1. **(a)** $\frac{1}{2}m\omega^2(A^2-x^2)=\frac{3}{4}\left(\frac{1}{2}m\omega^2A^2\right)$

$$\Rightarrow A^2-x^2=\frac{3}{4}A^2$$

$$\Rightarrow x^2=\frac{A^2}{4}$$

$$\Rightarrow x=\frac{A}{2}$$

2. **(a)** $\frac{Q}{A}=\frac{Q_0e^{-t/RC}}{A_0e^{-\lambda t}}$ (ratio is time independent)

$\lambda=\frac{1}{RC}$, $RC=\frac{1}{\lambda}=T_{\text{mean}}$ (mean life)

$$R=\frac{T_{\text{mean}}}{C}=\frac{30\times10^{-3}}{100\times10^{-6}}=300\Omega$$

3. **(d)** For collision between A and B

$m_A=m, m_B=2m, u_A=9m/s, u_B=0, v_B=?$

$$v_B=\frac{2m_Au_A}{m_A+m_B}-\left(\frac{m_A-m_B}{m_A+m_B}\right)u_B$$
$$=\frac{2\times m\times 9}{m+2m}-0=6m/s$$

For collision between B and C

$m_B=2m, m_C=2m, u_B=6m/s, u_C=0, v=?$

$m_Bu_B+m_Cu_C=(m_B+m_C)v$

$2m\times6+0=4mv$

$$\Rightarrow v=\frac{12}{4}=3\text{ m/s}$$

4. **(a)** $\frac{\lambda_2}{\lambda_1}=\frac{h/p}{h/mv}=\frac{mv}{p}=\frac{9.1\times10^{-31}\times10^{6}}{10^{-27}}=910$

5. **(b)** Moment of inertia of a disc about it's center perpendicular to the plane of the disc is $\frac{MR^2}{2}$

& Moment of inertia of a disc about one of its diameter is $\frac{MR^2}{4}$

Now both dices have same surface mass density (σ)(say)

then mass of 1st discs with radius $r=(\pi r^2)\sigma$

$\therefore$ M.I of disc 1 about center, $\perp$ to the plane $=\frac{mr^2}{2}=(\pi r^2)\sigma\frac{r^2}{2}$

$$=\frac{\pi\sigma r^4}{2}=I_1$$

mass of 2nd disc with radius $R = (\pi R^2)\sigma$

$\therefore$ M.I of 2nd disc about one of it's diameters is $\frac{mR^2}{4}$

$$= (\pi R^2)\sigma \frac{R^2}{4} = I_2 = \frac{\pi R^4 \sigma}{4}$$

$$\frac{I_1}{I_2} = \frac{\left(\frac{\pi \sigma r^4}{2}\right)}{\frac{\pi \sigma R^4}{4}}$$

$$= \frac{2r^4}{R^4}$$

6. **(b)** Both are in parallel, so p.d across them will be same.

$$Q_1 = C_1 V = (2C)V = 2CV$$

$$Q_2 = C_2 V = CV$$

Total charge = 3CV

When dielectric is inserted in it's capacitance becomes KC.

