

1. (b) $mv\ell = I\omega$

$$\omega = \frac{mv\ell}{I}$$

$$\frac{1}{2}I\omega^2 = (m + M)g\ell_{\text{com}}(1 - \cos\theta)$$

$$\frac{1}{2} \frac{(mv\ell)^2}{I} = (m + M)g\ell_{\text{com}}(1 - \cos\theta)$$

$$I = \left(\frac{M\ell^2}{3} + m\ell^2 \right)$$

$$I = \left(\frac{2}{3} + 1 \right) = \frac{5}{3}$$

$$\frac{36 \times 3}{2 \times 5} = \frac{3 \times 10 \times 2}{3} (1 - \cos\theta)$$

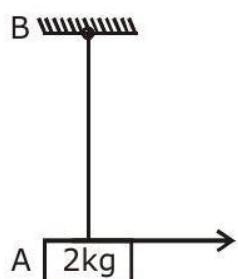
$$\frac{27}{50} = (1 - \cos\theta)$$

$$\cos\theta = 1 - \frac{27}{50}$$

$$\cos\theta = \frac{23}{50}$$

$$\theta = 63^\circ$$

2. (a)



$$\ell = 12\text{m}$$

$$m = 6\text{kg}$$

$$\mu = \frac{6}{12} = \frac{1}{2} \text{ kg/m}$$

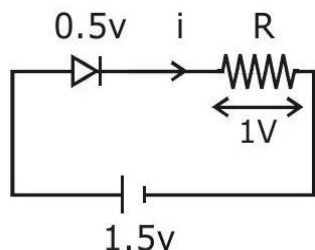
$$A \rightarrow V = \sqrt{\frac{T}{\mu}} = f\lambda$$

$$B \rightarrow \sqrt{\frac{T'}{\mu}} = f\lambda'$$

$$\sqrt{\frac{T}{T'}} = \frac{\lambda}{\lambda'}$$

$$\sqrt{\frac{20}{80}} = \frac{6}{\lambda'} = \lambda' = 12$$

3. (d)



$$V = iR$$

$$R = \frac{V}{i} = \frac{1}{10 \text{ mA}}$$

$$= \frac{1000}{10} = 100\Omega$$

4. (b) Least count = $\frac{0.1}{50} = \frac{1}{500} = 0.2 \times 10^{-2} = 0.002$

when we multiply by division no.'s

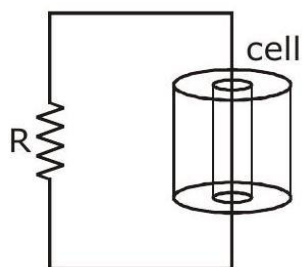
it must be even because L.C. is 0.002

5. (d) $dR = \rho \frac{dr}{2\pi r \ell}$

$$\int dR = \frac{\rho}{2\pi \ell} \int_a^b \frac{1}{r} dr$$

$$R = \frac{\rho}{2\pi \ell} [\ln(r)]_a^b$$

$$R = \frac{\rho}{2\pi \ell} \ln\left(\frac{b}{a}\right)$$



$$r = R$$

For Max heat transfer

6. (b) $U = \frac{f}{2} nRT$

$$U = \frac{6}{2} nRT$$

$$U = 3nRT \quad n = 1$$

$$U = 3RT$$

7. (b)

$$\phi = BA \cos \omega t$$

$$\epsilon = \frac{d\phi}{dt} = BA \omega \sin \omega t$$

$$\epsilon = B \pi a b \omega \sin \omega t$$

$$\epsilon_0 = B \pi a b \omega$$

$$P = \frac{\epsilon^2}{R} = \frac{B^2 \pi^2 a^2 b^2 \omega^2}{R} (\sin^2 \omega t)$$

$$P_{av} = \frac{B^2 \pi^2 a^2 b^2 \omega^2}{2R}$$

8. (d) irreversible adiabatic → Because Energy can not be restored.

9. (b) $KE_{\max} = \frac{hc}{\lambda_1} - \phi$

$$3KE_{\max} = \frac{hc}{\lambda_2} - \phi$$

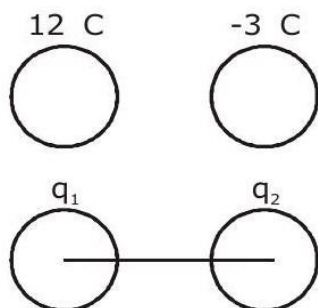
$$3 \left(\frac{hc}{\lambda_1} - \phi \right) = \frac{hc}{\lambda_2} - \phi$$

$$\frac{3hc}{\lambda_1} - \frac{hc}{\lambda_2} = 2\phi$$

$$\frac{3 \times 1240}{500} - \frac{1240}{200} = 2\phi$$

$$\phi = 0.6$$

10. (a)



$$q_1 + q_2 = 9\mu C$$

$$\frac{Kq_1}{\frac{2}{3}R} = \frac{Kq_2}{\frac{1}{3}R}$$

$$q_1 = 2q_2$$

$$3q_2 = 9\mu c$$

$$q_2 = 3\mu c$$

$$q_1 = 6\mu c$$

$$11. (a) N = N_0 e^{-\lambda t}$$

$$\left(\frac{N}{N_0}\right) = e^{-\lambda t}$$

$$\frac{9}{16} = e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t/2}$$

$$\frac{N}{N_0} = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$$

$$12. (d) M = d \cdot V$$

$$M = d\pi R^2 L$$

$$I = M \left(\frac{R^2}{4} + \frac{L^2}{12} \right)$$

$$I = M \left(\frac{M}{4d\pi L} + \frac{L^2}{12} \right)$$

$$I = \left(\frac{M^2}{4d\pi L} + \frac{ML^2}{12} \right)$$

$$\frac{dI}{dL} = \frac{M^2}{4d\pi} \left(\frac{-1}{L^2} \right) + \frac{2LM}{12} = 0$$

