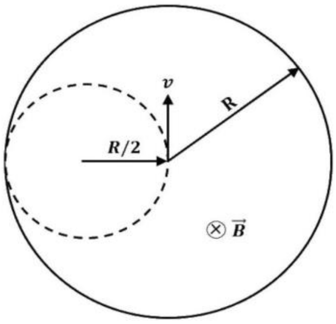
1. **(b)** Looking at the cross-section of the solenoid, $R_{\max}$ of the particle's motion has to be $\frac{R}{2}$ for it not to strike the solenoid.



$$qvB = \frac{mv^2}{\frac{R}{2}}$$

$$R_{\max} = \frac{R}{2} = \frac{mv_{\max}}{q\mu_0 in}$$

$$V_{\max} = \frac{Rq\mu_0 in}{2m}$$

2. **(a)** Maximum charge on capacitor = 5CV

is forward biased and (b) is reverse biased

For case (a)

$$q = q_{\max}\left(1 - e^{\frac{-t}{RC}}\right) = 5\text{CV}$$

For case (b)

$$Q_A = 5CVe^{-1}$$

$$Q_B = 5CV$$

3. **(d)**

	a	b	c	$a+b+c=d$	Round off
1.	220.1	20.4567	40.118	$d_1 = 280.6747$	280.7
2.	218.2	22.3625	40.372	$d_2 = 280.9345$	280.9
3.	221.2	20.2435	39.432	$d_3 = 280.8755$	280.9
4.	221.4	18.3625	40.281	$d_4 = 280.0435$	280.0

4. **(a)**

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

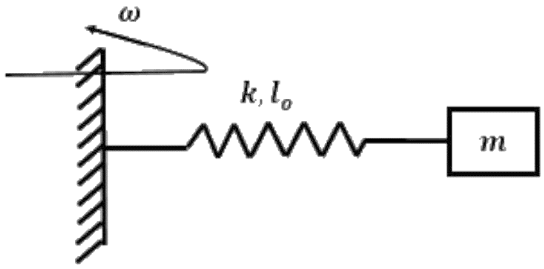
$$32 = 0 + \frac{1}{2} \times 4t^2 \rightarrow t = 4\text{sec}$$

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$= 3 \times 4 + \frac{1}{2} \times 6 \times 16$$

$$= 60 \text{ m}$$

5. (c)



Using Newton's second law of dynamics,

$$m\omega^2(l_o + x) = kx$$

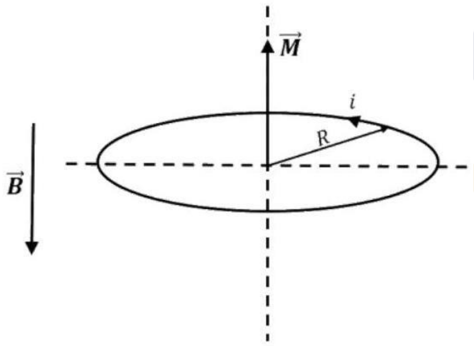
$$\left(\frac{l_o}{x} + 1\right) = \frac{k}{m\omega^2}$$

$$x = \frac{l_o m\omega^2}{k - m\omega^2}$$

$$k >> m\omega^2$$

So, $\frac{x}{l_o}$ is equal to $\frac{m\omega^2}{k}$

6. (b)



Considering the torque situation on the loop,

$$\tau = MB\sin\theta = -i\alpha$$

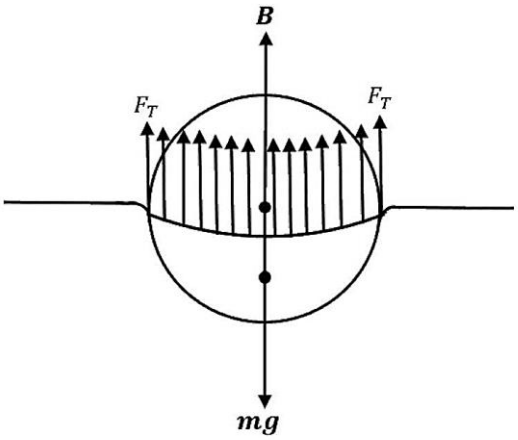
$$\pi R^2 iB\theta = -\frac{mR^2}{2}\alpha$$

The above equation is analogous to $\theta = -C\alpha$, where $C = \omega^2 = \frac{2\pi iB}{M}$

$$\omega = \sqrt{\frac{2\pi iB}{M}} = \frac{2\pi}{T}$$

$$T = \sqrt{\frac{2\pi M}{iB}}$$

7. (c)



In equilibrium, net external force acting on the sphere is zero.

$$mg = F_T + B$$

$$\rho V g = \sigma\left(\frac{V}{2}\right) g + T 2\pi R$$

$$\rho \frac{4}{3}\pi R^3 g = \sigma \frac{2}{3}\pi R^3 g + T 2\pi R$$

$$R = \sqrt{\frac{3T}{2\left(\rho - \frac{\sigma}{2}\right) g}}$$

8. (b) Key Idea: The difference of two consecutive resonant frequencies is the fundamental resonant frequency.