If V′ is final common potential, then since total charge remains unchanged

$$Q_{1f} + Q_{2f} = Q_1 + Q_2 = 3CV$$

$$2C(V') + (KC)V' = 3CV$$

$$V' = \left(\frac{3V}{2+k}\right)$$

7. **(a)** No, of molecules = no of moles $\times N_A$

$$= n \times N_A$$

$$n = \frac{PV}{RT} = \frac{PV}{N_A kT}$$

Given avg. kinetic energy $= 2 \times 10^{-9}$ J

$$\therefore \frac{3}{2}kT = 2 \times 10^{-9}$$

$$\therefore kT = \frac{4 \times 10^{-9}}{3}$$

Now given 2 atm pressure

$$\therefore P = 2 \times 1.013 \times 10^5 \text{ N/m}^2$$

Volume = 1 litre $= 10^{-3}$ m^3

No of molecules $= \frac{PV}{N_A kT} \times N_A$

$= \frac{PV}{kT}$

$= \frac{2 \times 1.013 \times 10^5 \times 10^{-3}}{4/3 \times 10^{-9}}$

$\approx \frac{3}{2} \times 10^{11}$

8. (d) $A \rightarrow B$ and $D \rightarrow C$ are isotherms

So. $T_A = T_B \& T_C = T_D$

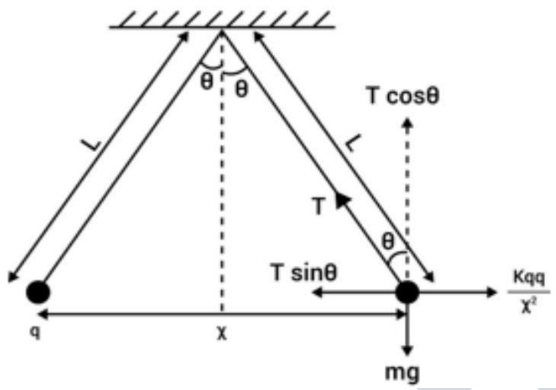
Now, $B \rightarrow C$ and $D \rightarrow A$ are adiabatic

$\therefore |W_{BC}| = \frac{nR}{r-1}(T_B - T_C)$

$|W_{AD}| = \frac{nR}{r-1}(T_A - T_D) - \frac{nR}{r-1}(T_B - T_C)$

$\therefore W_{BC} = W_{AD}$

9. (b)



$T \sin\theta = \frac{Kq^2}{x^2}$

$T \cos\theta = mg$

From eq (i) and (ii)

$\tan\theta = \frac{Kq^2}{x^2} \cdot \frac{1}{mg}$

$x = \sqrt{\frac{Kq^2}{mg \tan\theta}}$

$x = q\sqrt{\frac{\cot\theta}{4\pi\varepsilon_0 mg}}$

10. (b) $V = V_0\left(1 - e^{-\frac{t}{RC}}\right)$

$50 = 100\left(1 - e^{-\frac{t}{RC}}\right)$

$\frac{1}{2} = 1 - e^{-\frac{t}{RC}}$

$e^{-\frac{t}{RC}} = \frac{1}{2}$

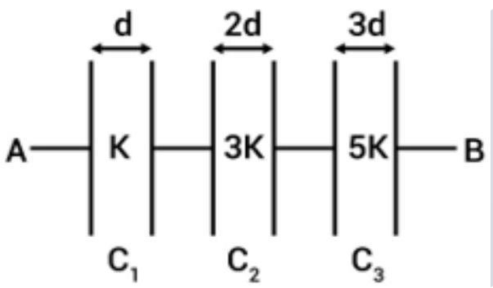
$\frac{t}{RC} = \ln 2$

$t = RC\ell n2$

$t = 100 \times 1 \times 10^{-6} \times 0.6$

$t = 60\mu s$

11. (c)



$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$\frac{1}{C_{eq}} = \frac{d}{\varepsilon_0 AK} + \frac{2d}{\varepsilon_0 A3K} + \frac{3d}{\varepsilon_0 A5K}$

$\frac{1}{C_{eq}} = \frac{d}{\varepsilon_0 AK}\left\{1 + \frac{2}{3} + \frac{3}{5}\right\}$

$\frac{1}{C_{eq}} = \frac{d}{\varepsilon_0 AK} \times \frac{34}{15}$

$C_{eq} = \frac{15\varepsilon_0 AK}{34d}$

12. (a) Young's modulus $Y = 0.5 \times 10^3$pa

$\ell = 0.1$ m

$A = 0.04 \times 10^{-4}$ m^2

$\Delta\ell = 0.001$ m

$m = 20g = 20 \times 10^{-3}$

Energy stored in the string due to extension $= \frac{1}{2}\sigma\varepsilon \times$ volume

$= \frac{1}{2} Y \varepsilon^2 \cdot$ volume

From energy conservation

$\frac{1}{2} m u^2 = \frac{1}{2} Y \varepsilon^2 \times$ volume

$$20 \times 10^{-3} \times u^2 = 0.5 \times 10^9 \times \left(\frac{0.001}{0.1}\right)^2 \times 0.04 \times 10^{-4} \times 0.1$$
$$2 \times 10^{-2} u^2 = 2 \times 10^{-2}$$
$$u^2 = 1$$
$$u = 1 \text{ m/s.}$$

