1. (b) $[P] = MLT^{-1} \leftarrow$ momentum

$[A] = M^0L^2T^0 \leftarrow$ Area

$[T] = M^0L^0\ T^1 \leftarrow$ Time

Let $[E] = P^{x}A^{r}T^{Z}$

$ML^2\ T^{-2} = [MLT^{-1}]^x[\ L^2]^y[\ T]^2$

$= M^xL^{x+2y}T^{z-x}$

Comparing both sides :-

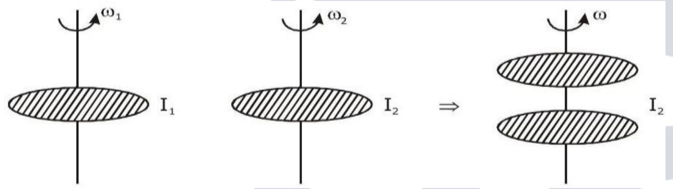
$x = 1$

$x + 2y = 2 \Rightarrow 1 + 2y = 2$ or, $y = \frac{1}{2}$

$z - x = -2 \Rightarrow z - 1 = -2$ or $z = -1$

$\therefore [E] = \left[P^1\ A^{1/2}\ T^{-1}\right]$

2. (d)



$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{0.1 \times 10 + 0.2 \times 5}{0.1 + 0.2} = \frac{1+1}{0.3} = \frac{2}{0.3}$$

$$\omega = \frac{20}{3}$$

Now find $KE = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2$

$$= \frac{1}{2}(I_1 + I_2)\omega^2 = \frac{1}{2} \times 0.3 \times \left(\frac{20}{3}\right)^2$$

$$= \frac{1}{2} \times \frac{3}{10} \times \frac{20}{3} \times \frac{20}{3}$$

$$\frac{0.3}{2} \times \frac{400}{9}$$

$$(\text{ K.E. })_f = \frac{20}{3}$$

3. (c) $P = \frac{h}{\lambda}$

$\lambda = \frac{h}{P}$

$\frac{\lambda_{Paricle}}{\lambda_e} = 1.878 \times 10^{-4}$

$\Rightarrow \frac{h}{P_{particle}} \times \frac{Pe}{h} = 13.878 \times 10^{-4} \Rightarrow \frac{Pe}{P_{particle}} = 1.878 \times 10^{-4}$

$\Rightarrow \frac{M_e \cdot V_e}{M_p \cdot V_p} = 1.878 \times 10^{-4} \Rightarrow M_p = \frac{M_e}{1.878 \times 10^{-4}} \times \left(\frac{V_e}{V_p}\right)$

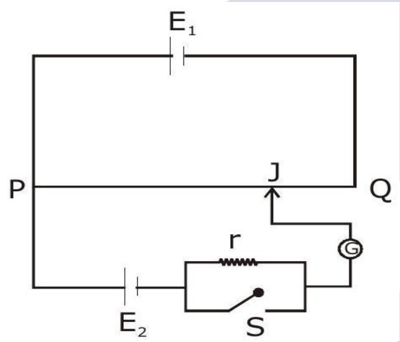
$= \frac{9.11 \times 10^{-31}}{1.878 \times 10^{-4}} \times \frac{1}{5}$

$= \frac{7.11 \times 10}{1.878 \times 10^{-4}} \times \frac{1}{5}$

$= 0.97 \times 10^{-27}$ kg

$= 9.7 \times 10^{-28}$ kg

4. (b)



1.02 V

PQ = 1 m

QJ = 49 cm

$\therefore$ PJ = 51 cm

$\frac{v}{\ell} = \frac{1.02}{51} = 0.02$v/cm

5. (c)
6. (c)

LOOP = ABCD

$\vec{M}_1 = (abI)\hat{k}$

Loop DEFA

$\vec{M}_2 = (abI)\hat{j}$

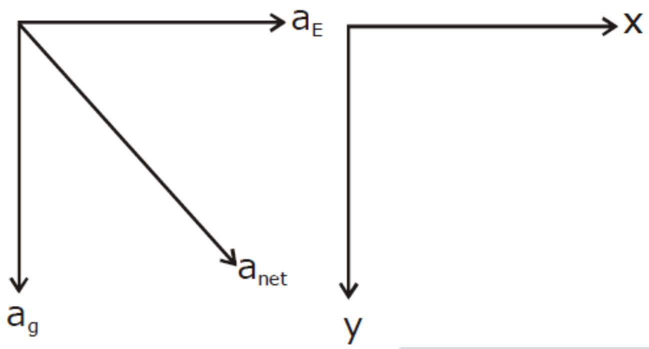
$\vec{M} = \vec{M}_1 + \vec{M}_2 = abI(\hat{j} + \hat{k})$

$|\vec{M}| = \sqrt{2}abI$

direction $=$ along $\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$

$\sqrt{2}$ abl, along $\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$

7. (c)



Since it is released from rest.

And a_{net} is constant.

If will have straight line path along net ' a '.

