1. (a) Statement (C) is correct because, the magnetic field outside the toroid is zero and they form closed loops inside the toroid itself.

Statement (E) is correct because we know that super conductors are materials inside which the net magnetic field is always zero and they are perfect diamagnetic.

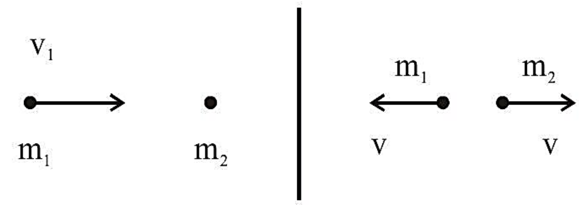
$\mu_r = 1 + \chi$

$\chi = -1$

$\mu_r = 0$

For superconductors.

2. (a)



$m_1 v_1 = -m_1 v + m_2 v$

$v_1 = -v + \frac{m_2}{m_1} v$

$\frac{(v_1 + v)}{v} = \frac{m_2}{m_1}$

$e = \frac{2v}{v_1} = 1$

$v = \frac{v_1}{2}$

$\frac{v_1 + v_1/2}{v_1/2} = \frac{m_2}{m_1}$

$3 = \frac{m_2}{m_1}$

3. (a) $PV\gamma =$ constant

Differentiating

$\frac{dP}{dV} = -\frac{\gamma P}{V}$

$\frac{dP}{P} = -\frac{\gamma dV}{V}$

4. (d)

$r = \frac{mv}{qB} = \frac{p}{qB}$

$$\frac{m_\alpha}{m_p} = 4$$

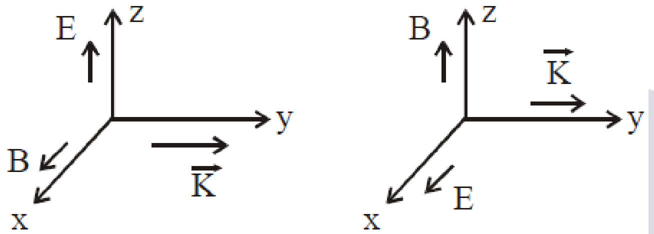
$$\frac{r_p}{r_\alpha} = \frac{p_p}{q_p}\frac{q_\alpha}{p_\alpha} = \frac{2}{1}$$

$$\frac{p_p}{p_\alpha} = \frac{2q_p}{q_\alpha} = 2\left(\frac{1}{2}\right)$$

$$\frac{p_p}{p_\alpha} = 1$$

$$\frac{K_p}{K_\alpha} = \frac{p_p^2}{p_\alpha^p}\frac{m_\alpha}{m_p} = (1)\,(4)$$

5. (c)



6. (c) $\pi r = L \Rightarrow r = \frac{L}{\pi}$

$$I = Mr^2 = \frac{ML^2}{\pi^2}$$

7. (c) $v = -\left(\frac{v_o}{x_0}\right)x + v_o$

$$a = \frac{vdv}{dx}$$

$$a = \left[-\left(\frac{v_0}{x_0}\right)x + v_0\right]\left[-\frac{v_0}{x_0}\right]$$

$$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

8. (d) $\alpha = \frac{I_C}{I_E}, \beta = \frac{I_C}{I_B}$

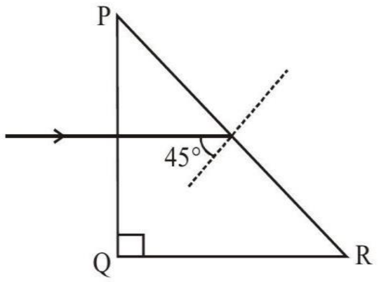
$$I_E = I_B + I_C$$

$$\alpha = \frac{I_C}{I_B + I_C} = \frac{1}{\frac{I_B}{I_C} + 1}$$

$$\alpha = \frac{1}{\frac{1}{\beta} + 1}$$

$$\alpha = \frac{\beta}{1 + \beta}$$

9. (b)



Assuming that the right-angled prism is an isoceles prism, so the other angles will be 45° each.

⇒ Each incident ray will make an angle of 45° with the normal at face PR.

⇒ The wavelength corresponding to which the incidence angle is less than the critical angle, will pass through PR.

⇒ θ_C = critical angle

⇒ $\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$

⇒ If $\theta_C \geq 45°$

the light ray will pass

⇒ $(\theta_C)_{Red} = \sin^{-1}\left(\frac{1}{1.27}\right) = 51.94°$

Red will pass.

⇒ $(\theta_C)_{Green} = \sin^{-1}\left(\frac{1}{1.42}\right) = 44.76°$

Green will not pass

⇒ $(\theta_C)_{Blue} = \sin^{-1}\left(\frac{1}{1.49}\right) = 42.15°$

Blue will not pass

⇒ So only red will pass through PR.

10. (d) For objects to float

$mg = m\omega^2 R$

ω = angular velocity of earth.

R = Radius of earth

$\omega = \sqrt{\frac{g}{R}}$

Duration of day = T

$T = \frac{2\pi}{\omega}$

$\Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$

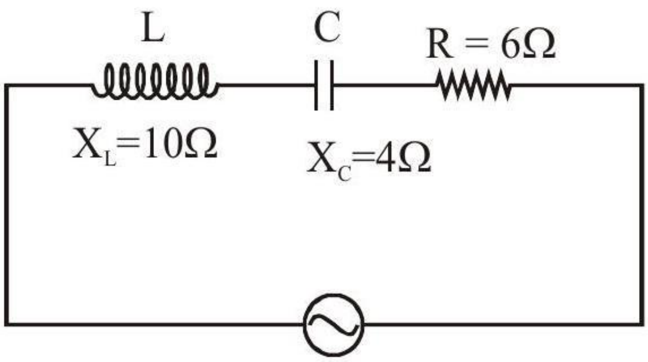
$= 2\pi\sqrt{\frac{6400 \times 10^3}{10}}$

$\Rightarrow \frac{T}{60} = 83.775$ minutes

$\simeq 84$ minutes

11. (b) It is possible only inside the nucleus and not otherwise.