13. (b) Fringe width $\beta = \frac{\lambda D}{d}$

$\lambda_{\text{orange}} > \lambda_{\text{blue}}$

$\beta_{\text{orange}} \propto \lambda_{\text{orange}}$

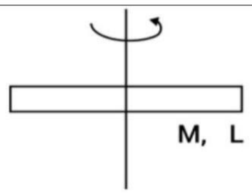
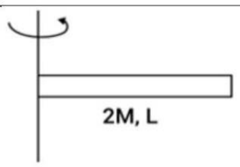
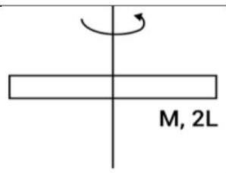
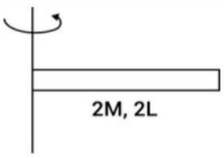
$\beta_{\text{blue}} \propto \lambda_{\text{blue}}$

From (i) and (ii)

$\frac{\beta_{\text{blue}}}{\beta_{\text{orange}}} = \frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}}$

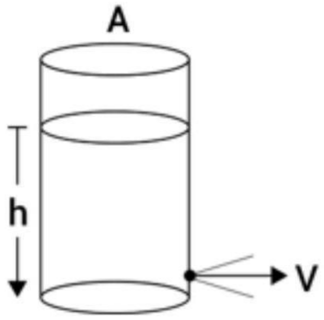
$\beta_{\text{blue}} < \beta_{\text{orange}}$.

14. (a)

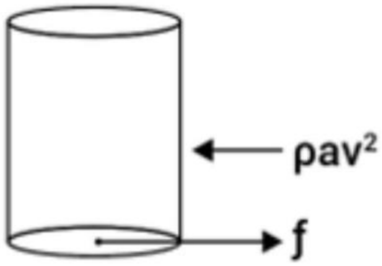
A	 $\rightarrow \frac{ML^2}{12}$
B	 $\rightarrow \frac{2ML^2}{3} = 2\frac{ML^2}{3}$
C	 $\rightarrow \frac{M(2L)^2}{12} = \frac{ML^2}{3}$
D	 $\rightarrow \frac{2M \cdot (2L)^2}{3} = \frac{8ML^2}{3}$

A(iv), B(iii), C(ii), D(i)

15. (a)



$v = \sqrt{2gh}$ as a < A



Thrust on container is $\rho a v^2$

where

$\rho \to$ density

$a \to$ area of cross section of orifice

$v \to$ speed of fluid

$f \to$ force of friction.

$f \leq f_L$

$f \leq \mu mg$

$f \leq \mu \rho A h \cdot g$

For

$\rho a v^2 \leq \rho A h g \times \mu$

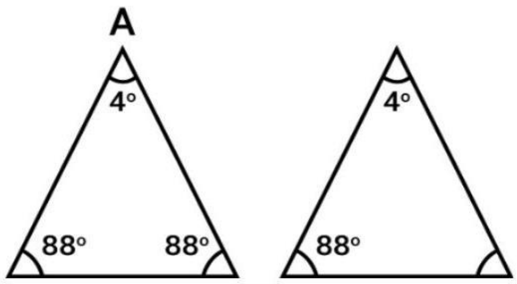
Container does not move

$\rho a \times 2gh \leq \rho A h g \times \mu$

$\frac{2a}{A} \leq \mu$

$\mu_{\min} = \frac{2a}{A}$

16. (b)



$\mu_1 = 1.2 + \dfrac{10.8 \times 10^{-14}}{\lambda^2}$

$\mu_2 = 1.45 + \dfrac{1.8 \times 10^{-14}}{\lambda^2}$

$\delta_1 + \delta_2 = 0$ for the system.

$\delta_1 = (\mu_1 - 1)A_1$

$\delta_2 = (\mu_2 - 1)A_2$

$\delta_1 + \delta_2 = \mu_1 A_1 - A_1 + \mu_2 A_2 - A_2 = 0$

$\left|\dfrac{(\mu_1 - 1)}{(\mu_2 - 1)}\right| = \dfrac{A_2}{A_1}$

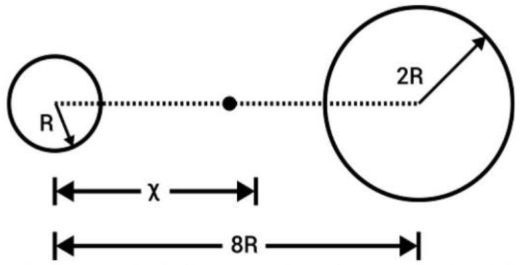
$$\frac{1.2 + \dfrac{10.8 \times 10^{-14}}{\lambda^2} - 1}{1.45 + \dfrac{1.8 \times 10^{-14}}{\lambda^2} - 1} = +\frac{A_2}{A_1}$$

