1. **(c)** Weight of the object at the pole, $W = mg = 196$ N

Mass of the object, $m = \frac{W}{g} = \frac{196}{10} = 19.6$ kg

Weight of object at the equator $(W') =$ Weight at pole - Centrifugal acceleration

$$W' = mg - m\omega^2 R$$

$$196 - (19.6)\left(\frac{2\pi}{24 \times 3600}\right)^2 \times 6400 \times 10^3 = 195.33N$$

2. **(b)** Total power consumption of the house (P) $=$ Number of appliances $\times$ Power rating of each appliance

$P = (15 \times 45) + (15 \times 100) + (15 \times 10) + (2 \times 1000) = 4325$ W

So, minimum fuse current $I = \frac{\text{Total power consumption}}{\text{Voltage supply}} = \frac{4325}{220}$ A $= 19.66$ A

3. **(b)** Relaxation time (τ) dependence on volume and temperature can be given by $(\tau) \propto \frac{V}{\sqrt{T}}$ Also, for an adiabatic process,

$$T \propto \frac{1}{V^{\gamma-1}}$$

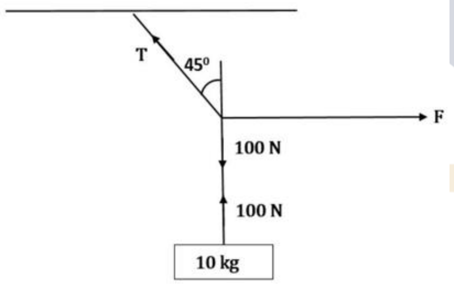
$$\Rightarrow \tau \propto V^{\frac{1+\gamma}{2}}$$

Thus,

$$\frac{\tau_f}{\tau_i} = \left(\frac{2V}{V}\right)^{\frac{1+\gamma}{2}}$$

$$\frac{\tau_f}{\tau_i} = (2)^{\frac{1+\gamma}{2}}$$

4. **(a)** Equating the vertical and horizontal components of the forces acting at point



$$\frac{T}{\sqrt{2}} = 100$$

$$\frac{T}{\sqrt{2}} = F$$

$F = 100$ N

5. **(a)**

$$\sigma = A + Br$$
$$\int dm = \int (A + Br)2\pi r dr$$
$$I = \int dmr^2$$
$$= \int_0^a (A + Br)2\pi r^3 dr$$
$$= 2\pi\left(A\frac{a^4}{4} + B\frac{a^5}{5}\right)$$
$$= 2\pi a^4\left(\frac{A}{4} + \frac{Ba}{5}\right)$$

6. (b) Heat input to 1st engine $= Q_H$

Heat rejected from 1st engine $= Q_L$

Heat rejected from 2nd engine $= Q_L$

Work done by 1st engine = Work done by 2nd engine

$$Q_H - Q_L = Q_L - Q_L$$
$$2Q_L = Q_H + Q_L$$
$$2 = \frac{T_1}{T} + \frac{T_2}{T}$$
$$T = \frac{T_1 + T_2}{2}$$

7. (b) Using the half-life equation,

$$\ln\frac{A_0}{A_t} = \lambda t$$

At half-life, $t = t_{\frac{1}{2}}$ and $A_t = \frac{A_0}{2}$

$$\Rightarrow \ln 2 = \lambda t_{\frac{1}{2}}$$

Also given

$$\ln\frac{500}{700} = \lambda(30)$$

Dividing the equations,

$$\frac{\ln 2}{\ln\left(\frac{7}{5}\right)} = \frac{t_1}{30}$$

$$\Rightarrow t_{\frac{1}{2}} = 61.8 \text{ minutes}$$

8. (a) Given,

Maximum diameter of pipe = 6.4 cm

Minimum diameter of pipe = 4.8 cm

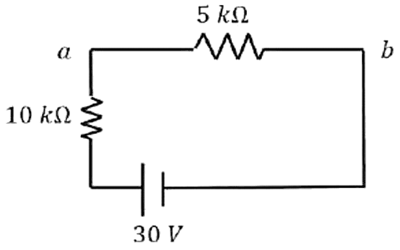
$$\beta = \lambda \frac{D}{d} = \frac{589 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}}$$
$$= 5.9 \text{ mm}$$

9. (b) Using equation of continuity

$$A_1V_1 = A_2V_2$$

$$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \left(\frac{4.8}{6.4}\right)^2 = \frac{9}{16}$$

10. (c) Diode is in forward bias, so it will behave as simple wire. So, the circuit effectively becomes

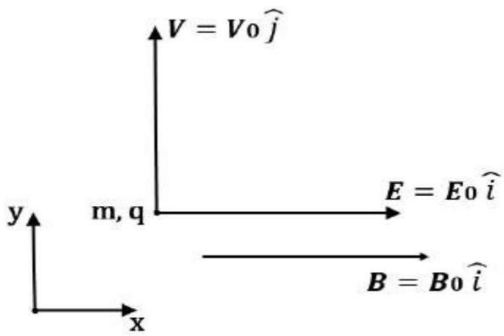


$$V_{ab} = \frac{30}{5+10} \times 5 = 10 \text{ V}$$

11. (a)

As $\vec{V} = V_o\hat{j}$ (magnitude of velocity does not change in y-z plane)

$$(2V_o)^2 = V_o^2 + V_x^2$$
$$V_x = \sqrt{3}V_o$$



$$\therefore \sqrt{3}V_x = 0 + \frac{qE}{m}t \Rightarrow t = \frac{mv_o\sqrt{3}}{qE}$$

