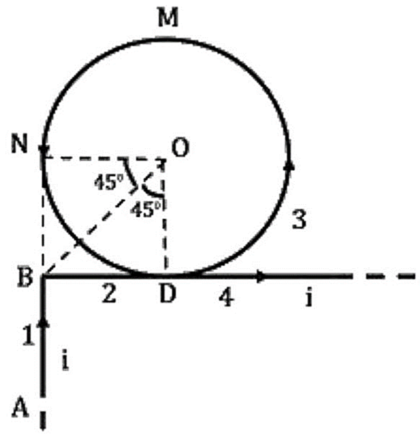
1. (d)



To get magnetic field at 0 , we need to find magnetic field due to each current carrying part 1, 2, 3 and 4 individually.

Let's take total magnetic field as B_T, then

$$\vec{B}_T = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$$

Since 2 and 4 are parts of same wire, hence

$$\vec{B}_T = \frac{\mu_o i}{4\pi R}(\sin 90^\circ - \sin 45^\circ)(-\hat{k}) + \frac{\mu_o i}{2R}\hat{k} + \frac{\mu_o i}{4\pi R}(\sin 90^\circ + \sin 45^\circ)\hat{k}$$

$$= \frac{-\mu_o i}{4\pi R}\left[1 - \frac{1}{\sqrt{2}}\right] + \frac{\mu_o i}{2R} + \frac{\mu_o i}{4\pi R}\left[1 + \frac{1}{\sqrt{2}}\right]\hat{k}$$

$$\vec{B}_T = \frac{\mu_o i}{4\pi R}[\sqrt{2} + 2\pi]\hat{k}$$

$$\vec{B}_T = \frac{\mu_o i}{2\pi R}\left[\frac{1}{\sqrt{2}} + \pi\right]\hat{k}$$

$\hat{k}$ denotes that direction of magnetic field is in the plane coming out of the plane of current.

2. (d)

$$\vec{r} = \cos\omega t\hat{\imath} + \sin\omega t\hat{\jmath}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \omega[-\sin\omega t\hat{\imath} + \cos\omega t\hat{\jmath}]$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2[\cos\omega t\hat{\imath} + \sin\omega t\hat{\jmath}]$$

$$\vec{a} = -\omega^2\vec{r}$$

Since there is negative sign in acceleration, this means that acceleration is in opposite direction of $\vec{r}$

For velocity direction we can take dot product of $\vec{v}$ and $\vec{r}$.

$$\vec{v}\cdot\vec{r} = \omega(-\sin\omega t\hat{\imath} + \cos\omega t\hat{\jmath})\cdot(\cos\omega t\hat{\imath} + \sin\omega t\hat{\jmath})$$

$$= \omega[-\sin\omega t\cos\omega t + \cos\omega t\sin\omega t] = 0$$

which implies that $\vec{v}$ is perpendicular to $\vec{r}$.

3. (b)

$$\frac{E_1}{E_2}=\frac{r_1}{r_2}$$

$$\frac{V_1}{V_2}=\frac{E_1r_1}{E_2r_2}=\frac{r_1}{r_2}\times\frac{r_1}{r_2}=\left(\frac{r_1}{r_2}\right)^2$$

4. (d)

$$V\propto\sqrt{T}$$

$$\frac{V_1}{V_2}=\sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow\frac{2V}{V}=\sqrt{\frac{2.06\times10^4}{T}}$$

$$\Rightarrow T=\frac{2.06\times10^4}{4}N$$

$$=5.15\times10^3\ \text{N}$$

5. (a) Time taken for the collision $t_1=\frac{h}{\sqrt{2gh}}$

After t_1

$$V_A=0-gt_1=-\frac{\sqrt{gh}}{2}$$

And $V_B=\sqrt{2gh}-gt_1=\sqrt{gh}\left[\sqrt{2}-\frac{1}{\sqrt{2}}\right]$

At the time of collision

$$\vec{P_l}=\vec{P_f}$$

$$\Rightarrow m\vec{V_A}+m\vec{V_B}=2m\vec{V_f}$$

$$\Rightarrow-\sqrt{\frac{gh}{2}}+\sqrt{gh}\left[\sqrt{2}-\frac{1}{\sqrt{2}}\right]=2\vec{V_f}$$

$$V_f=0$$

And height from the ground $=h-\frac{1}{2}gt_1^2=h-\frac{h}{4}=\frac{3h}{4}$.

So, time taken to reach ground after collision $=\sqrt{2\times\frac{\left(\frac{3h}{4}\right)}{g}}=\sqrt{\frac{3h}{2g}}$

6. (b) For Carnot engine using as refrigerator

Work done on engine is given by

$W=Q_1-Q_2\ ...$

where Q_1 is heat rejected to the reservoir at higher temperature and Q_2 is the heat absorbed from the reservoir at lower temperature.

It is given $\eta=1/10$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$\Rightarrow \frac{Q_2}{Q_1} = \frac{9}{10}$$

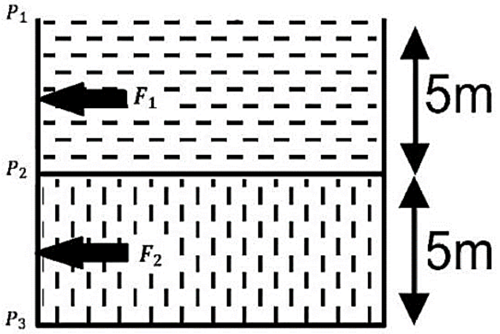
We are given, $W = 10$ J

Therefore, from equations (1) and (2),

$$Q_2 = \frac{10}{\frac{10}{9} - 1}$$

$$\Rightarrow Q_2 = 90 \text{ J}$$

7. (c)



The net force exerted on the wall by one type of liquid will be average value of pressure due to that liquid multiplied by the area of the wall.