$$\frac{0.2 + \dfrac{10.8 \times 10^{-14}}{\lambda^2}}{0.45 + \dfrac{1.8 \times 10^{-14}}{\lambda^2}} = +\frac{A_2}{A_1}$$

on solving

$\lambda = 600$ nm

17. (4)



We have to find the point where the gravitational field must be zero.This is because after that point particle itself pulled by greater mass, no need to give any amount of kinetic energy.

$E_G = 0$

$$\frac{GM}{x^2} = \frac{GM \times q}{(8R - x)^2}$$

$$\frac{(8R - x)^2}{x^2} = 9$$

$$\frac{8R - x}{x} = 3 \Rightarrow 8R - x = 3x$$

$$x = 2R$$

Potential at A

$$V_A = -\frac{GM}{R} - \frac{G9M}{7R} = -\frac{16GM}{7R}$$

Potential at point x distance apart from A

$$V_x = -\frac{GM}{2R} - \frac{G9M}{6R} = -\frac{12GM}{6R}$$

$$= -\frac{2GM}{R}$$

Potential difference is

$$\Delta V = V_x - V_A = -\frac{2GM}{R} + \frac{16GM}{7R} = \frac{2GM}{7R}$$

Applying conservation of energy principle

$$\Delta KE = \Delta U$$

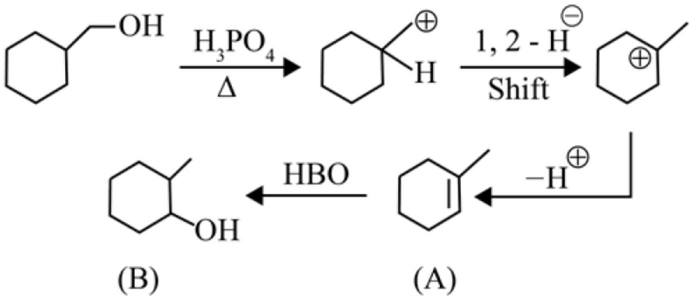
$$\frac{1}{2}mV^2 = m\Delta V$$

$$\frac{1}{2}mV^2 = \frac{2GMm}{7R}$$

$$V = \sqrt{\frac{4GM}{7R}}$$

$$a = 4$$

18. (b)



19. (a) Let the volume of solution = 1000 cm^3

$\therefore$ Weight of solution $= 1200$ g

Now, weight of water in the solution $= 1000$ g

Thus, weight of NaOH $= 200$ g

$\therefore$ Moles of NaOH $= \frac{200}{40} = 5$

Thus, molarity $= \frac{5}{1} = 5M$

20. **(b)** Electron is removed from anti-bonding of NO (bond order $= 2.5$) hence bond order increases by 0.5 hence NO^+has bond order 3 and so does CO. The difference is zero

21. **(b)** $5HI + \text{Conc. } HIO_3 \rightarrow 3I_2 + 3H_2O$

22. **(a)** Ellingham diagram does not give any information about rate of reaction