12. (a) $f_0\left(\frac{C}{C-V}\right) - f_0\left(\frac{C}{C+V}\right) = 2$

$$V = \frac{1}{4} \text{ m/s}$$

13. (d) λ_d for electron $= \frac{h}{\sqrt{2mE}}$

λ for photon $= \frac{hC}{E}$

Ratio $= \frac{h}{\sqrt{2mE}} \frac{E}{hC} = \frac{1}{C}\sqrt{\frac{E}{2m}}$

14. (a) $\omega = \frac{2\pi}{T} = \frac{\pi}{5}$

When $\omega t = \frac{\pi}{2}$

Then φ_{flux} will be minimum

$\therefore$ e will be maximum

$$t = \frac{\frac{\pi}{2}}{\frac{\pi}{5}} = 2.5\text{sec}$$

When $\omega t = \pi$

Then φ_{flux} will be maximum

$\therefore$ e will be minimum

$$t = \frac{\pi}{\frac{\pi}{5}} = 5\text{sec}$$

15. (b) Force due to electric field is in direction $-\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

Because at $t = 0, E = -\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}E_0$

Force due to magnetic field is in direction $q(\vec{v} \times \vec{B})$ and $\vec{v} \parallel \hat{k}$

$\therefore$ It is parallel to $\vec{E}$

$\therefore$ Net force is antiparallel to $\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}$.

16. (a) $\frac{1}{f_a} = \left(\frac{\mu_g}{\mu_g} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f_m} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\Rightarrow \frac{f_a}{f_m} = \frac{\left(\frac{\mu_g}{\mu_m} - 1\right)}{\left(\frac{\mu_g}{\mu_a} - 1\right)} = \frac{\left(\frac{1.50}{1.42} - 1\right)}{\left(\frac{1.50}{1} - 1\right)} = \frac{0.08}{(1.92)(0.5)}$$
$$\frac{f_m}{f_a} = \frac{(1.42)(0.5)}{0.08} = 8.875 = 9$$

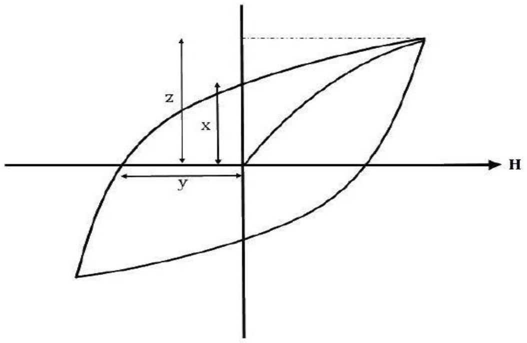
17. (a) Net force on motor will be

$$F_m = [920 + 68(10)]g + 6000$$
$$F_m = 22000N$$

So, required power for motor

$P_m = \overrightarrow{F_m} \cdot \vec{v}$
$= 22000 \times 3$
$= 66000$ W

18. (c)



x = retentivity

y = coercivity

z = saturation magnetization

19. (a)

$$i = i_o\left(1 - e^{\frac{-t}{L/R}}\right)$$
$$= \frac{20}{5}\left(1 - e^{\frac{-t}{0.01/5}}\right)$$
$$= 4(1 - e^{-500t})$$

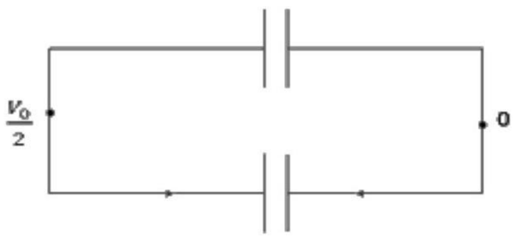
$$i_{40} = 4(1 - e^{-500\times 40}) = 4\left(1 - \frac{1}{(e^2)^{10000}}\right) = 4\left(1 - \frac{1}{7.29^{10000}}\right)$$

$\frac{i_\infty}{i_{40}} \approx 1$ (Slightly greater than one)

20. (a) Energy density in magnetic field $= \frac{B^2}{2\mu_o}$

$$= \frac{\text{Force} \times \text{displacement}}{(\text{displacement})^3} = \frac{MLT^{-2} \cdot L}{(L)^3} = ML^{-1}T^{-2}$$

21. (6)



$V_0 = 20$ V

Initial potential energy $U_i = \frac{1}{2}CV_0^2$

After connecting identical capacitor in parallel, voltage across each capacitor will be

$\frac{V_0}{2}$. Then, final potential energy $U_f = 2\left[\frac{1}{2}C\left(\frac{V_0}{2}\right)^2\right]$

Heat loss $= U_i - U_f$

$$= \frac{CV_0^2}{2} - \frac{CV_0^2}{4} = \frac{CV_0^2}{4} = \frac{60 \times 10^{-12} \times 20^2}{4} = 6 \times 10^{-9} = 6\ \text{nJ}$$

22. (40)

Here, heat absorbed by ice $= m_{ice}L_f + m_{ice}C_w(40-0)$

Heat released by steam $= m_{\text{steam}}\, L_v + m_{\text{steam}}\, C_w(100-40)$

Heat absorbed $=$ heat released

$m_{\text{ice}}\, L_f + m_{\text{ice}}\, C_w(40-0) = m_{\text{steam}}\, L_v + m_{\text{steam}}\, C_w(100-40)$

$\Rightarrow 200 \times 80\text{cal/g} + 200 \times 1\text{cal/g/}\,^\circ\text{C} \times (40-0)$

$= m \times 540\text{cal/g} + 540 \times 1\text{cal/g/}\,^\circ\text{C} \times (100-40)$