Fundamental frequency $= 490 - 420 = 70$ Hz

$$70 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow 70 = \frac{1}{2l}\sqrt{\frac{540}{6 \times 10^{-3}}}$$

$$\Rightarrow l = \frac{1}{2 \times 70}\sqrt{90 \times 10^{-3}} = \frac{300}{140}$$

$$\Rightarrow l \approx 2.14 \text{ m}$$

9. (a) EM wave is in direction $\frac{\hat{\imath}+\hat{\jmath}}{2}$

Electric field is in direction $\hat{k}$

Direction of propagation of EM wave is given by $\vec{E} \times \vec{B}$

10. **(b)** Mean free time $= \frac{1}{\sqrt{2} n \pi d^2}$

$$\frac{t_{Ar}}{t_{Xe}} = \frac{d_{Xe}^2}{d_{Ar}^2} = \left(\frac{0.1}{0.07}\right)^2 = \left(\frac{10}{7}\right)^2 = 2.04$$

11. **(a)**

$$\frac{dU}{dV} = \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y}$$
$$= \frac{1}{2} \times \frac{F^2}{A^2 Y}$$
$$\frac{dU}{dV} \propto \frac{1}{D^4}$$
$$\frac{\left(\frac{dU}{dV}\right)_1}{\left(\frac{dU}{dV}\right)_2} = \frac{D_2^4}{D_1^4} = \frac{1}{4}$$

$$\frac{D_1}{D_2} = (4)^{\frac{1}{4}}$$

$\therefore D_1 : D_2 = \sqrt{2} : 1$

12. **(c)** We know that the escape velocity is given by,

$$V_e = \sqrt{\frac{2GM}{R}}$$

Now

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{2GM}{R}}}{\sqrt{\frac{\frac{2GM}{2}}{\frac{R}{2}}}} = 1$$

We are given that $\frac{V_1}{V_2} = \frac{n}{4}$

$$\Rightarrow \frac{n}{4} = 1$$
$$\Rightarrow n = 4$$

13. **(b)** Here we take a small element along the length as dx at a distance x from the left end as shown.

$$x_{cm}=\frac{1}{M}\int_0^l x\cdot dm$$

$$\Rightarrow dM=\lambda\cdot dx=\left(a+b\left(\frac{x}{l}\right)^2\right)\cdot dx$$

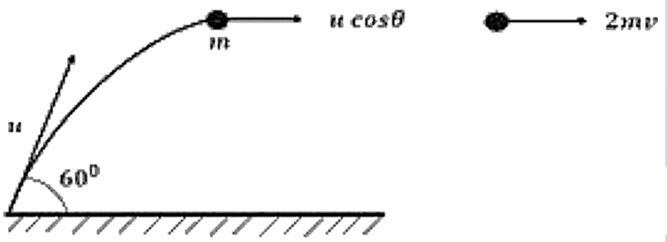
$$x_{cm}=\frac{\int x\lambda dx}{\int \lambda dx}=\frac{\int_0^l x\left(a+\frac{bx^2}{l^2}\right)dx}{\int_0^l\left(a+\frac{bx^2}{l^2}\right)dx}$$

$$=\frac{a\left(\frac{x^2}{2}\right)_0^l+\frac{b}{l^2}\left(\frac{x^4}{4}\right)_0^l}{a(x)_0^l+\frac{b}{l^2}\left(\frac{x^3}{3}\right)_0^l}$$

$$=\frac{\frac{al^2}{2}+\frac{bl^2}{4}}{al+\frac{bl}{3}}$$

$$=\frac{3l}{4}\left(\frac{2a+b}{3a+b}\right)$$

14. (b)



The only external force acting on the colliding system during the collision is the gravitational force. Since gravitational force is non-impulsive, the linear momentum of the system is conserved just before and just after the collision.

$$p_i=p_f$$

$$mu+mu\cos\theta=2mv$$

$$\Rightarrow v=\frac{u(1+\cos 60^\circ)}{2}=\frac{3}{4}u$$

So the horizontal range after the collision $= vt$

$$=v\sqrt{\frac{2H_{\max}}{g}}$$

$$=\frac{3}{4}u\sqrt{\frac{2u^2\sin^2 60^0}{2g^2}}$$

$$=\frac{3}{4}u^2\frac{\sqrt{\frac{3}{4}}}{g}=\frac{3\sqrt{3}u^2}{8g}$$

15. (a) Assume initial potential energy of the blocks to be zero. Initial kinetic energy is also zero since the blocks are at rest.

When block m_1 falls by h, m_2 goes up by h (because of length constraint)

Final P.E $= m_2gh - m_1gh$

Let the final speed of the blocks be v and angular velocity of the pulley be ω

Final K.E $= \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$

Total energy is conserved. Hence,

$$0 = m_2gh - m_1gh + \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$$

$v = \omega r$ (due to no slip condition)

$$\Rightarrow \frac{1}{2}(m_1 + m_2)\omega^2R^2 + \frac{1}{2}I\omega^2 = (m_1 - m_2)gh$$

$$\Rightarrow \omega^2\left[\frac{1}{2}(m_1 + m_2)R^2 + \frac{1}{2}I\right] = (m_1 - m_2)gh$$

$$\Rightarrow \omega^2 = \frac{2(m_1 - m_2)gh}{R^2\left[(m_1 + m_2) + \frac{I}{R^2}\right]}$$

$$\Rightarrow \omega = \frac{1}{R}\sqrt{\frac{2(m_1 - m_2)gh}{\left[(m_1 + m_2) + \frac{I}{R^2}\right]}}$$

16. (b)

$$\frac{hc}{\lambda} = (13.6\text{eV})Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
$$n_1 = 1$$

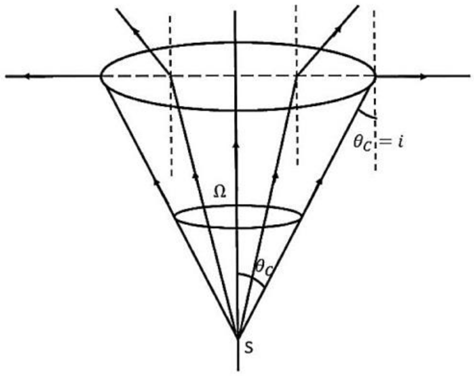
$n_2 = 3$

For an H-like atom, ionization energy is $(R)Z^2$.