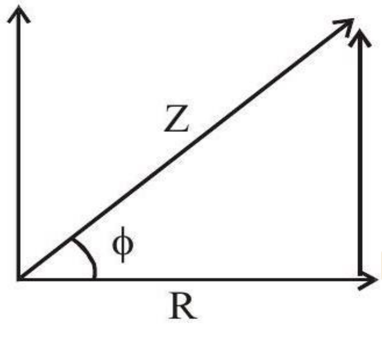
12. (c)



We know that power factor is $\cos\phi$, $\cos\phi = \frac{R}{Z}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$(\omega L - 1/\omega C)$

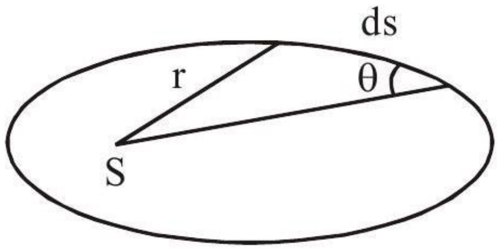


$\Rightarrow Z = \sqrt{6^2 + (10-4)^2}$

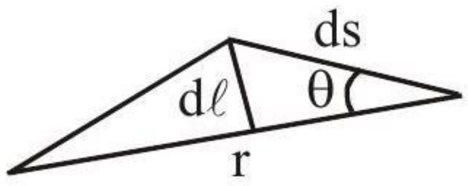
$\Rightarrow Z = 6\sqrt{2} \mid \cos\phi = \frac{6}{6\sqrt{2}}$

$\cos\phi = \frac{1}{\sqrt{2}}$

13. (d)



For small displacement ds of the planet its area can be written as



$dA=\frac{1}{2}rd\ell$

$=\frac{1}{2}rdssin\theta$

A.vel $=\frac{dA}{dt}=\frac{1}{2}rsin\theta\frac{ds}{dt}=\frac{Vrsin\theta}{2}$

$\frac{dA}{dt}=\frac{1}{2}\frac{mVrsin\theta}{m}=\frac{L}{2\,m}$

14. (d) Time period $T=\frac{2\pi}{\omega'}$

$\frac{\pi}{\omega}=\frac{2\pi}{\omega'}$

$\omega'=2\omega\rightarrow$ Angular frequency of SHM

Option (c)

$\sin^2\omega t=\frac{1}{2}(2\sin^2\omega t)=\frac{1}{2}(1-\cos2\omega t)$

Angular frequency of $\left(\frac{1}{2}-\frac{1}{2}\cos2\omega t\right)$ is 2ω

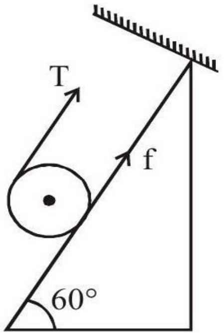
Option (d)

Angular frequency of SHM

$3\cos\left(\frac{\pi}{4}-2\omega t\right)$ is 2ω.

So option (c) & (d) both have angular frequency 2ω but option (d) is direct answer.

15. (c)



Let's take solid cylinder is in equilibrium $T + f = mg\sin 60$

$TR - fR = 0$

Solving we get

$$T = f_{req} = \frac{mg\sin\theta}{2}$$

But limiting friction < required friction

$$\mu mg\cos 60^\circ < \frac{mg\sin 60^\circ}{2}$$

$\therefore$ Hence cylinder will not remain in equilibrium

Hence $f =$ kinetic

$= \mu_k N$

$= \mu_k mg\cos 60^\circ$

$$= \frac{mg}{5}$$

16. (c) Magnetic energy $= \frac{1}{2}Li^2 = 25\%$

$$ME \Rightarrow 25\% \Rightarrow i = \frac{i_0}{2}$$

$i = i_0\left(1 - R^{-Rt/L}\right)$ for charging

$$t = \frac{L}{R}\ln 2$$

17. (a)

$$U = -\frac{C}{r}$$

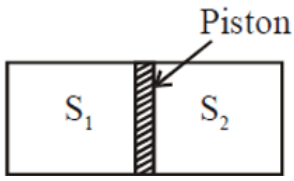
$$F = -\frac{dU}{dr} = -\frac{C}{r^2}$$

$$|F| = \frac{mv^2}{r}$$

$$\frac{C}{r^2} = \frac{mv^2}{r}$$

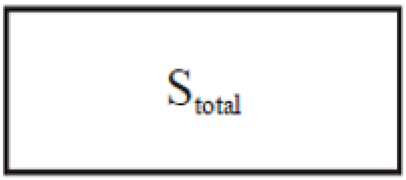
$$v^2 \propto \frac{1}{r}$$

18. (d)



$: S_1 > S_2$

After piston is removed



S_{total} ; $S_{total} = S_1 + S_2$

19. (c) $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$v_{avg} = \sqrt{\frac{8}{\pi}\frac{RT}{M}}$$

$$\frac{v_{rms}}{v_{arg}} = \sqrt{\frac{3\pi}{8}}$$

20. (a) Resolving power (RP) $\propto \frac{1}{\lambda}$ $\lambda = \frac{h}{P} = \frac{h}{mv}$

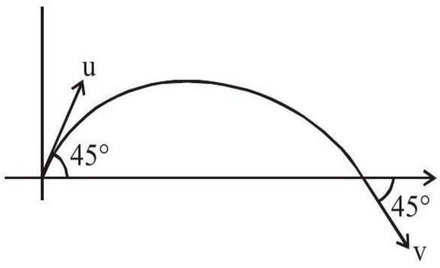
So (RP) $\propto \frac{mv}{h}$

RP $\propto$ P

RP $\propto$ mv

RP $\propto$ m

21. (5)



$|\vec{u}| = |\vec{v}|$

$\vec{u} = u\cos 45\hat{i} + u\sin 45\hat{j}$

$\vec{v} = v\cos 45\hat{i} - v\sin 45\hat{j}$

$|\overrightarrow{\Delta P}| = |m(\vec{v} - \vec{u})|$

$\Delta P = 2mu\sin 45^\circ$

$= 2 \times 5 \times 10^{-3} \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 50 \times 10^{-3}$

$= 5 \times 10^{-2}$

22. (6) Let's say the compression in the spring by: y. So, by work energy theorem we have

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}ky^2$

$\Rightarrow y = \sqrt{\frac{m}{k}} \cdot v$

$\Rightarrow y = \sqrt{\frac{4}{100}} \times 10$

$\Rightarrow y = 2 \text{ m}$

$\Rightarrow$ final length of spring

$= 8 - 2 = 6 \text{ m}$

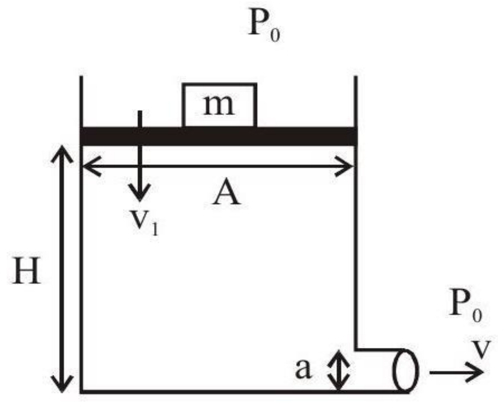
23. (200) $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2\times10^{-3}}{10\times10^{-6}}$