$\Rightarrow 200[80 + (40)1] = m[540 + (60)1]$

$m = 40\ \text{g}$

23. (50) F balances kinetic friction so that the block can move

So, $F = \mu mg$

For no toppling, the net torque about bottom right edge should be zero

i.e

$$F\left(\frac{a}{2} + b\right) \le mg\frac{a}{2}$$
$$\mu mg\left(\frac{a}{2} + b\right) \le mg\frac{a}{2}$$
$$F\mu\frac{a}{2} + \mu b \le \frac{a}{2}$$
$$0.2a + 0.4b \le 0.5a$$

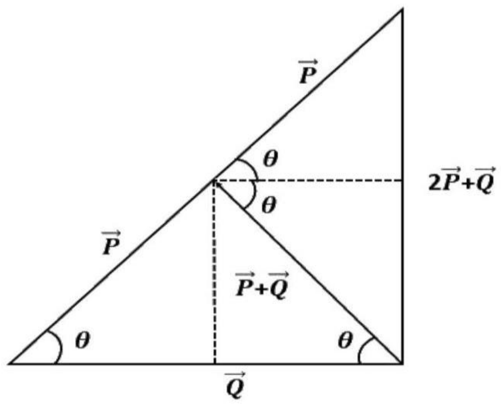
7^{th} *Jan (Shift 2, Physics)*

$$0.4b \le 0.3a$$
$$b \le \frac{3}{4}a$$

But, maximum value of b can only be $0.5a$

$\therefore$ Maximum value of $100\frac{b}{a}$ is 50

24. (90°)



25. (12) Let the emf of cell is ε internal resistance is ' r ' and potential gradient is x.

$\varepsilon = 560x$

After connecting the resistor

$$\frac{\varepsilon \times 10}{10 + r} = 500x$$

From (1) and (2)

$$\frac{560 \times 10}{10 + r} = 500s$$
$$56 = 540 + 5r$$
$$r = \frac{6}{5} = 1.2\Omega$$

n = 12

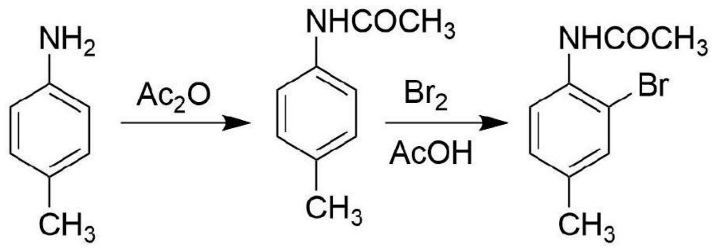
26. (b) In aryl halides, due to the partial double bond character generated by chlorine, the aryl cation is not formed.

Vinyl halides do not give Friedel-Crafts reaction, because the intermediate that is generated (vinyl cation) is not stable.

$H_2C=\overset{+}{C}H$

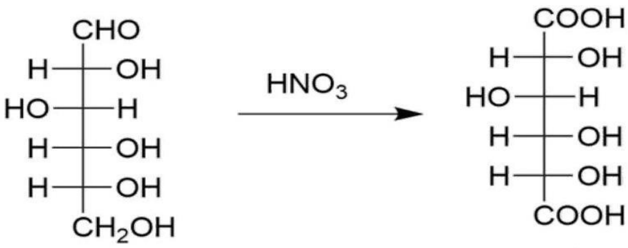
vinyl
cation

27. (a) During trisubstitution, the acetanilide group attached to the benzene ring is more electron donating than the methyl group attached, owing to +M effect, and therefore, the incoming electrophile would prefer ortho w.r.t the acetanilide group.



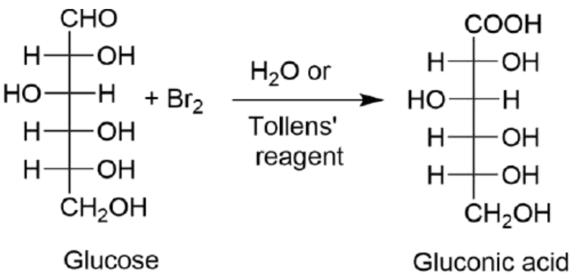
28. (b) The gluconic acid formed is a monocarboxylic acid which is formed during the partial oxidation of glucose

(a) Glucose on reaction with HNO_3 will give glucaric acid:

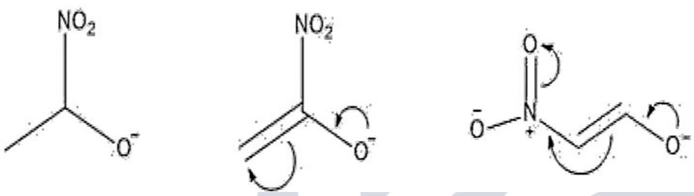


Glucose Glucaric acid

(b) Glucose on partial reduction will give gluconic acid:



29. (a) Higher the delocalization of the negative charge, more will be the stability of the anion.

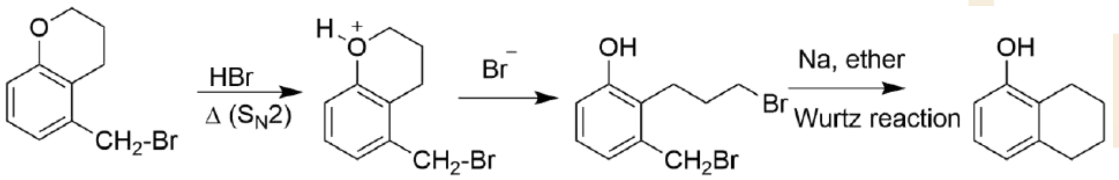


(A) The negative charge is stabilized only through -I effect exhibited by the $-NO_2$ group.

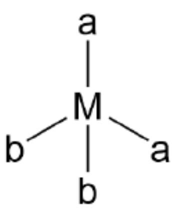
(B) The negative charge is stabilized by the delocalization of the double bond and the -I effect exhibited by the $-NO_2$ group.