23. **(a)** $H_4P_2O_7 : 4 + 2x - 14 = 0$

$\Rightarrow 2x - 10 = 0$

$\Rightarrow 2x = 10$

$\Rightarrow x = +5$

$H_4P_2O_6 : \Rightarrow 4 + 2x - 12 = 0$

$\Rightarrow 2x - 8 = 0$

$\Rightarrow 2x = 8$

$\Rightarrow x = +4$

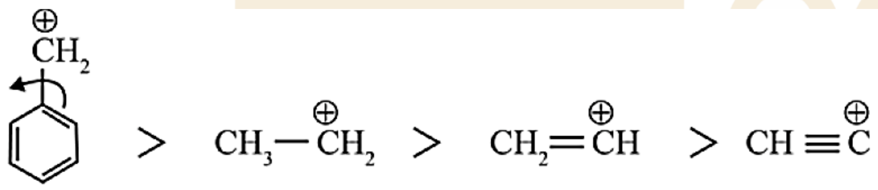
$H_4P_2O_5 : 4 + 2x - 10 = 0$

$\Rightarrow 2x - 6 = 0$

$\Rightarrow 2x = 6$

$\Rightarrow x = +3$

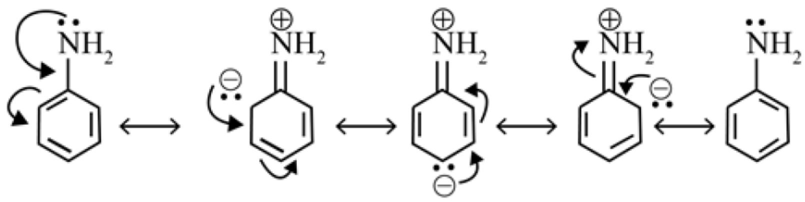
24. **(a)**



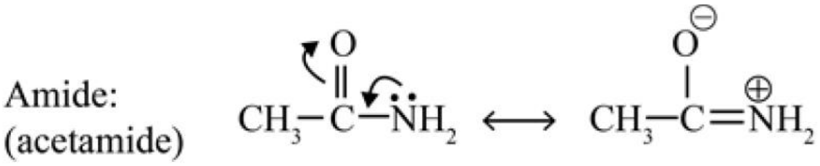
(i) +I and +H sp^2 sp

Resonance 3αH

25. **(c)** Acetamide is less basic then aniline because its lone pair of Nitrogen atoms involved in resonance with oxygen atom hence it is less available to donate



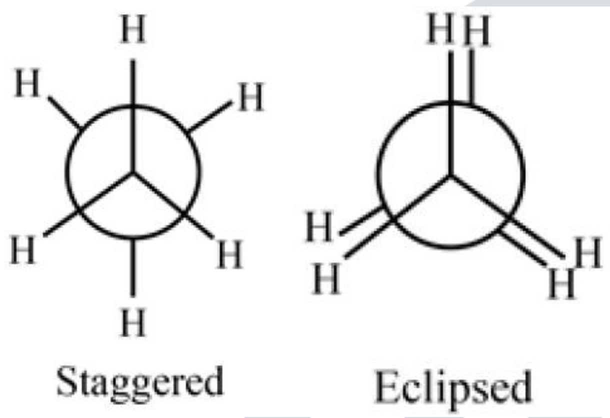
aniline



26. **(b)** Lithium halides are somewhat covalent. It is because of the high polarisation capability of lithium ion (The distortion of electron cloud of the anion by the cation is called polarisation). The Li^{+}ion is very small in size and has high tendency to distort electron cloud around the negative halide ion. Since anion with large size can be easily distorted, among halides, lithium iodide is the most covalent in nature. Polarisability is defined for anion

27. **(a)** Both given statement are correct.

28. (a) Rotamers are any of a number of isomers of a molecule which can be inter converted by rotation of part of the molecule about a particular bond.



Staggered and Eclipsed form of ethane are inter convertible by rotation of the molecule about a bond. Hence these are rotamers

