**1. (a)** Moment of Inertia of solid sphere $=\frac{2}{5} M\left(\frac{d}{2}\right)^2$

Distance of centroid (Point $P$ ) from centre of sphere $=\left(\frac{2}{3} \times \frac{\sqrt{3} d}{2}\right)=\frac{d}{\sqrt{3}}$

By Parallel axis theorem,

Moment of Inertia about $P=3\left[\frac{2}{5} M\left(\frac{d}{2}\right)^2+M\left(\frac{d}{\sqrt{3}}\right)^2\right]=\frac{13}{10} M d^2$

Moment of Inertia about $B=2\left[\frac{2}{5} M\left(\frac{d}{2}\right)^2+M(d)^2\right]+\frac{2}{5} M\left(\frac{d}{2}\right)^2=\frac{23}{10} M d^2$

Now ratio $=\frac{13}{23}$

**2. (c)** For solid sphere,

Field inside sphere, $\mathrm{E}=\frac{\rho r}{3 \epsilon_0}$ & Field outside sphere, $\mathrm{E}=\frac{\rho \mathrm{R}^3}{3 \mathrm{r}^2 \epsilon_0}$ where, r is distance from centre and R is radius of sphere

Electric field at A due to sphere of radius R (sphere 1 ) is zero and therefore, net electric field will be because of sphere of radius $\frac{\mathrm{R}}{2}$ (sphere 2 ) having charge density $(-\rho)$

$$\mathrm{E}_{\mathrm{A}}=\frac{-\rho \mathrm{R}}{2\left(3 \epsilon_0\right)}$$
$$\left|\mathrm{E}_{\mathrm{A}}\right|=\frac{\rho \mathrm{R}}{6 \varepsilon_0}$$

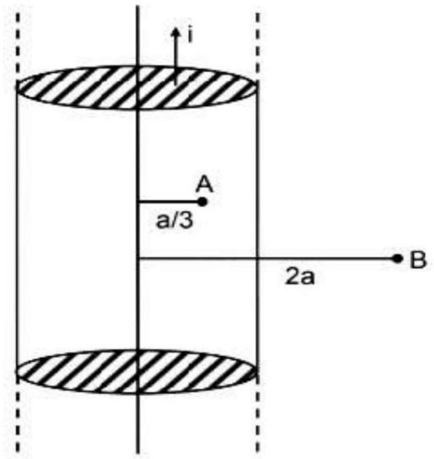
Similarly, Electric field at point $\mathrm{B}=\mathrm{E}_{\mathrm{B}}=\mathrm{E}_{1 \mathrm{B}}+\mathrm{E}_{2 \mathrm{B}}$

$\mathrm{E}_{1 \mathrm{B}}=$ Electric Field Due to solid sphere of radius $\mathrm{R}=\frac{\rho \mathrm{R}}{3 \varepsilon_0}$

$\mathrm{E}_{2 \mathrm{B}}=$ Electric Field Due to solid sphere of radius $\frac{\mathrm{R}}{2}$ which having charge density $(-\rho)$

$$=-\frac{\rho\left(\frac{\mathrm{R}}{2}\right)^3}{3\left(\frac{3 \mathrm{R}}{2}\right)^2 \varepsilon_0}=-\frac{\rho \mathrm{R}}{54 \varepsilon_0}$$
$$\mathrm{E}_{\mathrm{B}}=\mathrm{E}_{1 \mathrm{A}}+\mathrm{E}_{2 \mathrm{A}}=\frac{\rho \mathrm{R}}{3 \varepsilon_0}-\frac{\rho \mathrm{R}}{54 \varepsilon_0}=\frac{17 \rho \mathrm{R}}{54 \varepsilon_0}$$
$$\frac{\left|\mathrm{E}_{\mathrm{A}}\right|}{\left|\mathrm{E}_{\mathrm{B}}\right|}=\frac{9}{17}$$

**3. (b)**




$$B_A = \frac{\mu_0 ir}{2\pi a^2} = \frac{\frac{\mu_0 ia}{3}}{2\pi a^2} = \frac{\mu_0 i}{\pi a^2}\frac{a}{6} = \frac{\mu_0 i}{6\pi a}$$

$$B_B = \frac{\mu_0 i(2a)^2}{2\pi(2a)} = \frac{\mu_0 i}{4\pi a}$$

$$\frac{B_A}{B_B} = \frac{4}{6} = \frac{2}{3}$$

**4. (a)**

$$d\vec{s} = (dx\hat{\imath} + dy\hat{\jmath})$$
$$= (-x\hat{\imath} + y\hat{\jmath}) \cdot (dx\hat{\imath} + dy\hat{\jmath})$$
$$= \int_1^0 -xdx + \int_0^1 ydy$$

$$= -\frac{x^2}{2}\Big|_1^0 + \frac{y^2}{2}\Big|_0^1 = \frac{1}{2} + \frac{1}{2} = 1J$$

**5. (a)** For process A-B; Volume is constant;

For process $\mathrm{B-C}$, $\mathrm{PV}^{\gamma} = \mathrm{Constant}$ & $\mathrm{PV = nRT}$, Therefore $\mathrm{TV}^{\gamma-1} = \mathrm{Constant}$;

Therefore as V increases T increases.

For process $\mathrm{C-A}$; pressure is constant, Therefore $\mathrm{V = kT}$

From above, correct answer is option 1.

**6. (a)** Since, we have to find vector parallel to electric field at position $\vec{r}$

We have to find $\vec{p} \cdot \vec{r} = 0$

Since already in question, $\vec{p} \cdot \vec{r} = 0$ is given we need to find E such that

$\vec{E} = \lambda(\vec{p})$

where $\lambda$ is a arbitrary positive constant

On putting , $\lambda = -1$, we get, $\vec{E} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

**7. (a)** By conservation of linear momentum

$$\frac{m}{2}\frac{V}{2}+mV=\left(m+\frac{m}{2}\right)V_f$$

$$V_f=\frac{5\text{ V}}{6}$$

Escape velocity will be at $\sqrt{2}$ V and at velocity less than escape velocity path will be elleptical or part of ellipse except for velocity V where path will be circular.