$\beta = \frac{1}{5} \times 10^3$

$\beta = 2 \times 10^2$

$\beta = 200$

24. (3)



$m = 24$ kg

$A = 0.4\ m^2$

$a = 1\ cm^2$

$H = 40$ cm

Using Bernoulli's equation

$$\Rightarrow \left(P_0 + \frac{mg}{A}\right) + \rho gH + \frac{1}{2}/v_1^2$$
$$= P_0 + 0 + \frac{1}{2}\rho v^2$$
$\Rightarrow$ Neglecting v_1

$$\Rightarrow v = \sqrt{2gH + \frac{2mg}{A\rho}}$$

$$\Rightarrow v = \sqrt{8 + 1.2}$$

$$\Rightarrow v = 3.033\ m/s$$

$$\Rightarrow v \simeq 3\ m/s$$

25. (50) Range $= \sqrt{2Rh}$

Range (i) $= \sqrt{2Rh}$

Range (ii) $= \sqrt{2Rh} + \sqrt{2Rh'}$

where $h = 20$ m& $h' = 5$ m

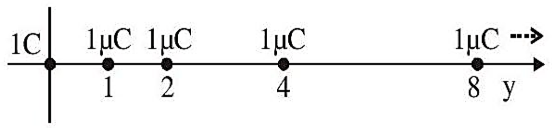
$$= \frac{\sqrt{2Rh'}}{\sqrt{2Rh'}} \times 100\% = \frac{\sqrt{5}}{\sqrt{20}} \times 100\% = 50\%$$

26. (34) $v = \frac{4}{3}\pi r^3$

taking log & then differentiate

$$\frac{dV}{V} = 3\frac{dr}{r}$$
$$= \frac{3 \times 0.85}{7.5} \times 100\% = 34\%$$

27. (12)



$$F = k(1C)(1\mu C)\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \cdots\right]$$

$$= 9 \times 10^3 \left[\frac{1}{1 - \frac{1}{4}}\right] = 12 \times 10^3 \text{ N}$$

28. (48) In Balanced conditions

$$\frac{12}{6} = \frac{x}{72 - x}$$

$x = 48$ cm

29. (4) in parallel

$$R_{net} = \frac{R_1 R_2}{R_1 + R_2} \underbrace{\square\, OA}_{\ell}$$
$$\frac{\rho\ell}{2\,A} = \frac{\rho_1 \frac{\ell}{A} \times \rho_2 \frac{\ell}{A}}{\rho_1 \frac{\ell}{A} + \rho_2 \frac{\ell}{A}}$$
$$\frac{\rho}{2} = \frac{6 \times 3}{6 + 3} = 2$$
$$\rho = 4$$

30. (6) $\frac{\Delta\lambda}{\lambda} c = v$

$$\Delta\lambda = \frac{v}{c} \times \lambda = \frac{286}{3 \times 10^5} \times 630 \times 10^{-9} = 6 \times 10^{-10}$$

31. (a) The oxidation states of Nitrogen in following molecules are as follows

$NO_3^- \rightarrow +5$

$NO_2 \rightarrow +4$

$NO \rightarrow +2$

$N_2O \rightarrow +1$

32. (d) In basic medium, oxidising action of H_2O_2.

$Mn^{2+} + H_2O_2 \rightarrow Mn^{+4} + 2OH$

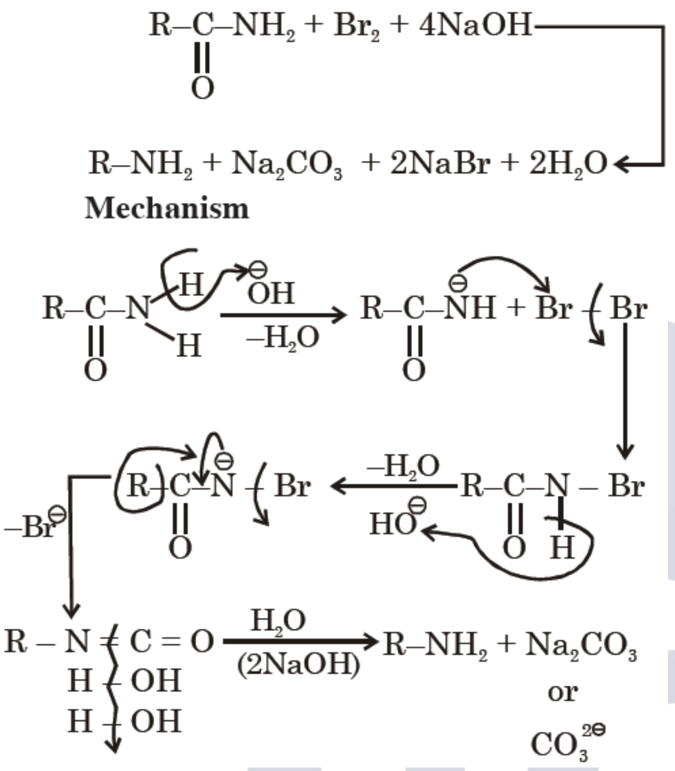
In basic medium, reducing action of H_2O_2

$I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

In acidic medium, oxidising action of H_2O_2.

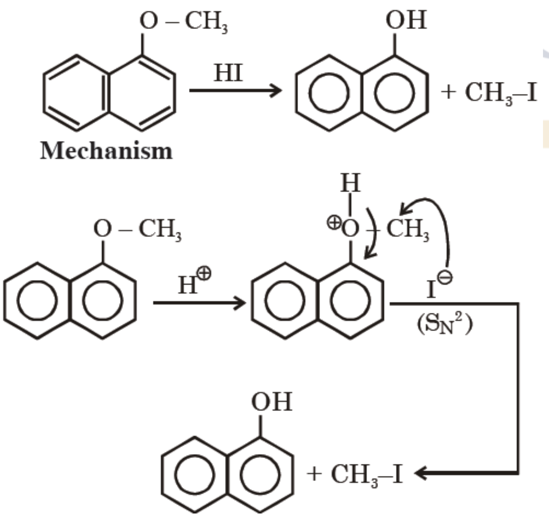
$PbS(s) + 4H_2O_2(aq) \rightarrow PbSO_4(s) + 4H_2O(\ell)$

33. (a)



34. (b) Mn_3O_4 shows magnetic properties.