29. **(c)** $PCl_5 \rightleftharpoons PCl_3 + Cl_2$

3 __ ___

$3 - x \; x \; x$

$\frac{x^2}{3-x} = 1.844$

$\Rightarrow x^2 = 5.532 - 1.844x$

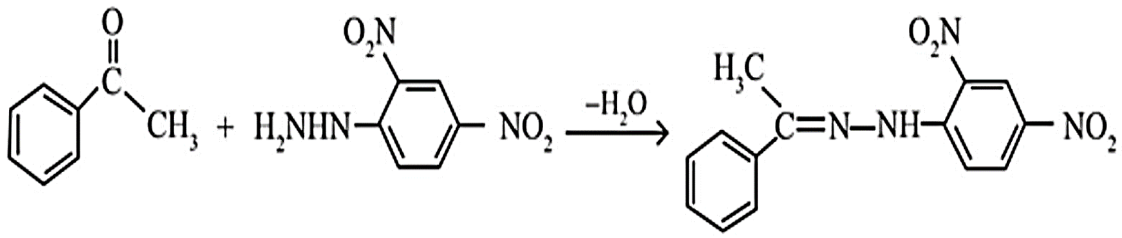
$\Rightarrow x^2 + 1.844x - 5.532 = 0$

$\Rightarrow x = 1.6M$

$\therefore$ Concentration of PCl_5 at equilibrium $= 3 - x = 3 - 1.6 = 1.4M$

30. **(b)** Decomposition of N_2O_5 is an example of first order reaction

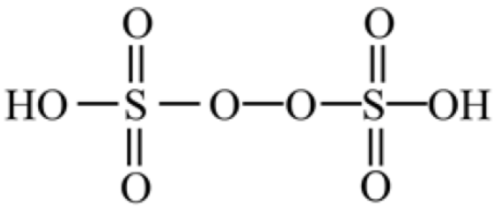
31. **(a)** 2,4-dinitrophenyl hydrazine solution is used to detect ketones and aldehydes. A positive test is confirmed by the formation of a yellow, orange or red precipitate.



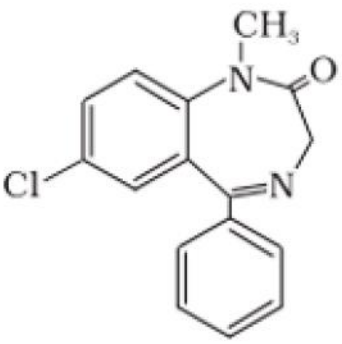
32. (a)

$2SO_4^{2-} \overset{\text{electrolysis}}{\rightarrow} S_2O_8^{2-} + 2e^-$

$H_2\ S_2O_8$:

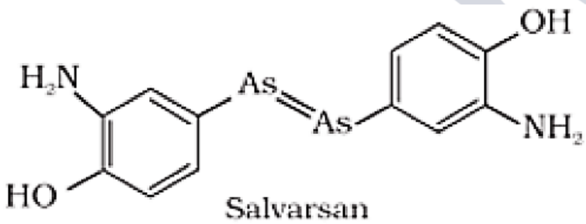


33. (a)



Valium

Tranquilizer



Salvarsan

Arsphenamine, known as a salvarsan used as Antibiotic

Antiseptics are applied to the living tissues such as wounds, cuts, ulcers and diseased skin surfaces. Examples are furacine, soframicine.

34. (b) Eutrophication is the process in which a water body becomes overly enriched with nutrients, leading to plentiful growth of simple plant life. The excessive growth (or bloom) of algae and plankton in a water body are indicators of this process. The excessive growth of algae in entrap, water is accompanied by the generation of a lame biomass of dead algae. These dead algae sink to the bottom of the water body where they are broken down by bacteria, which consume oxygen in the process.

35. (a) Given

$\alpha = \beta = 90^\circ, \gamma \neq 120^\circ$

$a \neq b \neq c$

Thus, crystal system is Monoclinic

36. (a) DNA contains four base adenine(A), guanine (G), cytosine (C) and thymine (T). RNA also contains four bases, the first bases are same as in DNA but the fourth one is uracil (U).

37. (a) $Be(OH)_2 \Rightarrow$ Amphoteric

$B(OH)_3 \Rightarrow$ Acidic

$NaOH \Rightarrow$ Basic

$Ca(OH)_2 \Rightarrow$ Basic

$Al(OH)_3 \Rightarrow$ Amphoteric

38. (c) Barfoed's test distinguishes monosaccharides from disaccharides. In this test, copper acetate in dilute acid is reduced in 30 seconds by monosaccharides whereas disaccharides take several minutes.

39. (3.00)

$[Co(NH_3)_3(NO_2)_3 \Rightarrow Ma_3\ b_3$

Total geometric isomerism $= 2$

$Cr(C_2O_4)_3 \Rightarrow M(AA)_3$

Total geometric isomerism $= 1$

Thus, total $= 3$

Please note that $Cr(C_2O_4)_3$ can exist in one form only and examiner might not consider it as geometric isomerism

40. (17.00)

$$\frac{(x/m)_1}{(x/m)_2} = \left(\frac{p_1}{p_2}\right)^{1/n}$$
$$\Rightarrow \frac{1}{64} = \left(\frac{1}{2}\right)^{1/n}$$
$$\Rightarrow \frac{1}{n} = 6$$
$$\Rightarrow n = \frac{1}{6} = 0.167 = 16.7 \times 10^{-2} = 17$$

