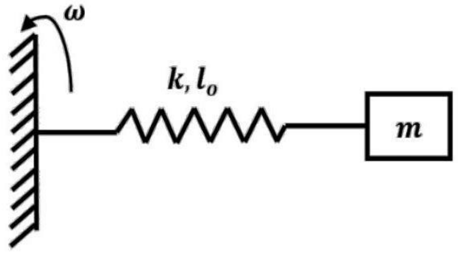
1. **(a)** The centripetal force is provided by the spring force.

$$m\omega^2(l_o + x) = kx$$

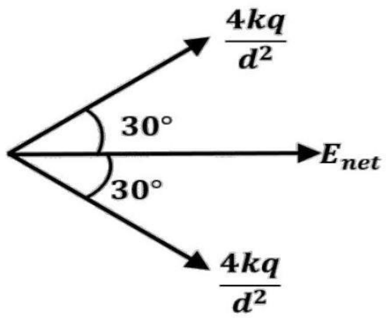
$$\left(\frac{l_o}{x} + 1\right) = \frac{k}{m\omega^2}$$

$$x = \frac{l_o m\omega^2}{k - m\omega^2}$$



2. **(c)** Applying superposition principle,

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$



By symmetry, net electric field along the x-axis.

$$|\vec{E}_{net}| = \frac{4kq}{d^2} \times 2\cos 30^\circ = \frac{q\sqrt{3}}{\pi\varepsilon_0 d^2}$$

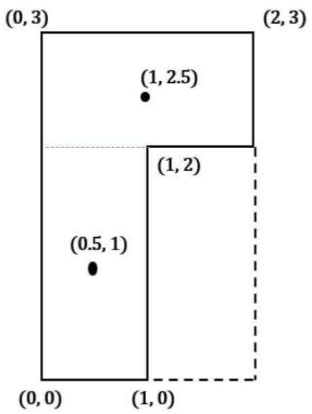
3. **(a)** For the given $V - T$ graph

For the process $x \to y; V \propto T; P = \text{constant}$

For the process $y \to z; V = \text{constant}$

Only ' a ' satisfies these two conditions.

4. **(a)** The Lamina can be divided into two parts having equal mass m each.

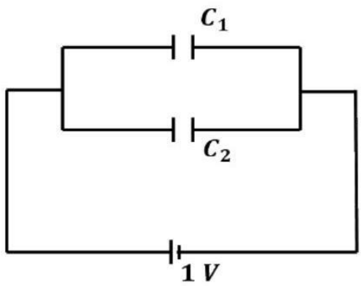


$$\vec{r}_{cm} = \frac{m \times \left(\frac{\hat{i}}{2} + \hat{j}\right) + m \times \left(\hat{i} + \frac{5\hat{j}}{2}\right)}{2m}$$

$$\vec{r}_{cm} = \frac{3}{4}\hat{i} + \frac{7}{4}\hat{j}$$

5. (a) $\tau \propto \frac{1}{\sqrt{T}}$

6. (a)



Given that,

$$C_1 + C_2 = 10\mu F$$

$$4\left(\frac{1}{2}C_1\,V^2\right) = \frac{1}{2}C_2\,V^2$$

$$\Rightarrow 4C_1 = C_2$$

From equations (i) and (ii)

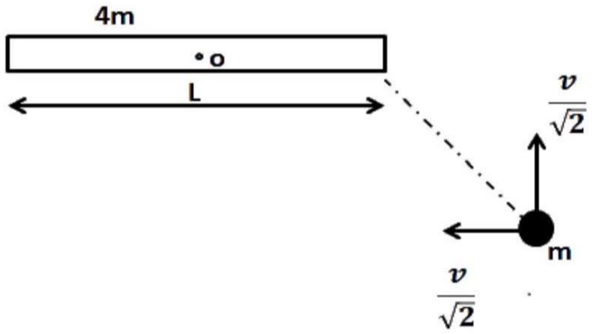
$$C_1 = 2\mu F$$

$$C_2 = 8\mu F$$

If they are in series

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 1.6\mu F$$

7. (a)



There is no external torque on the system about the hinge point.

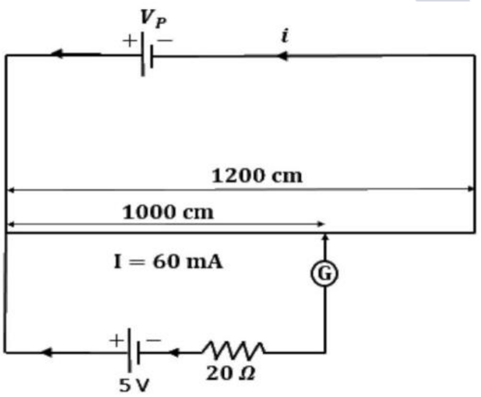
So,

$$\overrightarrow{L_1} = \overrightarrow{L_f}$$
$$\frac{mv}{\sqrt{2}} \times \frac{1}{2} = \left[\frac{4mL^2}{12} + \frac{mL^2}{4}\right] \times \omega$$
$$\omega = \frac{6\ V}{7\sqrt{2}L} = \frac{3\sqrt{2}\ V}{7L}$$

8. **(d)** $\lambda = \frac{h}{\sqrt{2(KE)m_B}} = \lambda \propto \frac{1}{\sqrt{KE}}$

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{KE_B}}{\sqrt{KE_A}}$$
$$\frac{1}{2} = \sqrt{\frac{T_A - 1.5}{T_A}}$$
$$T_A = 2eV$$
$$KE_B = 2 - 1.5 = 0.5eV$$
$$\emptyset_B = 4.5 - 0.5 = 4eV$$

9. **(b)**



Assume the terminal voltage of the primary battery as V_p. As long as this potentiometer is operating on balanced length, V_p will remain constant.