Here, since the pressure due a liquid of uniform density varies linearly with depth, its average will be just the mean value of pressure at the top and pressure at the bottom. So,

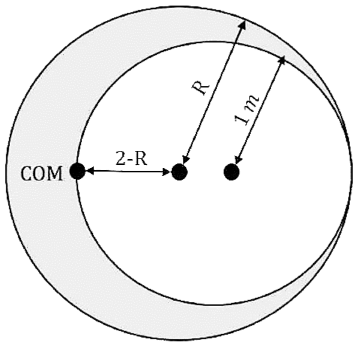
$$P_1 = 0$$
$$P_2 = \rho g \times 5$$
$$P_3 = 5\rho g + 2\rho g \times 5$$

$$F_1 = \left(\frac{P_1 + P_2}{2}\right) A$$
$$F_2 = \left(\frac{P_2 + P_3}{2}\right) A$$

So,

$$\frac{F_1}{F_2} = \frac{1}{4}$$

8. (a)



Let M be the mass of the sphere and M' be the mass of the cavity.

Mass of the remaining part of the sphere $= M - M'$

Mass moments of the cavity and the remaining part of sphere about the original CoM should add up to zero.

$(M - M')(2 - R) - M'(R - 1) = 0$

(Mass of the cavity to be taken negative)

$$\Rightarrow \frac{4}{3}\pi(R^3 - 1^3)\rho g(2 - R) = \frac{4}{3}\pi(1)^3\rho g(R - 1)$$
$$\Rightarrow (R^3 - 1^3)(2 - R) = (1^3)(R - 1)$$
$$\Rightarrow (R^2 + R + 1)(R - 1)(2 - R) = (R - 1)$$

(using identity)

$\Rightarrow (R^2 + R + 1)(2 - R) = 1$

9. **(b)** Applying work energy theorem,

$$qEx = \frac{1}{2}mv^2$$
$$v^2 \propto x$$

10. **(b)**

$$V_0 = i_g R_g = 0.1v$$
$$v = 10\text{ V}$$
$$R = R_g\left(\frac{v}{v_g} - 1\right)$$
$$= 100 \times 99 = 9.9\text{ K}\Omega$$

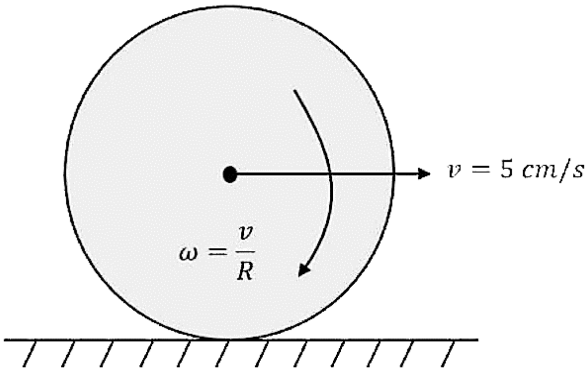
11. **(c)** Using formula

$$C_{v_{\text{mix}}} = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$$

$= \frac{n\times\frac{3R}{2}+2n\times\frac{5R}{2}}{3n} = \frac{13R}{6}$ ($\because$ He is monoatomic and O_2 is diatomic)

$$C_{p_{\text{mix}}} = C_{v_{\text{mix}}} + R = \frac{19R}{6}$$
$$\therefore \gamma_{\text{mix}} = \frac{C_{p_{\text{mix}}}}{C_{v_{\text{mix}}}} = \frac{19}{13}$$

12. (a)



Total K.E. = Translational K.E + Rotational K.E.

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

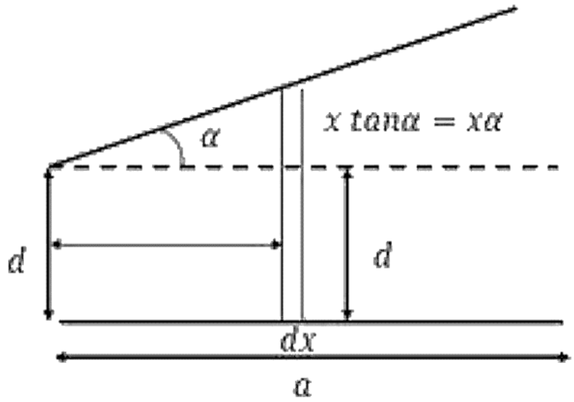
$$= \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$$

k is radius of gyration.

$$= \frac{1}{2}\times\frac{1}{2}\times\left(\frac{5}{100}\right)^2\left(1 + \frac{2}{5}\right)$$

$$= \frac{35}{4}\times 10^{-4}\ \text{J}$$

13. (b)



Let dC be the capacitance of the element of thickness dx

$$dc = \frac{\varepsilon_0 adx}{d + \alpha x}$$

These are effectively in parallel combination

So,

$$C=\int dc$$
$$C=\int_0^a \frac{\varepsilon_0 a dx}{d+\alpha x}$$
$$\Rightarrow c=\frac{\varepsilon_0 a}{\alpha}[\ln(d+\alpha x)]_0^a$$
$$=\frac{\varepsilon_0 a}{\alpha}\left[\ln\left(1+\frac{\alpha a}{d}\right)\right]$$
$$\approx\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$$