Hence the resultant mass will go on to an elliptical path

**8. (a)** Conserving linear momentum

$$mv\hat{\imath}+m\left(\frac{v}{2}\hat{\imath}+\frac{v}{2}\hat{\jmath}\right)=2m(v_1\hat{\imath}+v_2\hat{\jmath})$$

By equating $\hat{\imath}$ and $\hat{\jmath}$

$v_1=\frac{3v}{4}$ and $v_2=\frac{v}{4}$

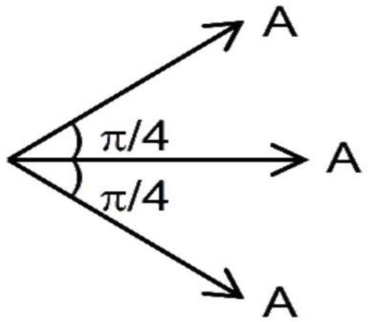
Initial K.E $=\frac{mv^2}{2}+\frac{m}{2}\times\left(\frac{v}{\sqrt{2}}\right)^2=\frac{3mv^2}{4}$

Final K.E $=\frac{2m}{2}\times\left(\frac{v\sqrt{10}}{4}\right)^2=\frac{mv^2}{8}$

Change in KE =

$$\frac{3mv^2}{4}-\frac{5mv^2}{8}=\frac{mv^2}{8}$$

**9. (a)**




Amplitudes can be vectorially added

$A_{\text{resultant}}=(\sqrt{2}+1)A$

Since, I $\propto A^2$

Therefore, $I_{\text{res}}=(\sqrt{2}+1)^2I_0=5.8I_0$

**10. (a)** Using equation of continuity

$\mathrm{V_A}\times$ Area $_A=\mathrm{V_B}\times$ Area $_\mathrm{B}$

$40\ \mathrm{V_A}=20\ \mathrm{V_B}$

$2\ \mathrm{V_A}=\mathrm{V_B}$

Using Bernoulli's equation

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

$$\Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right)$$

$$\Delta P = 500 \times \frac{3V_B^2}{4}$$

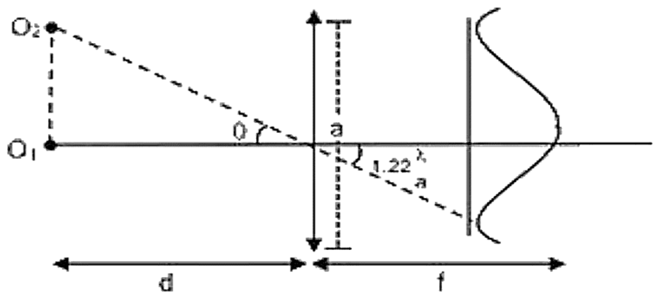
$$V_B = \sqrt{\frac{(\Delta P)\times 4}{1500}} = \sqrt{\frac{(700)\times 4}{1500}} = \sqrt{\frac{28}{15}}\ \mathrm{m/s}$$

Volume flow rate $= \mathrm{V_B} \times \mathrm{Area}\ _{\mathrm{B}} = 20 \times 100 \times \sqrt{\frac{28}{15}}\ \mathrm{cm^3/s} = 2732\ \mathrm{cm^3/s}$

**11.** **(b)** Pitch $= \frac{3}{6} = 0.5$ mm

Least count $= \frac{\text{Pitch}}{\text{Number of division}} = \frac{0.5\ \text{mm}}{50} = \frac{1}{100}$ mm $= 0.01$ mm $= 0.001$ cm

**12.** **(a)**




Minimum angle for clear resolution,

$$\theta = 1.22\frac{\lambda}{\mathrm{a}}$$

distance $= \mathrm{O_1O_2} = \mathrm{d}\theta$

$$= 1.22\frac{\lambda}{\mathrm{a}}\mathrm{d}$$

distance $= \mathrm{O_1O_2} = \frac{1.22\times 5893\times 10^{-10}\times 4\times 10^{8}}{5} \approx 57.52$ m

**13.** **(c)** From photoelectric equation,

$$\frac{\mathrm{hc}}{\lambda} = \mathrm{W} + \mathrm{K}\cdot\mathrm{E_{max}}$$

Where, hc $= 12400$eVÅ

$\Rightarrow \frac{12400}{6556} = \mathrm{W} + \mathrm{K} \cdot \mathrm{E}_{\max}$

$\Rightarrow 1.9\mathrm{eV} = \mathrm{W} + \mathrm{K.E}_{\max} - - - - - (1)$

Radius of charged particle moving in a magnetic field is given by

$\mathrm{r} = \frac{\mathrm{mv}}{\mathrm{qB}}$ where, $\frac{1}{2}\mathrm{mv}^2 = \mathrm{K} \cdot \mathrm{E}_{\max} = \mathrm{eV}$

$$\Rightarrow r = \frac{\sqrt{\frac{2\mathrm{eV}}{\mathrm{m}}} \times \mathrm{m}}{\mathrm{eB}} = \frac{1}{\mathrm{B}}\sqrt{\frac{2\mathrm{mV}}{\mathrm{e}}}$$

$$\Rightarrow 10^{-2} = \frac{1}{3 \times 10^{-4}}\sqrt{\frac{2 \times 9.1 \times 10^{-31} \times V}{1.6 \times 10^{-19}}}$$

$\Rightarrow \mathrm{V} = 0.8\ \mathrm{V}$

So, $\mathrm{K.E}_{\max} = 0.8\mathrm{eV}$

Substituting in (1),

$1.9 = \mathrm{W} + 0.8$

i.e. $\mathrm{W} = 1.1\mathrm{eV}$

**14. (a)**

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m(KE)}}$$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{KE}}$$

$$\frac{\lambda}{\lambda/2} = \sqrt{\frac{KE_f}{KE_i}}$$

$$4KE_i = KE_f$$

$\Rightarrow \Delta E = KE_f - KE_i = 4KE_i - KE_i = 3KE_i = 3E$

**15. (b)**

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow hc = E\lambda$$

Since, $[E] = [ML^2T^{-2}]$

Therefore,

$[hc] = [ML^3T^{-2}]$

$[c] = [LT^{-1}]$

$[G] = [[M^{-1}L^3T^{-2}]$

$$\left[\sqrt{\frac{hc^5}{G}}\right] = [ML^2T^{-2}]$$

**16. (a)** Apparent height as seen from liquid 1 (having refractive index $\mu_1 = \sqrt{2}$ ) to liquid 2

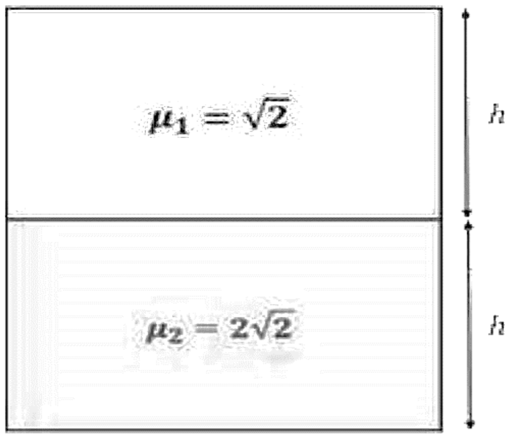
(refractive index $\mu_2 = 2\sqrt{2}$ )

$$\mathrm{D} = \frac{h\mu_1}{\mu_2} = \frac{h}{2}$$

Now, Actual height perceived from air, h $+ \frac{h}{2} = \frac{3h}{2}$

Therefore, apparent depth of bottom surface of the container (apparent depth as seen from air (having refractive index $\mu_0 = 1$ ) to liquid 1(having refractive index $\mu_1 = \sqrt{2}$ ) $= \frac{3h}{2} \times \frac{\mu_0}{\mu_1}$

$$= \frac{3h}{2} \times \frac{1}{\sqrt{2}} = \frac{3h}{2\sqrt{2}} = \frac{3\sqrt{2}h}{4}$$




**17. (b)** Since resistance $1\Omega$ and $4\Omega$ are in parallel

$$\therefore R' = \frac{4 \times 1}{4 + 1} = \frac{4}{5}$$

Similarly we can find equivalent resistance ( $R''$) for resistances $2\Omega$ and $3\Omega$

$$\Rightarrow R'' = \frac{6}{5}$$

And $R'$ and $R''$ are in series

$$\therefore R_{\text{eff}} = \frac{4}{5} + \frac{6}{5} = 2\Omega$$

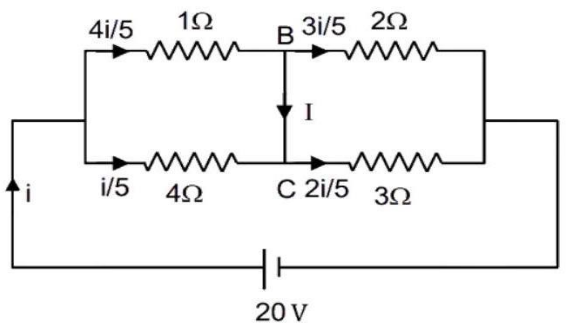
So total current flowing in the circuit ' $i$ ' can be given as

$$i = \frac{V}{R_{eff}} = \frac{20}{2} = 10 \text{ A}$$

Current will distribute in ratio opposite to resistance.