As we know, potential gradient $= \frac{5}{1000} = \frac{V_p}{1200}$

$V_p = 6$ V

And $R_p = \frac{V_p}{i} = \frac{6}{60\times10^{-3}} = 100\Omega$

10. (a)

$$m = \frac{f_0}{f_e} = 5$$
$$\Rightarrow f_0 = 5f_e$$
$$f_0 + f_e = 5f_e + f_e = 6f_e = \text{ length of the tube}$$
$$\Rightarrow 6f_e = 60 \text{ cm}$$
$$\Rightarrow f_e = 10 \text{ cm}$$

11. (a) Gravitation field will be maximum at the surface of a sphere. Therefore,

$$\frac{Gm_2}{2^2} = 3 \& \frac{Gm_1}{1^2} = 2$$

$$\Rightarrow \frac{m_2}{m_1} \times \frac{1}{4} = \frac{3}{2}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{1}{6}$$

12. (a) $K.E = 1\times10^6 eV = 1.6\times10^{-13}$ J

$$= \frac{1}{2} m_e v^2$$

Where m_e is the mass of the electron $= 1.6\times10^{-27}$ kg

$$\Rightarrow 1.6\times10^{-13} = \frac{1}{2}\times1.6\times10^{-27}\times v^2$$

$$\therefore v = \sqrt{2}\times10^7 \text{ m/s}$$

$$Bqv = m_e a$$

$$\therefore B = \frac{1.6\times10^{-27}\times10^{12}}{1.6\times10^{-19}\times\sqrt{2}\times10^7}$$

$$= 0.71\times10^{-3} \text{ T} = 0.71 \text{mT}$$

13. (c) The magnitude of the electric field is constant and the electric field must be along the area vector i.e. the surface is equipotential.

14. (d)

$V = K(h)^a(I)^b(G)^c(C)^d$ Unit of stopping potential is (V) Volt.

We know $[h] = ML^2T^{-1}$

$[I] = A$

$[G] = M^{-1}L^3T^{-2}$

$[C] = L^{-1}$

$[V] = M^2T^{-3}A^{-1}$

$ML^2T^{-3}A^{-1} = (M^2T^{-1})^a(A)^b(M^{-1}L^3T^{-2})^c(L^{-1})^d$

$ML^2T^{-3}A^{-1} = M^{a-c}L^{2a+3c+d}T^{-a-2c-d}A^b$

$a - c = 1$

$2a + 3c + d = 2$

$-a - 2c - d = -3$

$b = -1$

On solving,

$c = -1$

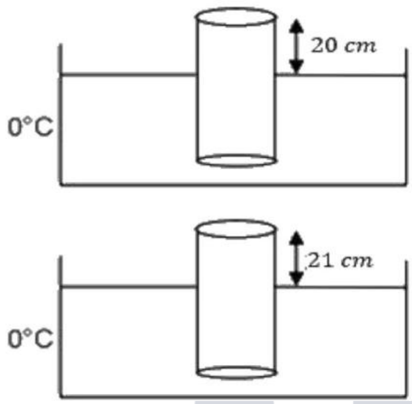
$a = 0$

$d = 5$

$b = -1$

$V = K(h)^0(I)^{-1}(G)^{-1}(C)^5$

15. (a)



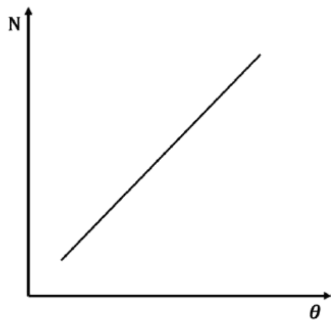
Since the cylinder is in equilibrium, it's weight is balanced by the Buoyant force.

$\text{mg} = \text{A}(80)(\rho_{0^\circ\text{C}})\text{g}$

$\text{mg} = \text{A}(79)(\rho_{4^\circ\text{C}})\text{g}$

$$\frac{\rho_{4^\circ\text{C}}}{\rho_{0^\circ\text{C}}} = \frac{80}{79} = 1.01$$

16. (c)



$$N \propto \frac{1}{\sin^4(\theta/2)}$$

17. (c) If the speed of light in the given medium is V then,

$$V = \frac{1}{\sqrt{\mu\epsilon}}$$

We know that, $\mathrm{n} = \frac{\mathrm{c}}{\mathrm{v}}$

$$\mathrm{n} = \sqrt{\mu_{\mathrm{r}}\epsilon_{\mathrm{r}}} = 2$$

$$\sin\theta_{\mathrm{c}} = \frac{1}{2}$$

$$\theta_{\mathrm{c}} = 30^{\circ}$$

18. (a)

$$\epsilon = \left|-\frac{\mathrm{d}\phi}{\mathrm{dt}}\right| = \left|-\frac{\mathrm{AdB}}{\mathrm{dt}}\right|$$

$$= (16 \times 4 - 4 \times 2)\frac{(1000 - 500)}{5} \times 10^{-4} \times 10^{-4}$$

$$= 56 \times \frac{500}{5} \times 10^{-8} = 56 \times 10^{-6}\ \mathrm{V}$$

19. (d) First part of figure shown OR gate and second part of figure shown NOT gate.