This gives $Z = 3$

$$\frac{hc}{\lambda} = (13.6\text{eV})(3^2)\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$
$$\Rightarrow \frac{hc}{\lambda} = (13.6\text{eV})(9) \times \frac{8}{9}$$
$$\text{wavelength } = \frac{1240}{8 \times 13.6}\text{ nm}$$
$$\lambda = 11.39\text{ nm}$$

17. (a)



The portion of light escaping into the air from the liquid will form a cone. As long as the angle of incidence on the liquid - air interface is less than the critical angle, i.e. $i < \theta_C$, the light rays will undergo refraction and emerge into the air.

For $i > \theta_C$, the light rays will suffer TIR. So, these rays will not emerge into the air.

The portion of light rays emerging into the air from the liquid will form a cone of half angle $= \theta_C$

$$\sin\theta_C = \frac{1}{\mu_{\text{Liq}}} = \frac{3}{4}, \cos\theta_C = \frac{\sqrt{7}}{4}$$

Solid angle contained in this cone is

$$\Omega = 2\pi(1 - \cos\theta_C)$$

Percentage of light that escapes from liquid $= \frac{\Omega}{4\pi} \times 100$

Putting values we get

Percentage $= \frac{4-\sqrt{7}}{8} \times 100 \approx 17\%$

18. (c)

$$\lambda_D = \frac{h}{mv}$$

where, $v = at$

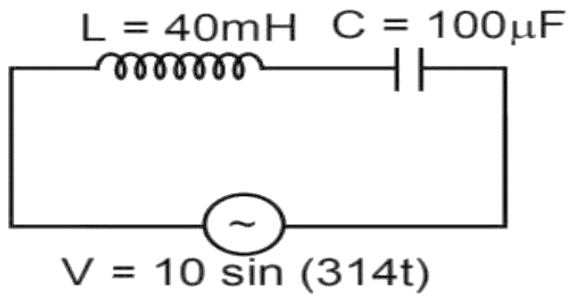
$$v = \frac{eE}{M}t \left(a = \frac{eE}{M}\right)$$

$$\lambda_D = \frac{h}{m\frac{eE}{M}t}$$

$$\lambda_D = \frac{h}{eEt}$$

$$\frac{d\lambda_D}{dt} = \frac{h}{|e|Et^2}$$

19. (b)



Impedance $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$$= \sqrt{(X_C - X_L)^2}$$

$$= X_C - X_L$$

$$= \frac{1}{\omega C} - \omega L$$

$$= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$$

$$= 31.84 - 12.56 = 19.28\Omega$$

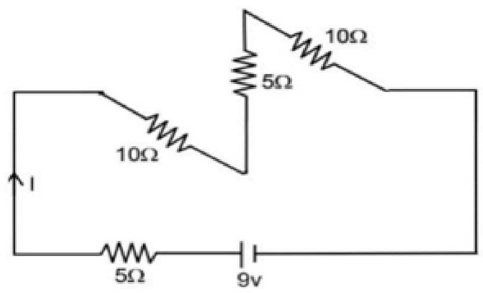
For $X_C > X_L$, current leads voltage by $\frac{\pi}{2}$

$$\therefore i = \frac{V}{Z} = \frac{10\sin\left(314t + \frac{\pi}{2}\right)}{19.28}$$

$$= 0.52\cos(314t)$$

20. (b) Since the diodes are reverse biased, they will not conduct.

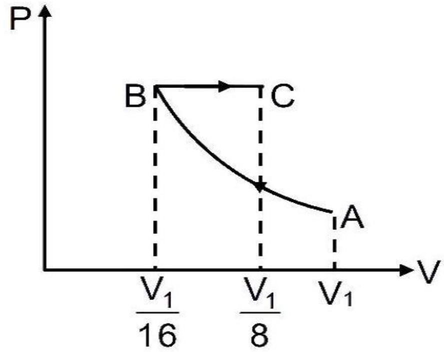
Hence, the circuit will look like



$R_{\text{eff}} = 5 + 10 + 5 + 10 = 30\Omega$

$$I = \frac{9}{30} = 0.3\text{ A}$$

21. (1819k)



$PV^{\gamma} =$ Constant

$TV^{(\gamma-1)} =$ constant

$$300(V_1)^{(1.4-1)} = T_B\left(\frac{V_1}{16}\right)^{\frac{2}{5}}$$

$$T_B = 300 \times 2^{\left(\frac{8}{5}\right)}$$

Now for BC process

$$\frac{V_B}{T_B} = \frac{V_C}{T_C}$$

$$T_C = \frac{V_C T_B}{T_B} = 2 \times 300 \times 2^{\left(\frac{8}{5}\right)}$$

$$T_C = 1819\text{ K}$$

22. (-40) Electric flux through a surface is $\emptyset = \int \vec{E} = d\vec{A}$

For surface $ABCD$,

$d\vec{A}$ is along $(-\hat{k})$

So, at all the points of this surface,

$\vec{E} \cdot d\vec{A} = 0$

Because, $\emptyset_{ABCD} = \emptyset_1 = 0$

For surface $BCEF$,

$d\vec{A}$ is along $(\hat{\imath})$

So,

$$\vec{E} \cdot d\vec{A} = E_x dA$$

$$\emptyset_{BCEF} = \emptyset_2 = 4x(2 \times 2)$$

If if $x = 3$

$$\emptyset_2 = 48\frac{N - m^2}{C}$$

Hence, $\emptyset_1 - \emptyset_2 = -48\frac{N-m^2}{C}$

23. (750)

If the length of the segment is y,

Then $y = n\beta$

$n =$ no. of fringes,

$\beta =$ fringe width

$$15 \times 500 \times \frac{D}{d} = 10 \times \lambda_2 \times \frac{D}{d}$$

$\lambda_2 = 15 \times 50$ nm
$\lambda_2 = 750$ nm

24. (40)

$$\frac{X}{R} = \frac{75}{25} = 3$$
$$R = \frac{\rho l}{A} = \frac{4\rho l}{\pi d^2}$$
$$R' = \frac{4\rho \frac{l}{2}}{\pi \left(\frac{d}{2}\right)^2} = 2R$$
$$\text{Then } \frac{X}{R'} = \left(\frac{100 - l}{l}\right)$$
$$\frac{100 - l}{l} = \frac{X}{2R} = \frac{3}{2}$$

$l = 40.00$ cm

25. (40)

$$i = \left(\frac{12 - 8}{200 + 200}\right) A = \frac{4}{400} = 10^{-2} A$$

Power loss in each diode $= (4)(10^{-2})\text{W} = 40$ mW

26. (c) $Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2$

$Zn + 2HCl \rightarrow ZnCl_2 + H_2$

So, the ratio of volume of H_2 released in both the cases is 1:1.