So, distribution will be as

So current in the branch BC will be




$$I = \frac{4i}{5} - \frac{3i}{5} = \frac{i}{5} = \frac{10}{5} = 2\ A$$

$$I = \frac{4i}{5} - \frac{3i}{5} = \frac{i}{5} = \frac{10}{5} = 2A$$

**18. (b)** Given that the magnetic field vectors are:

$$\vec{E}_1 = E_o\hat{j}\cos(kx - \omega t)$$
$$\vec{E}_2 = E_o\hat{k}\cos(ky - \omega t)$$

So the magnetic field vectors of the electromagnetic wave are given by

$$\vec{B}_1 = \frac{E_o}{c}\hat{k}\cos(kx - \omega t)$$
$$\vec{B}_2 = \frac{E_o}{c}\hat{\imath}\cos(ky - \omega t)$$

Then force is

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$
$$= q(\vec{E}_1 + \vec{E}_2) + q(\vec{v} \times (\vec{B}_1 + \vec{B}_2))$$

Now if we put the values of $\vec{E}_1, \vec{E}_2, \vec{B}_1$ and $\vec{B}_2$ we can get the net Lorentz force as

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

Putting values and solving we get

$$\vec{F} = qE_o[\cos(kx - \omega t)\hat{\jmath} + \cos(ky - \omega t)\hat{k} + 0.8\cos(kx - \omega t)\hat{\imath} - 0.8(\cos ky - \omega t)\hat{k}]$$

$$\vec{F} = qE_o[0.8\cos(kx - \omega t)\hat{\imath} + \cos(kx - \omega t)\hat{\jmath} + 0.2(\cos ky - \omega t)\hat{k}]$$

Now at $t = 0$ and $x = y = 0$ we get

$\vec{F} = (0.8\hat{\imath} + \hat{\jmath} + 0.2\hat{k})$

**19. (d)** We know that,

Molar heat capacity at constant volume, $C_V = \frac{fR}{2}$ (Where f is degree of freedom)

Since, $A$ is diatomic and rigid, degree of freedom for $A$ is 5

Therefore, Molar heat capacity of A at constant volume $(C_V)_A = \frac{5R}{2}$

Since, $B$ is diatomic and have extra degree of freedom because of vibration, degree of freedom for $B$ is $5 + 2 \times 1 = 7$ ( 1 vibration for each atom).

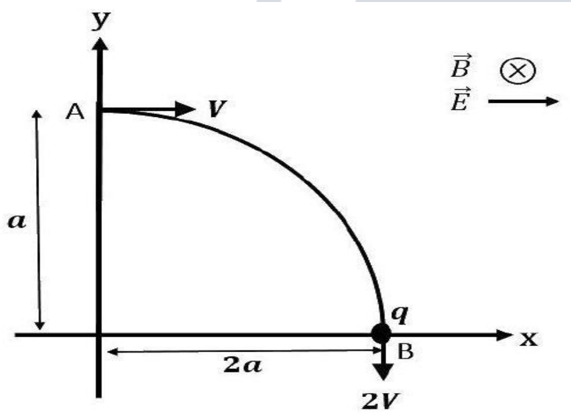
Therefore, Molar heat capacity of B at constant volume $(C_V)_B = \frac{7R}{2}$

Ratio of molar specific heat of A and B $= \frac{(C_V)_A}{(C_V)_B} = \frac{5}{7}$

**20. (12)** By applying Kirchhoff's Voltage Law in the loop ACBFA

$12.7 - 0.7 - V_{\mathrm{EF}} = 0$
$\Rightarrow V_{\mathrm{EF}} = 12\ \mathrm{V}$
$\Rightarrow \mathrm{V_E} = 12\ \mathrm{V}$

**21. (a)**




Considering statement A

By Work-Energy theorem

$$W_{\text{mag}} + W_{\text{ele}} = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$
$$\Rightarrow 0 + qE_o 2a = \frac{3}{2}mv^2$$
$$E_o = \frac{3}{4}\frac{mv^2}{qa}$$

So statement A is correct

Now considering statement $B$

Rate of change of work done at $A =$ Power of electric force

$$= 9E_o v$$
$$= \frac{3}{4}\frac{mv^3}{a}$$

So statement B is correct Coming to statement C

At B,

$\vec{E} \perp \vec{v}$

So, $\frac{dw}{dt} = 0$ for both forces

Coming to statement $D$.

Change in angular momentum about the origin is

$\Delta\vec{L} = \Delta\overrightarrow{L_B} - \Delta\overrightarrow{L_A}$

$\overrightarrow{L_B} = m(2v)(2a)$

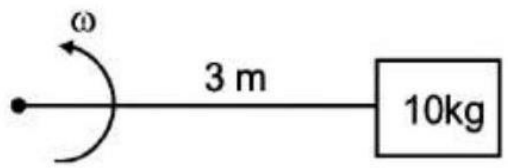
$\overrightarrow{L_A} = m(v)(a)$

Hence, $\Delta L = 3mva$

**22. (20)** By using KVL,

$$\mathrm{V} - \mathrm{L}\frac{dI}{dt} = 0$$
$$\Rightarrow 200 = \frac{L(0.25)}{0.025} \times 10^3$$
$$L = 200 \times 10^{-4}\mathrm{H}$$
$$= 20\mathrm{mH}$$

**23. (4)**




Breaking stress

$$\sigma = \frac{T}{A}$$
$$T = m\omega^2 \mathrm{l}$$
$$\Rightarrow \sigma = \frac{m\omega^2 \mathrm{l}}{A}$$
$$\Rightarrow \omega^2 = \frac{\sigma A}{ml} = \frac{4.8 \times 10^8 \times 10^{-6}}{10 \times 3} = 16$$
$$\Rightarrow \omega = 4\mathrm{rad/s}$$

**24. (3)** Let, $v$ be velocity, $\alpha$ be the acceleration then,

$x^2 = at^2 + 2bt + c$

$2xv = 2at + 2b$

$xv = at + b$

$\Rightarrow v = \dfrac{at + b}{x}$

Now, differentiating equation (1),

$$v^2 + \alpha x = a$$
$$\alpha x = a - \left(\frac{at + b}{x}\right)^2$$

$$\alpha = \frac{a(at^2 + 2bt + c) - (at + b)^2}{x^3}$$
$$\alpha = \frac{ac - b^2}{x^3}$$

$\alpha \propto x^{-3}$

**25. (15)** By using conservation of energy,

$mg\dfrac{1}{2}\sin 30^\circ = \dfrac{1}{2}\dfrac{ml^2}{3}\omega^2$

On solving

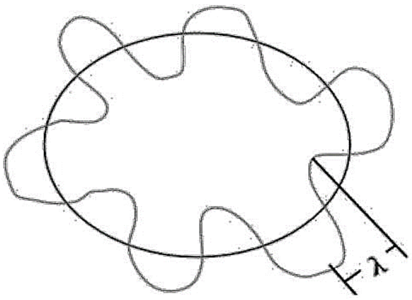
$\omega^2 = 15$

$\omega = \sqrt{15}$

Therefore, n = 15

**26. (c)** n = 4

Z=1
Λ=?




