1. **(c)** de Broglie wavelength $\lambda = \frac{h}{\sqrt{2mk}}$

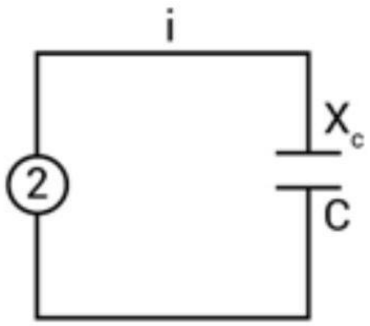
for electron, $\lambda_e = \frac{h}{\sqrt{2m_e k}}$ ... (i)

for proton, $\lambda_p = \frac{h}{\sqrt{2m_p k}}$ ... (ii)

from equation (i) and (ii)

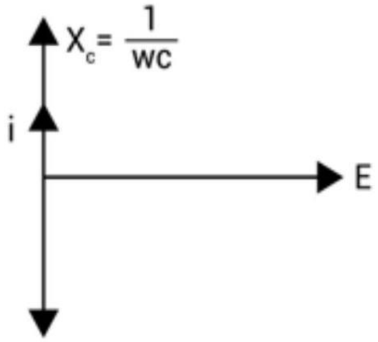
$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$$

2. **(a)**



$$i = \frac{\varepsilon}{x_C}\hat{j}$$

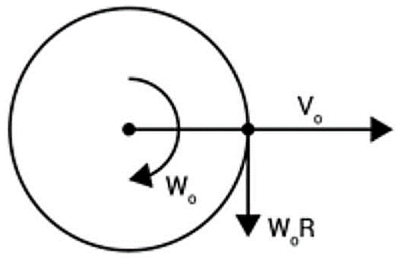
$$i = \frac{\varepsilon_0 \sin \omega t}{\frac{1}{\omega C}}\hat{j}$$



$$i = \varepsilon_0 \omega C \sin \omega \hat{j}$$

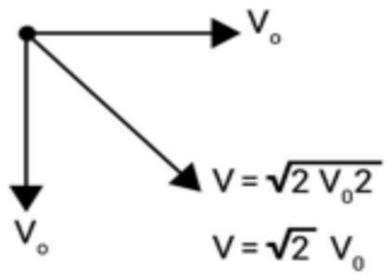
$$i = \varepsilon \omega C \hat{j}$$

3. **(b)**



In pure rolling

$V_0 = \omega_0 R$



Therefore x = 2.

4. (a) $X = A\sin\omega t + B\cos(\omega t)$

$$X = \sqrt{A^2 + B^2}\left\{\frac{A}{\sqrt{A^2 + B^2}}\sin\omega t + \frac{B}{\sqrt{A^2 + B^2}}\cos(\omega t)\right\}$$

$$X = \sqrt{A^2 + B^2}\{\cos\phi \sin\omega t + \sin\phi \cos\omega t\}$$

$$\left[\sin\phi = \frac{B}{\sqrt{A^2 + B^2}}\right.$$

$$\cos\phi = \frac{A}{\sqrt{A^2 + B^2}}$$

$$\tan\phi = \frac{B}{A}$$

$$\phi = \tan^{-1}\left(\frac{B}{A}\right)$$

$$X = \sqrt{A^2 + B^2}\sin(\omega t + \phi)$$

Therefore, $C = \sqrt{A^2 + B^2}, \phi\tan^{-1}\left(\frac{B}{A}\right)$

5. (d) $I = 0.92 w/m^2$

$$I = \frac{1}{2} C \frac{B_0^2}{u_0}$$

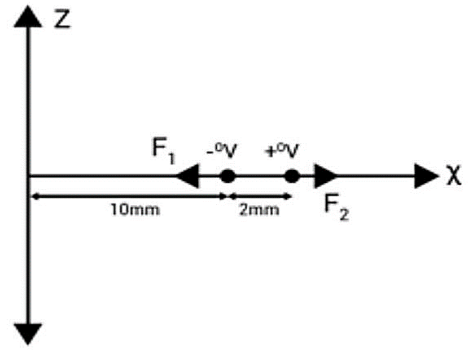
$$0.92 = \frac{1}{2} \times \frac{3 \times 10^8 \times B_0^2}{1.26 \times 10^{-6}}$$