(C) The negative charge is stabilized by extended conjugation

30. (c)



31. (c) Case 1: If M is sp^3 hybridized, the geometry will be tetrahedral. There will be a plane of symmetry and thus it does not show optical activity.



Case 2: If M is dsp^2 hybridized, the geometry will be square planar. Due to the presence of a plane of symmetry, it does not show optical activity.

32. (a) $CN^-$is a 14-electron system. The bond order and magnetism can be predicted using MOT.

The MOT electronic configuration of $CN^-$is:

$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\pi_{2p_x}^2\pi_{2p_y}^2\sigma_{2p_z}^2$

Bond order $= \frac{1}{2}\times\left(N_{bonding} - N_{antibonding}\right) = 3$

As CN^-does not have any unpaired electrons, and hence it is diamagnetic.

33. (c) $\Lambda_m^0 NaI - \Lambda_m^0 NaBr = \Lambda_m^0 NaBr - \Lambda_m^0 KBr$

$[\lambda_m^0 Na^+ + \lambda_m^0 I^-] - [\lambda_m^0 Na^+ + \lambda_m^0 Br^-] = [\lambda_m^0 Na^+ + \lambda_m^0 Br^-] - [\lambda_m^0\ K^+ + \lambda_m^0 Br^-]$

$\lambda_m^0 I^- - \lambda_m^0 Br^- \neq \lambda_m^0 Na^+ - \lambda_m^0\ K^+$

34. (a) $6NaOH + 3Cl_2 \rightarrow 5NaCl + NaClO_3 + 3H_2O$

$2Ca(OH)_2 + Cl_2 \rightarrow Ca(OCl)_2 + CaCl_2 + H_2O$

35. (a) Consider beaker I contains the solvent and beaker 2 contains the solution. Let the vapour pressure of the beaker I be P° and the vapour pressure of beaker II be P^S. According to Raoult's law, the vapour pressure of the solvent (P°) is greater than the vapour pressure of the solution (Ps^s) $(P^\circ > Ps^s)$

Due to a higher vapour pressure, the solvent flows into the solution. So volume of beaker II would increase. In a closed beaker, both the liquids on attaining equilibrium with the vapour phase will end up having the same vapour pressure. Beaker II attains equilibrium at a lower vapour pressure and so in its case, condensation will occur so as to negate the increased vapour pressure from beaker I, which results in an increase in its volume. On the contrary, since particles are condensing from the vapour phase in beaker II, the vapour pressure will decrease. Since beaker I at equilibrium attains a higher vapour pressure, there, evaporation will be favoured more so as to compensate for the decreased vapour pressure, as mentioned in the previous statement.

36. (d) Liquation is the process of refining a metal with a low melting point containing impurities of high melting point

37. (c) Decomposition of H_2O_2: $2H_2O_2(l) \rightarrow O_2(g) + 2H_2O(l)$

Industrially, H_2O_2 is prepared by the auto-oxidation of 2-alklylanthraquinols.

$2KClO_3 \xrightarrow{150-300^\circ C} 2KCl + 3O_2$

$2\ Pb(NO_3)_2 \xrightarrow{200-470^\circ C} 2PbO + 4NO_2 + O_2$

$2NaNO_3 \rightarrow 2NaNO_2 + O_2$

Synthesis of sodium perborate:

$Na_2\ B_4O_7 + 2NaOH + 4H_2O_2 \rightarrow 2NaBO_3 + 5H_2O$

38. (a) $N_2 + O_2 \xrightarrow{2000\ K} 2NO$: The oxidation state of N changes from 0 to +2 , and the oxidation state of O changes from 0 to -2

In all the remaining reactions, there is no change in oxidation states of the elements participating in the reaction.

$3O_2 \rightarrow 2O_3$

$2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O$ (Neutralisation reaction)

$AgNO_3 + NaCl \rightarrow NaNO_3 + AgCl$ (Double displacement)

39. (a)

$$C_{\text{RMS}} = \sqrt{\frac{3RT}{M}}$$

$$C_{\text{Average}} = \sqrt{\frac{8RT}{\pi M}}$$

$$C_{\text{MPS}} = \sqrt{\frac{2RT}{M}}$$

$$\sqrt{3} > \sqrt{\frac{8}{\pi}} > \sqrt{2}$$

$$C_{\text{RMS}} > C_{\text{Average}} > C_{\text{MPS}}$$

40. (a)

Element	First Electron gain enthalpy(kJ/mol)
Li	-60
Na	-53
F	-320
S	-200
Cl	-340
Se	-195

Despite F being more electronegative than Cl, due to the small size of F, Cl would have a more negative value of electron gain enthalpy because of inter-electronic repulsions.

As we go down group, the negative electron gain enthalpy decreases.

41. (a) Co^{3+} has d^6 electronic configuration. In the presence of strong field ligand, $\Delta_o > P$. Thus the splitting occurs as: t_{2g}^6, e_g^0; so the magnetic moment is zero.

According to the spectrochemical series, en is a stronger ligand than F and therefore promotes pairing. This implies that the Δ_o of en is more than the Δ_o of F.

$$\Delta_o = \frac{hc}{\lambda_{abs}}$$

$$\Delta_t = \frac{4}{9}\Delta_o = 8000\ \text{cm}^{-1}$$

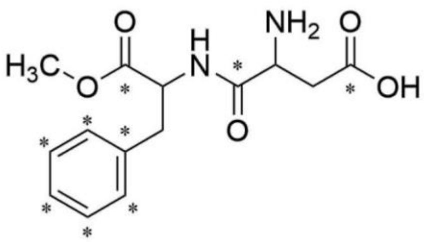
42. (a) Moles of urea $= \left(\frac{0.6}{60}\right) = 0.01$

$$\underset{0.01}{NH_2CONH_2 + 2NaOH} \rightarrow \underset{0.02}{Na_2CO_3 + 2NH_3}$$

0.02 moles of NH_3 reacts with 0.02 moles of HCl.