8. (a)

$\vec{E} \times \vec{B} = \hat{k} \times (2\hat{i} - 2\hat{j}) = 2\hat{k} \times \hat{i} - 2\hat{k} \times \hat{j} = 2\hat{j} + 2\hat{i}$

unit vector along $\vec{E} \times \vec{B} = \frac{1}{2\sqrt{2}}(2\hat{i} + 2\hat{j}) = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$

$$C = \frac{1}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})$$

9. (d) L-R circuit

$$\tan 45^\circ = \frac{x_L}{R}$$

$$1 = \frac{x_L}{R} \Rightarrow x_L = R$$

$$\text{Now } Z = \sqrt{R^2 + x_L^2}$$

$$\text{or } Z = \sqrt{x_L^2 + x_L^2} = \sqrt{2x_L^2} = \sqrt{2}x_L$$

$$100 = \sqrt{2}x_L$$

$$x_L = \frac{100}{\sqrt{2}}$$

$$\omega L=\frac{100}{\sqrt{2}}\Rightarrow L=\frac{100}{\sqrt{2}\omega}=\frac{100}{\sqrt{2}\times 2\pi f}=\frac{100}{\sqrt{2}\times 2\times 3.14\times 1000}$$

$$=1.1\times 10^{-2}\mathrm{H}$$

10. $(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C})$

(A) at $t=\frac{3\mathrm{T}}{4}$

Particle is at mean position

$a=0$

$\mathrm{F}=0$

(B) at $\mathrm{t}=\mathrm{T}$,

Particle is at extreme.

F is maximum

$\mathrm{a}=\max$

(C) at $\mathrm{t}=\frac{\mathrm{T}}{4}$; mean position

so, maximum velocity

(d) $KE=PE$

$$\frac{1}{2}\mathrm{k}(\mathrm{A}^2-\mathrm{x}^2)=\frac{1}{2}\mathrm{kx}^2$$

$$A^2-x^2=x^2$$

$$A^2=2x^2$$

$$A=\sqrt{2}x$$

$$x=\frac{A}{\sqrt{2}}=A\cos\omega t$$

$$\cos\omega\mathrm{t}=\frac{1}{\sqrt{2}}$$

$$\omega\mathrm{t}=\frac{\pi}{4}$$

$$\frac{2\pi}{\mathrm{T}}\cdot\mathrm{t}=\frac{\pi}{4}\Rightarrow\mathrm{t}=\frac{\mathrm{T}}{8}$$

11. (a) $y=\frac{D\lambda}{d}$

or $\mathrm{n}_1\frac{\mathrm{D}\lambda_1}{\mathrm{d}}=\mathrm{n}_2\frac{\mathrm{D}\lambda_2}{\mathrm{d}}$

$n_1\lambda_1 = n_2\lambda_2$

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}$$

$$n_2 = n_1 \cdot \frac{\lambda_1}{\lambda_2} \Rightarrow 16 \times \frac{700}{400} = 28$$

12. (a)

$$\eta = \frac{W}{\sum Q_+} = \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3}$$

$$0.5 = \frac{1915 - 40 + 125 - Q}{1915 + 125}$$

$$1020 = 1915 - 40 + 125 - Q$$

$$Q = 2000 - 1020 = 980\text{ J}$$

13. (c) $E_n = \frac{-Rhc}{n^2}$

$$E_{n+1} = \frac{-Rhc}{(n+1)^2}$$

$$\Delta E = E_{n+1} - E_n$$

$$h\nu = RhC\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$\nu = Rc\left[\frac{(n+1)^2 - n^2}{n^2 \cdot (n+1)^2}\right]$$

$$= Rc\left[\frac{1+2n}{n^2(n+1)^2}\right]$$

if $n >> 1$

$$\nu = \frac{2n}{n^2 \times n^2} = \frac{2n}{n^4} = \frac{2}{n^3}$$

$$\nu \propto \frac{1}{n^3}$$

14. (a)

$$\Delta\ell = \ell \propto \Delta t$$

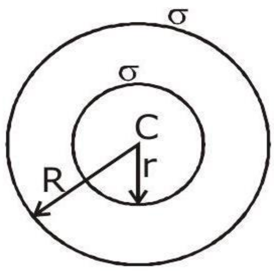
$$\alpha = \frac{\Delta\ell}{\ell\Delta t} = \frac{0.02}{100 \times 10} = 2 \times 10^{-5}$$

$$v = 3\alpha = 6 \times 10^{-5}$$

Now, $\frac{\Delta v}{v} \times 100 = \gamma.\Delta t.100 = 6 \times 10^{-5} \times 10 \times 100$

$= 6 \times 10^{-2} = 0.06$

15. (b)



$Q_1 = \sigma . 4\pi r^2$

$Q_2 = \sigma . 4\pi R^2$

$Q = 4\pi\sigma(R^2 + r^2)$

$\Rightarrow \sigma = \dfrac{Q}{4\pi(R^2 + r^2)}$

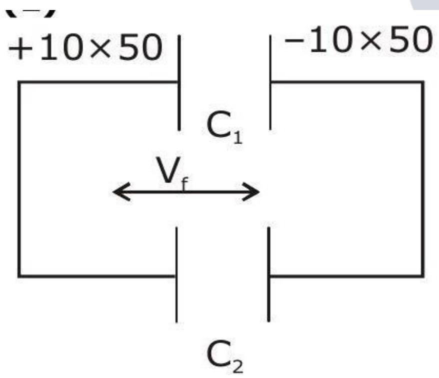
$v_c = \dfrac{KQ_1}{r} + \dfrac{KQ_2}{R}$

$= \dfrac{K\sigma 4\pi r^2}{r} + \dfrac{K\sigma 4\pi R^2}{R} = K\sigma 4\pi(r + R)$

$= 4\pi K \dfrac{Q}{4\pi(R^2 + r^2)}(r + Q) = \dfrac{KQ(r + Q)}{(R^2 + r^2)}$

$\dfrac{1}{4\pi E_0} \dfrac{(R + r)}{(R^2 + r^2)} \times Q$

16. (a)