Circumference $(2\pi r)$ = n$\lambda$

$\dfrac{2\pi a_0 n^2}{z} = n\lambda$

On solving, we get $8\pi a_o$

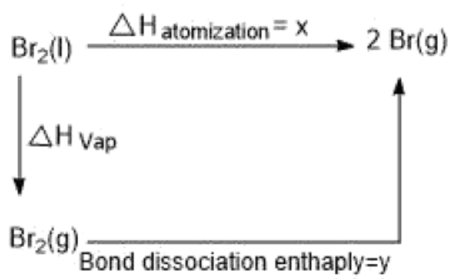
**27. (c)**

$O_2: \sigma_{1\,s^2}\sigma^*_{1\,s^2}\sigma_{2\,s^2}\sigma^*_{2\,s^2}\sigma_{2p_z^2}\pi_{2p_x^2} = \pi_{2py^2}\pi^*_{2p_x^1} = \pi^*_{2p_y^1}$

$O_2^-: \sigma_{1\,s^2}\sigma^*_{1\,s^2}\sigma_{2\,s^2}\sigma^*_{2\,s^2}\sigma_{2p_z^2}\pi_{2p_x^2} = \pi_{2py^2}\pi^*_{2p_x^2} = \pi^*_{2p_y^1}$

$O_2^+: \sigma_{1\,s^2}\sigma^*_{1\,s^2}\sigma_{2\,s^2}\sigma^*_{2\,s^2}\sigma_{2p_z^2}\pi_{2p_x^2} = \pi_{2py^2}\pi^*_{2p_x^1}$

**28. (a)**




$\Delta H_{\text{atomisation}} = \Delta H_{\text{vap}} + y$

$x - y = \Delta H_{\text{vap}}$

**29. (b)** Non-metallic oxides are acidic in nature metallic oxides are basic in nature and $Al_2O_3$ is amphoteric in nature

**30. (b)** Spin only magnetic moment = 3.8 B. M. This implies, $\mu = \sqrt{\ }(n(n+2))$ B.M.

($\sqrt{16} = 4$ implies that $\sqrt{15}$ should be less than four.

This means, $n = 3$ as $\sqrt{15} = \sqrt{\ }(3(3+2))$

$Cr(24) = [Ar]4\,s^1 3\,d^5$

(g.s)

For 3 unpaired electrons, the oxidation state of Cr should be +3

$Cr^{3+}$ can be attained if the complex has a structure that looks like: $[Cr(H_2O)_4Cl_2]Cl_2 2H_2O$

$[Cr(H_2O)_4Cl_2]Cl.2H_2O$ has the IUPAC name : Tetraaquadichloridochromium(III) chloride dihydrate

**31. (a)** Ce (58): $[Xe]6\,s^2 4f^2$

(g.s)

$Ce^{3+}: [Xe]4f^1$

$Eu(63): [Xe]6\,s^2 4f^7$

(g.s)

$Eu^{2+}: [Xe]4f^7$

**32. (a)**

Given $K_{sp}$ of $PbCl_2 = 1.6 \times 10^{-5}$

$Pb(NO_3)_2$: mmoles $= 300\text{ mL} \times 0.134\text{M} = 40.2$

NaCl: mmoles $= 100\text{ mL} \times 0.4\text{M} = 40$

This implies, $[Pb]^{2+} = \frac{40.2}{400} \approx 0.1\text{M}$

$[Cl]^- = \frac{40}{400} = 0.1\text{M}$

$Q_{sp} = [Pb^{2+}][2Cl^-]^2 = 4 \times 10^{-3} > K_{sp}$

**33. (c)** When the oxidation state is maximum it acts like a strong oxidising agent

When the oxidation state is minimum it acts like a strong reducing agent

When the oxidation state is between its maximum and minimum, it acts like both an oxidizing and as a reducing agent

In $H_3PO_4$, P has a +5 oxidation state and hence can act like a strong oxidising agent. In the rest, the oxidation state is between their maximum and minimum.

**34. (d)** Be (4): $1\ s^2 2\ s^2$

B (5): $1s^2 2s^2 2p^1$

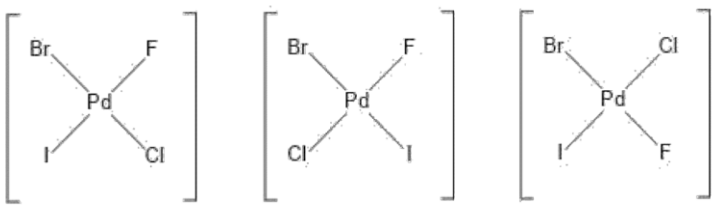
The electron in $2p^1$ can easily be extracted.

The penetrating power is of the order: $s > p > d > f$

The shielding power order: $s > p > d > f$

As we move along the period, the size decreases, as $Z_{eff}$ increases. Hence the radius of B is smaller than the radius of Be.

**35. (a)**




Number of geometrical isomers (n) = 3

$[Fe(CN)_6]^{n-6} = [Fe(CN)_6]^{3-6} = [Fe(CN)_6]^{-3}$

This implies, that Iron is in its +3 oxidation state.

$Fe^{3+}(26)$: $[Ar]3\ d^5$

$CN^-$is a strong ligand in $[Fe(CN)_6]^{-3}$ and causes pairing. Hence, according to CFT, the configuration will be $t_{2g}^5 e_g^0$.

Hence, there is only 1 unpaired electron, i.e, $n = 1$ in $\sqrt{n(n+2)} = \sqrt{3} = 1.73$ B.M

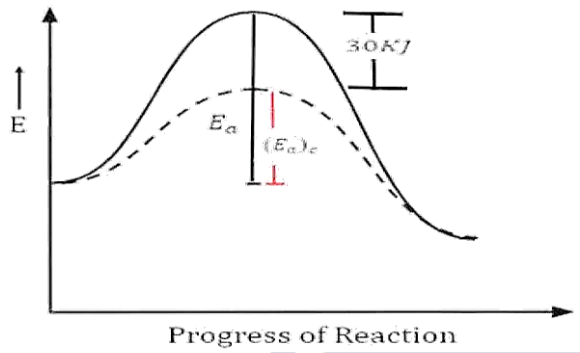
$$\text{CFSE} = (-0.4 \times n_{t2g} + 0.6 \times n_{eg})\Delta_0$$
$$= (-0.4 \times 5 + 0.6 \times 0)\Delta_0$$
$$= -2\Delta_0$$

**36.** **(a)** In Ellingham's diagram, the line of the element that lies below can reduce the oxide of the element which lies above it. Therefore, for A to reduce $BO_2$, the temperature when the line for element A is below that of $BO_2$, according to the graph when $T > 1400\ ^\circ C$.