So, $Y = \overline{A + B} = \bar{A}.\bar{B}$

20. (a) Let the mass of the sphere be m and the density of the liquid be ρ_L

$$\rho = \rho_o\left(1 - \frac{r^2}{R^2}\right), 0 < r \leq R$$

Since the sphere is floating in the liquid, buoyancy force (F_B) due to liquid will balance the weight of the sphere.

$$F_B = mg$$

$$\rho_L\frac{4}{3}\pi R^3 g = \int \rho(4\pi r^2 dr)g$$

$$\rho_L\frac{4}{3}\pi R^3 = \int \rho_o\left(1 - \frac{r^2}{R^2}\right)4\pi r^2 dr$$

$$\rho_L\frac{4}{3}\pi R^3 = \int_0^R \rho_o 4\pi\left(r^2 - \frac{r^4}{R^2}\right)dr = \rho_o 4\pi\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)_0^R$$

$$\rho_L = \frac{2}{5}\rho_o$$

21. (1) Mass of each object, $m_1 = m_2 = 0.1$ kg

Initial velocity of 1st object, $u_1 = 5$ m/s

Initial velocity of 2nd object, $u_2 = 3$ m/s

Final velocity of 1st object, $V_1 = 4\hat{\imath} + 4\hat{\jmath}\,\mathrm{m/s} = \sqrt{4^2 + 4^2} = 16\sqrt{2}$ m/s

For elastic collision, kinetic energy remains conserved

Initial kinetic energy (K_i) = Final kinetic energy (K_f)

$$\frac{1}{2}mu_1^2+\frac{1}{2}mu_2^2=\frac{1}{2}mV_1^2+\frac{1}{2}mV_2^2$$
$$\frac{1}{2}m(5)^2+\frac{1}{2}m(3)^2=\frac{1}{2}m(16\sqrt{2})^2+\frac{1}{2}mV_2^2$$
$$V_2=\sqrt{2}m/s$$

Kinetic energy of second object $=\frac{1}{2}mV_2^2=\frac{1}{2}\times 0.1\times \sqrt{2}^2=\frac{1}{10}$

$\Rightarrow x=1$

22. (60 cm)

Applying Lens makers' formula,

$$\frac{1}{\mathrm{f}}=(\mu-1)\left(\frac{1}{\mathrm{R}_1}-\frac{1}{\mathrm{R}_2}\right)$$
$$\mathrm{R}_1=\infty$$
$$\mathrm{R}_2=-30\text{ cm}$$
$$\frac{1}{\mathrm{f}}=(1.5-1)\left(\frac{1}{\infty}-\frac{1}{-30}\right)$$

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$$\frac{1}{\mathrm{f}}=\frac{0.5}{30}$$

$\mathrm{f}=60$ cm

23. (580 m/s)

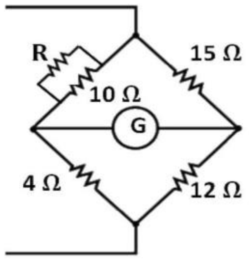
$$\mathrm{X_A}=-3\mathrm{t}^2+8\mathrm{t}+\mathrm{c}$$
$$\overrightarrow{\mathrm{v_A}}=(-6\mathrm{t}+8)\hat{\imath}$$
$$=2\hat{\imath}$$
$$\mathrm{Y_B}=10-8\mathrm{t}^3$$
$$\overrightarrow{\mathrm{v_B}}=-24\mathrm{t}^2\hat{\jmath}$$
$$|\overrightarrow{\mathrm{v}_{B/A}}|=|\overrightarrow{\mathrm{v_B}}-\overrightarrow{\mathrm{v_A}}|=|-24\hat{\jmath}-2\hat{\imath}|$$
$$\mathrm{v}=\sqrt{24^2+2^2}$$
$$\mathrm{v}=580\text{ m/s}$$

24. (105.75 Hz)

$$\mathrm{V}=\sqrt{\frac{\mathrm{B}}{\rho}}$$
$$\frac{\mathrm{V_{pipe}}}{\mathrm{V_{air}}}=\frac{\sqrt{\frac{B}{2\rho}}}{\sqrt{\frac{B}{\rho}}}=\frac{1}{\sqrt{2}}$$
$$\mathrm{V_{pipe}}=\frac{\mathrm{V_{air}}}{\sqrt{2}}$$
$$\mathrm{f_n}=\frac{(\mathrm{n}+1)}{2\mathrm{l}}\mathrm{V_{pipe}}$$

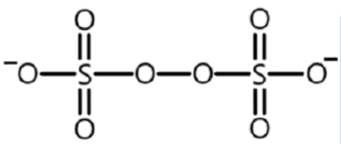
$$f_1 - f_0 = \frac{v_{pipe}}{2l} = \frac{300}{2\sqrt{2}} = 105.75 \text{ Hz}(\text{ if } \sqrt{2} = 1.41)$$
$$= 106.05 \text{ Hz}(\text{ if } \sqrt{2} = 1.414)$$

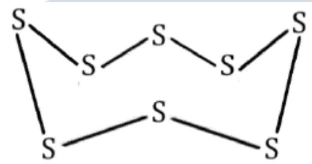
25. (**10Ω**)



$$\frac{10R}{10+R} \times 12 = 15 \times 4 \Rightarrow R = 10\Omega$$

26. **(a)** Solution: Here, we have to count $S-0$ single bonds as well as $S=0$ in $S_2O_8{}^{2-}$, as each double bond also has one sigma bond. The structure of $S_2O_8^{2-}$ and S_8 is shown below:





27. **(c)** London dispersion forces (also called as induced dipole - induced dipole interactions), exist because of the generation of temporary polarity due to collision of particles and for this very reason, they are present in all molecules and inert gases as well.

Because of the presence of a permanent dipole, there will be dipole-dipole interactions present here.

There is no H that is directly attached to an oxygen atom, so H-bonding cannot be present.

28. **(c)**

(A) is correct since the series studied in H-spectrum, including Balmer series, are de-excitation series or emission series. So, electrons get de-excited to $n = 2$ which means that $n_{lower} = 2$.
(B) It is possible to obtain I.E. from the formula above, but since the question has stated the formula for the Balmer series, nlower has been fixed as 2 . So, it is not possible to calculate I.E. from it. To calculate I.E., we'll have to put $n_{lower} = 1$, which isn't possible here.
(C) $\Delta E = hc/\lambda$

With $n_{\text{lower fixed as}}$ a, ΔE increases as n_{higher} is increased. So, the last line of the Balmer series, i.e. from infinity to $n = 2$, will have the maximum energy in the series and thus, the lowest wavelength. Similarly, the first line in the series, i.e. from $n = 3$ to $n = 2$ will have the lowest energy in the series and thus, the highest wavelength. Which makes this statement correct.
(D)As orbits with higher orbit number or those that are further away from the nucleus are considered, the energy gap in-between subsequent orbits decreases. Now, consider the following for example and with nlower fixed as 2 .