$$B_0 = \sqrt{\frac{0.92 \times 2 \times 1.26 \times 10^{-6}}{3 \times 10^8}}T$$

$$B_0 = 0.877 \times 10^{-7}T$$

$$B_0 = 8.77 \times 10^{-8}T$$

6. (b)



$$\lambda = 4 \times 10^{-5} c / m$$

Electric field due to infinitely long charged wire $\varepsilon = \frac{\lambda}{2\pi\epsilon_0 x}$

$$F_{net} = F_1 - F_2$$

$$4 = \frac{q\lambda}{2\pi\varepsilon_0 \times 10 \times 10^{-3}} - \frac{q\lambda}{2\pi\varepsilon_0 \times 12 \times 10^{-3}}$$

$$4 = q \times \left(\frac{2q}{4\pi\varepsilon_0 10^{-2}} - \frac{2\lambda}{4\pi\varepsilon_0 \times 12 \times 10^{-3}}\right)$$

$$4 = q \times \left(\frac{2 \times 4 \times 10^{-5} \times 2 \times 10^9}{10^{-2}} - \frac{2 \times 4 \times 10^{-5} \times 9 \times 10^3}{12 \times 10^{-3}}\right)$$

$$4 = q \times 12 \times 10^{-6}$$

$$q = \frac{1}{3} \times 10^{-6}\text{C}$$

$$q = 0.33\mu C$$

7. (a)

δ' is apparent dip, let α be the angle made by dip circle with magnetic meridian.

Now rotating dip circle by $90°$ from this position. It will now make angle $90 - \alpha$ with the magnetic meridian. In this case apparent dip is δ''

Now we have relation

$$\cot^2 \delta' + \cot^2 \delta'' = \cot^2 \delta \ldots \text{(i)}$$

Where, δ is true dip in magnetic meridian

From eq (i)

$\cot^2 \delta' < \cot^2 \delta$

or $\cot \delta' < \cot \delta$

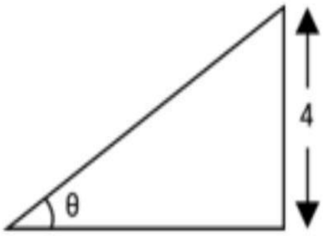
$\Rightarrow \delta' > \delta$

8. **(b)** The center of mass of the uniform semicircular ring is at $\frac{2R}{\pi}$ hence the value of x is 2

9. **(a)**

$$t = \sqrt{\frac{2h}{g\sin^2\theta}\left(1+\frac{k^2}{R^2}\right)}$$

$$t \propto \left(1+\frac{k^2}{R^2}\right)^{1/2}$$



For ring

$$\frac{k^2}{R^2} = 1$$

For solid cylinder

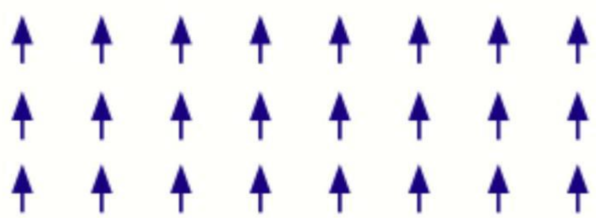
$$\frac{k^2}{R^2} = \frac{1}{2}$$

$$t_1 \propto (1+1)^{\frac{1}{2}}$$

$$t_2 \propto \left(1+\frac{1}{2}\right)^{\frac{1}{2}}$$

$$\frac{t_1}{t_2} = \sqrt{\frac{2}{\frac{3}{2}}} = \sqrt{\frac{4}{3}} \Rightarrow t_1 > t_2$$

10. **(a)**



Ferromagnetic material: All the molecular magnetic dipoles are pointed in the same direction When the ferromagnetic material heated, all magnetic dipoles are disturbed and get disoriented. Due to which the net magnetic dipole moment becomes very weak, so they behave as paramagnetic material.