$C_1 = 10\mu F$

$v_f = 20v$

$\dfrac{500}{C_1 + C_2} = 20$

$\dfrac{500}{10 + C_2} = 20$

$$10 + C_2 = \frac{500}{20} = 25$$

$C_2 = 15\mu F$

17. (a) $\lambda = \frac{1}{\sqrt{2}\left(\frac{N}{V}\right)\pi d^2}$

$\lambda \rightarrow$ Mean free path

N $\rightarrow$ No. 7 molecules

v = volume of container

d = diameter of molecule

$\because$ N and V are constant

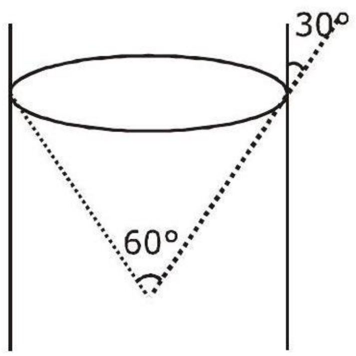
$\therefore$ Mean free path reamains unchanged.

Now, If T $\uparrow$ no. of collisions increases.

18. (c) $h = \frac{2\,T\cos\theta}{\rho g r}\{\theta = 30°\}$

$$= \frac{2 \times 0.05 \times \frac{1}{2}}{667 \times 10 \times 0.15 \times 10^{-3}}$$

= 0.087 m option (c)



19. (c) $\frac{g_0}{\left(1+\frac{h}{R}\right)^2} = g_0\left(1 - \frac{h}{R}\right)$

$$\frac{R^2}{(R+h)^2} = \frac{(R-h)}{R}$$

$R^3 = (R-h)(R+h)^2$

$R^3 = (R-h)(R^2 + 2h + h^2)$

on solving we get,

$$h = \frac{\sqrt{5}R - R}{2}$$

20. (c) $\ell = 10 \times \text{pitch}$

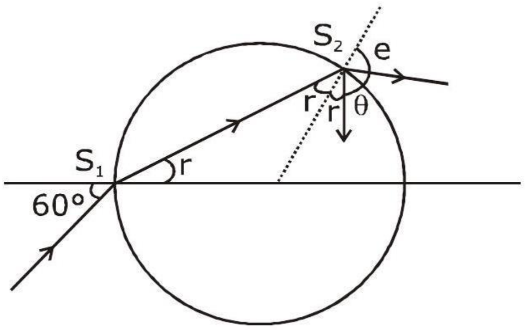
$= 10 \times v\cos 60^\circ \times \frac{2\pi m}{qB}$

$= 10 \times v \times \frac{1}{2} \times \frac{2\pi m}{qB}$

$\ell = \frac{10v\pi m}{qB} = \frac{10 \times 4 \times 10^5 \times 3.14 \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 0.3}$

$\simeq 0.44$ m

21. (c)



At S_1

$1 \times \sin 60^\circ = \sqrt{3}\sin r$

$r = 30^\circ$

$\therefore r_1 = 30^\circ$ {from geometry }

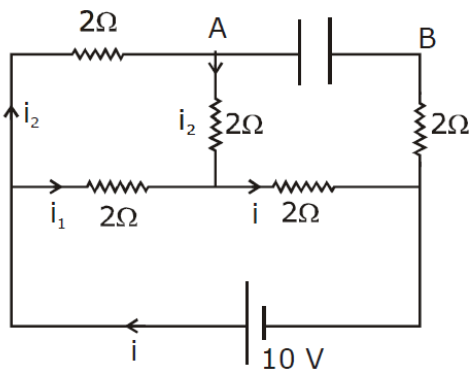
As $S_2 \sqrt{3}\sin r_1 = 1\sin e$

$e = 60^\circ$

Now, $r_1 + \theta + e = 180^\circ$

$\theta = 90^\circ$

22. (8)



$$i = \frac{10}{\frac{4}{3}+2} = \frac{10 \times 3}{10} = 3\text{Amp}$$

$i_1 = 2\text{Amp}$

$i_2 = 1\text{Amp}$

$V_{AB} = 1 \times 2 + 3 \times 2 = 8\text{ V}$

23. (23)

$$X_{cm} = \frac{\pi a^2 \times 0 - \frac{a^2}{4} \times \frac{a}{2}}{\pi a^2 - \frac{a^2}{4}}$$

$$= \frac{-a}{2(4\pi - 1)} = \frac{-a}{8\pi - 2}$$

$X = (8\pi - 2) = 8 \times 3.14 - 2$

$= 23.12$

Nearest Integer $= 23$

24. (10)

25. (35) $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{\rho A}} = \frac{1}{2\ell}\sqrt{\frac{Y\Delta\ell}{\rho\ell}}$

$$f = \frac{1}{2 \times 1}\sqrt{\frac{9 \times 10^{10} \times 4.9 \times 10^{-4}}{9000 \times 1}}$$

$= \frac{1}{2}\sqrt{49 \times 100} = 35\text{ Hz}$

26. (a)

27. (a)

$[XeF_5]^-$ 5BP + 2LP = 7VSEP $\Rightarrow sp^3\ d^3$ hybridisation

$XeO_3\ F_2$ 5BP + 0LP = 5VSEP $\Rightarrow sp^3$ d hybridisation

28. (b) $\Delta = \frac{hc}{\lambda_{\text{absorbedf(max)}}}$

$A \rightarrow NH_3$ comp (iii)

$B \rightarrow$ NCS comp (i)