For $T > 1400\ ^\circ C, \Delta G_r < 0$ for $A + BO_2 \rightarrow B + AO_2$

**37.** **(c)** $K = Ae^{\left(-\frac{E_a}{RT}\right)}$

$K_{catalyst} = K_{without\ catalyst}$




$$Ae^{\left(-\frac{(E_a)c}{RT_{500k}}\right)} = Ae^{\left(-\frac{(E_a)}{RT_{700k}}\right)}$$
$$e^{\left(-\frac{(E_a)c}{RT_{500k}}\right)} = e^{\left(-\frac{(E_a)}{RT_{700k}}\right)}$$
$$-\frac{(E_a)c}{RT_{500k}} = -\frac{(E_a)}{RT_{700k}}$$
$$(E_a)c = E_a - 30$$
$$-\frac{(E_a - 30)}{T_{500k}} = -\frac{(E_a)}{T_{700k}}$$

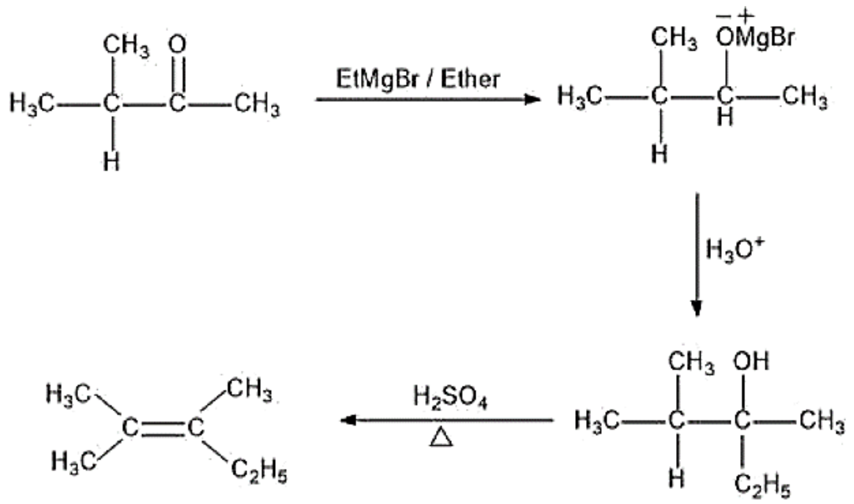
On Solving, $E_a = 105 kJmol^{-1}$

**38.** **(d)** $CCl_4$ is non polar and does not conduct in either solid or liquid state.

**39.** **(c)**

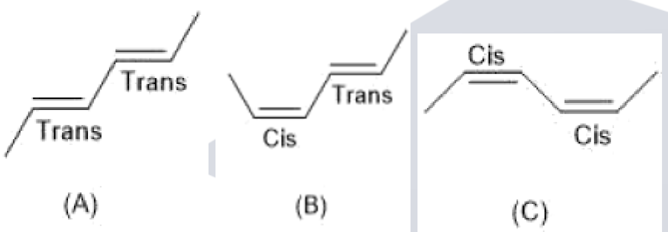
**40.** **(b)** Out of the four options given, only aniline and phenol show strong $+R$ effects, but as we know, aniline is a Lewis base and can react with a Lewis acid that is added during the reaction. Hence, Phenol gives the highest yield in Friedel-Craft's reaction.

**41.** **(a)**



**42. (c)** Heat of combustion $\alpha \frac{1}{\text{stability}}$

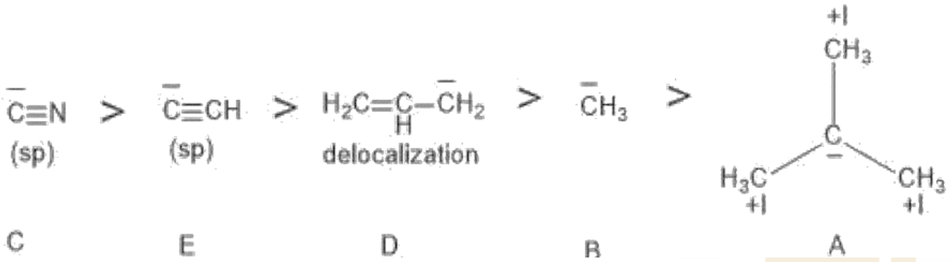
The trans-isomer is more stable than the cis-isomer. More the number of trans forms in a structure, higher the stability.




**43. (a)**

As we know weaker the conjugate base, stronger the acid.

The order of stability of conjugate base:

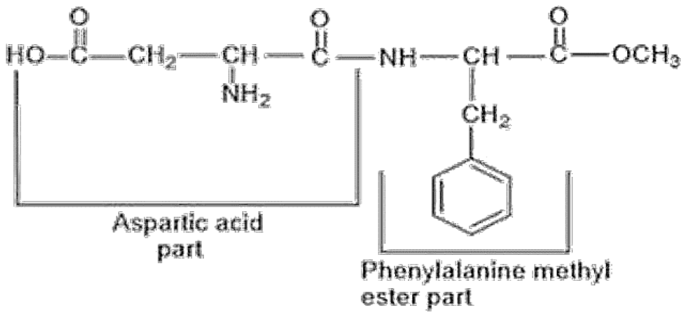



Hence, the order of basicity or acidic strength is:

A > B > D > E > C

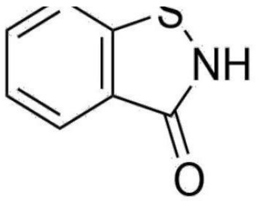
**44. (b)**

A - Aspartame



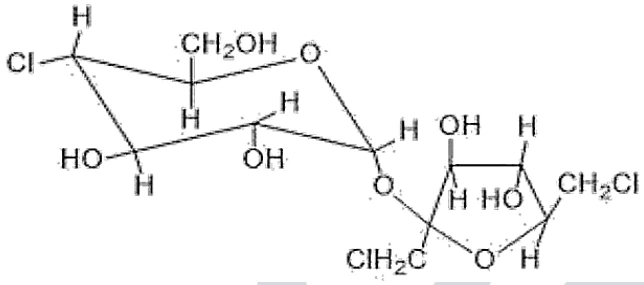

It has a free amine group and hence reacts with ninhydrin to give a purple colour known as Ruhemann's purple.

B- Saccharine



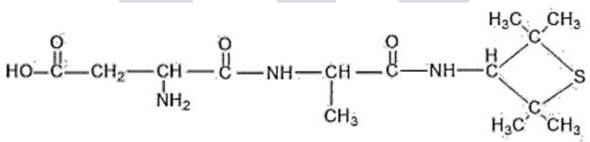
It has Sulphur, therefore, it will give a positive test with sodium nitroprusside.