Moles of HCl in option a $= 0.2 \times \frac{100}{1000} = 0.02$

43. (9)



The marked carbons are sp^2 hybridised.

44. (5.22) mmole of acetic acid in 20 mL = 2

mmole of HCl in 20 mL = 1

mmole of NaOH = 2.5

$HCl + NaOH \rightarrow NaCl + H_2O$

1 2.5

$-$ 1.5 1 1

$CH_3COOH + NaOH$ (remaining) $\longrightarrow CH_3COONa$ + water

2

1.5

0.5 0

1.5

$$pH = pK_a + \log\frac{1.5}{0.5} = 4.74 + \log 3 = 4.74 + 0.48 = 5.22$$

45. (0.3675 g)

For 1L sol 30mmol of HCl is required

For 1 Lsol15mmol of H_2SO_4 is required

For 250 mL of sol,

$\frac{15}{4} \times 98 \times 10^{-3}$ g of $H_2SO_4 = 0.3675$ g

46. (18)

$$NaCl \xrightarrow{K_2Cr_2O_7/\text{ Conc. }_2SO_4} CrO_2Cl_2 \xrightarrow{NaOH} Na_2CrO_4 + NaCl$$

$$Na_2CrO_4 \xrightarrow{\text{Dil. } H_2SO_4} Na_2Cr_2O_7 \xrightarrow{\text{Dil. } H_2O_2} CrO_5$$

$(A) = CrO_2Cl_2, (B) = Na_2CrO_4$ and $(C) = CrO_5$

47. (-85 kJ/mol)

$C_{(\text{graphite})} + O_2(g) \rightarrow CO_2(g) \Delta H_c{}^\circ = -393.5$ kJ/mol.

$H_2(g) + 0.5O_2(g) \rightarrow H_2O(l)\ \Delta H_c{}^\circ = -286\ \mathrm{kJ/mol}$

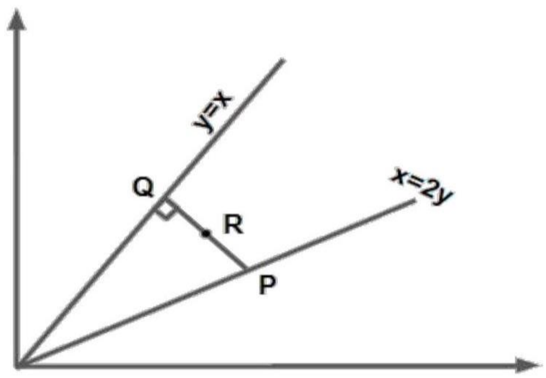
$C_2H_6(g) + 3.5O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l) \Delta H_c{}^\circ = -1560\ \mathrm{kJ/mol}$.

$2 \times (-393.5) + 3 \times (-286) - (-1560) = -85\ \mathrm{kJ/mol}$

By inverting (3) and multiplying (1) by 2 and (2) by 3 and adding, we get,

$2 \times (-393.5) + 3 \times (-286) - (-1560) = -85\ \mathrm{kJ/mol}$

48. (a)



Let R be the midpoint of PQ

PQ is perpendicular on line $y = x$

$\therefore$ Equation of the line PQ can be written as $y = -x + c$

$y = -x + c$ intersects $y = x$ at $Q: \left(\frac{c}{2}, \frac{c}{2}\right)$

$y = -x + c$ intersects $x = 2y$ at $P: \left(\frac{2c}{3}, \frac{c}{3}\right)$

$\therefore$ Midpoint $R: \left(\frac{7c}{12}, \frac{5c}{12}\right)$

Locus of $R: x = \frac{7c}{12}$

$$y = \frac{5c}{12}$$

$\Rightarrow 5x = 7y$

49. (d)

$$2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0,2\pi)$$
$$\Rightarrow 2\operatorname{cosec}^2\theta - 2 - 5\operatorname{cosec}\theta + 4 = 0$$
$$\Rightarrow 2\operatorname{cosec}^2\theta - 4\operatorname{cosec}\theta - \operatorname{cosec}\theta + 2 = 0$$
$$\Rightarrow \operatorname{cosec}\theta = 2 \text{ or } \frac{1}{2} \text{ (Not possible)}$$

As $\theta \in [0,2\pi)$,

$\theta_1=\frac{\pi}{6},\theta_2=\frac{5\pi}{6}$

$\Rightarrow \int_{\theta_1}^{\theta_2}\cos^2 3\theta d\theta=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\frac{(1+\cos 6\theta)}{2}d\theta$

$=\frac{1}{2}\left(\frac{5\pi}{6}-\frac{\pi}{6}\right)+\frac{\sin 6\theta}{12}\Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$

$=\frac{\pi}{3}$

50. (d) Coefficient of x^7 in $(1+x)^{10}+x(1+x)^9+x^2(1+x)^8+\cdots+x^{10}$

Applying sum of terms of G.P. $=\frac{(1+x)^{10}\left(1-\left(\frac{x}{1+x}\right)^{11}\right)}{\left(1-\frac{x}{1+x}\right)}=(1+x)^{11}-x^{11}$