Energy of a photon released on transition from $n = 100$ to $n = 2$ will have similar energy to that of the photon that gets released on transition from $n = 101$ to $n = 2$, because energy of the 100^{th} and the 101^{th} orbit will be very close in value. That means they will also have very close values of wavelengths, which further implies that these two lines will be situated quite close to each other on the photographic plate.

In a similar fashion, we can see that as the n_{higher} increases, the lines start to converge together. And since, increasing the n_{higher} will indeed lead to an increase in the energy of the photon released, it will end up releasing photons of shorter wavelengths. Combining these two statements we can easily see that as the wavelength decreases, the spectral lines start to converge.

29. (a) The expected order is Na < Mg < Al < Si.

But the actual/experimental order turns out to be Na < Al < Mg < Si, because of the fully filled ssubshell of magnesium and the s^2p^1 configuration of Al which makes it relatively easy for Al to lose its outermost electron.

30. (a)

(a) $XY_{(s)} \rightleftharpoons X^+ + Y^-$

$K_{sp} = [X^+][Y^-] = 2 \times 10^{-3} \times 10^{-3} = 2 \times 10^{-6}$

(b) $XY_{2(s)} \rightleftharpoons X^{2+} + 2Y^-$

$K_{sp} = [X^{2+}][Y^-]^2 = 2 \times 10^{-3} \times 10^{-6} = 2 \times 10^{-9}$

(c) $X_2Y_{(s)} \rightleftharpoons 2X^+ + Y^{2-}$

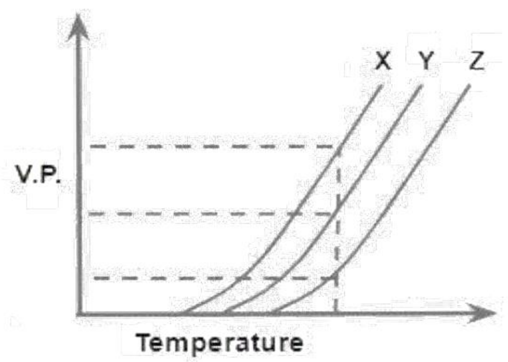
$K_{sp} = [X^+]^2[Y^{2-}] = 4 \times 10^{-6} \times 10^{-3} = 4 \times 10^{-9}$

(d) $XY_{2(s)} \rightleftharpoons X^{2+} + 2Y^-$

$K_{sp} = [X^{2+}][Y^-]^2 = 2 \times 10^{-3} \times 10^{-6} = 2 \times 10^{-9}$

31. (d) Facial and meridional geometrical isomerism is observed only in $[MA_3B_3]$ type complexes which is given in option d.

32. (c) As shown in the plot below, for the same T, the vapour pressure of X is the highest and of Z is the lowest. Now, that means with the same average K.E. of X, Y and Z molecules, the X molecules are able to compensate their respective intermolecular forces better. So, X molecules have the highest vapour pressure. Which implies that the intermolecular forces in X are the weakest among the three. The opposite could be said for Z as well.



33. (a) $K_1 = Ae^{-E_{a1}/RT}$

$K_2 = Ae^{-E_{a2}/RT}$

Dividing equation 1 with equation 2 , we get

$$\frac{K_1}{K_2} = e^{(E_{a2}-E_{a1})/RT}$$

$10^{-6} = e^{(E_{a2}-E_{a1})/RT}$

Taking $\log_e$ on both sides, we get

$\Delta E = E_{a2} - E_{a1} = -6 \times 2.303RT$

34. (C) $CaSO_4 \cdot 2H_2O \xrightarrow{393\ K} CaSO_4 \cdot \frac{1}{2}H_2O$

35. (c) Consider an element E

$E^{2+} \rightarrow E^{3+}$ would be the 3rd I.E. of the element E.

Electronic configuration of Mn is $[Ar]4\ s^2 3\ d^5$

Electronic configuration of Co is $[Ar]4\ s^2 3\ d^7$

Electronic configuration of Fe is $[Ar]4\ s^2 3\ d^6$

Electronic configuration of Ni is $[Ar]4\ s^2 3\ d^8$

Electronic configuration of Mn^{2+} is $[Ar]3\ d^5$

Electronic configuration of Co^{2+} is $[Ar]3\ d^7$

Electronic configuration of Fe^{2+} is $[Ar]3\ d^6$

Electronic configuration of Ni^{2+} is $[Ar]3\ d^8$

As it is evident from the above configurations of the E^{2+} for the given elements, Fe^{2+} would require the least amount of energy for removal of electron as it has the configuration $3d^6 4s^0$. That means that its E^{3+} form is the most stable among the four elements provided in their respective E^{3+} states, i.e., when compared, the next electron removal will require least amount of energy.

36. (d) The standard solution is always kept in burette. The oxalic acid is a primary standard solution while H_2SO_4 is a secondary standard solution.

37. (b) In E_1 mechanism, the rate determining step is formation of carbocation. So, stability of carbocation formed decides the rate.

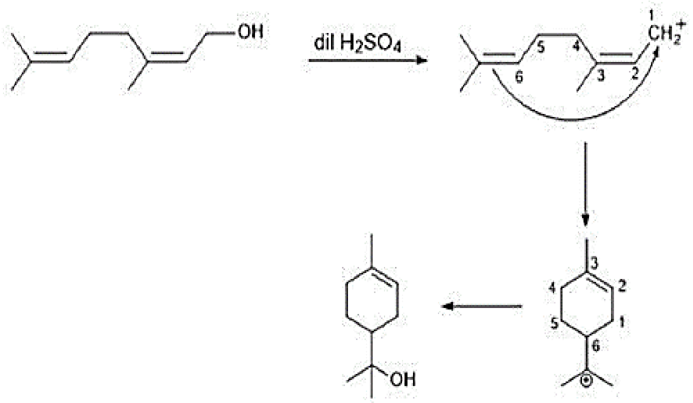
In option c, the cation formed is resonance stabilised.