11. (b) The average kinetic energy of a molecule is given by $\frac{1}{2}fkT$ Where f is degree of freedom For monoatomic gas $f = 3$ so average kinetic energy is $\frac{3}{2}kT$

12. (d)

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$$\vec{\alpha}_1 = \vec{A}\times\vec{B} = i(1-0) - j(1-0) + \hat{k}(1-0)$$
$$= -i - j + k$$
$$\vec{\alpha}_1 = \vec{A}\times\vec{B} = i(1-0) - j(1-0) + \hat{k}(1-0)$$
$$= -i - j + k$$
$$\vec{\alpha}_2 = \vec{B}\times\vec{C} = \begin{vmatrix} 0 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$
$$= \hat{\imath}(-1) - \hat{\jmath}(0+1) + \hat{k}(+1)$$
$$= -i - j + \hat{k}$$

Angle between $\vec{\alpha}_1 \& \vec{\alpha}_2$

$$\vec{\alpha}_1\cdot\vec{\alpha}_2 = (i - j + \hat{k})(-i - j + \hat{k})$$
$$= -1 + 1 + 1$$
$$\vec{\alpha}_1\cdot\vec{\alpha}_2 = 1$$
$$|\alpha_1| = \sqrt{3}$$
$$|\alpha_2| = \sqrt{3}$$

$$\vec{\alpha}_1\cdot\vec{\alpha}_2 = \alpha_1\alpha_2\cos\theta$$

$$\alpha_1\alpha_2\cos\theta = \vec{\alpha}_1\cdot\vec{\alpha}_2$$

$$(\sqrt{3})(\sqrt{3})\cos\theta = 1$$

$$\cos\theta = \frac{1}{3}$$

$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$

13. (a) Range $\mathrm{d} = 1500$ km, Radius of earth $\mathrm{R_e} = 6400$ km

$$d = \sqrt{2hR_e}$$

$$h = \frac{d^2}{2R_e} = \frac{(1500\times10^3)^2}{2\times6400\times10^3} \approx 175\ \text{km}$$

14. ($-60°$) When set at minimum deviation position, $A - 2i - d$

$$i - 45°, d - 30°$$

$$A - 2\times45 - 30 - 60°$$

15. (b) $T_1 = 75°\mathrm{C}, T_2 = 65°\mathrm{C}, t = 5\mathrm{min}, T_0 = 25°\mathrm{C}, T = ?$

$$\frac{T_1 - T_2}{t} = k\left(\frac{(T_1 + T_2)}{2} - T_0\right) \Rightarrow \frac{10}{t} = 45k$$

$$\frac{T_2 - T}{t} = k\left(\frac{(T + T_2)}{2} - T_0\right) \Rightarrow \frac{65 - T}{t} = k\left(\frac{(T + 65)}{2} - 25\right) \ldots$$

From (i) and (ii) $\frac{10}{65-T} = \frac{45}{\left(\frac{(T+65)}{2} - 25\right)}$

on solving for $T, T = 57^{\circ}\mathrm{C}$

16. (a)

$$\alpha = 10^{-6^{\circ}}\mathrm{C}^{-1}, \Delta T = 10^{\circ}\mathrm{C}, Y = 10^{11}\ \mathrm{N/m^2}, A = 0.01\ \mathrm{m^2} = 10^{-2}\ \mathrm{m^2}$$
$$\ell = \alpha L \Delta T \Rightarrow \frac{\ell}{L} = \alpha \Delta T \text{ (strain)}$$
$$= (10 \times 10^{-6})$$
$$E_{\text{per unil length}} = \frac{1}{2} \times Y \times (\text{ strain })^2 \times \text{ area}$$
$$= \frac{1}{2} \times 10^{11} \times (10 \times 10^{-6})^2 \times 10^{-2}$$
$$= 5 \times 10^{-2}\ \mathrm{J/m}$$