C - Sucralose



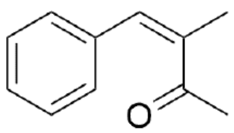
It has chlorine and hence it forms a precipitate with $AgNO_3$ in the Lassaigne's extract of the sugar.

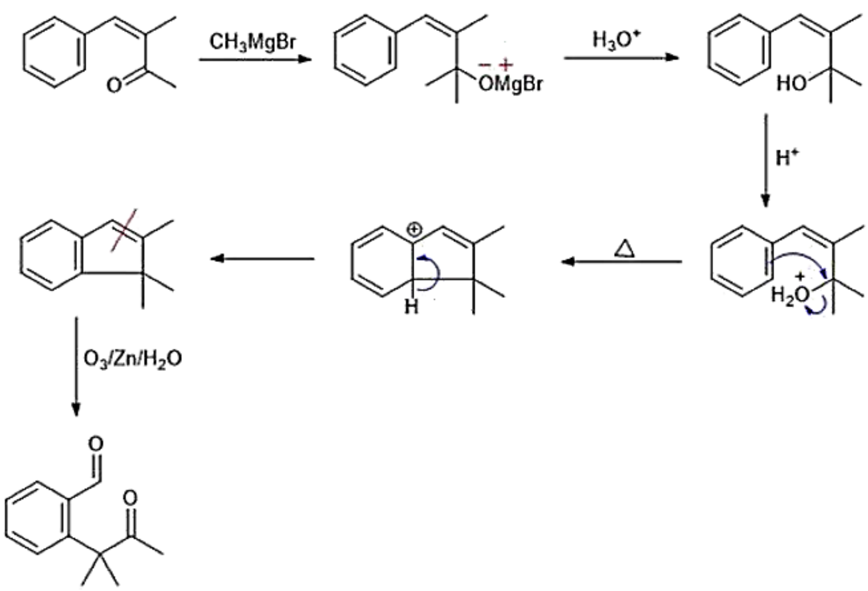
D - Alitame



It has a free amine group and hence reacts with ninhydrin to give purple colour known as Ruhemann's purple. Also, it has Sulphur, therefore, it will give positive test with sodium nitroprusside.

**45. (b)**

 is a methyl ketone, which gives positive Iodoform test




**46. (14.00)**

$\%\frac{w}{w} = 63\%$

$\rho = 1.4\text{ g/mL}$

$M = \frac{\left(\%\frac{w}{w} \times \rho \times 10\right)}{MM}$

$M = \frac{(63 \times 1.4 \times 10)}{63}$

$M = 14\text{ mol/L}$

**47. (100.00)**

Hardness of water is measured in ppm in terms $CaCO_3$.

$n_{CaCO_3} = n_{MgSO_4}$

ppm is the parts (in grams) present per million i.e, $10^6$

1000 mL has $10^{-3}$ moles of $MgSO_4$.

Grams of $CaCO_3$ in 1000 mL $= 10^{-3} \times 100$ grams

Grams of $CaCO_3$ in 1 mL $= \frac{10^{-3} \times 100}{1000\text{ mL}}$ grams

Hardness $= \frac{10^{-3} \times 100}{1000\text{ mL}} \times 10^6 = 100$

**48. (1.76)**

NaCl is strong electrolyte and gives 2 ions in the solution. This implies, i = 2.

Molatility $= \frac{w \times 1000}{58.5 \times 600}$

$\Delta T_f = 0.2^\circ C$

$\Delta T_f = i \times k_f \times m$

On solving we get,

$w = 1.76 grams$

**49. (5.68)**

On applying Faraday's 1 st law,

Moles of Ag deposited=108/108= 1 mol.

$Ag^+ + e^- \rightarrow Ag$

1Faraday is required to deposit 1 mole of Ag.

$H_2O \rightarrow 2H^+ + \frac{1}{2}O_2 + 2e^-$

$\frac{1}{2}$ moles of $O_2$ are deposited by 2 F of charge.

This implies, 1 F will deposit $\frac{1}{4}$ moles of $O_2$

Using PV=nRT

$P = 1$ bar

$T = 273$ K

$R = 0.0823 Lbarmol^{-1} K^{-1}$

On solving we get,

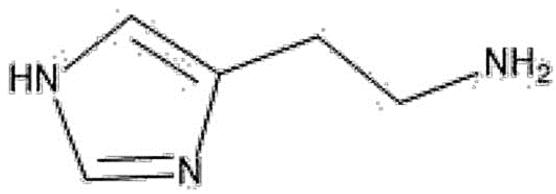
$V = 5.68$ L

**50. (37.84)**

Molecular mass of Histamine $= 111$

In Histamine, 3 Nitrogens are present (42g)

The percentage of Nitrogen by mass in Histamine $= \frac{42}{111} \times 100 = 37.84\%$




**51. (b)** Let thickness of ice be $x$ cm.

Therefore, net radius of sphere $= (10 + x)$cm

Volume of sphere $V = \frac{4}{3}\pi(10 + x)^3$

$\Rightarrow \frac{dV}{dt} = 4\pi(10+x)^2 \frac{dx}{dt}$

At $x = 5, \frac{dV}{dt} = 50 \text{ cm}^3/\text{min}$

$\Rightarrow 50 = 4\pi \times 225 \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{1}{18\pi} \text{cm/min}$

**52. (a)** $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$

$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$

$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

Let $e^x + \frac{1}{e^x} = u$

Then, $u^2 + u - 6 = 0$
$\Rightarrow u = 2, -3$

$u \neq -3$ as $u > 0 (\because e^x > 0)$

$\Rightarrow e^x + \frac{1}{e^x} = 2 \Rightarrow (e^x - 1)^2 = 0 \Rightarrow e^x = 1 \Rightarrow x = 0$

Hence, only one real solution is possible.

**53. (b)**

$$f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$$

$$f'(x) = \tan^{-1}\left(\frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}\right)$$

$$f'(x) = \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}\right]$$

$$f'(x) = \tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]$$

$$f'(x) = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

$$f'(x) = \frac{\pi}{4} + \frac{x}{2}$$

$$\Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$$

$$f(0) = 0 \Rightarrow c = 0$$

$$f(1) = \frac{\pi}{4} + \frac{1}{4} = \frac{\pi+1}{4}$$

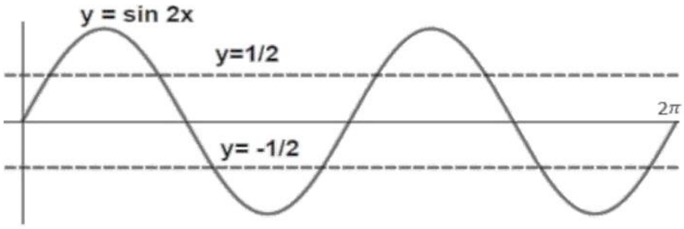
**54. (c)**

$$\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|, x \in [0,2\pi]$$

$$\Rightarrow \log_{\frac{1}{2}}|\sin x||\cos x| = 2$$

$$\Rightarrow |\sin x \cos x| = \frac{1}{4}$$

$$\therefore \sin 2x = \pm\frac{1}{2}$$




$\therefore$ We have 8 solutions for $x \in [0,2\pi]$

**55. (a)** $e_1 = \sqrt{1-\frac{4}{18}} = \frac{\sqrt{7}}{3} \& e_2 = \sqrt{1+\frac{4}{9}} = \frac{\sqrt{13}}{3}$

$\because (e_1, e_2)$ lies on the ellipse $15x^2 + 3y^2 = k$

$\therefore 15e_1^2 + 3e_2^2 = k$

$$\Rightarrow 15 \times \frac{7}{9} + 3 \times \frac{13}{9} = k \Rightarrow k = 16$$