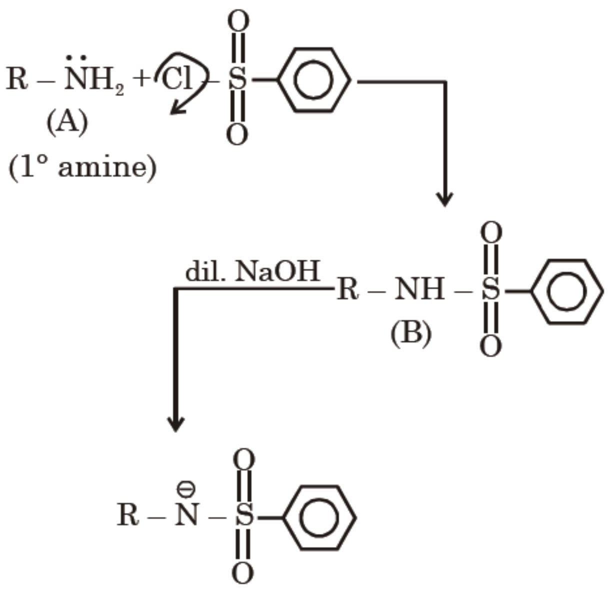
35. (b)



36. (a) Due to deficiency of Vitmain K causes increases in blood clotting time.

Note: Vitamin K related to blood factor.

37. (d) Hinsberg reagent (Benzene sulphonyl chloride) gives reaction product with 1° amine and it is soluble in dil. NaOH.



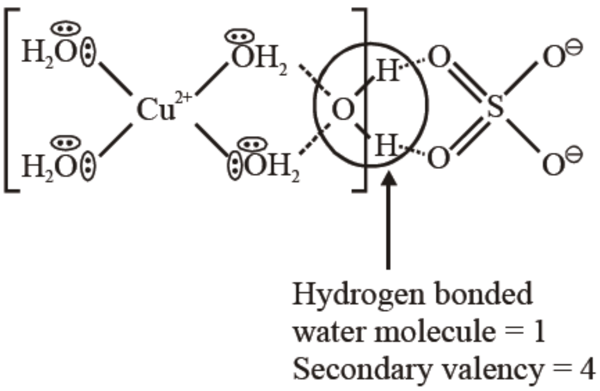
38. (c) The 1^{st} IE order of 3^{rd} period is

Na < Al < Mg < Si < S < P < Cl < Ar

X&Y are Ar&Cl

Z is sodium (Na).

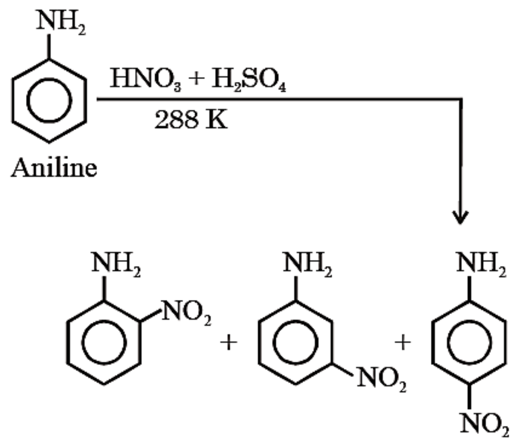
39. (b)



40. (b) Statement-I is false since Bohr's theory accounts for the stability and spectrum of single electronic species (eg: He^{+}, Li^{2+} etc)

Statement II is true.

41. (d)



(A) (B) (C)

(2%) (47%) (51%)

% yield order ⇒ C > B > A

42. (d)

43. (c)

(A) Antifertility drug → (iii) Nor ethindrone

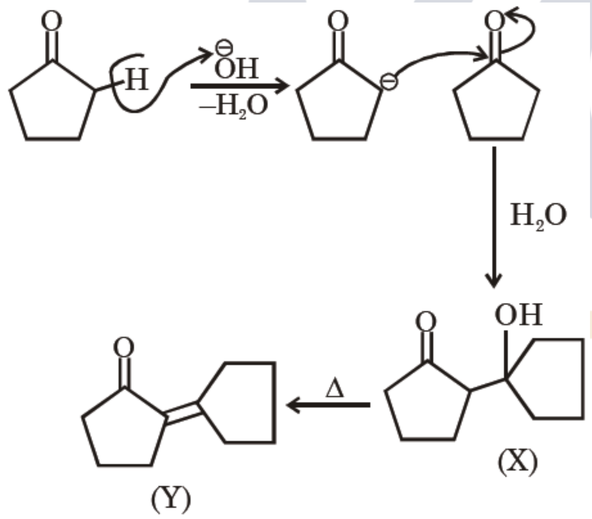
(B) Antibiotic → (iv) Salvarsan

(C) Tranquilizer → (i) Meprobamate

(D) Artificial sweetener → (ii) Alitame

A-iii, B-iv, C-i, D-ii

44. (c)



45. (b)

(a) Be → it is used in the Windows of X-ray tubes

(b) Mg → it is used in the Incendiary bombs and signals

(c) Ca → it is used in the Extraction of metals

(d) Ra → it is used in the Treatment of cancer

46. (a)

47. (d) Non-biodegradable wastes are generated by the thermal power plants which produces fly ash. Detergents which are biodegradable causes problem called eutrophication which kills animal life by deprieving it of oxygen.

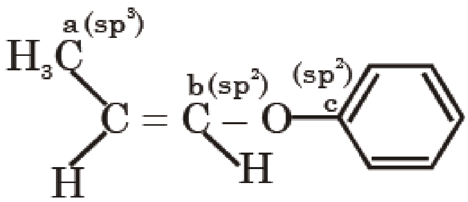
48. (c) (a) Mercury → Distillation refining

(b) Copper → Electrolytic refining

(c) Silicon → Zone refining

(d) Nickel → Vapour phase refining

49. (c)



50. (d) Covalent or network solid have very high melting point and they are insulators in their solid and molten form.

51. (10)

$$\cdot \frac{t_{99.9\%}}{t_{50\%}} = \frac{\frac{1}{K}\ln\frac{100}{0.1}}{\frac{1}{K}\ln 2}$$

$$= \frac{\ln 1000}{\ln 2} \times t_{50\%}$$

$$= \frac{3\ln 10}{\ln 2} \times 1$$

$$= \frac{3 \times 2.3}{0.69} = 10$$

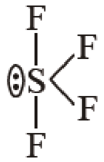
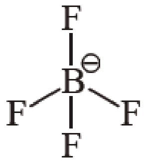
52. (288)

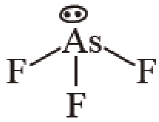
From Kohlrausch's law

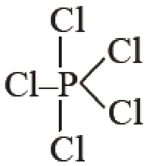
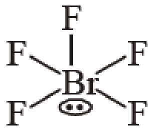
$$\Lambda_m^\infty(BaSO_4) = \lambda_m^\infty(Ba^{2+}) + \lambda_m^\infty(SO_4^{2-})$$
$$\Lambda_m^\infty(BaSO_4) = \Lambda_m^\infty(BaCl_2) + \Lambda_m^\infty(H_2SO_4)$$