$$\frac{M^2}{4d\pi L^2} = \frac{2LM}{12}$$

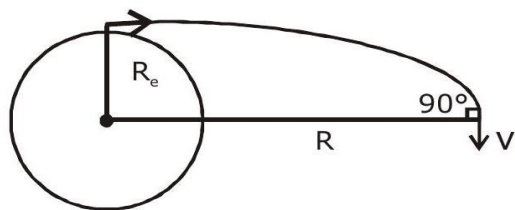
$$\frac{d\pi R^2 L}{4d\pi L^2} = \frac{L}{6}$$

$$\frac{R^2}{L^2} = \frac{2}{3}$$

$$\frac{R}{L} = \sqrt{\frac{2}{3}} \Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

$$13. (b)$$

$$V_0 = \sqrt{\frac{GM}{R_e}} \times \sqrt{\frac{3}{2}}$$



$$mV_0R_e = mVR$$

$$\sqrt{\frac{3}{2}} \sqrt{\frac{GM}{R_e}} R_e = VR$$

$$-\frac{GMm}{R_e} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$-\frac{GMm}{R_e} + \frac{1}{2}m \frac{3GM}{2R_e} = -\frac{GMm}{R} + \frac{1}{2}m \frac{3GM R_e^2}{2R_e R^2}$$

$$-\frac{1}{R_e} + \frac{3}{4R_e} = -\frac{1}{R} + \frac{3R_e}{4R^2}$$

$$-\frac{1}{4R_e} = -\frac{1}{R} + \frac{3R_e}{4R^2}$$

By further calculating

$$R = 3R_e$$

14. (d) $P_{in} = P_0 + \frac{4T}{R_1}$

$$1.01 = 1 + \frac{4T}{R_1}$$

$$0.01 = \frac{4T}{R_1}$$

$$0.02 = \frac{4T}{R_2}$$

$$\frac{1}{2} = \frac{R_2}{R_1}$$

$$\frac{V_1}{V_2} = \frac{8}{1}$$

15. (c) $B = \frac{\lambda D}{d}$

$$\theta = \frac{\beta}{D} = \left(\frac{\lambda}{d} \right)$$

$$\theta = \frac{500 \times 10^{-9}}{(0.05 \times 10^{-3})}$$

$$\theta = \frac{5 \times 10^{-2}}{5} = \frac{5}{100 \times 5}$$

$$\theta^\circ = \frac{5}{100} \times \frac{180}{5 \times \pi}$$

$$\theta^\circ = 0.57^\circ$$

16. (d)

$$f = 750\text{Hz}, V_{\text{rms}} = 20\text{V}, R = 100\Omega, L = 0.1803\text{H}$$

$$C = 10\mu\text{f}$$

$$S = 2 \text{ J/}^\circ\text{C}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$|Z| = \sqrt{(100)^2 + \left(2 \times 3.14 \times 750 \times 0.1803 - \frac{1}{2 \times 3.14 \times 750 \times 10^{-5}}\right)^2}$$

$$|Z| = 834\Omega$$

In AC

$$P = V_{\text{rms}} i_{\text{rms}} \cos \phi$$

$$P = \left(V_{\text{rms}} \cdot \frac{V_{\text{rms}}}{|Z|} \cdot \frac{R}{|Z|}\right)$$

$$P = \left(\frac{V_{\text{rms}}}{|Z|}\right)^2 R$$

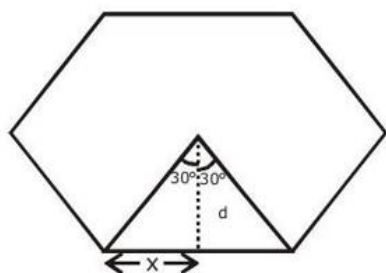
$$P = \left(\frac{20}{834}\right)^2 \times 100 = 0.0575 \text{ J/S}$$

$$P \times t = S\Delta\theta$$

$$t = \frac{2(10)}{0.0575}$$

$$t = 348\text{sec}$$

17. (c)



$$\tan 30 = \frac{x}{d}$$

$$d = \frac{x}{\tan 30}$$

$$d = \frac{5 \times 10^{-2}}{\frac{1}{\sqrt{3}}}$$

$$d = 5\sqrt{3} \times 10^{-2}$$

$$B = \frac{6 \times \mu_0 I N}{4\pi d} (\sin \theta_1 + \sin \theta_2)$$

$$B = \frac{\mu_0 I}{4\pi} \times \frac{50 \times 6}{5\sqrt{3} \times 10^{-2}} (\sin 30 + \sin 30)$$

$$B = \frac{10 \times 6 \times 100}{\sqrt{3} \times 4} \left(\frac{\mu_0 I}{\pi} \right)$$

$$B = 500\sqrt{3}$$

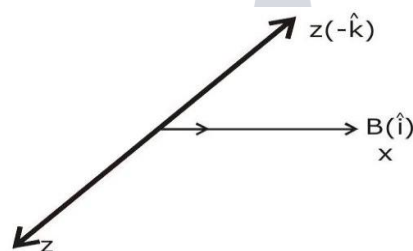
18. (a)

$$E = BC$$

$$E = 3 \times 10^{-8} \times 3 \times 10^8$$

$$E = 9$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{r}$$



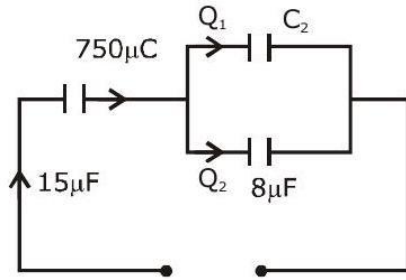
$$E = 9 \sin[200\pi(y + ct)(-\hat{k})]$$