17. (c)

$M \rightarrow X + \alpha$

$E = \frac{p^2}{2m}$

Let Total energy = **T**

$T = E_\alpha + E_X$

$M = M_\alpha + M_X$

$M_X = 180$

Conservation of momentum

$p_\alpha = p_X$

$ME_\alpha = M_x E_X$

$E_\alpha = \frac{M_x}{M} T$

$E_\alpha = \frac{184}{180} \times 5.5 = 5.4\mathrm{MeV}$

18. (b) $T = 2\pi\sqrt{\frac{L}{g}}$

$$T' = 2\pi\sqrt{\frac{L'}{g}}$$

$$L' = \frac{L}{16}$$

$$\therefore T' = 2\pi\sqrt{\frac{L}{16g}} = \frac{1}{4}\left(2\pi\sqrt{\frac{L}{g}}\right)$$

$$T' = \frac{T}{4}$$

19. (b) We know that

Projection of $\vec{a}$ on $\vec{b}$ is

$$= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

Given:

$$\vec{A} = \hat{\imath} + \hat{\jmath}$$

$$\vec{B} = \hat{\imath} + \hat{\jmath} + \hat{k}$$

$\therefore$ Projection $= \frac{(\hat{\imath}+\hat{\jmath})\cdot(\hat{\imath}+\hat{\jmath}+\hat{k})}{\sqrt{1^2+1^2+1^2}}$

$$= \frac{1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

20. (b) |Impulse|=|Change in momentum|

Mass of bullet $= 4\text{gm} = 4\times10^{-3}$Kg

Here initial velocity of bullet $= 0$

Here final velocity of bullet $= 50\ \text{m/s}$

$\therefore |\Delta\vec{P}| = m\left|\left(\vec{V}_f - \vec{V}_i\right)\right|$

$= 4\times10^{-3}(50-0)$

$= 2\times10^{-1}\text{kgm/s}$

$= 0.2\text{kgm/s}$

By conservation of linear momentum

$0 = (4\times10^{-3})(50) - 4(V)$

Here V = recoil velocity of gun

$\therefore V = 0.05\ \text{m/s} = 5\ \text{cm/s}$

21. (c) $m=80 \mathrm{~kg}$

$\Delta h=-80 \mathrm{~cm}=-0.8 m$ [height decreased]

$g=10 \mathrm{~m} / \mathrm{s}^2$

Work done=Change in potential energy

[K.E. doesn't change]

$W=\Delta U$

$=m g(\Delta h)$

$=80(10)(-0.8)$

$=-640 \mathrm{~J}$

22. (a) Total energy of body will remain zero

T.E $=0$

At a distance ' h ' from surface of earth

M=mass of earth

$M=$ mass of body

R $=$ Radius of earth

$\frac{1}{2} m v^2-G \frac{M m}{h}=0$

$v=\sqrt{\frac{2 G M}{h}}$

$\frac{d h}{d t}=\sqrt{\frac{2 G M}{h}}$

$\Rightarrow \sqrt{h} d h=\sqrt{2 G M} d t$

$\Rightarrow \int_R^{(R+h)} \sqrt{h} d h=\sqrt{2 G M} \int_0^t d t$

$\frac{2}{3}\left[h^{3 / 2}\right]_R^{R+h}=\sqrt{2 G M t}$

$\Rightarrow \frac{\frac{2}{3}\left[(R+h)^{3 / 2}-R^{3 / 2}\right]=t}{\sqrt{2 G M}}$

$\Rightarrow t=\frac{2}{3} \frac{\left[(R+h)^{3 / 2}-R^{3 / 2}\right]}{3 \sqrt{2 G M}}$

23. (15)

24. (b) As temperature increases, solubility of gas decreases.

