt{3}-1)h = \frac{3}{2} - \frac{1}{2}$

$\Rightarrow (\sqrt{3}-1)h = 1$

$h = \frac{1}{\sqrt{3}-1}$

61. (b)

$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$B = A + A^4$

$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

$B = \begin{bmatrix} (\cos\theta + \cos 4\theta) & (\sin\theta + \sin 4\theta) \\ -(\sin\theta + \sin 4\theta) & (\cos\theta + \cos 4\theta) \end{bmatrix}$

$|B| = (\cos\theta + \cos 4\theta)^2 + (\sin\theta + \sin 4\theta)^2$

$|B| = 2 + 2\cos 3\theta,\ \text{ when } \theta = \frac{\pi}{5}$

$|B| = 2 + 2\cos\frac{3\pi}{5} = 2(1 - \sin 18)$

$$|B| = 2\left(1 - \frac{\sqrt{5}-1}{4}\right) = 2\left(\frac{5-\sqrt{5}}{4}\right) = \frac{5-\sqrt{5}}{2}$$

62. (b) $f(x) = \frac{a-x}{a+x}\ x \in R - \{-a\} \to R$

$$f(f(x)) = \frac{a - f(x)}{a + f(x)} = \frac{a - \left(\frac{a-x}{a+x}\right)}{a + \left(\frac{a-x}{a+x}\right)}$$

$$f(f(x)) = \frac{(a^2-a) + x(a+1)}{(a^2+a) + x(a-1)} = x$$

$\Rightarrow (a^2 - a) + x(a+1) = (a^2+a)x + x^2(a-1)$

$\Rightarrow a(a-1) + x(1-a^2) - x^2(a-1) = 0$

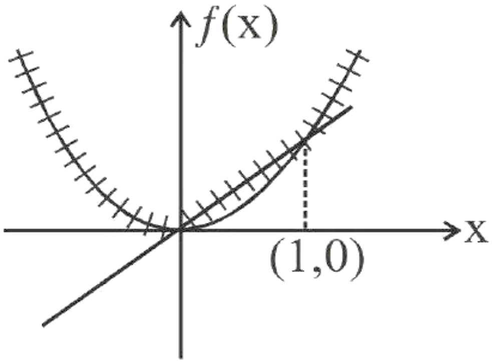
$\Rightarrow a = 1$

$f(x) = \frac{1-x}{1+x}$,

$$f\left(\frac{-1}{2}\right) = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$$

63. (a)

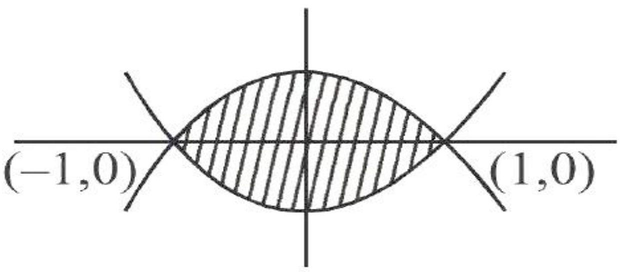
$f(x) = \max(x, x^2)$



Non-differentiable at $x = 0,1$

$S = \{0,1\}$

64. (b) $y = x^2 - 1$ and $y = 1 - x^2$



$A=\int_{-1}^{1}\left((1-x^2)-(x^2-1)\right)dx$

$A=\int_{-1}^{1}(2-2x^2)dx=4\int_{0}^{1}(1-x^2)dx$

$A=4\left(x-\frac{x^3}{3}\right)_0^1=4\left(\frac{2}{3}\right)=\frac{8}{3}$

65. (c) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$0.8=0.6+0.4-P(A\cap B)$
$P(A\cap B)=0.2$

$P(A\cup B\cup C)=\Sigma P(A)-\Sigma P(A\cap B)+P(A\cap B\cap C)$

$\alpha=1.5-(0.2+0.3+\beta)+0.2$

$\alpha=1.2-\beta\in[0.85,0.95]$

(where $\alpha\in[0.85,0.95]$)

$\beta\in[0.25,0.35]$

66. (c) $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$

$T_{r+1}={}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{-k}{x^2}\right)^r$
$T_{r+1}={}^{10}C_r\cdot x^{\frac{10-r}{2}}\cdot(-k)^r\cdot x^{-2r}$
$T_{r+1}={}^{10}C_r x^{\frac{10-5r}{2}}(-k)^r$