14. (b) In YDSE, the intensity at a point on the screen varies with the phase difference between the interfering light waves as:

$$I=I_0\cos^2\left(\frac{\Delta\emptyset}{2}\right)$$

Here, $\Delta\emptyset=$ phase difference between the interfering waves

$I_0=$ maximum intensity on the screen

$$\frac{I}{I_0}=\cos^2\left[\frac{\frac{2\pi}{\lambda}\times\Delta x}{2}\right]=\cos^2\left(\frac{\pi}{8}\right)$$
$$\frac{I}{I_0}=0.853$$

15. (a) This is standard $L-R$ growth of current circuit.

$$i(t)=\frac{\varepsilon}{R}e^{-\frac{t}{Tc}}$$

Substituting in the integral,

$$q=\int_0^{Tc}(i)dt$$
$$=\frac{\varepsilon}{R}\left[t-\frac{e^{\frac{-t}{Tc}}}{\frac{-1}{T_c}}\right]_0^{Tc}$$
$$\approx\frac{\varepsilon L}{eR^2}$$

16. (a) Since the radius of planet is much larger than 100 m, it's a uniformly accelerated motion.

So, Trapezium's area

$$s=\frac{g\left(\left[t-\frac{1}{2}+t\right]\right.}{2}\times\frac{1}{2}=19$$
$$\frac{1}{2}gt^2=100$$

Solving equations (i) and (ii), we get

$g = 8 \text{ m/s}^2$

17. (c)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$g = L \cdot \left(\frac{2\pi}{T}\right)^2$$

$$\frac{\Delta g}{g} = 2\frac{\Delta T}{T} + \frac{\Delta L}{L}$$

$$2\left(\frac{1}{50}\right) + \frac{0.1}{25} = 4.4\%$$

18. (b) Momentum of an electron

$$p = mv = \frac{h}{\lambda}$$

Initially $m(\sqrt{2}v_0) = \frac{h}{\lambda_0}$

Velocity as a function of time $= v_0\hat{\imath} + v_0\hat{\jmath} + \frac{eE_0}{m}t\hat{k}$

So, wavelength $\lambda = \frac{\mathrm{h}}{\mathrm{m}\sqrt{2\mathrm{v}_0^2 + \frac{\mathrm{e}^2\mathrm{E}_0^2}{\mathrm{m}^2}\mathrm{t}^2}}$

$$\Rightarrow \lambda = \frac{\lambda_0}{\sqrt{1 + \frac{\mathrm{e}^2\mathrm{E}_0^2}{\mathrm{m}^2\mathrm{v}_0^2}\mathrm{t}^2}}$$

19. (b)

$$Y = \overline{\overline{AB} \cdot A}$$
$$= \overline{\overline{AB}} + \bar{A}$$
$$= AB + \bar{A}$$
$$Y = 0 + 0 = 0$$

20. (486)

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda_1} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}.$$

$$\frac{1}{\lambda_2} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{36}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

$$\lambda_2 = \frac{20}{27} \times 6561\text{Å} = 4860\text{Å}$$

21. (15) $\frac{E}{B} = c$
$E = B \times c$

Given,

$\vec{B} = 5 \times 10^{-8} \hat{j} T$ and $C = 3 \times 10^{8}$ m/s

$E = 15 \frac{\text{volt}}{m}$

22. (50)

Since, all the containers have same material, specific heat capacity is the same for all.

$$V_1\theta_1 + 2\theta_2 = (V_1 + V_2)\theta_f$$
$$1\theta_1 + 1\theta_2 = (1 + 2)60$$
$$\theta_1 + 2\theta_2 = 180$$
$$0 \times \theta_1 + 1 \times \theta_2 + 2 \times \theta_3 = (1 + 2)30$$

$$\theta_2 + 2\theta_3 = 180$$
$$\theta_1 + 2\theta_2 + \theta_3 = (1 + 1 + 1)\theta$$

From equation. (1)+(2)+(3)

$3\theta_1 + 3\theta_2 + 3\theta_3 = 450$

Where, $\theta_1 + \theta_2 + \theta_3 = 150$

From (4) equation $150 = 30$

So, $\theta = 50°\text{C}$

23. (16) Taking, asteroid and earth as an isolated system conserving total energy.

$$KE_i + PE_i = KE_f + PE_f$$
$$\frac{1}{2}mu_0^2 + \left(-\frac{GMm}{10R}\right) = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$$
$$v^2 = u_0^2 + \frac{2GM}{R}\left[1 - \frac{1}{10}\right]$$
$$v = \sqrt{u_0^2 + \frac{9}{5}\frac{GM}{R}}$$

Since, escape velocity from surface of earth is $11.2 \frac{\text{km}}{\text{sec}^2} = \sqrt{\frac{2GM}{R}}$

$$= \sqrt{12^2 + \frac{9}{5}\frac{(11.2)^2}{2}}$$
$$= \sqrt{256.9} \approx 16 \text{ km/s}.$$

24. (30) If V_1 and V_2 are terminal voltage across the two batteries.

$$V_1 = 0$$
$$V_1 = \varepsilon_1 - i.r_1$$
$$0 = 10 - i \times 20$$
$$i = 0.5\text{ A}$$
$$V_2 = 10 - 0.5 \times 5$$
$$V_2 = 7.5\text{ V}$$
$$0.5 = \frac{7.5}{30} + \frac{7.5}{x}$$
$$\frac{7.5}{x} = 0.25$$
$$x = 30\Omega$$

25. **(c)** In C - F there is $2p - 2p$ overlapping involved, in $C - Cl$ the overlapping involved is $2p - 3p$ whereas for $C - Br$ and $C - I$ the overlappings involved are 2p − 4p and 2p − 5p, respectively. The bond length for the various type of overlappings can be given as:

$2p - 2p < 2p - 3p < 2p - 4p < 2p - 5p$.