25. (a)

$SF_4 - sp^3\ d$

$IF_5 - sp^3\ d^2$

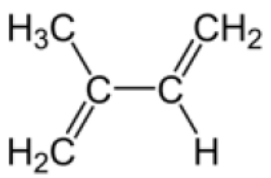
$NO_2{}^+ - sp$
$NH_4{}^+ - sp^3$

26. (b) $\underset{78\ g}{C_6H_6}\underset{AlCl_3}{CH_3Cl}C_6H_5 - CH_{3\,2\,g}$

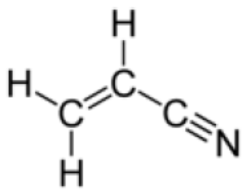
10 g benzene gives $= \frac{92}{78} \times 10$ g

% yield $= \frac{9.2}{920/78} \times 100 = 78\%$

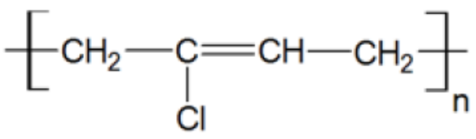
27. (d)



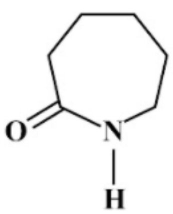
$\Rightarrow$ isoprene



$\Rightarrow$ acrylonitrile



$\Rightarrow$ neoprene



$\Rightarrow$ Caprolactam

28. (c) Half-life of tritium is 12.33 yrs

29. (b) When $AgNO_3$ is added to KI solution, Ag + is an excess so we get accumulation of Ag^+ions outside Agl, giving positively charged sol $AgNO_3 + KI \rightarrow AgI + KNO_3$

30. (b)

31. (d) $K_2SO_4 \rightarrow 2\ K^+ + SO_4{}^{2-}$ ......i = 3

$KI \rightarrow K^+ + I^-$ ......i = 2

$Al_2(SO_4)_3 \rightarrow 2Al^{3+} + 3SO_4{}^{2-}$ ......i = 5

$C_6H_{12}O_6 \rightarrow$ No dissociation i = 1

More the value of i, greater will be depression in freezing point

32. (d)

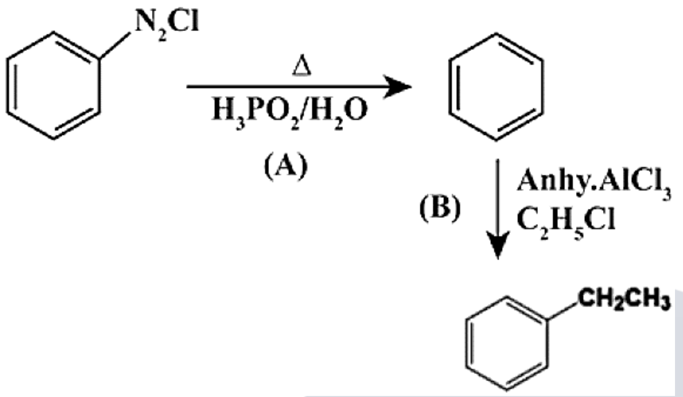
33. (a) Thiamine: Vitamin B_1

Pyridoxine: Vitamin B_6

34. (c) E-H bond energy decreases down the group. So, best reducing agent in group 15 is BiH_3

35. (c) $[Co(NH_3)_6]Cl_2$ contains 1 unpaired electron while $[Co(NH_3)_6]Cl_3$ contains 0 unpaired electron

36. (a)



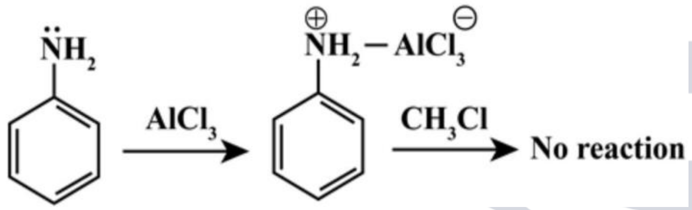
37. (b) Mn^{2+} and Cu^{2+} both are paramagnetic as well as coloured