19. (c) $\vec{F} = q(\vec{V} \times \vec{B})$

$$\vec{F} = 10^{-6}(2\hat{i} + 3\hat{j} + 4\hat{k}) \times (5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3}$$

$$\vec{F} = (-30\hat{i} + 32\hat{j} - 9\hat{k}) \times 10^{-9}$$

20. (c)



$$Q_2 = CV = 8 \mu f 20 = 160 \mu C$$

$$Q_1 = 750 - 160 = 590 \mu C$$

21. (9) $I_1 \omega_1 = \omega_2 I_2$

$$\left(\frac{MR^2}{2} + mR^2 \right) \omega_1 = \omega_2 \frac{MR^2}{2}$$

$$\left(1 + \frac{2mR^2}{MR^2} \right) \omega_1 = \omega_2$$

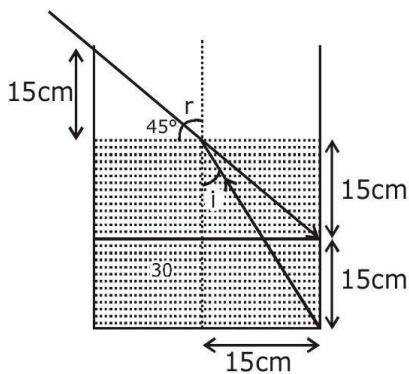
$$\left(1 + \frac{2 \times 80}{200} \right) \omega_1 = \omega_2$$

$$\omega_2 = \omega_1 1.8$$

$$2\pi f_2 = 2\pi f_1 \times 1.8$$

$$f_2 = 5 \times 1.8 = 9$$

22. (1.58)



$$\mu \sin i = 1 \sin 45$$

$$\mu \frac{15}{\sqrt{1125}} = \frac{1}{\sqrt{2}}$$

$$\mu = \frac{\sqrt{1125}}{15\sqrt{2}}$$

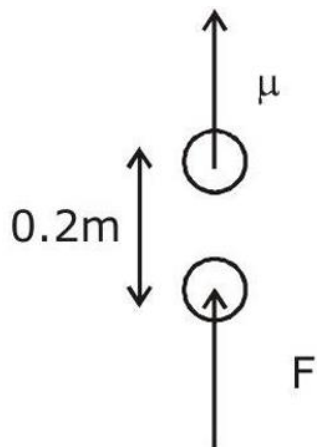
$$\mu = \sqrt{\frac{1125}{450}}$$

$$\mu = 1.58$$

$$\mu = \frac{N}{100} = 1.58$$

23. (150)

$$m = 0.15 \text{ kg}$$



$$\mu = \sqrt{2gh}$$

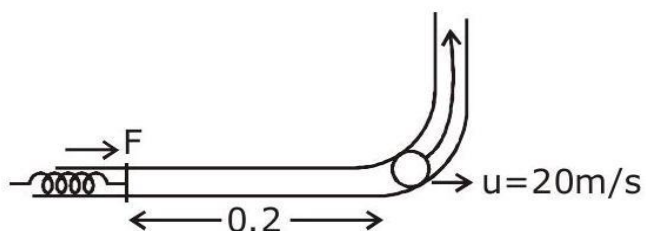
$$\mu = \sqrt{2 \times 10 \times 20}$$

$$\mu = 20 \text{ m/sec}$$

$$Fd = \frac{1}{2}mv^2$$

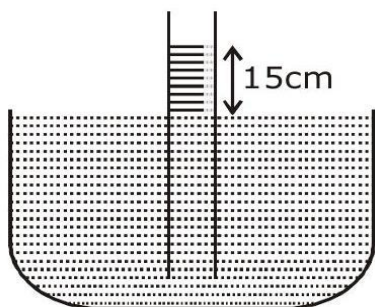
$$F \times 0.2 = \frac{1}{2} \times \frac{0.15}{100} \times 400$$

$$F = \frac{15 \times 200 \times 10}{100 \times 0.2} = 150$$



24. (101)

$$0.015$$



$$h = \frac{2T}{\rho g r}$$

$$T = \frac{h\rho g r}{2}$$

$$T = \frac{15 \times 10^{-2} \times 900 \times 10 \times 0.015 \times 10^{-2}}{2 \times 10^3}$$

$$T = 101 \text{ milliN/m}$$

25. (20)

$$\Delta V = v\gamma\Delta T$$

$$v_1\gamma_1 = v_2\gamma_2$$

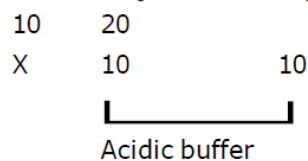
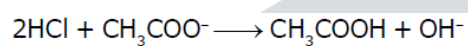
$$500 \text{cc} \times 6 \times 10^{-6} = V_m \times 1.5 \times 10^{-4}$$

$$V_m = \frac{500 \times 6 \times 10^{-6}}{1.5 \times 10^{-4}} = \frac{30}{1.5}$$