As we know that Bond energy $\alpha \frac{1}{\text{Bond length}}$

The order of bond energy comes out: $C - F > C - Cl > C - Br > C - I$

26. **(b)** The formula for Bohr's radius for any unielectronic species is: $r = \frac{a_0 n^2}{z}$ for Li^{2+}: $r = \frac{a_0 2^2}{3} = \frac{4a_0}{3}$

27. **(a)** As it is provided in the question that nitride is being formed so the option c and d can be eliminated. Amongst Mg and Na we already know that Mg can only form nitride so the correct choice is option a.

$3Mg + N_2 \rightarrow Mg_3N_2 \xrightarrow{H_2O} Mg(OH)_2 + NH_3$

28. **(a)** $[Pd(PPh_3)_2Cl_2]$: Here Pd is in +2 oxidation state and configuration of Pd^{2+} is $[Kr]4\ d^8$. As the CFSE value for Pd is very high so all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(CO)_4]$: Here Ni is in 0 oxidation state and configuration of Ni is $[Ar]3\ d^8 4\ s^2$. As here the ligand is carbonyl which is a strong field ligand, all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(CN)_4]^{2-}$: Here Ni is in +2 oxidation state and configuration of Ni^{2+} is $[Ar]3\ d^8$. As here the ligand is cyanide which is a strong field ligand, all the electrons will be paired and hence magnetic moment for this complex will be zero.

$[Ni(H_2O)_6]^{2+}$: Here Ni is in +2 oxidation state and configuration of Ni^{2+} is $[Ar]3\ d^8$. As here the ligand is water which is a weak field ligand, the electrons will not be paired and there are two unpaired electrons in this complex hence magnetic moment for this complex will be $\sqrt{8}$BM.

So the order of magnetic moment is A = B = C < D.

29. **(c)** Number of neutrons in protium = 0

Number of neutrons in deuterium = 1

Number of neutrons in tritium = 2

So, total number of neutrons = 3

30. **(a)** To avoid confusion, in this question we'll be denoting activation energy by E_x

$K = Ae^{-E_x/RT}$

$\log K = \log A - \frac{E_X}{2.303RT}$

Here, the graph given in the question is of a straight line and we know that the equation of straight line is

$y = mx + c$

Comparing equation 1 with 2 we get,

$\text{Slope} = \frac{-E_x}{2.303R}$

So, from the graph we can conclude that the line with the most negative slope will have the maximum activation energy value.

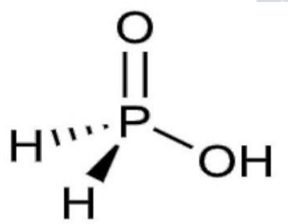
$E_b > E_c > E_d > E_a$

31. **(a)** The radius ratio for AgBr is intermediate. Thus, it shows both Frenkel and Schottky defects.

32. **(a)**

$P_4 + NaOH + H_2O \rightarrow PH_3 + NaH_2PO_2 \xrightarrow{HCl} H_3PO_2 + NaCl$

Here the product *B* which is mentioned in the question is H_3PO_2. The structure of H_3PO_2 can be given as:



As only 1 Hydrogen atom is attached to the oxygen, its basicity is one.

33. **(c)** In metallurgy of iron, CaO is used as flux which is used to remove the impurities of SiO_2, $CaO + SiO_2 \rightarrow CaSiO_3$

Also, here Fe_2O_3 is reduced by CO to Fe_3O_4 which is further reduced to FeO which is further reduced to Fe.

$3Fe_2O_3 + CO \rightarrow 2Fe_3O_4 + CO_2$

$Fe_3O_4 + CO \rightarrow 3FeO + CO_2$

$FeO + CO \rightarrow Fe + CO_2$

34. **(a)** Across the period size decreases, so the order that follows is: $C > O > N > F$

Down the group size increases and the order is: $Br > Cl > F$

Change in size down the group is much more significant as compared to across the period.

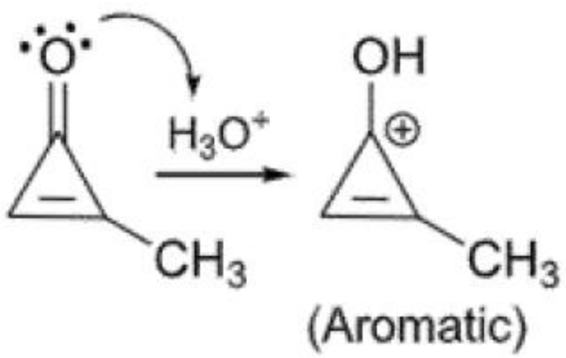
So, the overall order of radius of elements is: $Br > Cl > C > O > F$.

35. (a) The complexes of type Ma_4bc and $Ma_2\ b_2$ can show geometrical isomerism provided $Ma_2\ b_2$ is square planar. The compound given in B is Ma_4 bc type and compound in D is $Ma_2\ b_2$ type also in D, Ni is surrounded with strong field ligands which will result in dsp^2 hybridisation and hence square planar geometry.