Mn^{2+} – 2 unpaired electrons and light pink colour

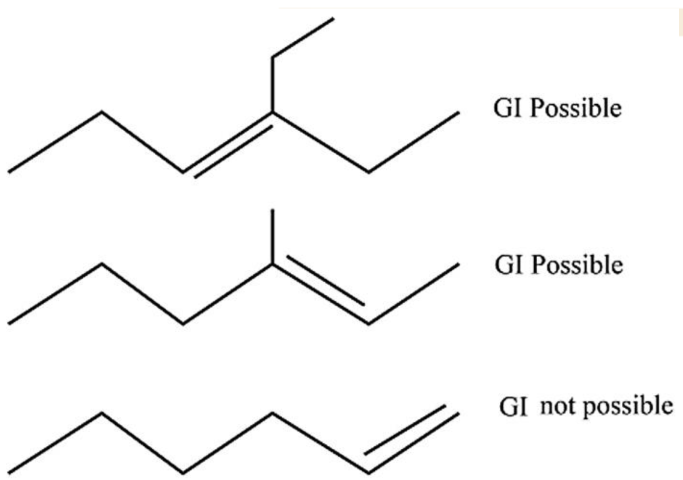
Cu^{2+} – 1 unpaired electrons and blue colour

38. (a) It is non-planar molecule in which B atoms are sp^3 hybridised. Molecule contains two 3c-2e bonds

39. (b) Lone pair of $\ddot{N}H_2$ combines with $AlCl_3$ due to which further reaction is not possible



40. (c)



41. (0.004) Molarity $= \frac{w}{M \times V}$

M of glucose $= 180$ g/mol

Molarity $= \frac{0.72}{180} = 0.004\text{M}$

42. (12.00)

${}_{23}V = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^3$

Total p electrons $= 12$

43. (2.00)

$\text{KP} = \text{K}_\text{C}(\text{RT})^{\Delta\text{ng}}$

$47.6 = \text{K}_C[0.0821 \times 288]$

$\text{K}_\text{C} = \frac{47.6}{23.65} = 2$

44. (d) $\sin^7 x + \cos^7 x = 1$

If $\sin x \neq 0$ and $\cos x \neq 0$

$\sin^7 x < \sin^2 x$ and $\cos^7 x < \cos^2 x$

Adding both we get

$\sin^7 x + \cos^7 x < \sin^2 x + \cos^2 x$

$\Rightarrow \sin^7 x + \cos^7 x < 1$

Hence $\sin^7 x + \cos^7 x = 1$ is only possible

When $\sin x = 1$ and $\cos x = 0$

or $\cos x = 1$ and $\sin x = 0$

$\Rightarrow x = 0, \frac{\pi}{2}, \frac{5\pi}{2}, 2\pi, 4\pi$

45. (d) $S_{10} = 5[2a + 9d] = 530 \Rightarrow 2a + 9d = 106$

$S_5 = \frac{5}{2}[2a + 4d] = 140 \Rightarrow 2a + 4d = 56$

$\Rightarrow d = 10$ and $a = 8$

$\therefore s_{20} - s_6 = 10[2a + 19d] - 3[2a + 5d]$

$= 14a + 175d = 1862$

46. (b) $f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$

$0 < \sin^{-1}\left(\frac{2x-1}{2}\right) \leq \frac{\pi}{2}$

$0 < \frac{2x-1}{2} \leq 1$

$0 < 2x - 1 \leq 2$

$1 < 2x \leq 3$

$\frac{1}{2} < x \leq \frac{3}{2}$

$x^2 - x + 1 = x^2 - x + \frac{1}{4} - \frac{1}{4} + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$

Also, $x^2 - x + 1 = x^2 - x + \frac{1}{4} - \frac{1}{4} + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$
$x^2 - x + 1 \leq 1$

$x^2 - x \leq 0$

$x(x-1) \leq 0$

$x \in [0,1]$

From (1) and (2)