$C \rightarrow F^-$comp (ii)

using spectrochemical series of ligand

$F^- < NCS^- < NH_3$ order of $\Delta + e$

crystal field spliting energy

So. NH_3 complex $\rightarrow$ A

F^-complex - C

NCS- complex $\rightarrow$ B

29. (a)

$$\text{Sucrose} \xrightarrow[\text{Cleavage (Hydrolysis)}]{\text{Glycosidic bond}} \text{Glucose} + \text{Fructose} \xrightarrow[\text{reagent}]{\text{Seliwanoff's}} \text{Red Colour}$$

30. (b) $2A + B$

Exp. (I) $6 \times 10^{-3} = K(0.1)^p(0.1)^q$

(II) $2.4 \times 10^{-2} = K(0.1)^p(0.2)^q$

(III) $1.2 \times 10^{-2} = K(0.2)^p(0.1)^q$

$$\frac{\exp(I)}{\exp(II)}\ \frac{1}{4} = \left(\frac{1}{2}\right)^q \Rightarrow q = 2$$

$$\frac{\text{Exp .II)}}{\text{Exp. (III)}}\ \frac{1}{2} = \left(\frac{1}{2}\right)^p \Rightarrow p = 1$$

exp. (I) ÷ exp (IV)

$$\frac{0.6 \times 10^{-2}}{7.2 \times 10^{-2}} = \left(\frac{0.1}{x}\right)^1 \cdot \left[\frac{0.1}{0.2}\right]^2$$

$$\frac{1}{12} = \frac{0.1}{x} - \frac{1}{4}$$

$[x] = 0.3$

exp(I) ÷ exp(V)

$$\frac{0.6 \times 10^{-2}}{2.88 \times 10^{-1}} = \left(\frac{0.1}{0.3}\right)^1 \times \left(\frac{0.1}{y}\right)^2$$

$$\frac{1}{48} = \frac{1}{3} \times \frac{10^{-2}}{y^2} \Rightarrow y^2 = 0.16$$

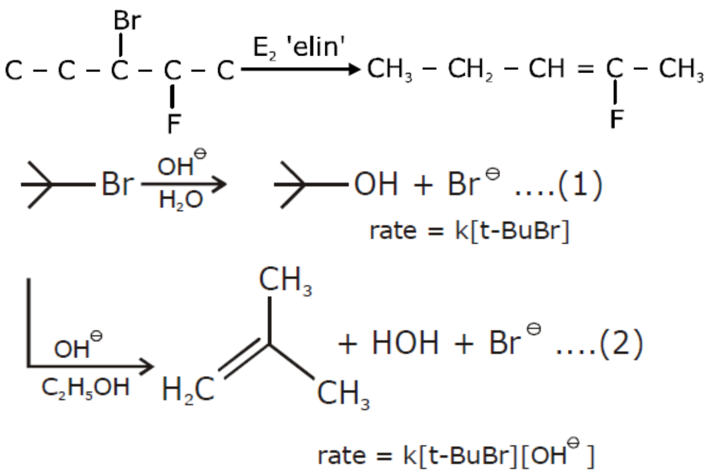
$y = 0.4$

31. (d) ion - ion $\alpha \frac{1}{r}$

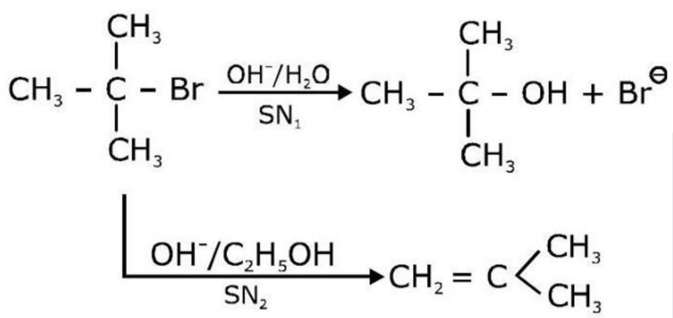
dipole - dipole $\alpha \frac{1}{r^3}$

Londong dispersion $\alpha \frac{1}{r^6}$

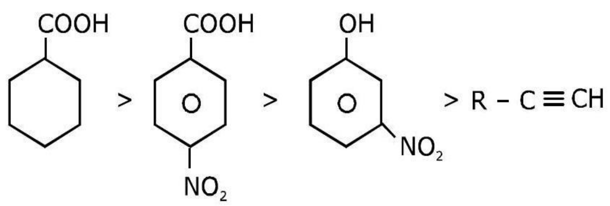
32. (a)



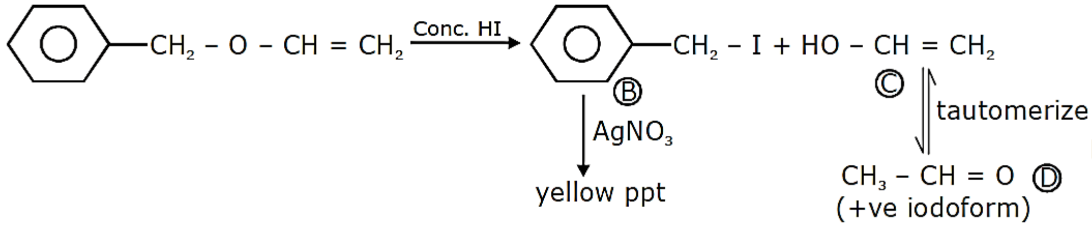
33. (a)



34. (b)
35. (b)
36. (b) Order of acidic strength



37. (a)



38. (d) $LiHCO_3$ & $Mg(HCO_3)_2$ does not exist in solid form but both forms nitrides with nitrogen gas
39. (c) n = 4