27. (a) $Cr(OH)_{3(s)} \rightarrow Cr^{3+}_{(aq.)} + 3OH^-_{(aq.)}$

$1 - S$ S 3 S

$K_{sp} = 27\ S^4$

$6 \times 10^{-31} = 27S^4$

$$S = \left[\frac{6}{27} \times 10^{-31}\right]^{1/4}$$

$$[OH^-] = 3\ S = 3 \times \left[\frac{6}{27} \times 10^{-31}\right]^{1/4} = (18 \times 10^{-31})^{1/4}\text{M}$$

28. (a) Only LiCl amongst the first group chlorides dissolve in pyridine because the solvation energy of lithium is higher than the other salts of the same group.

Lithium does not react with ethyne to form ethynilide due to its small size and high polarizability. Lithium and Magnesium both have very small sizes and very high ionization potentials so, they react slowly with water.

Amongst all the alkali metals, Li has the smallest size hence, the hydration energy for Li is maximum.

29. (a) The given data for ionization energies clearly shows that $IE_2 \gg IE_1$. So, the element belongs to the first group. Therefore, we can say that this element will be monovalent and hence forms a monoacidic base of the type MOH.

$\mathrm{MOH + HCl \rightarrow MCl + H_2O}$

$\mathrm{2MOH + H_2SO_4 \rightarrow M_2SO_4 + 2H_2O}$

So, from the above equation we can say that,

1 mole of metal hydroxide requires 1 mole of HCl and 0.5 mole of H_2SO_4.

30. (b) Let us assume the equation to be $A \rightleftharpoons B$,

Number of particles of $A = 6$

Number of particles of $B = 11$

$\mathrm{K} = \frac{11}{6} \approx 2$

31. (a) Complex (I) has the central metal ion as Fe^{2+} with strong field ligands.

Configuration of $Fe^{2+} = [Ar]3\ d^6$

Strong field ligands will pair up all the electrons and hence the magnetic moment will be zero.

Complex (II) has the central metal ion as Cr^{2+} with weak field ligands.

Configuration of $Cr^{2+} = [Ar]3\ d^4$

As weak field ligands are present, pairing does not take place. There will be 4 unpaired electrons and hence the magnetic moment $= \sqrt{24}$ B.M.

Complex (III) has the central metal ion as Co^{2+} with weak field ligands.

Configuration of $Co^{2+} = [Ar]3\ d^7$

As weak field ligands are present no pairing can occur. There will be 3 unpaired electrons and hence the magnetic moment $= \sqrt{15}$ B.M.

Complex (IV) has the central metal ion as Fe^{3+} with strong field ligands.

Configuration of $Fe^{3+} = [Ar]3\ d^5$

Strong field ligands will pair up the electrons but as we have a $[Ar]3\ d^5$ configuration, one electron will remain unpaired and hence the magnetic moment will be $\sqrt{3}$ B.M.

32. (a) Entropy is a function of temperature, at any temperature, the entropy can be given as:

$\mathrm{S_T} = \int_0^{\mathrm{T}} \frac{\mathrm{nCdT}}{\mathrm{T}}$

Change in entropy is also a function of temperature, at any temperature, the entropy change can be given as:

$\Delta S = \int \frac{dq}{T}$

33. (d)

$B_3N_3H_3Cl_3 + LiBH_4 \rightarrow B_3N_3H_6 + LiCl + BCl_3$

$B_3N_3H_3Cl_3 + 3CH_3MgBr \rightarrow B_3N_3H_3(CH_3)_3 + 3MgBrCl$

So, we can say that,

B is $B_3N_3H_6$

C is CH_3MgBr

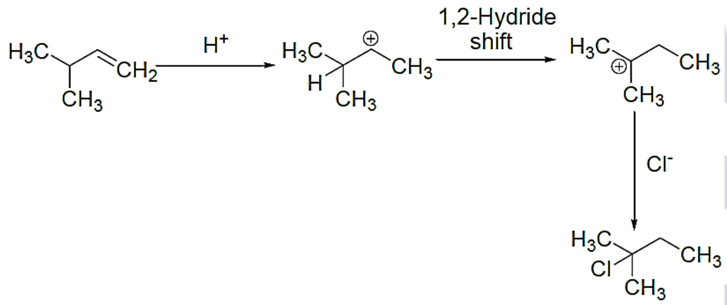
34. (c) As H_2, O_2 and CO gets adsorbed on the surface of charcoal, the pressure decreases. So, option (a) and (d) can be eliminated. After some time, as almost all the surface sites are occupied, the pressure becomes constant.

35. (a) In cis-isomer, similar ligands are at an angle of $90°$.

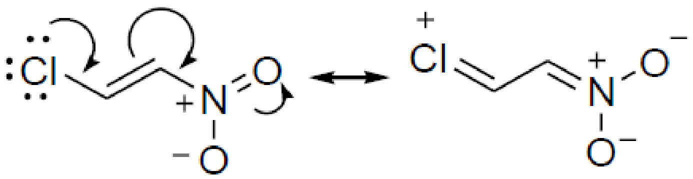
36. (a) In distilled water there are no ions present except H^+ and OH^- ions, both of which are immensely minute in concentration, that renders their collective conductivity negligible.