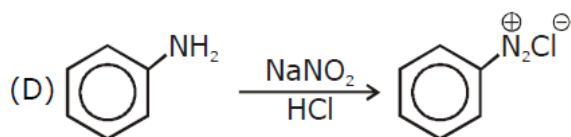
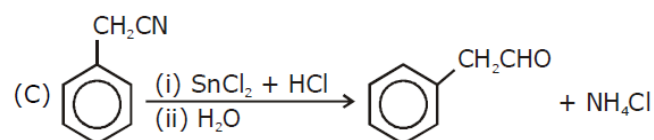
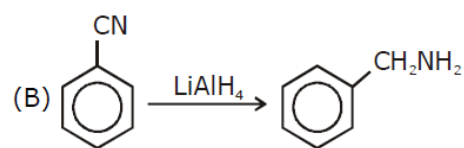
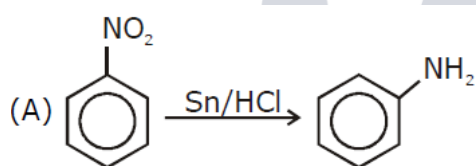
$$V_m = 20 \text{cc}$$

26. (c)

27. (a)



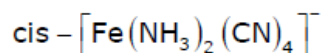
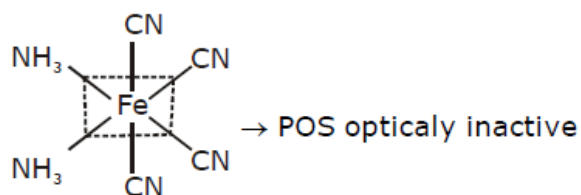
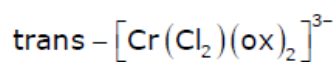
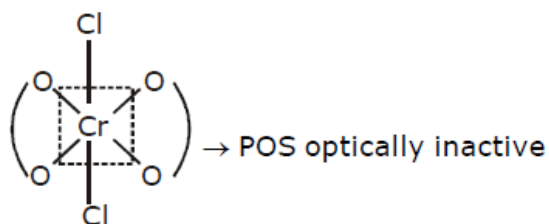
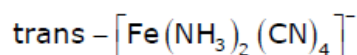
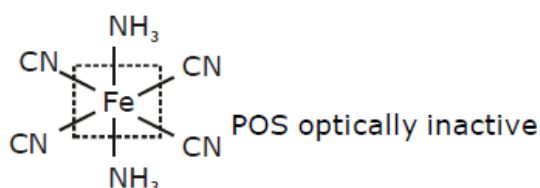
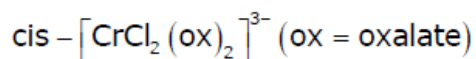
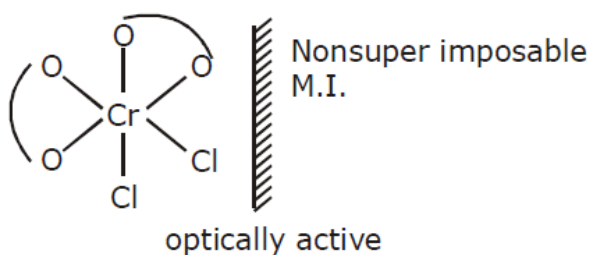
28. (c)



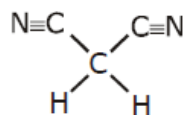
Diazo compound and inorganic nitrogen can't be estimated by kjeldal method.

29. (d) Less than 300 K (factual)

30. (a)

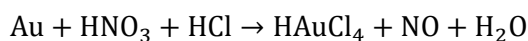


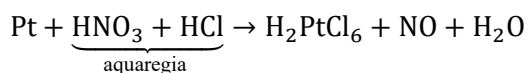
31. (c)



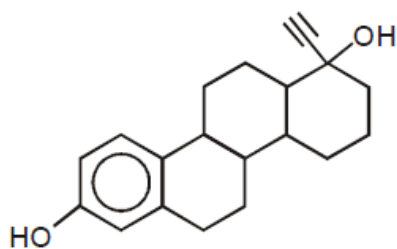
has most acidic hydrogen among given compound, this is due to strong -M effect of -CN group which stabilize -ve charge significantly.

32. (d)



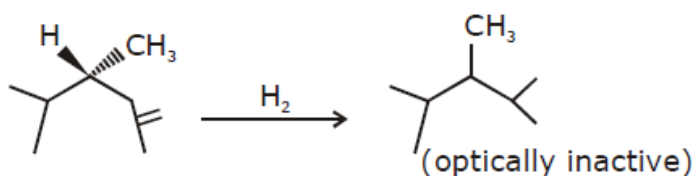


33. (a) Novestrol



It can react with Br_2/water due to presence of unsaturation, with ZnCl_2/HCl due to $-\text{OH}$ group and with FeCl_3 due to phenol.

34. (c)



35. (a) B.O. $\text{NO}^- = 2$

B.O. $\text{NO} + 3$

B.O. $2^+ = 2.5$

B.O. $= 2.5$

B. $O \propto \frac{1}{\text{B.L}}$

36. (b) conceptual Glycerol is separated in soap industries by distillation under reduced pressure

37. (d) Refer environmental chemistry

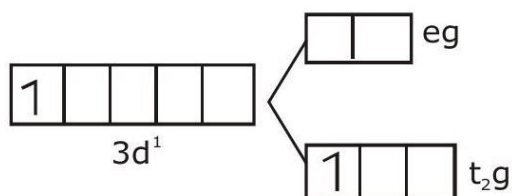
It emits CO_2 that combines with moisture of atmosphere and forms H_2CO_3 (carbonic acid)

38. (a) $p = K_H X$ mol fraction of gas in liquid.