Coefficient of $x^7 \Rightarrow {}^{11}C_7=330$

51. (a) Given α,β are the roots of $x^2-x-1=0$

$\Rightarrow \alpha+\beta=1\&\alpha\beta=-1$

$\Rightarrow \alpha^2=\alpha+1\&\beta^2=\beta+1$

$P_k=\alpha^{k-2}\alpha^2+\beta^{k-2}\beta^2$

$P_k=\alpha^{k-2}(\alpha+1)+\beta^{k-2}(\beta+1)$

$P_k=\alpha^{k-1}+\beta^{k-1}+\alpha^{k-2}+\beta^{k-2}$

$\Rightarrow P_k=P_{k-1}+P_{k-2}$

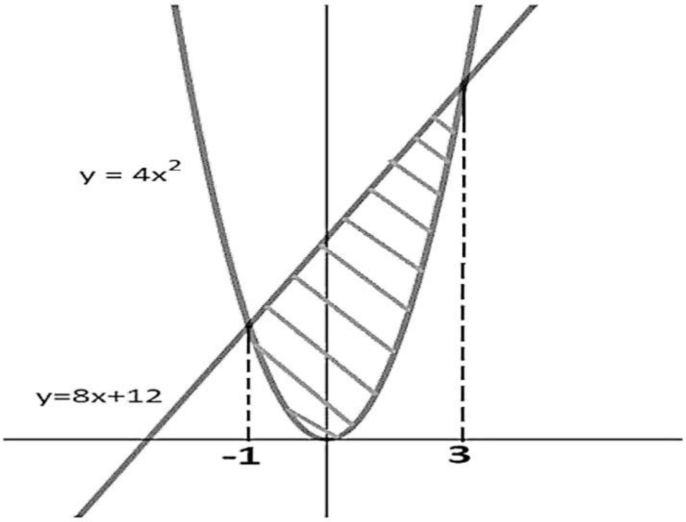
$\Rightarrow P_3=P_2+P_1=4$

$P_4=P_3+P_2=7$
$P_5=P_4+P_3=11$

$\therefore P_5\neq P_2P_3\&P_1+P_2+P_3+P_4+P_5=26$

$\&P_3=P_5-P_4$

52. (c)



For point of intersection,

$4x^2 = 8x + 12$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x = 3, -1$

Area bounded is given by

$A = \int_{-1}^{3} (8x + 12 - 4x^2) dx$

$$A = \left[\frac{8x^2}{2} + 12x - \frac{4x^3}{3}\right]_{-1}^{3}$$

$$A = (36 + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right)$$

$$A = 44 - \frac{4}{3} = \frac{128}{3}$$

53. (d) Given statements: $A \subset B$ and $B \subset C$

Let $A \subset B$ be p

$B \subset C$ be q

$C \subset D$ be r

Modified statement: $(p \wedge q) \Rightarrow r$

Contrapositive: $\sim r \Rightarrow \sim (p \wedge q)$

$\therefore r \vee (\sim p \vee \sim q)$

$\Rightarrow C \subset D$ or $A \not\subset B$ or $B \not\subset C$

54. (d) $S = \underline{3 + 4} + \underline{8 + 9} + 13 + 14 + \cdots$ ... 40 terms

$S = 7 + 17 + 27 + 37 + \cdots$ ... ... .20 terms

$S=\frac{20}{2}[14+(19)10]=20\times 102$

$\therefore m=20$

55. (d) using ${}^{36}C_{r+1}=\frac{36}{r+1}\times{}^{35}C_r$

$\frac{36}{r+1}\times{}^{35}C_r\times(k^2-3)={}^{35}C_r\times 6$

$k^2-3=\frac{r+1}{6}$

$k^2=\frac{r+1}{6}+3$

$k\in\mathbf{I}$

$r\rightarrow$ Non-negative integer $0\le r\le 35$

$r=5\Rightarrow k=\pm 2$

$r=35\Rightarrow k=\pm 3$

No. of ordered pairs $(r,k)=4$

56. (a) Given $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$

$\lim_{x\to 0}\frac{f(x)}{x^3}=2$

$\lim_{x\to 0}\frac{f(x)}{x^3}$ Limit exists and it is finite

$\therefore f(x)=ax^5+bx^4+cx^3$

$\Rightarrow \lim_{x\to 0}(ax^2+bx+c)=2$

$c=2$

Also $f'(x)=5ax^4+4bx^3+6x^2$

$f'(1)=5a+4b+6=0$

$f'(-1)=5a-4b+6=0$

$b=0,\ a=-\frac{6}{5}$

$f(x)=-\frac{6}{5}x^5+2x^3\ \Rightarrow f(x)$ is odd

$f'(x)=-6x^4+6x^2$

$f''(x)=-24x^3+12x\ (f''(1)<0, f''(-1)>0)$

At $x=-1$ local minima at $x=1$ local maxima

And $f(1)-4f(-1)=4$

57. (c) LMVT is applicable on $f(x)$ in [0,1], therefore it is continuous and differentiable in [0,1]

Now, $f(0)=11, f(1)=16$

$f'(x)=3x^2-8x+8$

$\therefore f'(c)=\frac{f(1)-f(0)}{1-0}=\frac{16-11}{1}$

$\Rightarrow 3c^2-8c+8=5$

$\Rightarrow 3c^2-8c+3=0$

$\Rightarrow c=\frac{8\pm 2\sqrt{7}}{6}=\frac{4\pm\sqrt{7}}{3}$

As $c\in(0,1)$

We get, $c=\frac{4-\sqrt{7}}{3}$

58. (b) $P(\text{ machine being faulty })=p=\frac{1}{4}$

$\therefore q=\frac{3}{4}$

$P(\text{ at most two machines being faulty })=P(\text{ zero machine being faulty })$

$+P$ (one machine being faulty) $+P$ (two machines being faulty)