37. (a) Benzene (C_6H_6) has $6sp^2$ hybridized carbons. Each carbon has 3σ-bonds and 1π-bond. 3σ-bonds means that there are $3sp^2$ hybrid orbitals for each carbon. Hence, the total number of sp^2 hybrid orbitals is 18.

38. (b)

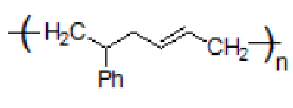
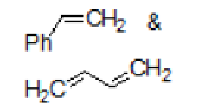
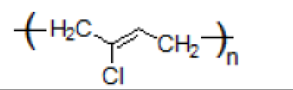
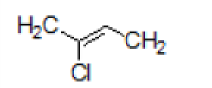
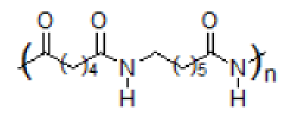
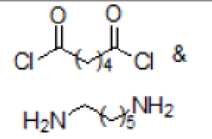
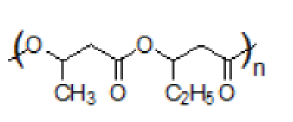
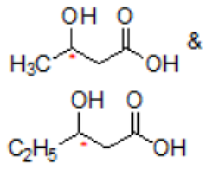


39. (d) There is extended conjugation present in option (d), which will reduce the length of C - Cl bond to the greatest extent which can be represented as follows:



40. (b) Biochemical oxygen demand (BOD) is the amount of dissolved oxygen used by microorganisms in the biological process of metabolizing organic matter in water.

41. (d)

	Polymers	Monomers
Buna-S		
Neoprene		
Nylon-6,6		
PHBV		

42. (a) Lactose, glucose and fructose gives positive Molisch's test.

Glucose gives positive Barfoed's test whereas sucrose gives a negative for Barfoed's test.

Albumin gives positive for Biuret test whereas alanine gives a negative Biuret test.

43. (b) The basicity of the compound depends on the availability of the lone pairs.

In compound IV, Nitrogen is sp^3 hybridized.

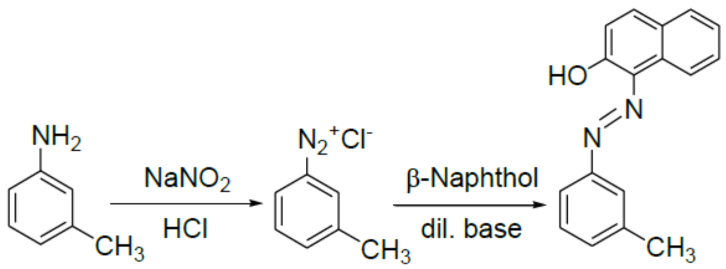
In compound III, Nitrogen is sp^2 hybridized and the lone pairs are not involved in resonance.

In compound I, Nitrogen is sp^2 hybridized and the lone pairs are involved in resonance.

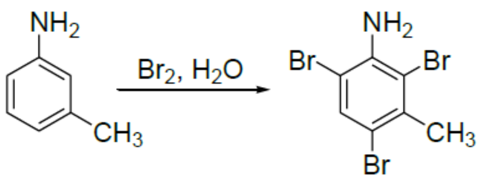
In compound II, Nitrogen is sp^2 hybridized and the lone pairs are involved in resonance such that, they are contributing to the aromaticity of the ring.

From the above points we can conclude that the basicity order should be IV > III > I > II.

44. (b)

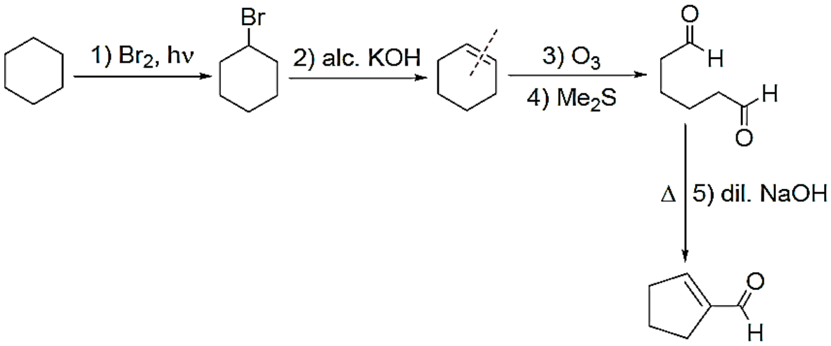


Coloured compound

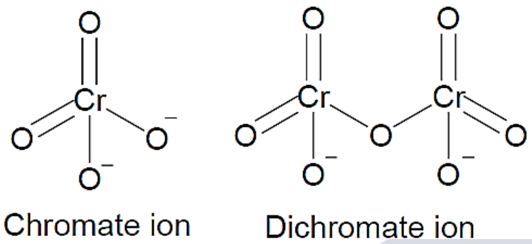


B ($C_7H_6NBr_3$)

45. (d)



46. (12)



47. (3.98) The generation time can be utilized to get an indication of the rate ratio. Let the amount generated be (x).

$$\text{Rate} = \frac{\text{Amount generated}}{\text{Time taken}}$$

$$\text{Rate}_{\ 300\text{ K}} = \frac{(x)}{60} \quad \text{Rate}_{\ 400\text{ K}} = \frac{(x)}{40}$$

$$\frac{\text{Rate}_{\ 300\text{ K}}}{\text{Rate}_{\ 400\text{ K}}} = \frac{40}{60}$$

For the same concentration (which is applicable here), the rate ratio can also be equaled to the ratio of rate constants.