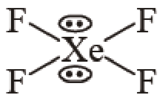
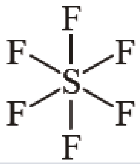
$$= 280 + 860 - 2(426)$$
$$= 288 Scm^2\ mol^{-1}(HCl)$$

53. (2)

SF_4 =  , $BF_4^{\ominus}$ = 

ClF_3 =  , AsF_3 = 

PCl_5 =  , BrF_5 = 

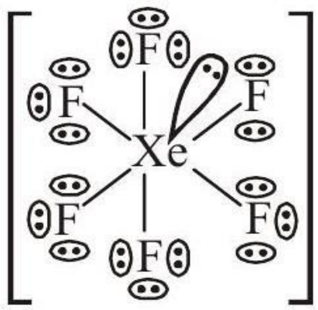
XeF_4 =  , SF_6 = 

Two 1.p. on central atom is $= ClF_3, XeF_4$

54. (19) $XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$

(A) (Limited water)

Structure of 'A'



Total 1.p. on (A) $= 19$

55. (166) Using formula

$\Delta_r G^0 = -RT\ln K_p$

$25200 = -2.3 \times 8.3 \times 400\log(K_p)$

$K_p = 10^{-3.3} = 10^{-3} \times 0.501$

$= 5.01 \times 10^{-4}\text{Bar}^{-1}$

$= 5.01 \times 10^{-9}\ \text{Pa}^{-1}$

$= \dfrac{K_C}{8.3 \times 400}$

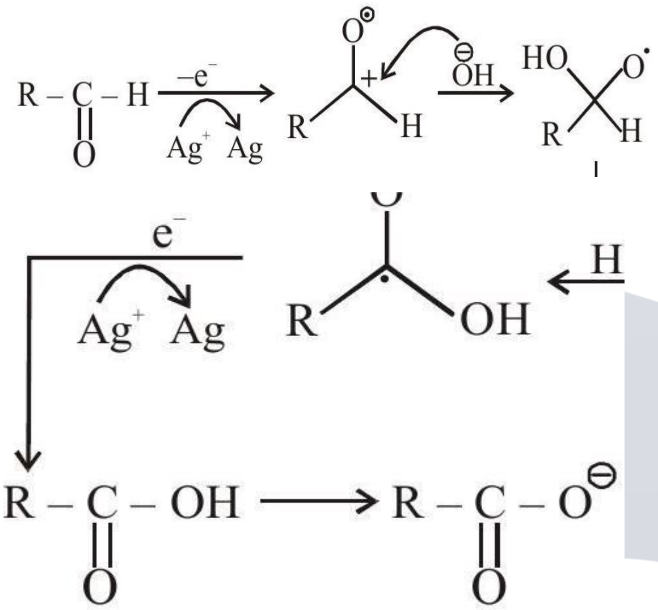
$K_C = 1.66 \times 10^{-5}\ m^3/mole$

$= 1.66 \times 10^{-2}\ L/mol$

56. (2) $AgNO_3 + NaOH \rightarrow AgOH + NaNO_3$

$2AgOH \rightarrow Ag_2O + H_2O$

$Ag_2O + 4NH_3 + H_2O \rightarrow 2Ag(NH_3)_2^+ + 2\overset{\ominus}{O}H$



Total $2e^-$ transfer to Tollen's reagent

57. (64) In pure water,

$K_{sp} = S^2 = (8 \times 10^{-4})^2$

$= 64 \times 10^{-8}$

In $0.01M H_2SO_4$

$H_2SO_{4(aq)} \rightarrow 2H^+_{(aq)} + SO_4{}^{2-}$ (aq.) 0.02 0.01

$BaSO_{4(s)} \rightleftharpoons Ba^{2+}{}_{(aq.)} + SO_4{}^{2-}{}_{(aq)}$

x (x + 0.01)

$K_{sp} = x(x + 0.01)$

$= 64 \times 10^{-8}$

$x + 0.01 \cong 0.01M$

So, $x(0.01) = 64 \times 10^{-8}$

$x = 64 \times 10^{-6} M$

58. (50) $\Delta T_b = T_b - T_b{}^0$

$100.52-100$

$=0.52°C$

$i=\left(1-\frac{\alpha}{2}\right)$

$\because \Delta T_b = iK_b \times m$

$0.52=\left(1-\frac{\alpha}{2}\right)\times 0.52\times 2$

$\alpha = 1$

So, percentage association $=100\%$.

59. (50) Most precise volume of HCl $=5$ml at equivalence point

Meq. of Na_2CO_3 = meq. of HCl.

Let molarity of Na_2CO_3

solution = M, then

$M\times 10\times 2 = 0.2\times 5\times 1$

$M = 0.05$ mol/L

$=0.05\times 1000$

$=50$mM

60. (78) Moles of Benzoic acid $=\frac{6.1}{122}$

= moles of m-bromobenzoic acid

So, weight of m-bromobenzoic acid

$=\frac{6.1}{122}\times 201$gm

$=10.05$gm

$\%$ yield $=\frac{\text{Actual weight}}{\text{Theoretical weight}}\times 100$

$=\frac{7.8}{10.05}\times 100$

$=77.61\%$

61. (a) Let $y+1=Y$

$\therefore \frac{dY}{dx}=Y^2e^{\frac{x^2}{2}}-xY$

Put $-\frac{1}{Y}=k$

$\Rightarrow \frac{dk}{dx} + k(-x) = e^{\frac{x^2}{2}}$

I.F. $= e^{-\frac{x^2}{2}}$

$\therefore k = (x + c)e^{x^2/2}$

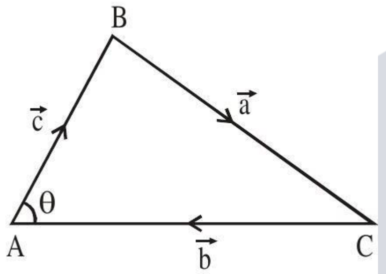
Put $k = -\frac{1}{y+1}$

$\therefore y + 1 = -\frac{1}{(x + c)e^{x^2/2}}$

when $x = 2, y = 0$, then $c = -2 - \frac{1}{e^2}$ diffentiate equation (i) & put $x = 1$

we get $\left(\frac{dy}{dx}\right)_{x=1} = -\frac{e^{3/2}}{(1+e^2)^2}$

62. (b)



$|\vec{a}| = 8, |\vec{b}| = 7, |\vec{c}| = 10$

$\cos\theta = \frac{|\vec{b}|^2 + |\vec{c}|^2 - |\vec{a}|^2}{2|\vec{b}||\vec{c}|} = \frac{17}{28}$