**56. (c)**

$$I = \int \frac{dx}{(x-3)^{\frac{6}{7}} \times (x+4)^{\frac{8}{7}}}$$

$$\Rightarrow I = \int \frac{(x+4)^{\frac{6}{7}} dx}{(x-3)^{\frac{6}{7}} \times (x+4)^2} = \int \left(\frac{x-3}{x+4}\right)^{-\frac{6}{7}} \times \frac{dx}{(x+4)^2}$$

Put $\frac{x-3}{x+4} = t \Rightarrow dt = 7\left(\frac{1}{(x+4)^2}\right) dx$

$$\Rightarrow I = \frac{\int t^{-\frac{6}{7}}}{7} dt = t^{\frac{1}{7}} + c = \left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + c$$

**57. (c)**

If $\left|\frac{z-i}{z+2i}\right| = 1 \& |z| = \frac{5}{2}$

$\Rightarrow |z-i| = |z+2i|$

$\Rightarrow x^2 + (y-1)^2 = x^2 + (y+2)^2$

$\Rightarrow y - 1 = \pm(y+2)$

$\Rightarrow y - 1 = -y - 2$

$$\Rightarrow y = -\frac{1}{2}$$

$$\Rightarrow |z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4}$$

$$\Rightarrow x^2 + \frac{1}{4} = \frac{25}{4}$$

$\Rightarrow x = \pm\sqrt{6}$

$$|z+3i| = \sqrt{x^2 + (y+3)^2}$$
$$\Rightarrow |z+3i| = \frac{7}{2}$$

**58. (c)**

$$2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \dots \infty = 2^{\frac{1}{4}} \times 2^{\frac{2}{16}} \times 2^{\frac{4}{48}} \dots \infty$$
$$\Rightarrow 2^{\frac{1}{4}} \times 2^{\frac{1}{8}} \times 2^{\frac{1}{16}} \dots \infty = 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots\infty}$$
$$\Rightarrow 2^{\left(\frac{\frac{1}{4}}{1-\frac{1}{2}}\right)} = \sqrt{2}$$

**59. (d)**

$$\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8}+\sin^3\frac{\pi}{8}\sin\frac{3\pi}{8}=\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8}-3\cos\frac{\pi}{8}\right]+\sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8}-4\sin^3\frac{\pi}{8}\right]$$
$$=4\left[\cos^6\frac{\pi}{8}-\sin^6\frac{\pi}{8}\right]+3\left[\sin^4\frac{\pi}{8}-\cos^4\frac{\pi}{8}\right]$$
$$=4\left[\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}\right]\left[\cos^4\frac{\pi}{8}+\sin^4\frac{\pi}{8}+\cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right]-3\left[\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}\right]$$
$$=\left[\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}\right]\left[4\left(1-\cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right)-3\right]$$
$$=\cos\frac{\pi}{4}\left[1-\sin^2\frac{\pi}{4}\right]=\frac{1}{2\sqrt{2}}$$

**60. (c)** Let $I=\int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$

$$I=\int_0^{2\pi}\frac{(2\pi-x)\sin^8(2\pi-x)}{\sin^8(2\pi-x)+\cos^8(2\pi-x)}dx$$

$$=\int_0^{2\pi}\frac{(2\pi-x)\sin^8 x}{\sin^8 x+\cos^8 x}dx$$

Adding (1) & (2), we get:

$$\Rightarrow 2I=2\pi\int_0^{2\pi}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx$$

$$I=\pi\int_0^{2\pi}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx$$

$$I=4\pi\int_0^{\frac{\pi}{2}}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx$$

$$I=4\pi\int_0^{\frac{\pi}{2}}\frac{\sin^8\left(\frac{\pi}{2}-x\right)}{\sin^8\left(\frac{\pi}{2}-x\right)+\cos^8\left(\frac{\pi}{2}-x\right)}dx=4\pi\int_0^{\frac{\pi}{2}}\frac{\cos^8 x}{\sin^8 x+\cos^8 x}dx$$

Adding (3) & (4), we get -

$$I=2\pi\int_0^{\frac{\pi}{2}}1dx=2\pi\times\frac{\pi}{2}=\pi^2$$

**61. (c)** $f(x)=a+bx+cx^2$

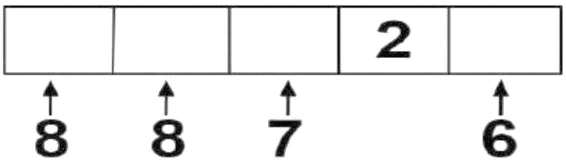
$f(0)=a, f(1)=a+b+c$

$$f\left(\frac{1}{2}\right)=\frac{c}{4}+\frac{b}{2}+a$$

$$\int_0^1 f(x)dx=\int_0^1(a+bx+cx^2)dx=a+\frac{b}{2}+\frac{c}{3}$$

$$=\frac{1}{6}(6a+3b+2c)=\frac{1}{6}(a+(a+b+c)+(4a+2b+c))$$
$$=\frac{1}{6}\left(f(0)+f(1)+4f\left(\frac{1}{2}\right)\right)$$

**62. (d)**




Total numbers that can be formed are

$= 8 \times 8 \times 7 \times 6$

$= 8 \times 336$

$\therefore k = 8$

**63. (a)** Coordinates of D are $\left(\frac{3+1+2}{3}, \frac{-1+3+4}{3}\right) = (2,2)$

Point of intersection of two lines

$\mathrm{x} + 3\mathrm{y} - 1 = 0$ and $3\mathrm{x} - \mathrm{y} + 1 = 0$

is P $\left(\frac{-1}{5}, \frac{2}{5}\right)$

Equation of line DP is $8\mathrm{x} - 11\mathrm{y} + 6 = 0$

Point $(-9, -6)$ lies on DP

**64. (d)** $\therefore c \in (a, b)$ and $f$ is twice differentiable and continuous function $(a, b)$

$\therefore$ LMVT is applicable

For $p \in (a, c),\ f'(p) = \frac{f(c)-f(a)}{c-a}$

For $q \in (c, b),\ f'(q) = \frac{f(b)-f(c)}{b-c}$

$\therefore f''(x) < 0 \Rightarrow f'(x)$ is decreasing

$f'(p) > f'(q)$

$\Rightarrow \dfrac{f(c)-f(a)}{c-a} > \dfrac{f(b)-f(c)}{b-c}$

$\Rightarrow \frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c}$ (as $f'(x) > 0 \Rightarrow f(x)$ is increasing)

**65. (d)** The given planes intersect in a line

$\therefore D = D_x = D_y = D_z = 0$

$D = 0$

$\Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 7\alpha + 25 - 4\alpha - 20 + 4 = 0$