$\ell = 0$

$m = 0$

$\ell = 1$

$m = -1, 0, +1$

$\ell = 2$

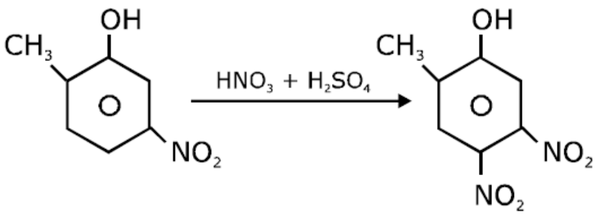
$m = -2, +2, -1, +1, 0$

$\ell = 3$

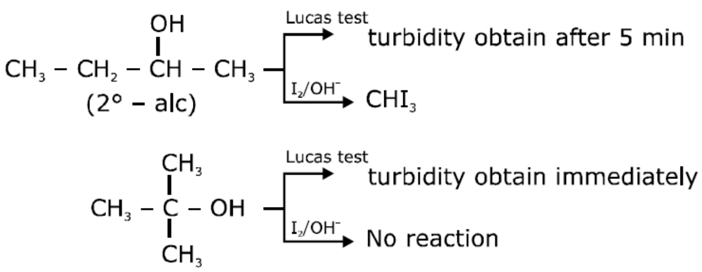
$\mathrm{m} = \pm 3, \pm 2, \pm 1, 0$

' 2 ' Subshells

40. (c)



41. (b)



42. (a)

43. (d) $[\mathrm{Ni(NH_3)_2Cl_2}]\mathrm{Ni}^{2+}$ is sp^3 hybridised & such tetrahedral complex does not show either of geometrical or optical isomerism

$[\mathrm{Ni(en)_3}]^{2+}$ shows only optical isomers while other three shows geometrical isomerism

44. (c)

45. (b) $\mathrm{SF_4}$ is Sp^3 d hybridised in which hybrid orbitals have TBP arrangement but its shape is sea-saw

46. Mass ratio of C: H is $4:1 \Rightarrow 12:3$

$\& C:O$ is $3:4 \Rightarrow 12:16$

So,

	mass	mole	moleratio
C	12	1	1
H	3	3	3
O	16	1	1

Empirical formula $\Rightarrow \mathrm{CH_3O}$ as compound is satured a cyclic so, molecular formula is $\mathrm{C_2H_6O_2}$.

$$\mathrm{C_2H_6O_2} + \underset{5\ \text{mole}}{\frac{5}{2}}\ \mathrm{O_{2(g)}} \longrightarrow 2\mathrm{CO_{2(g)}} + 3\mathrm{H_2O_{(g)}}$$

So, required moles of O_2 is $\Rightarrow 5$

47. (144)

$$2Cu^{+} \rightleftharpoons Cu(s) + Cu^{+2}$$
$$E^{O} = 0.52 - 0.16$$
$$= 0.36$$
$$EO = \frac{RT}{nF}\ln(k_{eq})$$
$$\ln(k_{eq}) = \frac{0.36}{0.025} \times \frac{1}{1}$$
$$= \frac{360}{25} = 14.4$$
$$= 144 \times 10^{-1}$$

48. (222)

$\phi = 4.41 \times 10^{-19}$ J

$\lambda = 300$ nm

$$KE_{max} = \frac{hc}{\lambda} - \phi$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{300 \times 10^{-9}} - 4.41 \times 10^{-19}$$

$$= 6.63 \times 10^{-19} - 4.41 \times 10^{-19}$$

$$= 222 \times 10^{-21}$$

49. (19)

$K_2Cr_2O_7 \qquad KMnO_4 \qquad K_2\left[\overset{+6}{Fe}OO_4\right]$

50. $\Delta H_c^0[C_2H_5OH] = -327$kcal

$$C_2H_5OH(I) + 3O_2(g) \longrightarrow 2CO_2(g) + 3(H_2O)(l)$$

$$\Delta E_c^0 = \Delta H_c^0 - \Delta ngRT$$

$$= -327 \times 1000 - (-1) \times 2 \times 300$$

$$= -327000 + 600$$

$$= -326400$$

51. (b) $f(a) = 2; f(x+y) = f(x) + f(y)$

$x = y = 1 \Rightarrow f(b) = 2 + 2 = 4$

$x = 2, y = 1 \Rightarrow f(c) = 4 + 2 = 6$

$g(n) = f(a) + f(b) + \cdots \ldots \ldots + f(n-1)$

$= 2 + 4 + 6 + \cdots \ldots . . +2(n-1)$

$= 2\sum(n-1)$

$= 2\frac{(n-1)\cdot n}{2}$

$= n^2 - n$

Given $g(n) = 20 \Rightarrow n^2 - n = 20$

$n^2 - n - 20 = 0$

$n = 5$

52. (b)

$\sum_{k=1}^{11} a_k = 0 \Rightarrow 11a + 55d = 0$

$a + 5d = 0$

Now $a_1 + a_3 + \cdots . + a_{23} = ka_1$

$12a + d(2 + 4 + 6 + \cdots .. + 22) = ka$

$12a + 2\, d. 66 = ka$

$12(a + 11d) = ka$

$12\left(a + 11\left(-\frac{a}{5}\right)\right) = ka$

$12\left(1 - \frac{11}{5}\right) = k$

$k = -\frac{72}{5}$

53. (c) $P(E_2^c \cap E_3^c / E_1) = \frac{P(E_2^c \cap E_3^c \cap E_1)}{P(E_1)}$

$= \frac{P(E_1) - P(E_1 \cap E_2) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)}{P(E_1)}$