Projection of $\vec{c}$ on $\vec{b}$

$= |\vec{c}|\cos\theta$

$= 10 \times \frac{17}{28}$

$= \frac{85}{14}$

63. (a) For non-trivial solution

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2\mu - 6\lambda + \lambda\mu = 12$

when $\mu = 6,\ 12 - 6\lambda + 6\lambda = 12$

which is satisfied by all λ

64. (c) $f(x) = y = \frac{x-2}{x-3}$

$$\therefore x = \frac{3y-2}{y-1}$$

$$\therefore f^{-1}(x) = \frac{3x-2}{x-1}$$

$$\& \ g(x) = y = 2x - 3$$

$$\therefore x = \frac{y+3}{2}$$

$$\therefore g^{-1}(x) = \frac{x+3}{2}$$

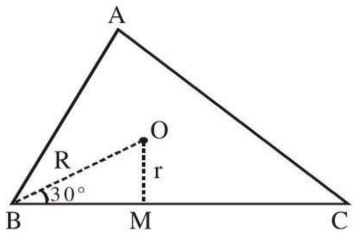
$$\because f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$$

$$\therefore x^2 - 5x + 6 = 0 < \begin{matrix} x_1 \\ x_2 \end{matrix}$$

$\therefore$ sum of roots

$$x_1 + x_2 = 5$$

65. (a)

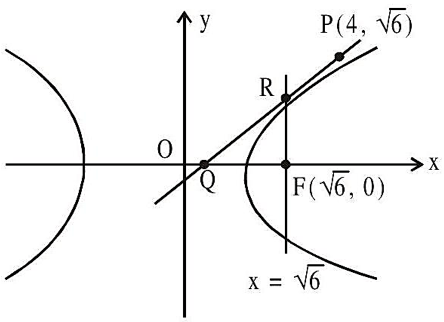


$$r = OM = \frac{3}{\sqrt{2}}$$

$$\& \sin 30^\circ = \frac{1}{2} = \frac{r}{R} \Rightarrow R = \frac{6}{\sqrt{2}}$$

$$\therefore r + R = \frac{9}{\sqrt{2}}$$

66. (c)



$\frac{x^2}{4}-\frac{y^2}{2}=1$

$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{3}{2}}$

$\therefore$ Focus $F(ae,0) \Rightarrow F(\sqrt{6},0)$

equation of tangent at P to the hyperbola is

$2x-y\sqrt{6}=2$

tangent meet x-axis at $Q(1,0)$

& latus rectum $x=\sqrt{6}$ at $R\left(\sqrt{6},\frac{2}{\sqrt{6}}(\sqrt{6}-1)\right)$

$\therefore$ Area of $\Delta_{QFR}=\frac{1}{2}(\sqrt{6}-1)\cdot\frac{2}{\sqrt{6}}(\sqrt{6}-1)$

$=\frac{7}{\sqrt{6}}-2$

67. (b) LHS of all the options are some i.e.

$((P\to Q)\wedge\sim Q)$
$\equiv(\sim P\vee Q)\wedge\sim Q$
$\equiv(\sim P\wedge\sim Q)\vee(Q\wedge\sim Q)$
$\equiv\sim P\wedge\sim Q$

(A) $(\sim P\wedge\sim Q)\to Q$

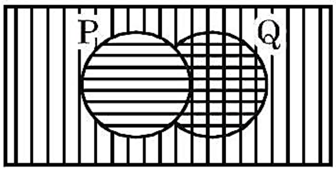
$\equiv\sim(\sim P\wedge\sim Q)\vee Q$

$\equiv(P\vee Q)\vee Q\neq$ tautology

(B) $(\sim P\wedge\sim Q)\to\sim P$

$\equiv\sim(\sim P\wedge\sim Q)\vee\sim P$

$\equiv(P\vee Q)\vee\sim P$



$\Rightarrow$ Tautology

(C) $(\sim P\wedge\sim Q)\to P$
(D) $(\sim P\wedge\sim Q)\to(P\wedge Q)$

$\equiv(P\vee Q)\vee(P\wedge Q)\neq$ Tautology

Aliter :

P	Q	P ∨ Q	P ∨ Q	~ P	(P ∨ Q) ∨~ P
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	T	T

68. **(c)** $\frac{1}{3} \le f(t) \le 1 \forall t \in [0,1]$

$0 \le f(t) \le \frac{1}{2} \forall t \in (1,3]$

Now, $g(c) = \int_0^3 f(t)dt = \int_0^1 f(t)dt + \int_1^3 f(t)dt$

$\because \int_0^1 \frac{1}{3}dt \le \int_0^1 f(t)dt \le \int_0^1 1.dt$

and $\int_1^3 0dt \le \int_1^3 f(a)dt \le \int_1^3 \frac{1}{2}dt$

Adding, we get

$$\frac{1}{3} + 0 \le g(c) \le 1 + \frac{1}{2}(3-1)$$
$$\frac{1}{3} \le g(c) \le 2$$

69. **(d)** $S_{2n} = \frac{2n}{2}[2a + (2n-1)d], S_{4n} = \frac{4n}{2}[2a + (4n-1)d]$

$\Rightarrow S_2 - S_1 = \frac{4n}{2}[2a + (4n-1)d] - \frac{2n}{2}[2a + (2n-1)d]$

$= 4an + (4n-1)2nd - 2na - (2n-1)dn$

$= 2na + nd[8n - 2 - 2n + 1]$

$\Rightarrow 2na + 2n[6n-1] = 1000$

$2a + (6n-1)d = \frac{1000}{n}$

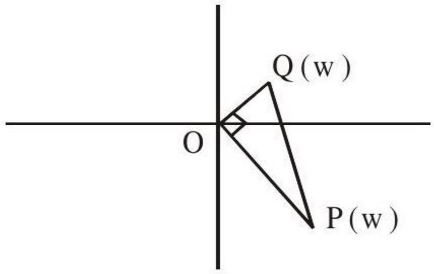
Now, $S_{6n} = \frac{6n}{2}[2a + (6n-1)d]$

$= 3n \cdot \frac{1000}{n} = 3000$

70. **(b)** $w = 1 - \sqrt{3}.i \Rightarrow |w| = 2$

Now, $|z| = \frac{1}{|w|} \Rightarrow |z| = \frac{1}{2}$

and $amp(z) = \frac{\pi}{2} + amp(w)$



$\Rightarrow$ Area of triangle $= \frac{1}{2}.\mathrm{OP}.\mathrm{OQ}$

$$= \frac{1}{2} \cdot 2 \cdot \frac{1}{2} = \frac{1}{2}$$

71. (a) Let observations are denoted by xi for $1 \le i < 2n$

$$\bar{x} = \frac{\sum x_i}{2n} = \frac{(a + a + \cdots + a) - (a + a + \cdots + a)}{2n}$$