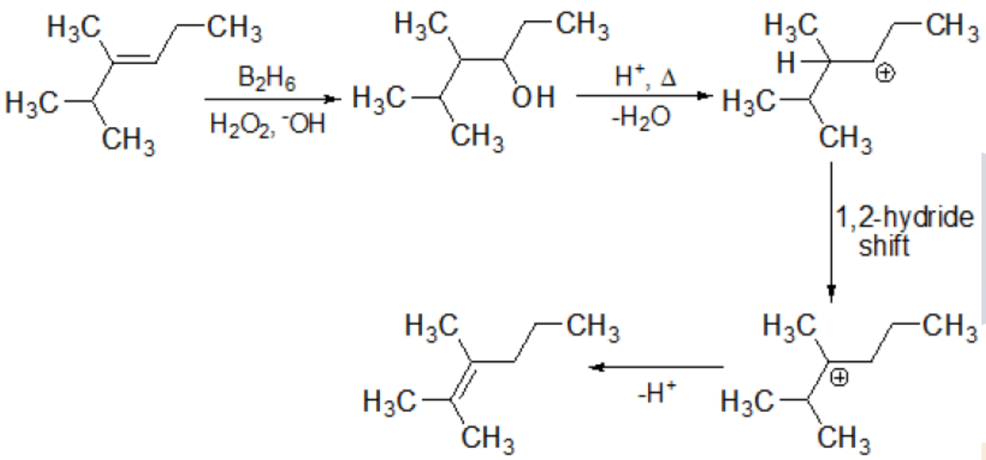
36. (d) $H_2O \rightarrow H^+ + OH^-$ is an endothermic process. On increasing the temperature, the value of K_w increases which will result in decrease in pK_w. So we can say that pH of water will decrease on increasing temperature because pH for water $= \frac{1}{2} pK_w$.

37. (a) Group 7-9 elements of the periodic table show variable valencies so they have maximum activity because of the increase in adsorption rate.

38. (b)



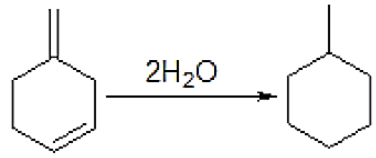
39. (a)

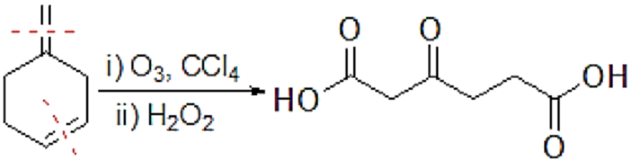


40. (b) In option b compound A has extensive inter-molecular hydrogen bonding because of the 3 $-OH$ groups while in compound B there are $-OCH_3$ groups present and no inter-molecular hydrogen bonding is possible.

41. (a) Kjeldahl method cannot be used for the estimation of nitrogen in the compounds in which nitrogen is involved in nitro, diazo groups or is present in the ring, as nitrogen atom can't be converted to ammonium sulphate under the reaction conditions.

42. (c)





3-oxohexanedioic acid

43. (a)

44. (a)

45. (6.25) $\Delta U = nC_v\Delta T$

$5000 = 4 \times Cv(500 - 300)$

$Cv = 6.25 JK^{-1}\ mol^{-1}$

46. (2.15)

Anodic half: $Sn \rightarrow Sn^{2+} + 2e^-$

Cathodic half: $Pb^{2+} + 2e^- \rightarrow Pb$

Net reaction: $Sn + Pb^{2+} \rightarrow Pb + Sn^{2+}$

$E^0_{cell} = E^0_{cathode} - E^0_{anode}$

$E^0_{cell} = 0.01\ V$

$E_{cell} = E^0_{cell} - \frac{0.06}{2} \log Q$

At equilibrium state $E_{cell} = 0$

So,

$0 = 0.01 - \frac{0.06}{2} \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$

$0.01 = \frac{0.06}{2} \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$

$\log \frac{[Sn^{2+}]}{[Pb^{2+}]} = \frac{1}{3}$

$\frac{[Sn^{2+}]}{[Pb^{2+}]} = 10^{\frac{1}{3}} = 2.154$

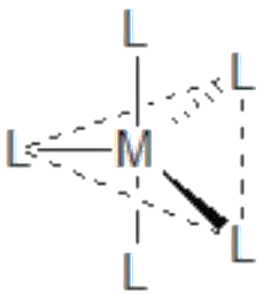
47. (2.13) Mol of $NaClO_3$ = mol of O_2

Mol of $O_2 = \frac{PV}{RT} = \frac{1 \times 492}{0.082 \times 300} = 20$ mol

Molar mass of $NaClO_3$ is 106.5

So, mass $= 20 \times 106.5 = 2130$ g $= 2.13$Kg

48. (20)



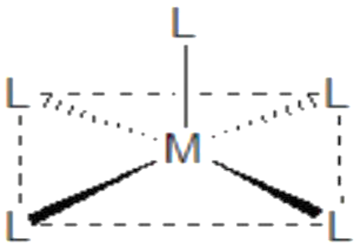
For trigonal bipyramidal geometry

Total number of $180° L-M-L$ bond angles $= 1$

Total number of $90° L-M-L$ bond angles $= 6$

Total number of $120° L-M-L$ bond angles $= 3$

Total $= 10$



For square pyramidal geometry

Total number of $180° L-M-L$ bond angles $= 2$

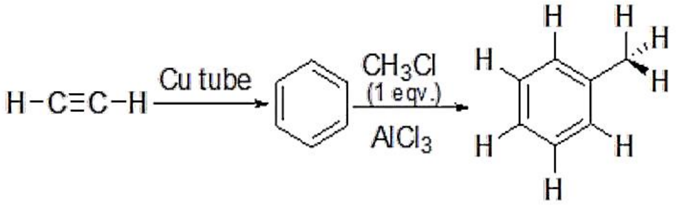
Total number of $90° L-M-L$ bond angles $= 8$