41. (111.00) Triethylamine is a base, like ammonia. Also like ammonia, it has a trigonal pyramidal structure. The C-N-C bond angle is 110.9°, compared with 107.2° in NH_3, presumably due to greater repulsion between the ethyl groups

42. (a)

$$\lim_{x\to 2}\frac{x^2f(2)-4f(x)}{x-2}$$
$$=\lim_{x\to 2}\frac{2xf(x)-4f'(x)}{1}$$
$$=2\times 2f(2)-4f'(2)$$
$$=2\times 2\times 4-4\times 1$$
$$=16-4=12$$

43. (a) Coeff. of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ = coeff. of x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$

$${}^{11}C_6\frac{1}{b^5}={}^{11}C_5\frac{1}{b^6}\Rightarrow b=1$$

44. (a)

$$I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{(1+e^{x\cos x})(\sin^4 x+\cos^4 x)}=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{e^{x\cos x}dx}{(\sin^4 x+\cos^4 x)(1+e^{x\cos x})}$$
$$\therefore I=\int_0^{\frac{\pi}{4}}\frac{dx}{\sin^4 x+\cos^4 x}=\int_0^{\frac{\pi}{4}}\frac{(\tan^2 x+1)\sec^2 x dx}{(\tan^4 x+1)}$$
$$I=\int_0^1\frac{t^2+1}{t^4+1}dt=\int_0^1\frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2}dt$$
$$I=\int_{-\infty}^0\frac{du}{u^2+2}=\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)\right]_{-\infty}^0=\frac{\pi}{2\sqrt{2}}$$

45. (b)

$$x^2+(20)^{\frac{1}{4}}x+(5)^{\frac{1}{2}}=0$$
$$\alpha+\beta=-(20)^{\frac{1}{4}},\alpha\beta=(5)^{\frac{1}{2}}\Rightarrow\alpha^2+\beta^2=(20)^{\frac{1}{2}}-2(5)^{\frac{1}{2}}=0$$
$$\alpha^4+\beta^4=-2\times 5=-10$$
$$\Rightarrow\alpha^8+\beta^8=100-2\times 25=50$$

46. (a) $f(0^-)=f(0)+f(0^+)\Rightarrow e^{3a}=b=e^{\frac{1}{2}}$

$\therefore 3a=\frac{1}{2}$ and $b=e^{\frac{1}{2}}\Rightarrow 6a+b^2=1+e$

47. (b) $\sin\theta+\cos\theta=\frac{1}{2}$

$$\Rightarrow\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\frac{1}{4}$$

$$\Rightarrow 1+\sin 2\theta=\frac{1}{4}$$

$$\Rightarrow\sin 2\theta=\frac{-3}{4}$$

$$\cos 4\theta=1-2\sin^2 2\theta=1-2\left(\frac{-3}{4}\right)^2$$

$$= 1 - 2\left(\frac{9}{16}\right) = 1 - \frac{9}{8} = -\frac{1}{8}$$

$$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$$

$$= 3 \times \left(\frac{-3}{4}\right) - 4\left(\frac{-3}{4}\right)^3$$

$$= \frac{-9}{4} + \frac{27}{16} = \frac{-36+27}{16} = \frac{-9}{16}$$

$$16(\sin 2\theta + \cos 4\theta + \sin 6\theta)$$

$$= 16\left[\frac{-3}{4} + \left(\frac{-1}{8}\right) + \left(\frac{-9}{16}\right)\right]$$

$$= -12 - 2 - 9 = -23$$

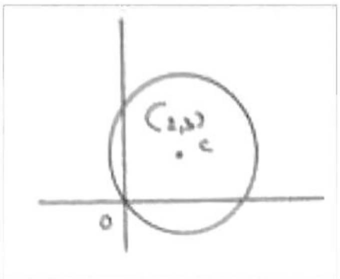
48. (c) Total case $= \{10,11,12, \ldots .99\} = 90$

$(2^n - 2) = 3k \Rightarrow n = \{11,13,15, \ldots .99\} = 45$

$\therefore$ Required probability $= \frac{45}{90} = \frac{1}{2}$

49. (b)