$\Rightarrow \bar{x} = 0$

and $\sigma_x^2 = \frac{\sum x_i^2}{2n} - (\bar{x})^2 = \frac{a^2+a^2+\cdots+a^2}{2n} - 0 = a^2$

$\Rightarrow \sigma_x = a$

Now, adding a constant b then $\bar{y} = \bar{x} + b = 5$

$\Rightarrow b = 5$

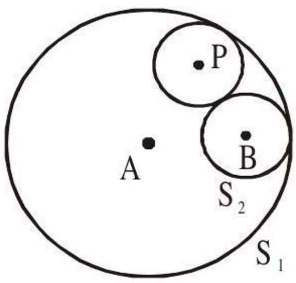
and $\sigma_y = \sigma_x$ (No change in S.D.) $\Rightarrow a = 20$

$\Rightarrow a^2 + b^2 = 425$

72. (c) $S_1 : x^2 + y^2 = 9 <$ $\begin{matrix} r_1 = 3 \\ A(0,0) \end{matrix}$

$S_2 : (x-2)^2 + y^2 = 1\chi_{B(2,0)}^{r_2=1}$

$\because c_1c_2 = r_1 - r_2$



$\therefore$ given circle are touching internally

Let a veriable circle with centre P and radius

$\Rightarrow PA = r_1 - r$ and $PB = r_2 + r$

$\Rightarrow PA + PB = r_1 + r_2$

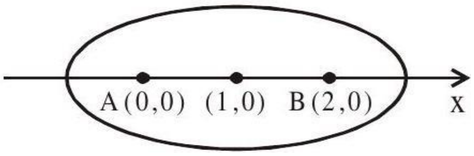
$\Rightarrow PA + PB = 4\ (> AB)$

$\Rightarrow$ Locus of P is an ellipse with foci at A(0,0) and B(2,0) and length of major axis is $2a = 4$,

$e = \frac{1}{2}$

$\Rightarrow$ centre is at (1,0) and $b^2 = a^2(1 - e^2) = 3$

if x-ellipse



$$\Rightarrow E: \frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$$

which is satisfied by $\left(2, \pm\frac{3}{2}\right)$

73. (b) $|\vec{a}| = |\vec{b}|, |\vec{a} \times \vec{b}| = |\vec{a}|, \vec{a} \perp \vec{b}$

$|\vec{a} \times \vec{b}| = |\vec{a}| \Rightarrow |\vec{a}||\vec{b}|\sin 90^\circ = |\vec{a}| \Rightarrow |\vec{b}| = 1 = |\vec{a}|$

$\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors.

Let $\vec{a} = \hat{\imath}, \vec{b} = \hat{\jmath} \Rightarrow \vec{a} \times \vec{b} = \hat{k}$

$$\cos\theta = \frac{(\hat{\imath} + \hat{\jmath} + \hat{k}).\hat{\imath}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

74. (a) $P(X = 1) = {}^5C_1 \cdot p \cdot q^4 = 0.4096$

$P(X = 2) = {}^5C_2 \cdot p^2 \cdot q^3 = 0.2048$

$\Rightarrow \frac{q}{2p} = 2$

$\Rightarrow q = 4p$ and $p + q = 1$

$\Rightarrow p = \frac{1}{5}$ and $q = \frac{4}{5}$

Now

$$P(X = 3) = {}^5C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^2 = \frac{10 \times 16}{125 \times 25} = \frac{32}{625}$$

75. (c) Equation of tangent be

$$\frac{x\cos\theta}{3\sqrt{3}} + \frac{y \cdot \sin\theta}{1} = 1,\ \theta \in \left(0, \frac{\pi}{2}\right)$$

intercept on x-axis

$OA = 3\sqrt{3}\sec\theta$

intercept on y-axis

$OB = \operatorname{cosec}\theta$

Now, sum of intercept

$= 3\sqrt{3}\sec\theta + \operatorname{cosec}\theta = f(\theta)$ let

$f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta$

$= 3\sqrt{3}\dfrac{\sin\theta}{\cos^2\theta} - \dfrac{\cos\theta}{\sin^2\theta}$

$= \underbrace{\dfrac{\cos\theta}{\sin^2\theta}\cdot 3\sqrt{3}}_{\oplus}\left[\tan^2\theta - \dfrac{1}{3\sqrt{3}}\right] = 0 \Rightarrow \theta = \dfrac{\pi}{6}$

$\dfrac{\Theta^{\downarrow},\ \oplus}{\theta = \frac{\pi}{6}}$

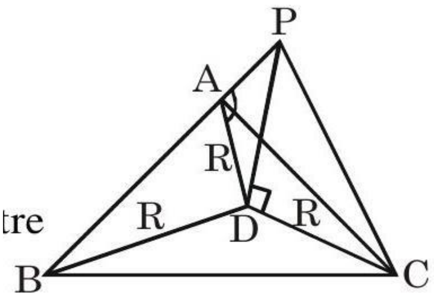
$\Rightarrow$ at $\theta = \frac{\pi}{6}, f(\theta)$ is minimum

76. (c)
77. (b) Let $PD = h, R = 2$

As angle of elevation of top of pole from

A, B, C are equal So

D must be circumcentre of $\triangle ABC$



$\tan\left(\dfrac{\pi}{3}\right) = \dfrac{PD}{R} = \dfrac{h}{R}$

$h = R\tan\left(\dfrac{\pi}{3}\right) = 2\sqrt{3}$

78. (d) $15\sin^4\alpha + 10\cos^4\alpha = 6$

$15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$

$(3\sin^2\alpha - 2\cos^2\alpha)^2 = 0$

$\tan^2\alpha = \frac{2}{3} \cdot \cot^2\alpha = \frac{3}{2}$

$\Rightarrow 27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$

$= 27(\sec^6\alpha)^3 + 8(\operatorname{cosec}^6\alpha)^3$

$= 27(1+\tan^2\alpha)3 + 8(1+\cot^2\alpha)^3$

$= 250$

79. (c)

80. (d) f(x) is continuous at x = 0

$$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x)$$
$$f(0) = b$$
$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(\frac{\sin(a+1)x}{2x} + \frac{\sin 2x}{2x}\right)$$
$$= \frac{a+1}{2} + 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}$$

$$= \lim_{x\to 0^+} \frac{(x+bx^3-x)}{bx^{5/2}\left(\sqrt{x+bx^3}+\sqrt{x}\right)}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{x}\left(\sqrt{1+b^2}+1\right)} = \frac{1}{2}$$