In option d, the cation formed is a 2° carbocation.

In option a and b, the carbocations formed are 1° but there is a chance of rearrangement in option b and after the rearrangement, the carbocation formed in option b will be allylic. So, the order of reaction is as follows:

C > D > B > A.

38. (c)



39. (b) In methanol, there is no resonance. In phenol, there is resonance. In p-Ethoxyphenol, there is resonance involved but the involvement of lone pair of oxygen in OH group is poor as compared with phenol due to the presence of lone pair oxygen in OCH_3 group which are also involved in resonance.

So, partial double bond character develops in $C-OH$ bond of phenol and p-Ethoxyphenol but in case of p-Ethoxyphenol, resonance is poor as compared to phenol. So, bond length follows the order: $A > C > B$

40. (d) CO_2, O_3, H_2O vapours and CFC' 's are green house gases.

41. (b) When the difference between the B.P. of the two liquids is less than around $40^{\circ}C$, fractional distillation is more efficient. The difference between the boiling points of isohexane and 3-methylpentane is only 3 degrees. So, fractional distillation is the best suitable method. Since, isohexane has a lower boiling point, it comes out first.

42. (b) Glucose exists in two crystalline forms α and β which are anomers of each other.

Glucose does not react with Schiff's reagent because after the internal cyclisation, it forms either α anomer or β-anomer. In these forms, free aldehydic group is not present.

Glucose forms open chain structure in aqueous solution which contains aldehyde at chain end. This aldehydic group reacts with NH_4OH to form oxime. On the other hand, glucose penta acetate being a cyclic structure even in aqueous form does not have terminal carbonyl group. Therefore it will not react with NH_4OH.

43. (b) B_2H_6 does not reduce amide, carbonyl group and cyanide. It selectively reduces carboxylic acid to alcohol. So, for this conversion, it is the best suitable reagent.

44. (26.92) To react completely with one mole of $[ML_6]Cl_3$, 3 moles of $AgNO_3$ is required.

0.3 $g[ML_6]Cl_3$ means $\frac{0.3}{267.46}$ moles of $[ML_6]Cl_3$.

So, moles of $AgNO_3$ required will be $\frac{0.3\times 3}{267.46}$ moles

To find the volume,

$$\frac{0.3\times 3}{267.46} = 0.125\times V(L)$$
$$V(L) = 0.02692$$
$$V(mL) = 26.92$$

45. (-0.93)

$$2H_2O \rightarrow O_2 + 4H^+ + 4e^-;\ E^{\circ} = -1.23\text{ V}$$

$$E = -1.23 - \frac{0.0591}{4}\log[H^+]^4$$

$= -1.23 + (0.0591 \times \text{pH}) = -1.23 + 0.0591 \times 5$

$= -1.23 + 0.2955 = -0.9345 \text{ V} = -0.93 \text{ V}$

46. (4.96) 10 ppm of Fe means 10 g of Fe in 10^6 g of wheat. So, for 100 kg i.e., 10^5 g of wheat. Fe needed is 1 g. So, for 1 g of Fe, the mass of $FeSO_4 \cdot 7H_2O$ required is $\frac{278}{56} = 4.96$ g.

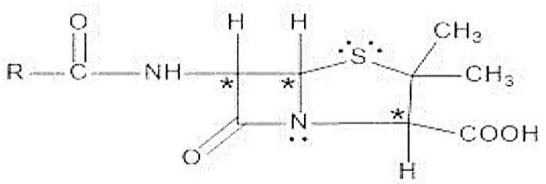
47. (48) Work done by the gas

= The area under the curve

= (Area of the square) + (Area of the triangle)

= 48 J

48. (3) The structure of penicillin is shown below:



So, the number of chiral centres = 3

49. (b) We know that, ${}^{n}C_r$ is maximum when $r = \begin{cases} \frac{n}{2}, n \text{ is even} \\ \frac{n+1}{2} \text{ or } \frac{n-1}{2}, n \text{ is odd} \end{cases}$

Therefore, $\max\left({}^{19}C_p\right) = {}^{19}C_9 = a$

$\max\left({}^{20}C_q\right) = {}^{20}C_{10} = b$

$\max\left({}^{21}C_r\right) = {}^{21}C_{11} = c$

$\therefore \frac{a}{{}^{19}C_9} = \frac{b}{\frac{20}{10} \times {}^{19}C_9} = \frac{c}{\frac{21}{11} \times \frac{20}{10} \times {}^{19}C_9}$

$\frac{a}{1} = \frac{b}{2} = \frac{c}{\frac{42}{11}}$

$\Rightarrow \frac{a}{11} = \frac{b}{22} = \frac{c}{42}.$

50. (c) If X and Y are independent events, then

$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X)P(Y)}{P(Y)} = P(X)$

Therefore, $P\left(\frac{A}{B}\right) = P(A) = \frac{1}{3} \Rightarrow P\left(\frac{A}{B'}\right) = P(A) = \frac{1}{3}.$

51. (d) $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}} = \frac{8^{4x} - 1}{8^{4x} + 1}$

Put $y = \frac{8^{4x}-1}{8^{4x}+1}$

Applying componendo-dividendo on both sides

$$\frac{y+1}{y-1} = \frac{2 \times 8^{4x}}{-2}$$
$$\frac{y+1}{y-1} = -8^{4x} \Rightarrow 8^{4x} = \frac{1+y}{1-y}$$
$$\Rightarrow x = \frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)$$
$$f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right).$$

52. (c) Given $x^2 + bx + 45 = 0, b \in \boldsymbol{R}$, let roots of the equation be $p \pm iq$