On increasing temp, K'_H increases

Hence solubility $\downarrow$

39. (d) $[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$ $\text{Ti}^{3+} 3d^1$ in octahedral field of ligand



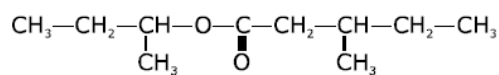
$$\text{CFSE} = -0.4\Delta_0$$

$$\text{CFSE} = \frac{-0.4 \times 20300}{83.7}$$

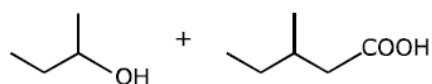
$$= 97 \text{ kJ mol}^{-1}$$

40. (a)

41. (b)

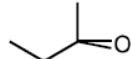
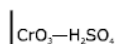


(A)



(C)

(B)



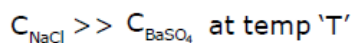
($\neq$ B)

42. (b)

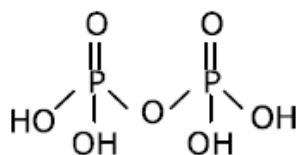
43. (d) Diameter of dispersed particles should not be much smaller than wavelength of light used. Refer topic surface chemistry

44. (d)

Ionic



45. (b)



P – OH bonds = 4

P = O bonds = 2

P – O – P linkage = 1

46. (47%)

$$x_{\text{Glucose}} = 0.1$$

$$\begin{aligned}\text{mass \% of glucose} &= \frac{0.1 \times 180}{0.1 \times 180 + 0.9 \times 18} \times 100 \\ &= \frac{1800}{18 + 16.2} \\ &= \frac{1800}{34.2} \% \\ &= 52.63\% \\ &= 53\%\end{aligned}$$

$$\therefore \text{mass \% of H}_2\text{O} = 47\%$$

47. (100)

$$\text{Vol. strength} = \frac{8.9}{2} \times \frac{0.821 \times 273}{1}$$

$$= 99.73$$

$$= 100$$

48. (143) Density = $\frac{Z \times \text{GMM}}{N_A \times a^3}$

$$2.7 \times 10^3 = \frac{Z \times 2.7 \times 10^{-2}}{6.023 \times 10^{23} \times (405 \times 10^{-12})^3}$$

$$Z = 6.023 \times 405 \times 405 \times 405 \times 10^{23-36+3+2}$$

$$Z = 6.023 \times 405 \times 405 \times 405 \times 10^{-8}$$

$$Z = 4$$

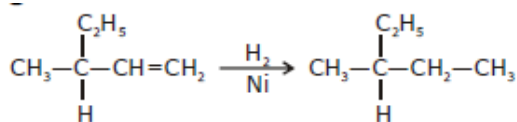
FCC

$$4R = \sqrt{2} \times a$$

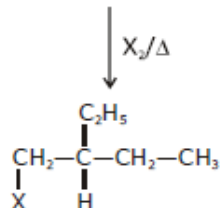
$$R = \frac{405}{2\sqrt{2}} \times 10^{-12} = 143.21 \times 10^{-12} \text{ m}$$

$$= 143$$

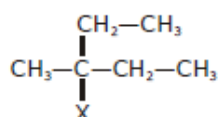
49. (8)



(Simplest optically active alkene)

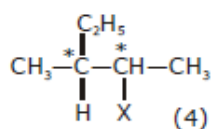


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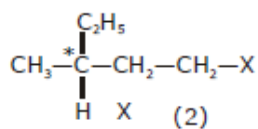


+

Total 8 organic products are possible



+



50. (58)

$$\text{Energy of photon} = 2.3 - E_{\text{cell}} \{ \text{for Na} \}$$

$$\text{Energy of photon} = 2.25 - E_{\text{cell}} \{ \text{for K} \}$$

$$E_{\text{cell}} \{ \text{for 'Na'} \} + 0.05 = E_{\text{cell}} \{ \text{for 'K'} \}$$

$$0.22 + 0.06 \log [\text{H}^+][\text{Cl}^-] + 0.05 = 0.22 + 0.06 \log [\text{H}^+][\text{Cl}^-]$$

$$6 \log (10^{-2}) + 5 = 6 \log [\text{H}^+][\text{Cl}^-]$$

$$\log (10^{-12}) + \log (10^5) = \log \{ [\text{H}^+][\text{Cl}^-] \}^6$$

$$\{ [\text{H}^+][\text{Cl}^-] \}^6 = 10^{-7}$$

$$[\text{H}^+]^{12} = 10^{-7}$$

$$\text{pH} = \frac{7}{12} = 0.58$$

$$= 58 \times 10^{-2} = 58 \text{ Ans}$$

51. (b)

$$2. {}^1P_0 = 2$$

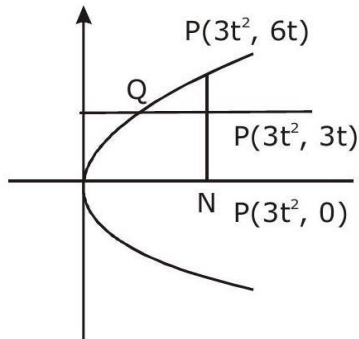
$$3. {}^2P_1 = 3$$

$$4. {}^3P_2 = 4$$

$$([2 - [3 + [4 - [5 + \dots \dots \dots 52]) + ([1 - [2 + [3 - [4 \dots \dots + \square 51)$$

$$= [52 + 1$$

52. (d)