Total number of $120° L-M-L$ bond angles $= 0$

Total $= 10$

Total for both the structures $= 20$

49. (13)



50. (a)

$3^x(3^x-1)+2=|3^x-1|+|3^x-2|$

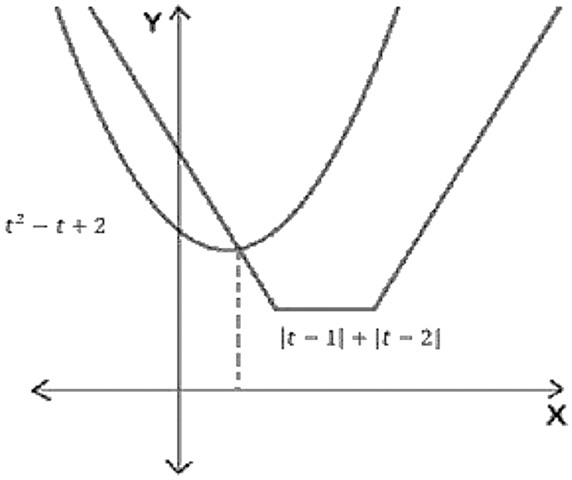
Let $3^x = t$

$t(t-1)+2 = |t-1|+|t-2|$

$\Rightarrow t^2 - t + 2 = |t-1|+|t-2|$

We plot $t^2 - t + 2$ and $|t-1|+|t-2|$

As 3^x is always positive, therefore only positive values of t will be the solution.



Therefore, we have only one solution.

51. (c) $\sim(p \vee \sim q) \rightarrow (p \vee q)$

$= (p \vee \sim q) \vee (p \vee q)$

$= (p \vee p) \vee (q \vee \sim q)$

$= p \vee T$

$= T$

52. (b) Vertex of hyperbola is $(\pm a, 0) \equiv (\pm 6, 0) \Rightarrow a = 6$

Let the equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{b^2} = 1$$

As $P(10,16)$ lies on the hyperbola.

$$\frac{100}{36} - \frac{256}{b^2} = 1$$

$$\Rightarrow \frac{64}{36} = \frac{256}{b^2} \Rightarrow b^2 = 144$$

Equation of hyperbola becomes $\frac{x^2}{36} - \frac{y^2}{144} = 1$

Equation of normal is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$

$\Rightarrow \frac{36x}{10} + \frac{144y}{16} = 180$

$\Rightarrow \frac{x}{50} + \frac{y}{20} = 1$

$\Rightarrow 2x + 5y = 100$

53. (d) For circle, $x^2 + y^2 = 1$

$2x + 2yy' = 0 \Rightarrow y' = -\frac{x}{y}$

Slope of tangent to $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -1$

$\Rightarrow$ Slope of tangent to $(x-3)^2 + y^2 = 1$ is $1 \Rightarrow m = 1$

Tangent to $(x-3)^2 + y^2 = 1$ is $y = x + c$

Perpendicular distance of tangent $y = x + c$ from centre $(3,0)$ is equal to radius $= 1$

$\left|\frac{3+c}{\sqrt{2}}\right| = 1$

$\Rightarrow c + 3 = \pm\sqrt{2}$

$\Rightarrow c^2 + 6c + 9 = 2$

$\Rightarrow c^2 + 6c + 7 = 0$

54. (c) $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$

$\Rightarrow \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$

$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$

$\Rightarrow -(\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$

$\Rightarrow -4\vec{c} = 6(\hat{\imath} - \hat{\jmath} + \hat{k}) - 4(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$\Rightarrow \vec{c} = -\frac{1}{2}(\hat{\imath} + \hat{\jmath} + \hat{k})$

$\therefore \vec{b} \cdot \vec{c} = -\frac{1}{2}.$

55. (d)

$$\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$$
$$=2[\,^6C_0x^6+\,^6C_2x^4(x^2-1)+\,^6C_4x^2(x^2-1)^2+\,^6C_6(x^2-1)^3]$$
$$=2[32x^6-48x^4+18x^2-1]$$
$$\Rightarrow \alpha=-96,\ \beta=36$$
$$\Rightarrow \alpha-\beta=-132$$

56. (a) $x^2=4b(y+b)$

Differentiating both the sides w.r.t. x, we get

$\Rightarrow 2x=4by'$

$\Rightarrow b=\dfrac{x}{2y'}$

Putting the value of b in (1), we get

$\Rightarrow x^2=\dfrac{2x}{y'}\left(y+\dfrac{x}{2y'}\right)$

$\Rightarrow x^2=\dfrac{2xy}{y'}+\dfrac{x^2}{y'^2}$

$\Rightarrow xy'^2=2yy'+x$

$\Rightarrow x\left(\dfrac{dy}{dx}\right)^2=2y\left(\dfrac{dy}{dx}\right)+x$

57. (a) Image of point $P(1,2,3)$ w.r.t. a plane $ax+by+cz+d=0$ is $Q\left(-\frac{7}{3},-\frac{4}{3},-\frac{1}{3}\right)$

Direction ratios of PQ: $-\frac{10}{3},-\frac{10}{3},-\frac{10}{3}=1,1,1$

Direction ratios of normal to plane is 1,1,1

Mid-point of PQ lies on the plane

$\therefore$ The mid-point of $PQ=\left(-\frac{2}{3},\frac{1}{3},\frac{4}{3}\right)$