Then, sum of roots $= 2p = -b$

Product of roots $= p^2 + q^2 = 45$

As $p \pm iq$ lies on $|z+1| = 2\sqrt{10}$, we get

$(p+1)^2 + q^2 = 40$

$\Rightarrow p^2 + q^2 + 2p + 1 = 40$
$\Rightarrow 45 - b + 1 = 40$

$\Rightarrow b = 6$

$\Rightarrow b^2 - b = 30.$

53. (a) Rolle's theorem is applicable on $f(x)$ in [3,4]

$\Rightarrow f(3) = f(4)$

$$\Rightarrow \ln\left(\frac{9+\alpha}{21}\right) = \ln\left(\frac{16+\alpha}{28}\right)$$

$$\Rightarrow \frac{9+\alpha}{21} = \frac{16+\alpha}{28}$$

$\Rightarrow 36 + 4\alpha = 48 + 3\alpha$

$\Rightarrow \alpha = 12$

Now,

$$f(x) = \ln\left(\frac{x^2+12}{7x}\right) \Rightarrow f'(x) = \frac{7x}{x^2+12} \times \frac{7x \times 2x - (x^2+12) \times 7}{(7x)^2}$$

$$f'(x) = \frac{x^2-12}{x(x^2+12)}$$

$f'(c) = 0 \Rightarrow c = 2\sqrt{3}$

$$f''(x) = \frac{-x^4 + 48x^2 + 144}{x^2(x^2+12)^2}$$

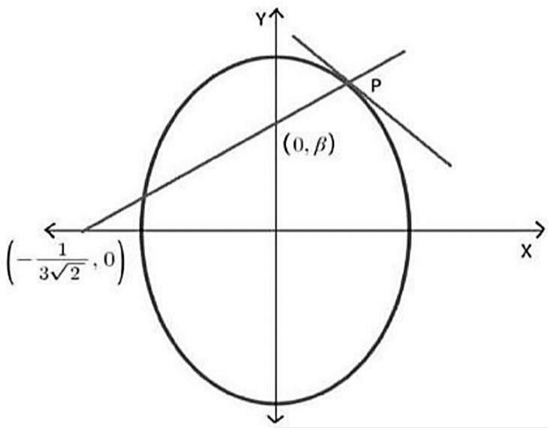
$f''(c) = \frac{1}{12}$.

54. (c)

$$f(x) = x\cos^{-1}(\sin(-|x|))$$
$$\Rightarrow f(x) = x\cos^{-1}(-\sin|x|)$$
$$\Rightarrow f(x) = x[\pi - \cos^{-1}(\sin|x|)]$$
$$\Rightarrow f(x) = x\left[\pi - \left(\frac{\pi}{2} - \sin^{-1}(\sin|x|)\right)\right]$$
$$\Rightarrow f(x) = x\left(\frac{\pi}{2} + |x|\right)$$
$$\Rightarrow f(x) = \begin{cases} x\left(\frac{\pi}{2} + x\right), & x \geq 0 \\ x\left(\frac{\pi}{2} - x\right), & x < 0 \end{cases}$$
$$\Rightarrow f'(x) = \begin{cases} \left(\frac{\pi}{2} + 2x\right), & x \geq 0 \\ \left(\frac{\pi}{2} - 2x\right), & x < 0 \end{cases}$$

Therefore, $f'(x)$ is decreasing $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$.

55. (c)



Let $P \equiv (x_1, y_1)$

$2x^2 + y^2 = 1$ is given equation of ellipse.

$\Rightarrow 4x + 2yy' = 0$

$$\Rightarrow y'|_{(x_1,y_1)} = -\frac{2x_1}{y_1}$$

Therefore, slope of normal at $P(x_1, y_1)$ is $\frac{y_1}{2x_1}$

Equation of normal at $P(x_1, y_1)$ is

$(y-y_1)=\frac{y_1}{2x_1}(x-x_1)$

It passes through $\left(-\frac{1}{3\sqrt{2}},0\right)$

$\Rightarrow -y_1=\frac{y_1}{2x_1}\left(-\frac{1}{3\sqrt{2}}-x_1\right)$

$\Rightarrow x_1=\frac{1}{3\sqrt{2}}$

$\Rightarrow y_1=\frac{2\sqrt{2}}{3}$ as P lies in first quadrant

Since $(0,\beta)$ lies on the normal of the ellipse at point P, hence we get

$\beta=\frac{y_1}{2}=\frac{\sqrt{2}}{3}$

56. (a) Area of triangle is

$$A=\frac{1}{2}\begin{vmatrix}0 & 2 & 1\\ 1 & -1 & 1\\ x' & y' & 1\end{vmatrix}=\pm 5$$
$$-2(1-x')+(y'+x')=\pm 10$$
$$-2+2x'+y'+x'=\pm 10$$
$$3x'+y'=12 \text{ or } 3x'+y'=-8$$
$$\Rightarrow \lambda=3 \text{ or } -2$$

57. (a)

$$\overrightarrow{AB}=-3\hat{\imath}-7\hat{\jmath}+6\hat{k}-(3\hat{\imath}+8\hat{\jmath}+3\hat{k})=-6\hat{\imath}-15\hat{\jmath}+3\hat{k}$$

$\vec{p}=\hat{\imath}+4\hat{\jmath}+22\hat{k}$

$\vec{q}=\hat{\imath}+\hat{\jmath}+7\hat{k}$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 4 & 22\\ 1 & 1 & 7\end{vmatrix}=6\hat{\imath}+15\hat{\jmath}-3\hat{k}$$

Shortest distance $=\frac{|\overrightarrow{AB}\cdot(\vec{p}\times\vec{q})|}{|\vec{p}\times\vec{q}|}=\frac{|36+225+9|}{\sqrt{36+225+9}}=3\sqrt{30}$.