$x \in \left(\frac{1}{2}, 1\right]$

$\alpha = \frac{1}{2}, \beta = 1$

$\alpha + \beta = \frac{1}{2} + 1 = \frac{3}{2}$

47. (96) $f(x) = \left(\frac{10}{11}\right)^x + \left(\frac{9}{11}\right)^x$

$f'(x) = \left(\frac{10}{11}\right)^x \ln^{\frac{10}{11}} + \left(\frac{9}{11}\right) \ln^{\frac{8}{11}} < 0$

$f(x)$ is decreasing

$f(4) > 1$ and $f(5) < 1$

Hence $x = 1,2,3,4$ will not satisfy

$x = 5,6,7, \ldots, 100$ will satisfy

Number of integers $= 96$

48. (96) Number of numbers formed $= 4 \times 4 \times 3 \times 2 \times 1 = 96$

49. $(\mathbf{5^5})$

Let $B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because AB = BA$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow d = b, e = a, f = c, g = h$$

So, i can be selected in 5 ways and each pair by 5 , so total ways $= 5^5$

50. $\mathbf{\frac{80}{81}}$

Let $a = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

For A to be singular, $ad = bc$

$\therefore a, d = 1{,}6$ and $b, c = 3{,}2 \Rightarrow 8$

$a, d = 2{,}6$ and $b, c = 4{,}3 \Rightarrow 8$

So, total case of singular matrix $= 8 + 8 = 16$

$\therefore$ Total non-singular matrix $= 6 \times 5 \times 4 \times 3 - 16$

$\therefore$ Probability $= \frac{344}{360} = \frac{43}{45}$

51. $\boldsymbol{e = \frac{\sqrt{5}-1}{2}}$

52. (100,181)

$$36x^2 + 36y^2 - 108x + 120y + C = 0$$

$$x^2 + y^2 - \frac{108x}{36} + \frac{120y}{36} + \frac{C}{36} = 0$$

$$x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$$

Circle does not touch or cut axes

$\Rightarrow g^2 - C < 0$ and $f^2 - C < 0$

$\Rightarrow \left(\frac{3}{2}\right)^2 - \frac{C}{36} < 0$ and $\left(\frac{10}{6}\right)^2 - \frac{C}{36} < 0$

$\Rightarrow \frac{9}{4} - \frac{C}{36} < 0$ and $C > 36\left(\frac{100}{36}\right)$

$\Rightarrow C > \frac{36\times5}{4}$ and $\Rightarrow C > 100$

$\Rightarrow C > 81$

Also $g^2 + f^2 - C > 0$

$\Rightarrow \left(\frac{3}{2}\right)^2 + \left(\frac{3}{3}\right)^2 - \frac{C}{36} > 0$

$\Rightarrow \frac{C}{36} < \frac{9}{4} + \frac{25}{9}$

$\Rightarrow C < 81 + 100$

$\Rightarrow C < 181$

From (1), (2) and (3)

$C \in (100,181)$

53. $(\mathbf{3}! \times \mathbf{7}\ !)$

$f(1) + f(2) + f(3) = 3$

Only possible combination is $0 + 1 + 2$

So total bijective function $= 3! \times 7\ !$

54. **(a)** $2x + y = k$ is tangent to $x^2 - y^2 = 3$

$y = -2x + k$

$c^2 = a^2m^2 - a^2$

$\Rightarrow k^2 = 3(-2)^2 - 3$

$\Rightarrow k^2 = 9$

$\Rightarrow k = \pm 3$

$y = -2x + 3$ and $y = -2x - 3$

If $y = mx + c$ is tangent to $y^2 = 4ax$ then $c = \frac{a}{m}$

$\Rightarrow 3 = \frac{\alpha}{4(-2)}$ or $-3 = \frac{\alpha}{4(-2)}$

$\Rightarrow \alpha = -24$ or $\alpha = 24$

55. **(1/2)**

$z = x + iy$

$(x + iy)^2 + 3(x - iy) = 0$

$x^2 - y^2 + 2ixy + 3x - 3iy = 0$

$x^2 - y^2 + 3x + i(2xy - 3y) = 0$

$\Rightarrow x^2 - y^2 + 3x = 0$ and $y(2x - 3) = 0$

If $y = 0\ x^2 + 3x = 0 \Rightarrow y = 0$ or $x = \frac{3}{2}$

$\Rightarrow x = 0, -3$

And if $x = \frac{3}{2}$

$y^2 = \left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right)$

$= \frac{9}{4} + 3\left(\frac{3}{2}\right) = \frac{9}{4} + \frac{9}{2}$

$= \frac{27}{4}$

Hence 3 solutions

$\sum_{k=0}^{1} \frac{1}{3^k} = \frac{1}{3} + \frac{1}{3^2} + \cdots \infty$

$= \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2}$

56. (c)