$\therefore$ Equation of plane is $x+\frac{2}{3}+y-\frac{1}{3}+z-\frac{4}{3}=0$

$\Rightarrow x+y+z=1$

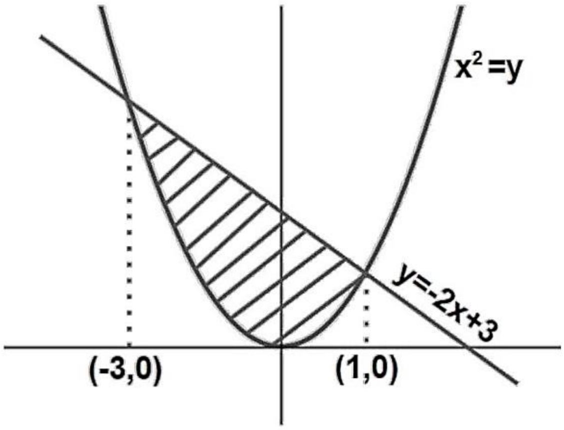
$(1,1,-1)$ satisfies the equation of the plane.

58. (b) $\lim_{x\to 0}\frac{\int_0^x t\sin 10t\,dt}{x}$

Applying L'Hospital's Rule:

$=\lim_{x\to 0}\dfrac{x\sin 10x}{1}=0$

59. (d) We have $x^2\le y\le -2x+3$



For point of intersection of two curves -

$x^2 + 2x - 3 = 0$

$\Rightarrow x = -3, 1$

$\Rightarrow$ Area $= \int_{-3}^{1} ((-2x + 3) - x^2) dx$

$= \left[-x^2 + 3x - \frac{x^3}{3}\right]_{-3}^{1} = \frac{32}{3}$ sq. units.

60. (d)

$$f(x) = \frac{x[x]}{x^2 + 1}$$

$$\Rightarrow f(x) = \begin{cases} \frac{x}{x^2 + 1} : 1 < x < 2 \\ \frac{2x}{x^2 + 1} : 2 \le x < 3 \end{cases}$$

$\Rightarrow$ Range of $f(x)$ is $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$.

61. (b) $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$

$A^{-1} = -\frac{1}{10}\begin{bmatrix} 4 & -2 \\ -9 & 2 \end{bmatrix}$

$\Rightarrow 10A^{-1} = \begin{bmatrix} -4 & 2 \\ 9 & -2 \end{bmatrix}$

$\Rightarrow 10A^{-1} = A - 6I$

62. (b)

Mean $= 10 \Rightarrow \frac{\sum x_i}{20} = 10 \Rightarrow \sum x_i = 200$

Variance $= 4 \Rightarrow \frac{\sum x\_i^2}{20} - 100 = 4 \Rightarrow \sum x_i^2 = 2080$

New mean $= \frac{200 - 9 + 11}{20} = \frac{202}{20} = 10.1$

New variance $= \frac{2080 - 81 + 121}{20} - (10.1)^2$

$= 106 - 102.01$

$= 3.99$

63. (c)

$$D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = 18\lambda - 5\lambda^2 - 24\lambda + 40 + 4\lambda^2 - 24$$

$\Rightarrow D = -\lambda^2 - 6\lambda + 16$

Now, $D = 0$

$\Rightarrow \lambda^2 + 6\lambda - 16 = 0$

$\Rightarrow \lambda = -8$ or 2

For $\lambda = 2$

$$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix} = 40 + 4 - 28 \neq 0$$

∴ Equations have no solution for $\lambda = 2$.

64. (a)

$T_{10} = \frac{1}{20}, T_{20} = \frac{1}{10}$

$T_{20} - T_{10} = 10d$

$\Rightarrow \frac{1}{20} = 10d$

$d = \frac{1}{200}$

$\therefore a = \frac{1}{200}$

$$S_{200} = \frac{200}{2}\left[2\left(\frac{1}{100}\right) + 199\left(\frac{1}{200}\right)\right]$$

$= 100\frac{1}{2}$

65. (a) $\alpha = \frac{-1+i\sqrt{3}}{2} = \omega$

$a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$

$\Rightarrow a(1+\alpha)[1+\alpha^2+\alpha^4+\cdots.+\alpha^{200}]$

$$\Rightarrow a = (1+\alpha)\left[\frac{1-(\alpha^2)^{101}}{1-\alpha^2}\right]$$

$$\Rightarrow a = \left[\frac{1-(\omega^2)^{101}}{1-\omega}\right] = \left[\frac{1-\omega}{1-\omega}\right] = 1$$

$b = \sum_{k=0}^{100} \alpha^{3k} = 1+\alpha^3+\alpha^6+\cdots..+\alpha^{300}$

$\Rightarrow b = 1+\omega^3+\omega^6+\cdots.+\omega^{300}$

$\Rightarrow b = 101$

Required equation is $x^2 - 102x + 101 = 0$

66. (d)

Let the polynomial be

$$\begin{aligned}
&f(x) = ax^3+bx^2+cx+d\\
&\Rightarrow f'(x) = 3ax^2+2bx+c\\
&\Rightarrow f''(x) = 6ax+2b\\
&f''(1) = 0 \Rightarrow 6a+2b = 0 \Rightarrow b = -3a\\
&f'(-1) = 0 \Rightarrow 3a-2b+c = 0\\
&\Rightarrow c = -9a\\
&f(-1) = 10 \Rightarrow -a+b-c+d = 10\\
&\Rightarrow -a-3a+9a+d = 10\\
&d = -5a+10\\
&f(1) = 6 \Rightarrow a+b+c+d = 6\\
&\Rightarrow a-3a-9a-5a+10 = 6\\
&\Rightarrow a = \frac{1}{4}\\
&\therefore f'(x) = \frac{3}{4}x^2-\frac{6}{4}x-\frac{9}{4} = \frac{3}{4}(x^2-2x-3)
\end{aligned}$$