58. (b) Let $\sin x=t \Rightarrow \cos x dx=dt$

$$\therefore \int\frac{dt}{t^3(1+t^6)^{\frac{2}{3}}}=\int\frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

Let $1+\frac{1}{t^6}=u \Rightarrow -6t^{-7}dt=du$

$$\Rightarrow \int\frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}=-\frac{1}{6}\int\frac{du}{u^{\frac{2}{3}}}=-\frac{3}{6}u^{\frac{1}{3}}+c=-\frac{1}{2}\left(1+\frac{1}{t^6}\right)^{\frac{1}{3}}+c$$

$= -\frac{(1+\sin^6 x)^{\frac{1}{3}}}{2\sin^2 x} + c = f(x)(1+\sin^6 x)^{\frac{1}{\lambda}}$

$\therefore \lambda = 3$ and $f(x) = -\frac{1}{2\sin^2 x}$

$\Rightarrow \lambda f\left(\frac{\pi}{3}\right) = -2.$

59. (c) $\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$

$\Rightarrow \sin^{-1} y + \sin^{-1} x = c$

If $x = \frac{1}{2}, y = \frac{\sqrt{3}}{2}$ then,

$\sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{2} = c$

$\therefore \frac{\pi}{3} + \frac{\pi}{6} = c \Rightarrow c = \frac{\pi}{2}$

$\Rightarrow \sin^{-1} y = \frac{\pi}{2} - \sin^{-1} x = \cos^{-1} x$

$\therefore \sin^{-1} y = \cos^{-1}\frac{1}{\sqrt{2}}$

$\Rightarrow \sin^{-1} y = \frac{\pi}{4}$

$\Rightarrow y\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

60. (a) Let $L = \lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ [Intermediate form 1^∞]

$\therefore L = e^{\lim_{x\to 0}\frac{1}{x^2}\left(\frac{3x^2+2}{7x^2+2}-1\right)}$

$= e^{\lim_{x\to 0}\frac{1}{x^2}\left(-\frac{4x^2}{7x^2+2}\right)}$

$= e^{-2}.$

61. (c) Number of ways to select at most 3 red balls $= P(0$ red balls $) + P(1$ red ball $) + P(2$ red balls $) + P(3$ red balls $)$

$= {}^7C_4 + {}^5C_1 \times {}^7C_3 + {}^5C_2 \times {}^7C_2 + {}^5C_3 \times {}^7C_1$

$= 35 + 175 + 210 + 70 = 490.$

62. (b)

$f(x) = [\sin(\tan^{-1} x) + \sin(\cot^{-1} x)]^2 - 1$

$\Rightarrow f(x) = [\sin(\tan^{-1} x) + \cos(\tan^{-1} x)]^2 - 1$

$\Rightarrow f(x) = \sin(2\tan^{-1} x)$

Now, $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\sin^{-1}(f(x)))$

$\Rightarrow 2y = \sin^{-1}(f(x)) + c$

If $x = \sqrt{3}, y = \frac{\pi}{6}$

$$\therefore \frac{\pi}{3} = \sin^{-1}\left(\sin\left(2\tan^{-1}\sqrt{3}\right)\right) + c$$

$$\Rightarrow \frac{\pi}{3} = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + c$$
$$\Rightarrow \frac{\pi}{3} = \frac{\pi}{3} + c \Rightarrow c = 0$$
$$\Rightarrow 2y = \sin^{-1}\sin(2\tan^{-1} x)$$

When $x = -\sqrt{3}$

$$2y = \sin^{-1}\left(\sin\left(2\tan^{-1}(-\sqrt{3})\right)\right) = \sin^{-1}\left(\sin\left(-\frac{2\pi}{3}\right)\right) = -\frac{\pi}{3}$$

$\Rightarrow y = -\frac{\pi}{6}$.

63. (c)

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix}$$

$R_3 \rightarrow R_3 - 2R_1 + 2R_2$

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

For inconsistent system, one of D_x, D_y, D_z should not be equal to 0

$$D_x = \begin{vmatrix} \mu & 4 & 5 \\ 1 & 2 & 3 \\ \delta & 4 & 4 \end{vmatrix} \; D_y = \begin{vmatrix} 3 & \mu & 5 \\ 1 & 1 & 3 \\ 4 & \delta & 4 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 3 & 4 & \mu \\ 1 & 2 & 1 \\ 4 & 4 & \delta \end{vmatrix}$$

For inconsistent system, $2\mu \neq \delta + 2$

$\therefore$ The system will be inconsistent for $\mu = 4, \delta = 3$.

64. (c) Volume of parallelepiped $= [\vec{u} \quad \vec{v} \quad \vec{w}]$

$$\Rightarrow \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = \pm 1$$

$\Rightarrow \lambda = 2$ or 4

For $\lambda = 4$,

$\cos\theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$.

65. (c) $2^{1-x} + 2^{1+x}, f(x), 3^x + 3^{-x}$ are in A.P.

$$\therefore f(x) = \frac{3^x + 3^{-x} + 2^{1+x} + 2^{1-x}}{2} = \frac{(3^x + 3^{-x})}{2} + \frac{2^{1+x} + 2^{1-x}}{2}$$

Applying A.M. ≥ G.M. inequality, we get

$$\frac{(3^x + 3^{-x})}{2} \geq \sqrt{3^x \cdot 3^{-x}}$$

$$\Rightarrow \frac{(3^x + 3^{-x})}{2} \geq 1$$

Also, Applying A.M. ≥ G.M. inequality, we get

$$\frac{2^{1+x} + 2^{1-x}}{2} \geq \sqrt{2^{1+x} \cdot 2^{1-x}}$$

$$\Rightarrow \frac{2^{1+x} + 2^{1-x}}{2} \geq 2$$

Adding (1) and (2), we get

$f(x) \geq 1 + 2 = 3$

Thus, minimum value of $f(x)$ is 3 .