$\Rightarrow \alpha = -3$

$D_z = 0$

$\Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 35 - 5\beta - 20 + 4\beta - 2 = 0$

$\Rightarrow \beta = 13$

$\therefore \alpha + \beta = 10$

**66. (d)**

$\sum_{i=1}^{10}(x_i - 5) = 10 \Rightarrow \sum_{i=1}^{10} x_i - 50 = 10$

$\Rightarrow \sum_{i=1}^{10} x_i = 60$

$\lambda = \frac{\sum_{i=1}^{10}(x_i - 3)}{10} = \frac{\sum_{i=1}^{10} x_i - 30}{10} = 3$

Variance is unchanged, if a constant is added or subtracted from each observation

$\mu = \mathrm{Var}(x_i - 3) = \mathrm{Var}(x_i - 5) = \frac{\sum_{i=1}^{10} (x_i - 5)^2}{10} - \left(\frac{\sum (x_i - 5)}{10}\right)^2$

$= \frac{40}{10} - \left(\frac{10}{10}\right)^2 = 3$

**67. (a)** Here $P(A) = P(B) = \frac{1}{2}$

Then, these following cases are possible $\rightarrow AA, BAA, ABA, ABBA, BBAA, BABA$

So, the required probability is $= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

**68. (b)** $p: \sqrt{5}$ is an integer

$q$: 5 is an irrational number

Given statement: $p \vee q$

Required negation statement: $\sim (p \vee q) = \sim p \wedge \sim q$

'$\sqrt{5}$ is not an integer and 5 is not an irrational number'

**69. (c)**

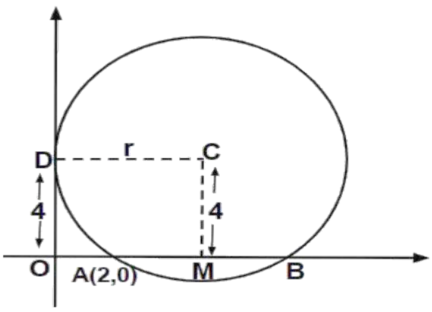
$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 13 + 1 - 8 = 6$

$B = \mathrm{adj}(A) \Rightarrow |\mathrm{adj}\, B| = |\mathrm{adj}(\mathrm{adj}\, A)| = |A|^4 = 6^4$

$|C| = |3A| = 3^3|A| = 3^3 \times 6$

$\frac{|\mathrm{adj}\, B|}{|C|} = \frac{6^4}{3^3 \times 6} = \frac{2^3 \times 3^3}{3^3} = 8$

**70. (d)**




$OD^2 = OA \times OB \Rightarrow 16 = 2 \times OB \Rightarrow OB = 8$

$\therefore AB = 6$

$\therefore AM = 3, CM = 4 \Rightarrow CA = 5$

$\therefore OM = 5$

Centre will be $(5,4)$ and radius is 5

Now checking option $(d)$

$3x + 4y - 6 = 0$

$$\frac{15 + 16 - 6}{\sqrt{3^2 + 4^2}} = 5(p = r)$$

**71. (3)**

$$(1 + x)\frac{dy}{dx} = [(1 + x)^2 + (y - 3)]$$

$$\Rightarrow (1 + x)\frac{dy}{dx} - y = (1 + x)^2 - 3$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{(1 + x)}y = 1 + x - \frac{3}{1 + x}$$

I. F. $= e^{-\int \frac{1}{1+x}dx} = \frac{1}{1+x}$

$$y \times \frac{1}{1 + x} = \int 1 - \frac{3}{(1 + x)^2}dx$$
$$\frac{y}{1 + x} = x + \frac{3}{1 + x} + c$$
$\Rightarrow y = x(1 + x) + 3 + c(1 + x)$
At $x = 2, y = 0$, we get
$0 = 6 + 3 + 3c$
$\Rightarrow c = -3$
$\Rightarrow$ At $x = 3$,
$y = x^2 - 2x = 9 - 6 = 3$
$\Rightarrow y(3) = 3$

**72. (0)**

$f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0^-} f(x) = b = \lim_{x\to 0^+} f(x)$

$$b = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{(h+3h^2)^{\frac{1}{3}} - h^{\frac{1}{3}}}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h\to 0} \frac{h^{\frac{1}{3}}\left[(1+3h)^{\frac{1}{3}} - 1\right]}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h\to 0} \frac{(1+3h)^{\frac{1}{3}} - 1}{h}$$

$$\Rightarrow b = \lim_{h\to 0} \frac{1}{3}(1+3h)^{-\frac{2}{3}} \times 3$$

or, $b = 1$

$$\lim_{x\to 0^-} f(x) = 1 \Rightarrow \lim_{h\to 0} \frac{\sin(a+2)(-h)) + \sin(-h)}{-h} = 1$$

$\Rightarrow a + 3 = 1 \Rightarrow a = -2$

$\Rightarrow a + 2b = 0$

**73. (615)**

General term of the given expression is given by $\frac{10!}{p!q!r!} x^{q+2r}$

Here, $q + 2r = 4$

For $p = 6, q = 4, r = 0$, coefficient $= \frac{10!}{6!\times 4!} = 210$

For $p = 7, q = 2, r = 1$, coefficient $= \frac{10!}{7!\times 2!\times 1!} = 360$

For $p = 8, q = 0, r = 2$, coefficient $= \frac{10!}{8!\times 2!} = 45$

Therefore, sum $= 615$

**74. (1)** As $\vec{P}, \vec{Q}, \vec{R}$ are coplanar,

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 3a+1 & 3a+1 & 3a+1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$(3a+1)\begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$(3a+1)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 0$$

$$3a + 1 = 0$$

$$\Rightarrow a = -\frac{1}{3}$$

$$\vec{P} = \frac{1}{3}(2\hat{\imath} - \hat{\jmath} - \hat{k}),\ \vec{Q} = \frac{1}{3}(-\hat{\imath} + 2\hat{\jmath} - \hat{k}),\ \vec{R} = \frac{1}{3}(-\hat{\imath} - \hat{\jmath} + 2\hat{k})$$

$$\vec{R} \times \vec{Q} = \frac{1}{9}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{vmatrix}$$

$$\vec{R} \times \vec{Q} = \frac{1}{9}(-3\hat{\imath} - 3\hat{\jmath} - 3\hat{k}) = -\frac{1}{3}(\hat{\imath} + \hat{\jmath} + \hat{k})$$

$$|\vec{R} \times \vec{Q}|^2 = \frac{1}{3}$$

$$\vec{P} \cdot \vec{Q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$$

$$3(\vec{P} \cdot \vec{Q})^2 - \lambda|\vec{R} \times \vec{Q}|^2 = 0$$

$$\Rightarrow \frac{1}{3} - \lambda \times \frac{1}{3} = 0 \Rightarrow \lambda = 1$$

**75. (8)** $\overrightarrow{AB} = \hat{\imath} - 3\hat{\jmath} + 8\hat{k}$

$\overrightarrow{CD} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

Projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ is $= \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{CD}|} = \frac{4+12+56}{\sqrt{4^2+4^2+7^2}} = \frac{72}{9} = 8$