$Q(h, 3t)$ lie on

Parabola

$$9t^2 = 12h$$

$$h = \frac{3t^2}{4}$$

$$Q = \left(\frac{3t^2}{4}, 3t\right)$$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3t^2}{4} - 3t^2\right)} (x - 3t^2)$$

$$y = \frac{-4t}{3t^2} (x - 3t^2)$$

put $x = 0$

$$y = \frac{-4}{3t} (-3t^2) = 4t$$

$$4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$$

$$PN = 6t = 6 \cdot \frac{1}{3} = 2$$

$$M = \left[\frac{1}{3}, 1\right], Q \left[\frac{1}{12}, 1\right]$$

$$MQ = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$$

53. (c)

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

$$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 3R_1$$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 1 & -x+1 & x-5 \end{vmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 0 & -1 & 3x-8 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

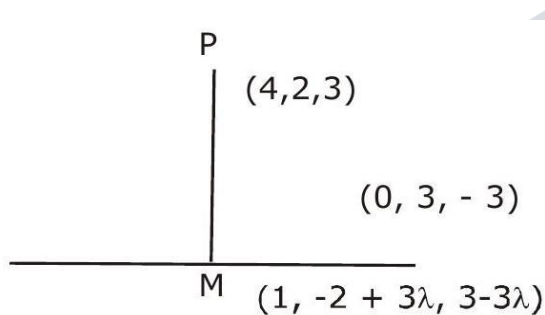
$$\Rightarrow -1[(3-2x)(x-2) - (3x-4)] + (3x-8)[(x-2)(-x+2) - (2x-3)] = Ax^3 + Bx^2 + Cx + D$$

$$\Rightarrow 3x - 2x^2 - 6 + 4x - 3x + 4 + (3x-8)[-x^2 + 4x - 4 - 2x + 3] = Ax^3 + Bx^2 + Cx + D$$

$$A = -3, B = 12, C = -15$$

$$B + C = -3$$

54. (c) $\vec{r} = (1, -2, 3) + \lambda(0, 3, -3)$



$$\overrightarrow{PM} \perp \vec{b}$$

$$\overrightarrow{PM} \cdot \vec{b} = 0$$

$$(-3, 3\lambda - 4, -3\lambda) \cdot (0, 3, -3) = 0$$

$$\Rightarrow 0 + 9\lambda - 12 + 9\lambda = 0 \Rightarrow \lambda = \frac{12}{18} = \frac{2}{3}$$

$$m = (1, 0, 1) \text{ are on } 2x + y - z = 1$$

55. (d) $2yy' + 2(-\tan x) = y'$

diff. w.r.t.x

$$2yy'' + 2(y')^2 - 2\sec^2 x = y''$$

Put $x = 0$ in given equation we get $y = 0, 1$

$$\text{from (a) } x = 0, y = 0 \Rightarrow y'(0) = 0$$

$$x = 0, y = 1, \Rightarrow y'(0) = 0$$

$$\text{from (b) } x = 0, y = 0, y'(0) = 0 \Rightarrow y''(0) = -2$$

$$x = 0, y = 1, y'(0) = 0 \Rightarrow y''(0) = 2$$

$$|y''(0)| = 2$$

56. (b)

$$2\pi - \left[\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\frac{16}{63} \right]$$

$$2\pi - \tan^{-1}\left(\frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}}\right) - \tan^{-1}\left(\frac{16}{63}\right)$$

$$\Rightarrow 2\pi - \tan^{-1}\left(\frac{48 + 15}{36 - 20}\right) - \tan^{-1}\left(\frac{16}{63}\right)$$

$$\Rightarrow 2\pi - \left[\tan^{-1}\left(\frac{63}{16}\right) + \cot^{-1}\left(\frac{63}{16}\right) \right]$$

$$\Rightarrow 2\pi - \frac{\pi}{2}$$

$$= \frac{3\pi}{2}$$

57. (a) $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$$b_1^2 = a_1^2(1 - e_1^2)$$

$$3 = 4(1 - e_1^2)$$

$$e_1 = \frac{1}{2}$$

$$\text{focus} = (\pm a_1 e_1, 0)$$

$$= (\pm 1, 0)$$

$$\text{Length of transverse axis } 2a_2 = \sqrt{2} \rightarrow a_2 = \frac{1}{\sqrt{2}}$$

$$a_2 e_2 = 1$$

$$= e_2 = \sqrt{2}$$

$$b_2^2 = a_2^2(e_2^2 - 1)$$

$$b_2^2 = \frac{1}{2}(2 - 1) = \frac{1}{2}$$

equation of Hyperbola

$$x^2 - y^2 = \frac{1}{2}$$

58. (c) $\sigma^2 \leq \frac{1}{4}(M - m)^2$

(M = upper bound of value of any random variable,

m = Lower bound of value of any random variable)

$$\sigma^2 \leq \frac{1}{4}(10 - 0)^2$$

$$\sigma^2 < 25$$

$$-5 < \sigma < 5$$

$$\sigma \neq 6$$

59. (d) Total Possibilities = (1,3), (3,1), (2,2),

(2,6), (6,2)(4,4)

(3,5), (5,3)(6,6)

fav. = 1 = (4,4)

$$\text{prob.} = \frac{1}{9}$$

60. (c)

$$T_{r+1} = {}^nC_r \left(3^{\frac{1}{2}}\right)^{n-r} \left(5^{\frac{1}{8}}\right)^r$$

$$\frac{n-r}{2} \rightarrow n-r = 0, 2, 4, 6, 8, \dots$$

$$\frac{r}{8} \rightarrow r = 0, 8, 16, 24, \dots$$

common $r = 0, 8, 16, 24$

no. of integral term = 33.