Constant term : $\frac{10-5r}{2}=0\Rightarrow r=2$

$T_3={}^{10}C_2\cdot(-k)^2=405$

$k^2=\frac{405}{45}=9$

$k=\pm3\Rightarrow|k|=3$

67. (c) $\int_1^2 e^x\cdot x^x(2+\log_e x)dx$

$\int_1^2 e^x(2x^x+x^x\log_e x)dx$

$\int_1^2 e^x(\underbrace{x^x}_{f(x)}+\underbrace{x^x(1+\log_e x)}_{f'(x)})dx$

$(e^x\cdot x^x)_1^2=4e^2-e$

68. (c)

$L:\frac{x}{3}+\frac{y}{1}=1\Rightarrow x+3y-3=0$

Image of point $(-1,-4)$

$$\frac{x+1}{1}=\frac{y+4}{3}=-2\left(\frac{-1-12-3}{10}\right)$$
$$\frac{x+1}{1}=\frac{y+4}{3}=\frac{16}{5}$$

$(x,y)\equiv\left(\frac{11}{5},\frac{28}{5}\right)$

69. (a) $y=\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x$

$$\frac{dy}{dx}=\frac{2}{\pi}\operatorname{cosec}x-\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x\cot x$$

$$\frac{dy}{dx}=\frac{2\operatorname{cosec}x}{\pi}-y\cot x$$

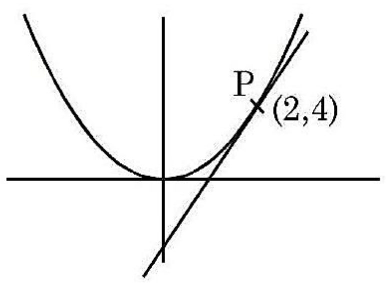
using equation (a)

$$\frac{dy}{dx}+y\cot x=\frac{2\operatorname{cosec}x}{\pi}$$

$$\frac{dy}{dx}+p(x)\cdot y=\frac{2\operatorname{cosec}x}{\pi}\quad x\in\left(0,\frac{\pi}{2}\right)$$

Compare: $p(x)=\cot x$

70. (b)



$y=x^2$

$$\left.\frac{dy}{dx}\right|_P=4$$

$(y-4)=4(x-2)$

$4x-y-4=0$

Circle: $(x-2)^2+(y-4)^2+\lambda(4x-y-4)=0$ passes through $(0,1)$

$$4+9+\lambda(-5)=0\Rightarrow\lambda=\frac{13}{5}$$

Circle : $x^2+y^2+x(4\lambda-4)+y(-\lambda-8)+(20-4\lambda)=0$

Centre : $\left(2-2\lambda,\frac{\lambda+8}{2}\right)\equiv\left(\frac{-16}{5},\frac{53}{10}\right)$

71. (3.00)

$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$

$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$

$2x+(3\lambda+1)y+(3\lambda-3)z=0$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$R_1 \rightarrow R_1 - R_2 \& R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} 0 & 3-\lambda & \lambda-3 \\ \lambda-3 & \lambda-3 & -2(\lambda-3) \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$$(\lambda-3)^2\begin{vmatrix} 0 & -1 & 1 \\ 1 & 1 & -2 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$(\lambda-3)^2[(3\lambda+1)+(3\lambda-1)]=0$

$6\lambda(\lambda-3)^2=0 \Rightarrow \lambda=0,3$

Sum $=3$

72. (5) $f(x+y)=f(x)f(y)$

put $x=y=1$ $f(b)=(f(a))^2=3^2$

put $x=2, y=1$ $f(c)=(f(a))^3=3^3$

Similarly, $f(x)=3^x$

$\sum_{i=1}^{n} f(i)=363 \Rightarrow \sum_{i=1}^{n} 3^i=363$

$(3+3^2+\cdots+3^n)=363$

$$\frac{3(3^n-1)}{2}=363$$

$3^n-1=242 \Rightarrow 3^n=243$

$\Rightarrow n=5$

73. (120) LETTER

vowels = EE, consonant = LTTR

_L_T_T_R –

$$\frac{4!}{2!}\times {}^5C_2 \times \frac{2!}{2!}=12\times 10=120$$

74. (6.00)
75. (1.00)

$|\vec{x}+\vec{y}|=|\vec{x}|$

$\sqrt{|\vec{x}|^2+|\vec{y}|^2+2\vec{x}\cdot\vec{y}}=|\vec{x}|$

$|\vec{y}|^2+2\vec{x}\cdot\vec{y}=0$

Now $(2\vec{x}+\lambda\vec{y})\cdot\vec{y}=0$

$2\vec{x}\cdot\vec{y}+\lambda|\vec{y}|^2=0$

from (a)

$$-|\vec{y}|^2+\lambda|\vec{y}|^2=0$$
$$(\lambda-1)|\vec{y}|^2=0$$

given $|\vec{y}|\neq 0 \ \Rightarrow \lambda=1$