$= \frac{P(E_1) - P(E_1)\cdot P(E_2) - P(E_1)\cdot P(E_3) + 0}{P(E_1)} = 1 - P(E_2) - P(E_3)$

$= P(E_3^c) - P(E_2)$

54. (b) $\cos^4\theta + \sin^4\theta + \lambda = 0$

$\lambda = -\left\{1 - \frac{1}{2}\sin^2 2\theta\right\}$

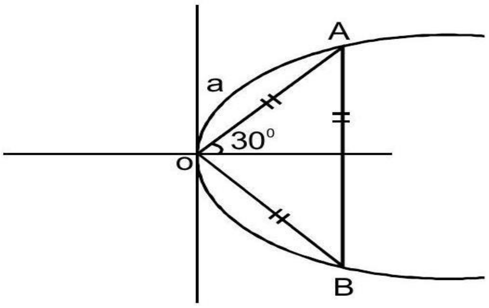
$2(\lambda + 1) = \sin^2 2\theta$

$0 \leq 2(\lambda + 1) \leq 1$

$0 \le \lambda + 1 \le \frac{1}{2}$

$-1 \le \lambda \le -\frac{1}{2}$

55. (b)



A : $(a\cos 30°, a\sin 30°)$

lies on parabola

$\frac{a^2}{4} = 8 \cdot \frac{a \cdot \sqrt{3}}{2}$

$a = 16\sqrt{3}$

Area of equilateral $\Delta = \frac{\sqrt{3}}{4}a^2$

$\Delta = \frac{\sqrt{3}}{4} \cdot 16 \cdot 16 \cdot 3$

$\Delta = 192\sqrt{3}$

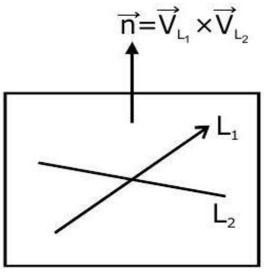
56. (b) $(3 + 2i\sqrt{54})^{1/2} - (3 - 2i\sqrt{54})^{1/2}$

$= \left(9 + 6i^2 + 2.3i\sqrt{6}\right)^{1/2} - \left(9 + 6i^2 - 2.3i\sqrt{6}\right)^{1/2}$

$= \left((3 + \sqrt{6}i)^2\right)^{1/2} - \left((3 - \sqrt{6}i)^2\right)^{1/2}$

$= \pm(3 + \sqrt{6}i) \mp (3 - \sqrt{6}i) = -2\sqrt{6}i$

57. (c)



$\vec{n}_p = (-4,5,7)$

Equation of plane:

P: $-4(x-3)+5(y-1)+7(z-1)=0$

P: $-4x+5y+7z+12-5-7=0$

$P: 4x-5y-7z=0$

Pass $(\alpha,-3,5)$

$4\alpha+15-35=0$

$4\alpha=20$

$\alpha=5$

58. (c) $X=\begin{bmatrix}x\\y\\z\end{bmatrix}\&x^2+y^2+z^2=1$

$Px=0$

$$\begin{bmatrix}1&2&1\\-2&3&-4\\1&9&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$x+2y+z=0.$

$-2x+3y-4z=0$

$x+9y-z=0.$

from (a) & (c)

$\Rightarrow 2x+11y=0$

from (a) & (b)

$\Rightarrow 2x+11y=0$
from (b) & (c)

$-6x-33y=0$

$\Rightarrow 2x+11y=0$

put in (a)

$-7y+2z=0$

Now $\left(\frac{11y}{2}\right)^2+y^2+\left(\frac{7y}{2}\right)^2=1$

$y^2(121+1+49)=4$

$y^2(171)=4$

$y=\pm\frac{2}{\sqrt{171}}\Rightarrow x=\pm\frac{7}{\sqrt{171}}\Rightarrow z=\mp\frac{11}{\sqrt{171}}\Rightarrow$ Only two pair possible

59. (c) at $x=0 \Rightarrow y=1+\cos^2(0)=2$

$p:(0,2)$

Now $y^1=(1+x)^{2y}\left\{\frac{2y}{1+x}+\ln(1+x)\cdot 2y\right\}-\sin 2(\sin^{-1}x)\cdot\frac{1}{\sqrt{1-x^2}}$

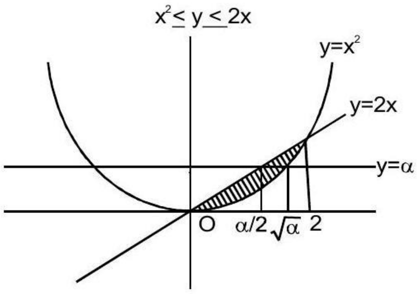
$y|_{(0,2)}=4-0$

$\mathrm{N_0}: \mathrm{y}-2=-\frac{1}{4}(\mathrm{x}-0)$

$N_0: 4y-8=-x$

$N_0: x+4y=8$

60. (b)



$A=\text{Area}=\int_0^2(2x-x^2)dx=4-\frac{8}{3}=\frac{4}{3}$

Now $\int_0^{\alpha/2}(2x-x^2)dx+\int_{\alpha/2}^{\sqrt{\alpha}}(\alpha-x^2)dx=\frac{1}{2}A$

$$\frac{\alpha^2}{4}-\frac{\alpha^3}{24}+\alpha(\sqrt{\alpha}-\alpha/2)-\left(\frac{\alpha\sqrt{\alpha}}{3}-\frac{\alpha^3}{24}\right)=\frac{4}{6}$$

$$\frac{\alpha^2}{4}+\alpha\sqrt{\alpha}-\frac{\alpha^2}{2}-\frac{\alpha\sqrt{\alpha}}{3}=\frac{4}{6}$$