$$L = 0 + (33 - 1) \times 8 \rightarrow L = 32 \times 8$$

$$= 256$$

61. (a)

$$\int_{-\pi}^{\pi} |\pi - |x|| dx$$

even function

$$2 \int_0^{\pi} |\pi - x| dx$$

$$= 2 \int_0^{\pi} (\pi - x) dx \Rightarrow 2 \left[\pi x - \frac{x^2}{2} \right]_0^{\pi}$$

$$= 2 \left[\frac{\pi^2}{2} \right] = \pi^2$$

62. (a) $D \geq 0$

$$(m + 1)^2 - 4(m + 4) \geq 0$$

$$\Rightarrow m^2 - 2m - 15 \geq 0$$

$$(m - 5)(m + 3) \geq 0$$

$$m \in (-\infty, -3] \cup [5, \infty)$$

$$A = (-\infty, -3] \cup [5, \infty)$$

$$B = [-3, 5)$$

$$A - B = (-\infty, -3) \cup [5, \infty)$$

$$A \cup B = R$$

63. (d)

$$\sim (p \wedge \sim q) \rightarrow \sim p \vee q$$

$$p \rightarrow (\sim p \vee q)$$

$$\Rightarrow \sim p \vee (\sim p \vee q)$$

$$\Rightarrow \sim p \vee q$$

64. (c)

$$f(x) = (3x - 7) \cdot \frac{2}{3x^{\frac{1}{3}}} + x^{\frac{2}{3}} \cdot 3$$

$$= \frac{6x - 14 + 9x}{3x^{\frac{1}{3}}}$$

$$= \frac{15x - 14}{3x^{\frac{1}{3}}}$$

+	-	+
0		14/15

$$f(x) > 0 \uparrow \Rightarrow x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$$

65. (a)

$$a = 3$$

$$\frac{25}{2}[2a + 24d] = \frac{15}{2}[2 \times (a + 25d) + 14d]$$

$$\Rightarrow 50a + 600d = 15[2a + 50d + 14d]$$

$$\Rightarrow 20a + 600d = 960d$$

$$\Rightarrow 60 = 360d$$

$$d = \frac{1}{6}$$

66. (d)

$$\int \left(\frac{1+y^2}{y^2} \right) dy = \int \left(\frac{1}{1+e^{-x}} \right) dx$$

$$\int \frac{1}{y^2} dy + \int dy = \int \left(\frac{e^x}{e^x+1} \right) dx$$

$$\Rightarrow \frac{-1}{y} + y = \ln|e^x+1| + c$$

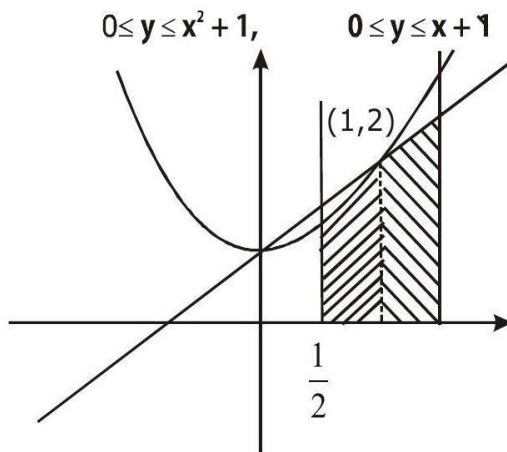
$$x=0, y=1$$

$$\Rightarrow -1+1 = \ln 2 + c \Rightarrow c = -\ln 2$$

$$\Rightarrow \frac{-1}{y} + y = \ln|e^x+1| - \ln 2$$

$$\Rightarrow y^2 = 1 + y \left[\ln \left(\frac{e^x+1}{2} \right) \right]$$

67. (d)



$$A = \int_{\frac{1}{2}}^1 (x^2 + 1) dx + \int_1^2 (x + 1) dx$$

$$\left(\frac{x^3}{3} + x \right)_{\frac{1}{2}}^1 + \left(\frac{x^2}{2} + x \right)_1^2$$

$$= \left(\frac{1}{3} + 1 \right) - \left(\frac{1}{24} + \frac{1}{2} \right) + \left((2+2) - \left(\frac{3}{2} \right) \right)$$

$$= \left(\frac{4}{3} - \frac{13}{24} \right) + \left(\frac{5}{2} \right)$$

$$= \left(\frac{32-13}{24} \right) + \left(\frac{5}{2} \right) = \frac{19+60}{24} = \frac{79}{24}$$

68. (c)

$$\alpha + \beta = -p, \alpha\beta = 2$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = -q, \frac{1}{\alpha\beta} = \frac{1}{2}$$

$$\frac{\alpha + \beta}{\alpha\beta} = -q \Rightarrow \frac{-p}{2} = -q$$

$$\Rightarrow p = 2q$$

$$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\alpha\beta} + 2$$

$$= 2 + \frac{1}{2} + 2 = \frac{9}{2}$$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right) = \alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}$$

$$= 2 + \frac{1}{2} - \left[\frac{\alpha^2 + \beta^2}{\alpha\beta}\right]$$

$$= \frac{5}{2} - \left[\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}\right]$$

$$= \frac{5}{2} - \left[\frac{p^2 - 4}{2}\right]$$

$$= \frac{9 - p^2}{2}$$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \left(\frac{9 - p^2}{2}\right)\left(\frac{9}{2}\right)$$

$$= \frac{9}{4}(9 - p^2)$$

69. (a) $\frac{2}{1} \neq \frac{0}{1} \neq \frac{1}{-1} \rightarrow$ lines are intersecting

$$\vec{r} = (1 + 2l)\hat{i} - j + l\hat{k}$$

$$\vec{r} = (2 + m)\hat{i} + (m - 1)\hat{j} - m\hat{k}$$

compare coeff. of $\hat{i}, \hat{j}, \hat{k}$

$$1 + 2l = 2 + m \quad \left| \begin{matrix} -1 = m - 1 \\ m = 0 \end{matrix} \right| \quad l = 0$$