$$\ln\left[\frac{K_{at400\text{ K}}}{K_{at300\text{ K}}}\right] = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln\frac{60}{40} = \frac{E_a}{8.3}\left[\frac{1}{300} - \frac{1}{400}\right]$$

$$E_a = 0.4 \times 8.3 \times 1200 = 3984\ \text{J/mol} = 3.98\ \text{kJ/mol}$$

48. (10.00) $\text{Ppm} = \frac{w_{\text{Solute}}}{w_{\text{Solution}}} \times 100$

Using the density of the solution and its volume ($1\text{ L} = 1000\text{ mL} = 1000\text{ cm}^3$), the weight of the solution can be calculated.

$$W_{\text{solution}} = 1.03 \times 1000 = 1030\text{ g}$$

Thus, $\text{ppm} = \frac{10.3 \times 10^{-3}\text{ g}}{1030\text{ g}} \times 100$

49. (2.18)

$K_f = 2$

Molality, ' m ' = 0.5

$\Delta T_f = K_f \cdot m$

$= (0.5 \times 2) = 1$

So, the initial temperature now becomes 272 K. Further using the given value of moles and initial volume of the gas and the calculated initial temperature value, we can find out the initial pressure of the ideal gas contained inside the piston.

$$P_{\text{gas}} = \frac{nRT}{V_1}$$
$$= (0.1)(0.08)(272) = 2.176 \text{ atm}$$

Now, on releasing the piston against an external pressure of 1 atm, the gas will expand until the final pressure of the gas, i.e. P_2 becomes equal to 1 atm. During this expansion, since no reaction is happening and the temperature of the gas is not changing as well, the boyle's law relation can be applied.

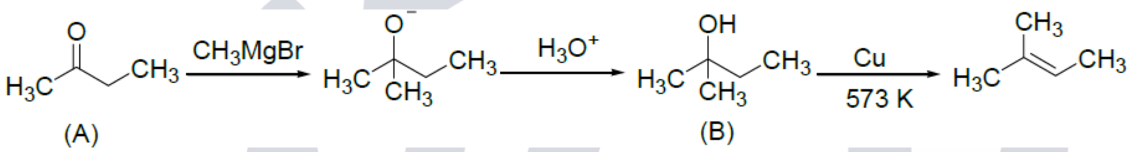
$P_1 V_1 = P_2 V_2$

$2.176 \times 1 = 1 \times V_2$

50. (66.67)

Compound A is $CH_3(CO)CH_2CH_3(C_4H_8O)$

The percentage of carbon in compound A by weight is $\frac{w_{\text{Carbon}}}{w_{\text{Compound}}} = \frac{12\times4}{72} = 66.67$



51. (a) Given $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

$a - 2b + c = 1$

Applying $R_1 \to R_1 - 2R_2 + R_3$

$$f(x) = \begin{vmatrix} a-2b+c & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

Using $a - 2b + c = 1$

$\therefore f(x) = (x+3)^2 - (x+2)(x+4)$

$\Rightarrow f(x) = 1$

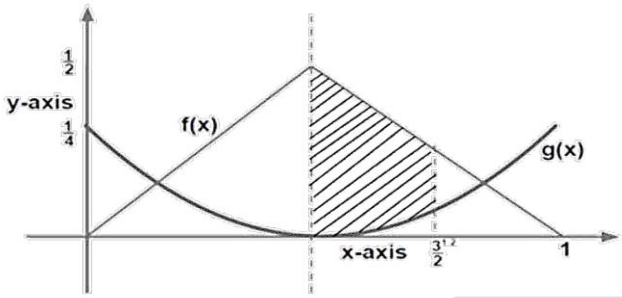
$\Rightarrow f(50) = 1$

$\Rightarrow f(-50) = 1$

52. (a) Given $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1 - x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = \left(x - \frac{1}{2}\right)^2$

The area between $f(x)$ and $g(x)$ from $x = \frac{1}{2}$ to $= \frac{\sqrt{3}}{2}$:



Points of intersection of $f(x)$ and (x) :

$$1 - x = \left(x - \frac{1}{2}\right)^2$$
$$\Rightarrow x = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

Required area $= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} (f(x) - g(x))dx$

$$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \left(1 - x - \left(x - \frac{1}{2}\right)^2\right) dx$$

$$= x - \frac{x^2}{2} - \frac{1}{3}\left(x - \frac{1}{2}\right)^3 \Big|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$

53. (c)

54. (c) Let $I = \int \frac{d\theta}{\cos^2\theta(\sec 2\theta + \tan 2\theta)}$

$$I = \int \frac{\sec^2\theta d\theta}{\left(\frac{1 + \tan^2\theta}{1 - \tan^2\theta}\right) + \left(\frac{2\tan\theta}{1 - \tan^2\theta}\right)}$$

$$I = \int \frac{(1 - \tan^2\theta)(\sec^2\theta)d\theta}{(1 + \tan\theta)^2}$$

Let $\tan\theta = k \Rightarrow \sec^2\theta d\theta = \mathrm{d}k$

$I=\int\frac{(1-k^2)}{(1+k)^2}dk=\int\frac{(1-k)}{(1+k)}dk$

$I=\left(\frac{2}{1+k}-1\right)dk$

$I=2\ln|1+k|-k+c$

$I=2\ln|1+\tan\theta|-\tan\theta+c$

Given $I=\lambda\tan\theta+2\log f(x)+c$

$\therefore \lambda=-1, f(x)=|1+\tan\theta|$

55. (a) a_n is a positive term of GP.