$= {}^5C_0p^0q^5+{}^5C_1p^1q^4+{}^5C_2p^2q^3$

$=q^5+5pq^4+10p^2q^3$

$=\left(\frac{3}{4}\right)^5+5\times\frac{1}{4}\left(\frac{3}{4}\right)^4+10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$

$=\left(\frac{3}{4}\right)^3\left[\frac{9}{16}+\frac{15}{16}+\frac{10}{16}\right]$

$=\left(\frac{3}{4}\right)^3\times\frac{34}{16}=\left(\frac{3}{4}\right)^3\times\frac{17}{8}$

$\therefore k=\frac{17}{8}$

59. (d) $a_1+a_2=4\Rightarrow a+ar=4\Rightarrow a(1+r)=4$

$a_3+a_4=16\Rightarrow ar^2+ar^3=16\Rightarrow ar^2(1+r)=16\Rightarrow 4r^2=16$

$\Rightarrow r=\pm 2$

If $r=2, a=\frac{4}{3}$ which is not possible as $a_1<0$

If $r=-2, a=-4$

$\sum_{i=1}^{9} a_i = \frac{a(r^9-1)}{r-1} = \frac{(-4)[(-2)^9-1]}{-3} = \frac{4}{3}(-512-1) = 4(-171)$

$\lambda = -171$

60. (b) $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$

Differentiating w.r.t. x on both the sides, we get:

$$y'\sqrt{1-x^2} + y \times \frac{1}{2\sqrt{1-x^2}} \times (-2x) = -\sqrt{1-y^2} - x \times \frac{1}{2\sqrt{1-y^2}} \times (-2y)y'$$

$$\Rightarrow y'\sqrt{1-x^2} - \frac{xy}{\sqrt{1-x^2}} + \sqrt{1-y^2} - \frac{xy}{\sqrt{1-y^2}}y' = 0$$

$$\Rightarrow y'\left[\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}}\right] = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$$

Putting $x = \frac{1}{2}, y = -\frac{1}{4}$

$$\Rightarrow y'\left[\frac{\sqrt{3}}{2} + \frac{\frac{1}{8}}{\frac{\sqrt{15}}{4}}\right] = -\frac{\frac{1}{8}}{\frac{\sqrt{3}}{2}} - \frac{\sqrt{15}}{4}$$

$$\Rightarrow y'\left[\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{15}}\right] = -\frac{1}{4\sqrt{3}} - \frac{\sqrt{15}}{4}$$

$$\Rightarrow y'\left[\frac{\sqrt{45}+1}{2\sqrt{15}}\right] = -\frac{1+\sqrt{45}}{4\sqrt{3}}$$

$$\Rightarrow y' = -\frac{\sqrt{5}}{2}$$

61. (c)

$b_{ij} = \lambda^{i+j-2}a_{ij}, \lambda = 3$

$$B = \begin{bmatrix} 3^0a_{11} & 3a_{12} & 3^2a_{13} \\ 3a_{21} & 3^2a_{22} & 3^3a_{23} \\ 3^2a_{31} & 3^3a_{32} & 3^4a_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 3^0a_{11} & 3a_{12} & 3^2a_{13} \\ 3a_{21} & 3^2a_{22} & 3^3a_{23} \\ 3^2a_{31} & 3^3a_{32} & 3^4a_{33} \end{vmatrix}$$

Taking 3^2 Common each from C_3&R_3

$$|B| = 81\begin{vmatrix} a_{11} & 3a_{12} & a_{13} \\ 3a_{21} & 3^2a_{22} & 3a_{23} \\ a_{31} & 3a_{32} & a_{33} \end{vmatrix}$$

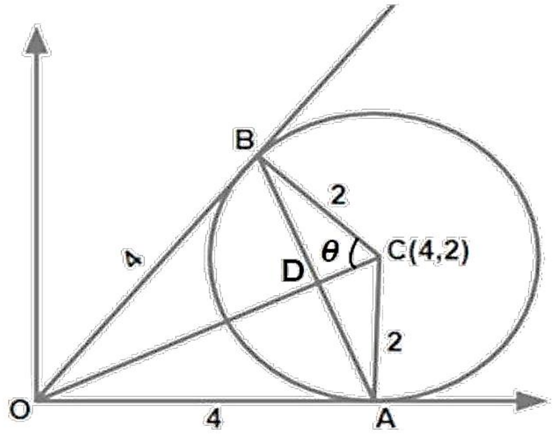
Taking 3 common each from C_2&R_2

$|B| = 81(9)\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Given $|B| = 81$
$\Rightarrow 81 = 81(9)|A|$
$\Rightarrow |A| = \dfrac{1}{9}$

62. (d) $x^2 + y^2 - 8x - 4y + 16 = 0$

$(x-4)^2 + (y-2)^2 = 4 \Rightarrow$ Centre $(4,2)$, radius (2)



$OA = 4 = OB$

In $\triangle OBC$

$\tan\theta = \dfrac{4}{2} = 2 \Rightarrow \sin\theta = \dfrac{2}{\sqrt{5}}$

In $\triangle BDC$

$\sin\theta = \dfrac{BD}{2} \Rightarrow BD = \dfrac{4}{\sqrt{5}}$

Length of chord of contact $(AB) = \frac{8}{\sqrt{5}}$

Alternative

(l) length of tangent $= 4$

(r) radius $= 2$

$\Rightarrow$ Length of chord of contact $= \frac{2lr}{\sqrt{(l^2+r^2)}}$

Square of length of chord of contact $= \frac{64}{5}$

63. (a) $(y^2 - x)\frac{dy}{dx} = 1$

$\frac{dx}{dy} + x = y^2$

$xe^y = \int y^2 e^y dy$

$x = y^2 - 2y + 2 + ce^{-y}$

Given $y(0) = 1$

$\Rightarrow c = -e$

$\therefore$ Solution is $x = y^2 - 2y + 2 - e^{-y+1}$

$\therefore$ The value of x where the curve cuts the x-axis will be at $x = 2 - e$