$-3\alpha^2+8\alpha\sqrt{\alpha}=8$

$3\alpha^2-8\alpha\sqrt{\alpha}+8=0$

61. (d)

$$f(x)=\frac{1}{x}\ln(1+x)$$

$$f'=\frac{x-\frac{1}{1+x}-\text{In}(1+x)}{x^2}$$

$$f'=\frac{1-\frac{1}{1+x}-\ln(1+x)}{x^2}$$

$f'<0\forall x\in(-1,\infty)$

62. (b)

$\sim p$	$p \vee q$	$\sim p \wedge (p \vee q)$	$\sim p \wedge (p \vee q) \to q$
F	T	F	T
F	T	F	T
T	T	T	T
T	F	F	T

63. (c) Let $f(x) = a(x-3)(x-\alpha)$

$f(-1) + f(b) = 0$

$a[(-1-3)(-1-\alpha) + (2-3)(2-\alpha)] = 0$

$a[4 + 4\alpha - 2 + \alpha] = 0$

$5\alpha + 2 = 0$

$\alpha = -\frac{2}{5}$

64. (b)

$S = \{x + ka\} + \{x^2 + (k+2)a\} + \{x^3 + (k+4)a\}$ up to 9 term

$S = (x + x^2 + \cdots .. + x^9) + a\{k + (k+2) + (k+4) + \cdots$ ... up to 9 term $)\}$

$S = \frac{x(1-x^9)}{1-x} + a\{9k + 2.36\}$

$S = \frac{x^{10}-x}{x-1} + 9ak + 72a$

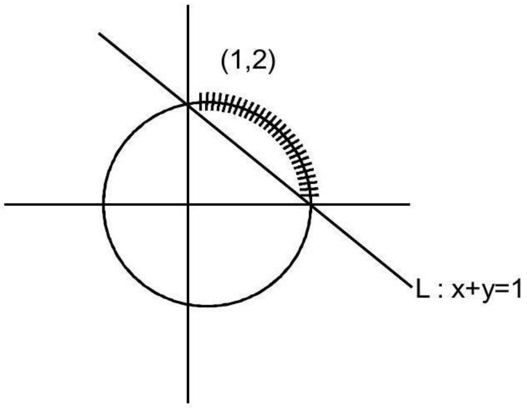
$S = \frac{x^{10} - x + 45a(x-1)}{x-1} = \frac{x^{10} - x + (9k+72)a(x-1)}{x-1}$

$= 45 = 9k + 72$

$9k = -27$

$k = -3$

65. (b)

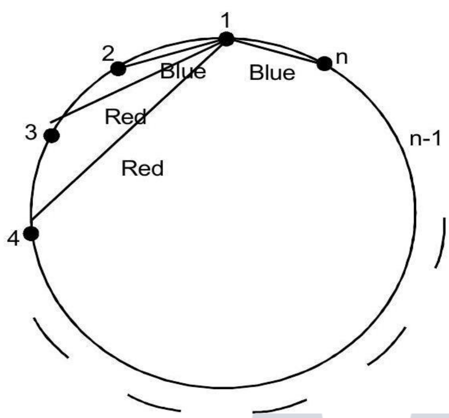


$(\sin\theta, \cos\theta)$ lie on $x^2 + y^2 = 1$

Shaded points satisfy

$\Rightarrow \theta \in (0, \pi/2)$

66. (a)



Red line = 99 blue line

$^nC_2 - n = 99n$

$$\frac{n(n-1)}{2} = 100n$$

$n - 1 = 200$

$n = 201$

67. (c) $2\frac{dy}{dx} = 2\frac{y}{x} + \left(\frac{y}{x}\right)^2 \rightarrow$ HDE

$\because y = vx$

$$2\left(v + x\frac{dv}{dx}\right) = 2v + v^2$$

$$2\frac{dv}{v^2} = \frac{dx}{x}$$

$$-\frac{2}{v} = \ln x + c$$

$$-\frac{2x}{y} = \ln x + c$$

$\downarrow (1,2)$

$c = -1$

$$c: \ln x + \frac{2x}{y} = 1$$

For $f(1/2) \Rightarrow \ln\left(\frac{1}{2}\right) + \frac{2}{2y} = 1$

$$y = \frac{1}{1 + \ln 2}$$

68. (a) $\mathrm{H}: \mathrm{x}^2 - \mathrm{y}^2 \sec^2\theta = 10$

$E: x^2 \sec^2\theta + y^2 = 5$

$$\sqrt{1 + \frac{10\cos^2\theta}{10}} = \sqrt{5}\sqrt{1 - \frac{5\cos^2\theta}{5}}$$

$1 + \cos^2\theta = 5 - 5\cos^2\theta$

$6\cos^2\theta = 4$

$$\cos\theta = \pm\sqrt{\frac{2}{3}}$$

I(LR) of ellipse $= \frac{2.5\cos^2\theta}{\sqrt{5}}$

$$= 2\sqrt{5} \cdot \frac{2}{3} = \frac{4\sqrt{5}}{3}$$

69. (b) $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4} + x\right)\right)^{1/x} (1^\infty) = \mathrm{e}^L$

$$L = \lim_{x\to 0}\frac{\tan\left(\frac{\pi}{4} + x\right) - 1}{x}$$

$$L = \lim_{x\to 0}\frac{1 + \tan x}{1 - \tan x} - 1$$

$$L = \lim_{x\to 0} 2\left(\frac{\tan x}{x}\right) \cdot \left(\frac{1}{1 - \tan x}\right)$$