For $f'(x) = 0 \Rightarrow x^2-2x-3 = 0 \Rightarrow x = 3, -1$

Minima exists at $x = 3$

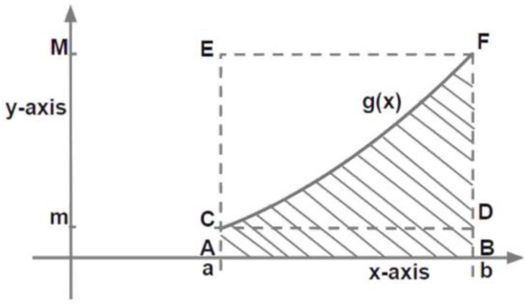
67. (a) $f(x) = \frac{1}{\sqrt{2x^3-9x^2+12x+4}}$

Differentiating w.r.t x

$$\begin{aligned}
f'(x) &= -\frac{1}{2}\times\frac{(6x^2-18x+12)}{(2x^3-9x^2+12x+4)^{3/2}}\\
&= \frac{-6(x-1)(x-2)}{2(2x^3-9x^2+12x+4)^{3/2}}
\end{aligned}$$

Here $f(x)$ is increasing in $(1,2)$

At $x=1, f(1)=\frac{1}{3}$ and $x=2, f(2)=\frac{1}{\sqrt{8}}$



Let $g(x)$ be a function such that it is increasing in (a,b) and $m \leq g(x) \leq M$, then

$$\operatorname{ar}(ABCD) < \int_a^b g(x)dx < \operatorname{ar}(ABEF)$$

$$m(b-a) < \int_a^b g(x)dx < M(b-a)$$

$$\text{Thus, } \frac{1}{3} < \int_1^2 f(x)dx < \frac{1}{\sqrt{8}}$$

$$\frac{1}{3} < I < \frac{1}{\sqrt{8}}$$

$$\text{or } \frac{1}{9} < I^2 < \frac{1}{8}$$

68. (a) Given curve: $x^2+2xy-3y^2=0$

$\Rightarrow x^2+3xy-xy-3y^2=0$

$\Rightarrow (x+3y)(x-y)=0$

Equating we get,

$x+3y=0$ or $x-y=0$

$(2,2)$ lies on $x-y=0$
$\therefore$ Equation of normal will be $x+y=\lambda$

It passes through $(2,2)$

$\therefore \lambda=4$

$L: x+y=4$

Distance of L from the origin $=\left|\frac{-4}{\sqrt{2}}\right|=2\sqrt{2}$

69. (b) $P($ exactly one of A or $B)=\frac{2}{5}$

$\Rightarrow P(A)-P(A\cap B)+P(B)-P(A\cap B)=\frac{2}{5}$

$\Rightarrow P(A\cup B)-P(A\cap B)=\frac{2}{5}$

$\Rightarrow P(A \cap B) = \frac{1}{10}$

70. (2454)

Word "EXAMINATION" consists of $2A, 2I, 2N, E, X, M, T, O$

Case I: All different letters are selected

Number of words formed $= {}^8C_4 \times 4! = 1680$

Case II: 2 letters are same and 2 are different

Number of words formed $= {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$

Case III: 2 pair of letters are same

Number of words formed $= {}^3C_2 \times \frac{4!}{2! \times 2!} = 18$

Total number of words formed $= 1680 + 756 + 18 = 2454$

71. (0.5) Let the co-ordinates of P be (t^2, t)

Equation of tangent at $P(t^2, t)$ is $y - t = \frac{1}{2t}(x - t^2)$

Therefore, co-ordinates of Q will be $(-t^2, 0)$

Area of $\triangle OPQ = 4$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix} = \pm 4$$

$\Rightarrow t^3 = 8 \Rightarrow t = \pm 2 \Rightarrow t = 2$ as $t > 0$

$m = \frac{1}{t} = \frac{1}{2}$

72. (504) $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$

$= \frac{1}{4}\sum_{n=1}^{7}(2n^3 + 3n^2 + n)$

$= \frac{1}{4}[2\sum_{n=1}^{7} n^3 + 3\sum_{n=1}^{7} n^2 + \sum_{n=1}^{7} n]$

$$= \frac{1}{4}\left[2 \times \left(\frac{7 \times 8}{2}\right)^2 + 3 \times \frac{7 \times 8 \times 15}{6} + \frac{7 \times 8}{2}\right]$$

$= \frac{1}{4}[2 \times 784 + 420 + 28] = 504$

73. (1)

$$\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}\Rightarrow\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha}=\frac{1}{7}\Rightarrow\tan\alpha=\frac{1}{7}$$

$$\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}\Rightarrow\frac{\sqrt{2}\sin\beta}{\sqrt{2}}=\frac{1}{\sqrt{10}}\Rightarrow\sin\beta=\frac{1}{\sqrt{10}}\Rightarrow\tan\beta=\frac{1}{3}$$

$$\tan 2\beta=\frac{2\tan\beta}{1-\tan^2\beta}=\frac{\frac{2}{3}}{1-\frac{1}{9}}=\frac{3}{4}$$

$$\tan(\alpha+2\beta)=\frac{\tan\alpha+\tan 2\beta}{1-\tan\alpha\tan 2\beta}=\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}=1$$