Let GP be $a, ar, ar^2, ...$

$\sum_{n=1}^{100} a_{2n+1}=a_3+a_5+\cdots\ldots+a_{201}$

$200=ar^2+ar^4+\cdots\ldots.+ar^{201}$

$200=\frac{ar^2(r^{200}-1)}{r^2-1}$

Also, $\sum_{n=1}^{100} a_{2n}=100$

$100=a_2+a_4+\cdots\ldots..+a_{200}$

$100=ar+ar^3+\cdots\ldots..+ar^{199}$

$100=\frac{\mathrm{ar}(r^{200}-1)}{r^2-1}$

From (1) and (2), $r=2$

And $\sum_{n=1}^{100} a_{2n+1}+\sum_{n=1}^{100} a_{2n}=300$

$\Rightarrow a_2+a_3+a_4\ldots\ldots.+a_{200}+a_{201}=300$

$\Rightarrow ar+ar^2+ar^3+\cdots\ldots.+ar^{200}=300$

$\Rightarrow r(a+ar+ar^2+\cdots..+ar^{199})=300$

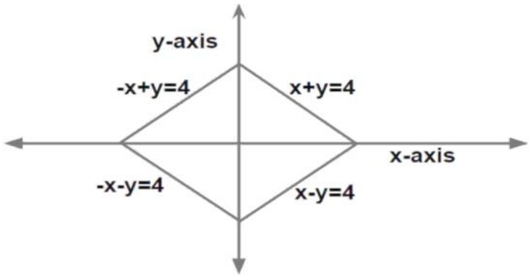
$\Rightarrow 2(a_1+a_2+a_3+\cdots\ldots+a_{200})=300$

$\sum_{n=1}^{200} a_n=150$

56. (b) $|\mathrm{Re}(z)|+|\mathrm{Im}(z)|=4$

Let $z=x+iy$

$\Rightarrow |x|+|y|=4$



$\therefore z$ lies on the rhombus.

Maximum value of $|z| = 4$ when $z = 4, -4, 4i, -4i$

Minimum value of $|z| = 2\sqrt{2}$ when $z = 2 \pm 2i, \pm 2 + 2i$

$|z| \in [2\sqrt{2}, 4]$

$|z| \in [\sqrt{8}, \sqrt{16}]$

$|z| \neq \sqrt{7}$

57. (b) $F(x) = x^2 g(x)$

Put $x = 1$
$\Rightarrow F(1) = g(1) = 0$

Now $F''(x) = 2xg(x) + g'(x)x^2$

$F''(1) = 2g(1) + g'(1)$

$\{\because g'(x) = f(x)\}$

$F''(1) = f(1) = 3$

From (1) and (2), $F(x)$ has local minimum at $x = 1$

58. (a) $\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$$

$$\frac{dy}{dx} = \frac{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}$$

$$\frac{dy}{dx} = \cot\frac{3\theta}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{2}\operatorname{cosec}^2\frac{3\theta}{2}\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \left(-\frac{3}{2}\operatorname{cosec}^2\frac{3\theta}{2}\right)\frac{1}{(2\cos\theta - 2\cos 2\theta)}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=\pi} = \frac{3}{8}$$

59. (d) $f(g(x)) = x$

$f'(g(x))g'(x) = 1$

Put $x = b$

$f'(g(b))g'(b) = 1$

$f'(a) \times 5 = 1$

$$f'(a) = \frac{1}{5}$$

60. (d) If $2b = \frac{4}{\sqrt{3}}$

$$b = \frac{2}{\sqrt{3}}$$

Comparing $y = -\frac{x}{6} + \frac{8}{6}$ with $y = mx \pm \sqrt{a^2m^2 + b^2}$

$m = -\frac{1}{6}$ and $a^2m^2 + b^2 = \frac{16}{9}$

$$\frac{a^2}{36} + \frac{4}{3} = \frac{16}{9}$$

$$\Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{3}$$

$\Rightarrow a^2 = 16$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{\frac{11}{12}}$$

61. (c) Let PQ be the focal chord of the parabola $y^2 = 8x$

$\Rightarrow P(t_1) = (2t_1^2, 4t_1) \& Q(t_2) = (2t_2^2, 4t_2)$

$\Rightarrow t_1t_2 = -1$

$\because \left(\frac{1}{2}, -2\right)$ is one of the ends of the focal chord of the parabola

Let $\left(\frac{1}{2}, -2\right) = (2t_2^2, 4t_2)$

$$\Rightarrow t_2 = -\frac{1}{2}$$

⇒ Other end of focal chord will have parameter $t_1 = 2$

⇒ The co-ordinate of the other end of the focal chord will be (8,8)

∴ The equation of the tangent will be given as → $8y = 4(x + 8)$

⇒ $2y - x = 8$

62. (b)

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$

As the system of equations are Homogeneous ⇒ the system is consistent.

$$\Rightarrow \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0$$

⇒ Infinite solutions exist (both trivial and non-trivial solutions)

When $y = 2z$

Let's take $y = 2, z = 1$

When $(x, 2,1)$ is substituted in the system of equations

⇒ $7x + 10 = 0$

$3x + 10 = 0$

$x - 10 = 0$ (which is not possible)

∴ $y = 2z$ ⇒ Infinite non-trivial solutions does not exist.

For $x = 2z$, lets take $x = 2, z = 1, y = y$

Substitute $(2, y, 1)$ in system of equations

⇒ $y = -2$

∴ For each pair of (x, z), we get a value of y.

Therefore, for $x = 2z$ infinite non-trivial solution exists.