Use (b), (c) & (d) in (a)

$\frac{1}{2} = \text{b} = \frac{\text{a}+1}{2} + 1$

$\Rightarrow \text{b} = \frac{1}{2}, \text{a} = -2$

$a + b = \frac{-3}{2}$

81. (0) $\text{P(x)} = f(\text{x}^3) + \text{xg}(\text{x}^3)$

$\text{P(a)} = f(\text{a}) + \text{g(a)}$

Now P(x) is divisible by $\text{x}^2 + \text{x} + 1$

$\Rightarrow \text{P(x)} = \text{Q(x)}(\text{x}^2 + \text{x} + 1)$

$\text{P(w)} = 0 = \text{P}(\text{w}^2)$ where w, w^2 are non-real cube roots of units

$\text{P(x)} = f(\text{x}^3) + \text{xg}(\text{x}^3)$

$\text{P(w)} = f(\text{w}^3) + \text{wg}(\text{w}^3) = 0$

$f(a) + wg(a) = 2$

$P(w^2) = f(w^6) + w^2\, g(w^6) = 0$

$f(a) + w^2\, g(a) = 0$

(b) + (c)

$\Rightarrow 2f(a) + (w + w^2)g(a) = 0$

$2f(a) = g(a)$

(b) −(c)

$\Rightarrow (w - w^2)g(a) = 0$

$g(a) = 0 = f(a)$ from (d)

from (a) $P(a) = f(a) + g(a) = 0$

82. (6) $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$

$P^2 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$

$P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \Rightarrow P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n$

$\Rightarrow n = 6$

83. (160)

$\sum_{r=1}^{10} r!\{(r+1)(r+2)(r+3) - 9(r+1) + 8\}$

$= \sum_{r=1}^{10} [\{(r+3)! - (r+1)!\} - 8\{(r+1)! - r!\}]$

$= (13! + 12! - 2! - 3!) - 8(11! - 1)$

$= (12.13 + 12 - 8).11! - 8 + 8$

$= (160)(11)\,!$

84. (210)

$\left(\left(x^{1/3} + 1\right) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$\left(x^{1/3} - x^{-1/2}\right)^{10}$

$T_{r+1} = {}^{10}C_r\left(x^{1/3}\right)^{10-r}\left(-x^{-1/2}\right)^r$

$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$

$\Rightarrow r = 4$

$T_5 = {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$

85. (8) Let $p'(x) = a(x-1)(x+1) = a(x^2-1)$

$p(x) = a\int (x^2-1)dx + c$

$= a\left(\frac{x^3}{3} - x\right) + c$

Now $p(-3) = 0$

$\Rightarrow a\left(-\frac{27}{3} + 3\right) + c = 0$

$\Rightarrow -6a + c = 0$

Now $\int_{-1}^{1}\left(a\left(\frac{x^3}{3} - x\right) + c\right)dx = 18$

$= 2c = 18 \Rightarrow c = 9$

$\Rightarrow$ from (a) & (b) $\Rightarrow -6a + 9 = 0 \Rightarrow a = \frac{3}{2}$

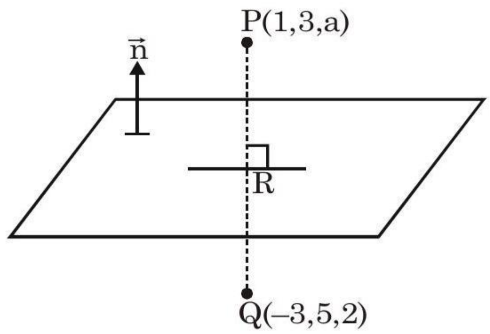
$\Rightarrow p(x) = \frac{3}{2}\left(\frac{x^3}{3} - x\right) + 9$

sum of coefficient

$= \frac{1}{2} - \frac{3}{2} + 9$

$= 8$

86. (1)



plane $= 2x - y + z = b$

$R \equiv \left(-1,4,\frac{a+2}{2}\right) \rightarrow$ on plane

$\therefore -2 - 4 + \frac{a+2}{2} = b \Rightarrow a + 2 = 2b + 12 \Rightarrow a = 2b + 10$

$< PQ > = < 4, -2, a-2 >$

$\therefore \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2}$

$\Rightarrow a - 2 = 2 \Rightarrow a = 4, b = -3$

$\therefore |a + b| = 1$

87. (3) If $f(x + y) = f(x) \cdot f(y) \& f'(0) = 3$ then

$f(x) = a^x \Rightarrow f'(x) = a^x \cdot \ln a$

$\Rightarrow f'(0) = \ell\text{na} = 3 \Rightarrow a = e^3$

$\Rightarrow f(x) = (e^3)^x = e^{3x}$

$\lim_{x\to 0} \frac{f(x) - 1}{x} = \lim_{x\to 0} \left(\frac{e^{3x} - 1}{3x} \times 3\right) = 1 \times 3 = 3$

88. (19) Instead of ${}^nC_k$ it must be ${}^{10}C_k$ i.e.

$\sum_{k=0}^{10} (2^2 + 3k)\ {}^{10}C_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$

$\text{LHS} = 4\sum_{k=0}^{10}\ {}^{10}C_k + 3\sum_{k=0}^{10} k \cdot \frac{10}{k} \cdot\ {}^9C_{k-1}$

$= 4.2^{10} + 3.10.2^9$

$= 19.2^{10} = \alpha.3^{10} + \beta.2^{10}$

$\Rightarrow \alpha = 0, \beta = 19 \Rightarrow \alpha + \beta = 19$

89. (38) Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$

Now $(1, -1, \alpha)$ lies on it so

$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0 \Rightarrow 5\alpha + 38 = 0 \Rightarrow |5\alpha| = 38$

90. (4) $xdy - ydx = \sqrt{x^2 - y^2}dx$

$\Rightarrow \frac{xdy - ydx}{x^2} = \frac{1}{x}\sqrt{1 - \frac{y^2}{x^2}}dx$

$\Rightarrow \int \frac{d\left(\frac{y}{x}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x}$

$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ln|x| + c$

at $x = 1, y = 0 \Rightarrow c = 0$

$y = x\sin(\ell nx)$

$A = \int_1^{e^{\pi}} x\sin(\ell nx)dx$

$x = e^t, dx = e^t dt \Rightarrow \int_0^{\pi} e^{2t}\sin(t)dt = A$

$$\alpha e^{2\pi} + \beta = \left(\frac{e^{2t}}{5}(2\sin t - \cos t)\right)_0^{\pi} = \frac{1 + e^{2\pi}}{5}$$

$\alpha = \frac{1}{5}, \beta = \frac{1}{5}$ so $10(\alpha + \beta) = 4$