$\mathrm{L} = +2$

e^2

70. (d)

$a^3 + b^3 + c^3 = 2$

$A^{\top}A = I$

$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= a^2 + b^2 + c^2 = 1$

$\& ab + bc + ca = 0$

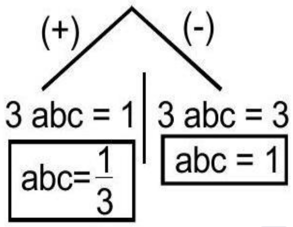
Now $(a + b + c)^2 = \sum a^2 + 2\sum ab$

$(\Sigma \mathbf{a})^2 = 1 + 0 \Rightarrow (\sum a)^2 = 1 \Rightarrow \Sigma \mathbf{a} = \pm 1$

Now $\sum a^3 - 3abc = (\sum a)(\sum a^2 - \sum ab)$

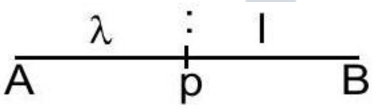
$2 - 3\text{abc} = \pm 1(1 - 0)$

$2 - 3abc = \pm 1$



71. (0.8)

$\overrightarrow{\text{OA}} = \langle 1,1,1 \rangle, \overrightarrow{\text{OB}} = \langle 2,1,3 \rangle$



$$\overrightarrow{\text{OP}} = \left(\frac{2\lambda + 1}{\lambda + 1}, 1, \frac{3\lambda + 1}{\lambda + 1}\right)$$

$$\overrightarrow{\text{OB}} \cdot \overrightarrow{\text{OP}} = \frac{2(2\lambda + 1)}{\lambda + 1} + 1 + \frac{3(3\lambda + 1)}{\lambda + 1}$$

$$= \frac{14\lambda + 6}{\lambda + 1}$$

$$|\overrightarrow{OA}\times\overrightarrow{OP}|^2 = |\overrightarrow{OA}|^2|\overrightarrow{OP}|^2 - (\overrightarrow{OA}\cdot\overrightarrow{OP})^2$$

$$3.\left(\frac{(2\lambda+1)^2+(\lambda+1)^2+(3\lambda+1)^2}{(\lambda+1)^2}\right)-\left(\frac{2\lambda+1+\lambda+1+3\lambda+1}{\lambda+1}\right)^2$$

$$=\frac{1}{(\lambda+1)^2}\{3(14\lambda^2+12\lambda+3)-(6\lambda+3)^2\}$$

$$=\frac{1}{(\lambda+1)^2}\{6\lambda^2\}$$

Now $\frac{14\lambda+6}{\lambda+1}-3\left(\frac{6\lambda^2}{(\lambda+1)^2}\right)=6$

$$(14\lambda+6)(\lambda+1)-18\lambda^2=6(\lambda+1)^2$$

$$-4\lambda^2+20\lambda+6=6\lambda^2+12\lambda+6$$

$$10\lambda^2-8\lambda=0$$

$$\lambda(10\lambda-8)=0$$

$$\because \lambda>0$$

$$\lambda=.8$$

72. (1)

$$\int_1^2 |2x-[3x]|dx$$

$$3x=t$$

$$=\frac{1}{3}\int_3^6\left|\frac{2t}{3}-[t]\right|dt$$

$$=\frac{1}{9}\left[\int_3^6 |2t-3[t]|\right]dt$$

$$=\frac{1}{9}\left[\int_3^4 |2t-9|+\int_4^5 |2t-12|+\int_5^6 |2t-15|\right]dt$$

$$=\frac{1}{9}\left[\int_3^4 (9-2t)+\int_4^5 (12-2t)+\int_5^6 (15-2t)\right]dt$$

$$=\frac{1}{9}[9.1+12.1+15.1-[4^2-3^2]-[5^2-4^2]-[6^2-5^2]]$$

$$=\frac{1}{9}[36-[4^2-3^2+5^2-4^2+6^2-5^2]]$$

$$=\frac{1}{9}[36-36+9]=1$$

Q. 23 If $y=\sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx-\frac{4}{5}\sin kx\right\}$, then $\frac{dy}{dx}$ at $x=0$ is

Sol. 91

$y=\sum_{k=1}^{6} k\cos^{-1}\{\cos(kx+\theta)\}$

where $\tan\theta=\frac{4}{3}$

$y=\cos^{-1}(\cos(x+\theta)+2\cos^{-1}(\cos(2x+\theta))\ldots\ldots+6\cos^{-1}(\cos(6x+\theta))$

$$\left.\frac{dy}{dx}\right|_{x=0}=\frac{\sin(\theta)}{\sqrt{1-\cos^2\theta}}+$$

$= 1.1 + 2.2 + 3.3 + \cdots .. + 6.6$

$= \sum 6^2 = \dfrac{6.7 \cdot 13}{6} = 91$

73. (3) $\mathrm{Var}(x) = \frac{\sum bi^2}{11} - \left(\frac{\sum bi}{11}\right)^2$

$$90 = \frac{a^2 + (a+d)^2 + (a+2d)^2 + \cdots . + (a+10d)^2}{11}$$

$$\left(\frac{a + a + d + a + 2d + \cdots + (a+10d)}{11}\right)^2$$

$10890 = 11\{11a^2 + 385d^2 + 110ad\} - \{11a + 55d\}^2$

$10890 = 1210d^2$

d = 3

74. (118)