63. (c) $ax^2 - 2bx + 5 = 0$ has both roots as α

$$\Rightarrow 2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$$

And $\alpha^2 = \frac{5}{a}$

⇒ $b^2 = 5a (a \neq 0)$

$\Rightarrow \alpha+\beta=2b \& \alpha\beta=-10$

$\alpha=\frac{b}{a}$ is also a root of $x^2-2bx-10=0$

$\Rightarrow b^2-2ab^2-10a^2=0$

$\because b^2=5a \Rightarrow 5a-10a^2-10a^2=0$

$\Rightarrow a=\frac{1}{4} \Rightarrow b^2=\frac{5}{4}$

$\Rightarrow \alpha^2=20, \beta^2=5$

$\Rightarrow \alpha^2+\beta^2=25$

64. (b) $A=\{x: x\in(-2,2)\}$

$B=\{x: x\in(-\infty,-1]\cup[5,\infty)\}$

$A\cap B=\{x: x\in(-2,-1]\}$

$B-A=\{x: x\in(-\infty,-2]\cup[5,\infty)\}$

$A-B=\{x: x\in(-1,2)\}$

$A\cup B=\{x: x\in(-\infty,2)\cup[5,\infty)\}$

65. (b) We know that $\sum_{x_i=1}^{5} P_i=1$

$\Rightarrow k^2+2k+k+2k+5k^2=1$

$\Rightarrow k=-1,\frac{1}{6} \therefore k=\frac{1}{6}$

$P(x_i>2)=P(x_i=3)+P(x_i=4)+P(x_i=5)$
$\quad =k+2k+5k^2=\frac{23}{36}$

66. (b) We find the point of intersection of the two lines, and the distance of given plane from the two lines is the distance of plane from the point of intersection.

$\therefore (2p-1, 2p+3, 8p-1)=(2q-3, q-2, \lambda q+1)$

$p=-2$ and $q=-1$

$\lambda=18$

Point of intersection is $(-5,-3,-17)$

$\therefore \frac{k}{\sqrt{633}}=\left|\frac{-115+30+34+48}{\sqrt{633}}\right| \Rightarrow k=3$

67. (b) $y=1+\cos^2\theta+\cos^4\theta+\cdots$

$\Rightarrow y=\frac{1}{1-\cos^2\theta} \Rightarrow \frac{1}{y}=\sin^2\theta$

$x = 1 - \tan^2\theta + \tan^4\theta - \cdots$

$\Rightarrow x = \dfrac{1}{1-(-\tan^2\theta)} = \cos^2\theta$

$\therefore x + \dfrac{1}{y} = 1 \Rightarrow y(1-x) = 1$

68. (b) $f(x) = [x^2]\sin\pi x$

It is continuous $\forall x \in \mathbf{Z}$ as $\sin\pi x \to 0$ as $x \to \mathbf{Z}$.

$f(x)$ is discontinuous at points where $[x^2]$ is discontinuous i.e. $x^2 \in \mathbf{Z}$ with an exception that $f(x)$ is continuous as x is an integer.

$\therefore$ Points of discontinuity for $f(x)$ would be at

$x = \pm\sqrt{2}, \pm\sqrt{3}, \pm\sqrt{5}, \ldots \ldots$

Also, it is given that $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$ (indeterminate form $(0\times\infty)$)

$\Rightarrow \lim_{x\to 0} x\left(\dfrac{4}{x} - \left\{\dfrac{4}{x}\right\}\right) = A$

$\Rightarrow 4 - \lim_{x\to 0}\left\{\dfrac{4}{x}\right\} = A$

$\Rightarrow A = 4$
$\sqrt{A+5} = 3$

$\sqrt{A+1} = \sqrt{5}$

$\sqrt{A+21} = 5$

$\sqrt{A} = 2$

$\therefore$ Points of discontinuity for $f(x)$ is $x = \sqrt{5}$

69. (36) Two circles touch each other if $C_1C_2 = |r_1 \pm r_2|$

$\sqrt{k} + 1 = 5$ or $|\sqrt{k} - 1| = 5$

$\Rightarrow k = 16$ or 36

Maximum value of k is 36

70. (51)

$S = {}^{25}C_0 + 5\,{}^{25}C_1 + 9\,{}^{25}C_2 + \cdots + 97\,{}^{25}C_{24} + 101\,{}^{25}C_{25} = 2^{25}k$

Reverse and apply property ${}^nC_r = {}^nC_{n-r}$ in all coefficients

$S = 101\,{}^{25}C_0 + 97\,{}^{25}C_1 + \cdots + 5\,{}^{25}C_{24} + {}^{25}C_{25}$

Adding (1) and (2), we get

$2S = 102[\,^{25}C_0 + \,^{25}C_1 + \cdots + \,^{25}C_{25}]$

$S = 51 \times 2^{25}$

$\Rightarrow k = 51$

71. (14) First common term is 23

Common difference $= \text{LCM}(7,4) = 28$

$23 + (n-1)28 \leq 407$

$n - 1 \leq 13.71$

$n = 14$

72. (30) $|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}||\vec{b} \times \vec{c}|\sin\theta$ where θ is the angle between $\vec{a}$ and $\vec{b} \times \vec{c}$

$\theta = \frac{\pi}{2}$ given

$$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3}|\vec{b} \times \vec{c}| = \sqrt{3}|\vec{b}||\vec{c}|\sin\frac{\pi}{3}$$

$$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3} \times 5 \times |\vec{c}| \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \frac{15}{2}|\vec{c}|$$

Now, $|\vec{b}||\vec{c}|\cos\theta = 10$

$$5|\vec{c}|\frac{1}{2} = 10$$

$|\vec{c}| = 4$

73. (16) $T_{r+1} = \,^{16}C_r \left(\frac{x}{\sin\theta}\right)^{16-r} \left(\frac{1}{x\cos\theta}\right)^r$

For term independent of x,

$16 - 2r = 0 \Rightarrow r = 8$

$$T_9 = \,^{16}C_8 \left(\frac{1}{\sin\theta\cos\theta}\right)^8 = \,^{16}C_8 2^8 \left(\frac{1}{\sin 2\theta}\right)^8$$

$L_1 = \,^{16}C_8 2^8$ at $\theta = \frac{\pi}{4}$

$$L_2 = \,^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = \,^{16}C_8 2^{12} \text{ at } \theta = \frac{\pi}{8}$$

$$\frac{L_2}{L_1} = 16$$