66. (a)

$$\begin{aligned} & (p \wedge (p \rightarrow q)) \rightarrow q \\ &= (p \wedge (\sim p \vee q)) \rightarrow q \\ &= [(p \wedge \sim p) \vee (p \wedge q)] \rightarrow q \\ &= (p \wedge q) \rightarrow q \\ &= \sim (p \wedge q) \vee q \\ &= \sim p \vee \sim q \vee q \\ &= T \end{aligned}$$

67. (c) $\operatorname{tr}(AA^T) = 3$

$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$\operatorname{tr}(AA^T) = a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{33}^2 = 3$

So out of 9 elements (a_{ij})'s, 3 elements must be equal to 1 or -1 and rest elements must be 0 .

So, the total possible cases will be

When there is $6(0's)$ and $3(1's)$ then the total possibilities is ${}^9C_6$

For $6(0's)$ and $3(-1's)$ total possibilities is ${}^9C_6$

For $6(0's), 2(1's)$ and $1(-1's)$ total possibilities is ${}^9C_6 \times 3$

For $6(0's), 1(1's)$ and $2(-1's)$ total possibilities is ${}^9C_6 \times 3$

$\therefore$ Total number of cases $= {}^9C_6 \times 8 = 672.$

68. (b) If mean $\bar{x}$ is multiplied by p and then q is subtracted from it,

then new mean $\bar{x}' = p\bar{x} - q$

$\therefore \bar{x}' = \frac{1}{2}\bar{x}$ and $\bar{x} = 10$

$\Rightarrow 10 = 20p - q$

If standard deviation is multiplied by p, new standard deviation (σ') is p times of the initial standard deviation (σ).

$\sigma' = |p|\sigma$

$\Rightarrow \frac{1}{2}\sigma = |p|\sigma \Rightarrow |p| = \frac{1}{2}$

If $p = \frac{1}{2}, q = 0$

If $p = -\frac{1}{2}, q = -20.$

69. (b) Let point P be $(2t, t^2)$ and Q be (h, k).

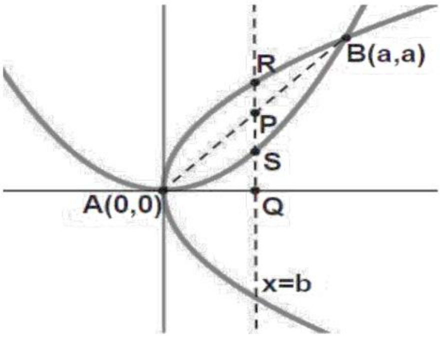
$$h = \frac{2t}{3}, k = \frac{-2 + t^2}{3}$$

Now, eliminating t from the above equations we get:

$$3k + 2 = \left(\frac{3h}{2}\right)^2$$

Replacing h and k by x and y, we get the locus of the curve as $9x^2 = 12y + 8$.

70. (c)



Given, $\text{ar}(\triangle APQ) = \frac{1}{2}$

$\Rightarrow \frac{1}{2} \times b \times b = \frac{1}{2}$

$\Rightarrow b = 1$

As per the question

$\Rightarrow \int_0^1 \left(\sqrt{ax} - \frac{x^2}{a}\right) dx = \frac{1}{2} \int_0^a \left(\sqrt{ax} - \frac{x^2}{a}\right) dx$

$\Rightarrow \frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$

$\Rightarrow 2a\sqrt{a} - 1 = \frac{a^3}{2}$
$\Rightarrow 4a\sqrt{a} = 2 + a^3$
$\Rightarrow 16a^3 = 4 + a^6 + 4a^3$
$\Rightarrow a^6 - 12a^3 + 4 = 0.$

71. (1540)

$= \sum_{k=1}^{20} \frac{k(k+1)}{2}$

$= \frac{1}{2} \sum_{k=1}^{20} k^2 + k$

$= \frac{1}{2}\left[\frac{20(21)(41)}{6} + \frac{20(21)}{2}\right]$

$= \frac{1}{2}[2870 + 210]$

$= 1540.$

72. (8)

$\because 2x^2 + (a-10)x + \frac{33}{2} = 2a, a \in \mathbf{Z}^+$ has real roots

$\Rightarrow D \geq 0$

$\Rightarrow (a-10)^2 - 4 \times 2 \times \left(\frac{33}{2} - 2a\right) \geq 0$

$\Rightarrow (a-10)^2 - 4(33-4a) \geq 0$

$\Rightarrow a^2 - 4a - 32 \geq 0 \Rightarrow a \in (-\infty, 4] \cup [8, \infty)$

Thus, minimum value of ' $a' \forall a \in \mathbf{Z}^+$is 8

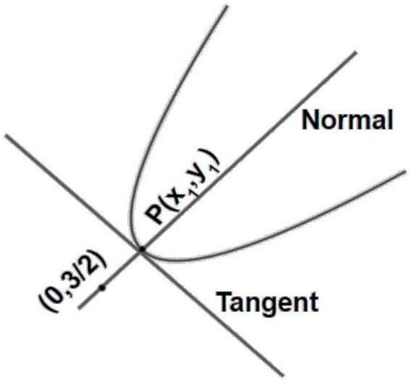
73. **(4)** Let co-ordinate of P be (x_1, y_1)

Differentiating the curve w.r.t x

$2yy' - 6x + y' = 0$

Slope of tangent at P

$\Rightarrow y' = \dfrac{6x_1}{1+2y_1}$



$\because m_{\text{normal}} = \left(\dfrac{y_1 - \frac{3}{2}}{x_1 - 0}\right)$

$\because m_{\text{normal}} \times m_{\text{tangent}} = -1$

$\Rightarrow \dfrac{\frac{3}{2} - y_1}{-x_1} \times \dfrac{6x_1}{1+2y_1} = -1$

$\Rightarrow y_1 = 1$

$\Rightarrow x_1 = \pm 2$

Slope of tangent $= \pm\frac{12}{3} = \pm 4$

$\Rightarrow |n| = 4$