64. (c)

$$4\alpha \int_{-1}^{2} e^{-\alpha|x|} dx = 5$$
$$4\alpha \left[\int_{-1}^{0} e^{-\alpha|x|} dx + \int_{0}^{2} e^{-\alpha|x|} dx\right] = 5$$
$$= 4\alpha \left[\int_{-1}^{0} e^{\alpha x} dx + \int_{0}^{2} e^{-\alpha x} dx\right] = 5$$
$$= 4\alpha \left[\left(\frac{1 - e^{-\alpha}}{\alpha}\right) + \left(\frac{e^{-2\alpha} - 1}{-\alpha}\right)\right] = 5$$
$$= 4[1 - e^{-2\alpha} - e^{-\alpha} + 1] = 5$$

Let $e^{-\alpha} = t$

$\Rightarrow -4t^2 - 4t + 3 = 0$

$\Rightarrow t = \frac{1}{2} = e^{-\alpha} \Rightarrow \alpha = \ln 2$

65. (c) Given $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = \overrightarrow{|0|}^2$

$\overrightarrow{|a|}^2 + \overrightarrow{|b|}^2 + \overrightarrow{|c|}^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\lambda = -\frac{3}{2}$

Also $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$

$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{b} \times (-\vec{a} - \vec{b}) + (-\vec{a} - \vec{b}) \times \vec{a}$

$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$

$\Rightarrow \vec{d} = 3(\vec{a} \times \vec{b}) = 3\vec{a} \times \vec{b}$

66. (b)

$3x + 4y = 12\sqrt{2}$ is tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$

Equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ is $y = mx + \sqrt{a^2m^2 + 9}$

Now, $3x + 4y = 12\sqrt{2} \Rightarrow y = -\frac{3}{4}x + 3\sqrt{2}$

$\Rightarrow m = -\frac{3}{4}$ and $\sqrt{a^2m^2 + 9} = 3\sqrt{2}$

$$\Rightarrow a^2\left(-\frac{3}{4}\right)^2 + 9 = 18$$

$$\Rightarrow a^2 \times \frac{9}{16} = 9$$

$\Rightarrow a^2 = 16 \Rightarrow a = 4$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

Distance between foci is $2ae = 2 \times 4 \times \frac{\sqrt{7}}{4} = 2\sqrt{7}$

67. (124)

Mean $= 10 \Rightarrow \frac{61+x+y}{8} = 10$
$\Rightarrow x + y = 19$

Variance $= \frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$$\Rightarrow 25 = \frac{2^2 + 3^2 + 16^2 + 20^2 + 13^2 + 7^2 + x^2 + y^2}{8} - 100$$

$\Rightarrow 1000 = 887 + x^2 + y^2$

$\Rightarrow x^2 + y^2 = 113$

$\Rightarrow (x + y)^2 - 2xy = 113$

$\Rightarrow 361 - 2xy = 113$

So, $xy = 124$

68. (4) Direction ratios of line L: $\left(\alpha - \frac{5}{3}, 7 - \frac{7}{3}, 1 - \frac{17}{3}\right)$

$$= \left(\frac{3\alpha - 5}{3}, \frac{14}{3}, -\frac{14}{3}\right)$$

Direction ratios of PQ: $\left(-\frac{2}{3}, -\frac{7}{3}, -\frac{8}{3}\right)$

As line L is perpendicular to PQ

So, $\left(\frac{3\alpha-5}{3}\right)\left(-\frac{2}{3}\right)+\left(\frac{14}{3}\right)\left(-\frac{7}{3}\right)+\left(-\frac{14}{3}\right)\left(-\frac{8}{3}\right)=0$

$\Rightarrow -6\alpha+10-98+112=0$

$\Rightarrow 6\alpha=24 \Rightarrow \alpha=4$

69. (13) The system of equations has more than 2 solutions

$\therefore D=D_3=0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix}=0 \Rightarrow 2\lambda-6-\lambda+9+2-6=0$$

$\Rightarrow \lambda=1$

$$\begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix}=0 \Rightarrow 2\mu-20-\mu+30-24=0$$

$\Rightarrow \mu=14$

So, $\mu-\lambda^2=13$

70. (5) As $f(x)$ is continuous

$\Rightarrow \lim_{x\to 0} f(x)=f(0)=k$

$$\Rightarrow \lim_{x\to 0}\left(\frac{1}{x}\right)\log_e\left(\frac{1+3x}{1-2x}\right)=k$$

$$\Rightarrow \lim_{x\to 0}\frac{3\log(1+3x)}{3x}-\lim_{x\to 0}\frac{(-2)\log(1-2x)}{(-2x)}=k$$

$\Rightarrow 3+2=k \Rightarrow k=5$

71. (29)

$A=\{x: x \text{ is multiple of } 2\}=\{2,4,6,8,\ldots\}$

$B=\{x: x \text{ is multiple of } 7\}=\{7,14,21,\ldots\}$

$X=\{x: 1\le x\le 50, x\in \mathbf{N}\}$

Smallest subset of X which contains elements of both A and B is a set with multiples of 2 or 7 less than 50 .

$P=\{x: x \text{ is a multiple of 2 less than or equal to } 50\}$

$Q=\{x: x \text{ is a multiple of 7 less than or equal to } 50\}$

$n(P\cup Q)=n(P)+n(Q)-n(P\cap Q)$

$=25+7-3$

$=29$