Lines do not intersect

70. (b) $\lim_{x \rightarrow 0} \left| \frac{1-x+|x|}{\lambda-x+[x]} \right| = L$

$$\lim_{h \rightarrow 0} \left| \frac{1-h+h}{\lambda-h+[h]} \right|$$

$$\lim_{h \rightarrow 0} \left| \frac{1}{\lambda-h+0} \right| = \left| \frac{1}{\lambda} \right| [x] = 0$$

$$\lim_{h \rightarrow 0} \left| \frac{1+h+h}{\lambda+h+[-h]} \right|$$

$$= \left| \frac{1}{\lambda-1} \right| [-h] = -1$$

$$\because |\lambda| = |\lambda - 1|$$

$$\lambda^2 = \lambda^2 - 2\lambda + 1 \Rightarrow \lambda = \frac{1}{2}$$

$$L = 2$$

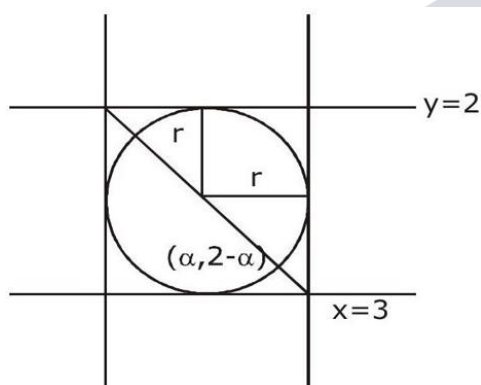
71. (8)

$$\lim_{x \rightarrow 0} \frac{\left(1 - \cos \frac{x^2}{2}\right) \left(1 - \cos \frac{x^2}{4}\right) \cdot \left(\frac{x^2}{2}\right) \cdot \left(\frac{x^2}{4}\right)^2}{\left(\frac{x^2}{2}\right)^2 \left(\frac{x^2}{4}\right)^2 \cdot x^8}$$

$$\lim_{x \rightarrow 0} \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{16} \Rightarrow \frac{1}{256} = 2^{-k}$$

$$2^{-8} = 2^{-k} \Rightarrow k = 8$$

72. (2)



$$p = r$$

$$\text{for } y = 2$$

$$r = \left| \frac{2 - \alpha - 2}{1} \right| = |\alpha|$$

$$\text{for } x = 3$$

$$r = \left| \frac{\alpha - 3}{1} \right| = |\alpha - 3|$$

$$|\alpha| = |\alpha - 3|$$

$$\Rightarrow \alpha^2 + \alpha^2 - 6\alpha + 9 \Rightarrow \alpha = \frac{3}{2}$$

$$2\alpha = 3 = 2r$$

73. (4)

$$\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \dots \infty = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2}$$

$$\log_{2.5} \left(\frac{1}{2} \right) \Rightarrow \log_{\frac{5}{2}} \frac{1}{2}$$

$$.16 = \frac{16}{100} = \frac{4}{25} = \left(\frac{2}{5} \right)^2$$

$$\Rightarrow \left(\frac{2}{5} \right)^{2 \log_{\frac{5}{2}} \frac{1}{2}} = \left(\frac{5}{2} \right)^{-2 \log_{\frac{5}{2}} \frac{1}{2}}$$

$$\Rightarrow \left(\frac{5}{2} \right)^{\log_{\frac{5}{2}} \left(\frac{1}{2} \right)^{-2}}$$

$$= 4$$

74. (10)

$$A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} X & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} X^2 + 1 & x \\ X & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} X^2 + 1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x^3 + x + x & x^2 + 1 \\ x^2 + 1 & x \end{bmatrix}$$

$$A^4 = \begin{bmatrix} X^3 + 2x & x^2 + 1 \\ x^2 + 1 & x \end{bmatrix} \begin{bmatrix} X & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x^4 + 2x^2 + x^2 + 1 & x^3 + 2x \\ x^3 + x + x & x^2 + 1 \end{bmatrix}$$

$$a_{11} \Rightarrow x^4 + 3x^2 + 1 = 109$$

$$x^4 + 3x^2 - 108 = 0$$

$$\Rightarrow (x^2 + 12)(x^2 - 9) = 0$$

$$x = \pm 3$$

$$a_{11} = x^2 + 1 = 10$$

75. (4)

$$\left[\frac{(1+i)(1+i)}{(1+i)(1-i)} \right]^{\frac{m}{2}} = \left[\left(\frac{1+i}{-1+i} \right) \left(\frac{-1-i}{-1-i} \right) \right]^{\frac{n}{3}} = 1$$

$$= \left(\frac{2i}{2} \right)^{\frac{m}{2}} = 1 \mid \left(\frac{-1-i-i+1}{1+1} \right)^{\frac{n}{3}} = 1$$

$$m = 8$$

$$(-i)^{n/3} = 1$$

$$n = 12$$

greatest common divisor of m & n is 4