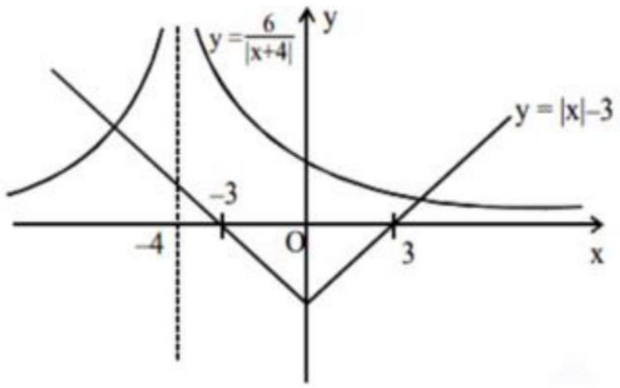
$r_1, r_2, r_3 \in \{1,2,3,4,5,6\}$

r_1, r_2, r_3 are of the form $3k, 3k+1, 3k+2$

Required Probability $= \frac{3! \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{6 \times 6 \times 6} = \frac{6 \times 8}{216} = \frac{2}{9}$

57. (c)

$x \neq -4$
$(|x| - 3)(|x + 4|) = 6$
$\Rightarrow |x| - 3 = \frac{6}{|x + 4|}$



Number of solutions $= 2$

58. (2/9)

$I = \int \frac{e^x(2 - x^2)}{1 - \sqrt{1 - x^2}}$

$$= \int e^x \left(\frac{1}{(1-x)\sqrt{1-x^2}} + \frac{\sqrt{1+x}}{\sqrt{1-x}} \right) dx$$

Let $\sqrt{\frac{1+x}{1-x}} = f(x)$

Then $f'(x) = \frac{1}{(1-x)\sqrt{1-x^2}}$

$\Rightarrow I = \int e^x (f(x) + f'(x)) dx$

$= e^x f(x) + c$

$$\Rightarrow I = e^x \sqrt{\frac{1+x}{1-x}} + c$$

$r_1, r_2, r_3 \in \{1,2,3,4,5,6\}$

r_1, r_2, r_3 are of the form $3k, 3k+1, 3k+2$

Required Probability $= \frac{3! \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{6 \times 6 \times 6} = \frac{6 \times 8}{216} = \frac{2}{9}$

59. (c) $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}} dx = 100 \int_0^{\pi} \frac{\sin^2 x}{e^{\frac{x}{\pi}}} dx$

Let $\frac{x}{\pi} = t \Rightarrow dx = \pi dt$

$I = 100\pi \int_0^1 e^{-t} \sin^2 \pi t dt = 50\pi \int_0^1 (1 - \cos 2\pi t) e^{-t} dt$

$$I = 50\pi \left[(-e^{-t})^1 - \left(\frac{e^{-t}}{1+4\pi^2} \{-\cos 2\pi t + 2\pi \sin 2\pi t\} \right)_0^1 \right]$$

$$= 50\pi \left[\left(1 - \frac{1}{e}\right) - \frac{1}{(1+4\pi^2)} \left\{ \frac{-1}{e} + 1 \right\} \right]$$

$$= 50\pi \left(1 - \frac{1}{e}\right) \left(\frac{4\pi^2}{1+4\pi^2} \right) = \frac{200\pi^3}{1+4\pi^2} (1 - e^{-1})$$

$\therefore \alpha = 200(1 - e^{-1})$