50. (-1,5) (5,1)



$$(x-2)^2 + (y-3)^2 = 2^2 + 3^2$$

$$(x-2)^2 + (y-3)^2 = 13$$

$$m_{oc} = \frac{3}{2}\ m_{cp} = \frac{-2}{3}$$

Equation of $cp = y - 3 = \frac{-2}{3}(x-2)$

$$\Rightarrow 3y - 9 = -2x + 4$$

$$\Rightarrow 3y + 2x - 13 = 0$$

$$\Rightarrow x = \frac{13-3y}{2}$$

Substituting in circle we get

$$\left(\frac{13-3y}{2}-2\right)^2+(y-3)^2=13$$
$$\left(\frac{9-3y}{2}\right)^2+(y-3)^2=13$$
$$81+9y^2-54y+4(y^2-6y+9)=52$$
$$81+9y^2-54y+4y^2-24y+36=52$$
$$13y^2-78y+117=52$$
$$13y^2-xy+65=0$$

$$y^2-6y+5=0$$
$$y=1,5$$
$$x=\frac{13-3y}{2}=5,-1$$
$$(x,y)\equiv(5,1)\text{ and }(-1,5)$$

51. (16.00)

$$\bar{x}=\frac{60+a+b}{8}=9\Rightarrow a+b=12$$
$$\sigma=\frac{642+a^2+b^2}{8}-81=\frac{37}{4}\Rightarrow a^2+b^2=80$$
$$\therefore (a+b)^2=a^2+b^2+2ab\Rightarrow ab=32$$
$$(a-b)^2=a^2+b^2-2ab=16$$

52. $\left(\mathbf{1+2\log\left(\frac{3}{2}\right)}\right)$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\frac{2\left(\frac{j}{n}\right)+8-\left(\frac{1}{n}\right)}{2\left(\frac{j}{n}\right)+4-\left(\frac{1}{n}\right)}\left\{\because \lim_{n\to\infty}\frac{1}{n}=0\right\}$$
$$\Rightarrow\int_0^1\frac{2x+8}{2x+4}dx=1+4\int_0^1\frac{dx}{2x+4}=[1+2\log(2x+4)]_0^1$$
$$=1+2\log\left(\frac{3}{2}\right)$$

53. (3.00)

$\log_3 2,\log_3 2^x-5,\log_3 2^x-\frac{7}{2}\Rightarrow$ AP

$$\Rightarrow 2\log_3 2^x-5=\log_3 2+\log_3 2^x-\frac{7}{2}$$

$$\Rightarrow (2^x-5)^2=2\left(2^x-\frac{7}{2}\right)$$

Let $2^x=t$

$$\Rightarrow t^2-10t+25=2t-7$$

$$\Rightarrow t^2-12t+32=0$$

$$\Rightarrow (t-8)(t-4)=0$$

$\Rightarrow t = 8{,}4$

But $2^x = 5$

$\Rightarrow 2^x = 8$

$\Rightarrow x = 3$

54. (6)

$$f(x) = \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}$$

$R_2 \to R_2 - R_1; R_3 \to R_3 - R_1$

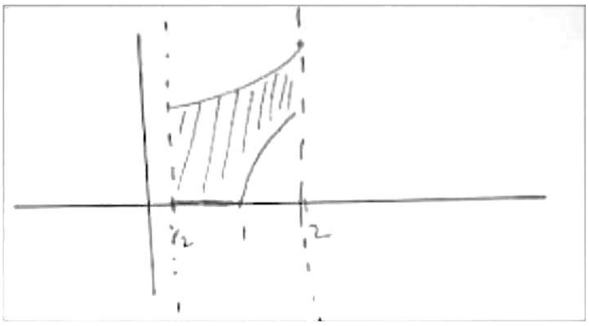
$$f(x) = \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix}$$

$$f(x) = 2\sin^2 x + 4 - 2\cos^2 x + 4\cos 2x = 4 + 2\cos^2 x$$
$$f(x)_{\max} = 6$$

55. (5.00)
56. (9/2)

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}; A^2 = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 10 \\ -5 & 14 \end{bmatrix}$$
$$\therefore I = \alpha A + \beta A^2 \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha-\beta & 2\alpha+10\beta \\ -\alpha-5\beta & 4\alpha+14\beta \end{bmatrix}$$
$$\therefore \alpha - \beta = 1, \alpha + 5\beta = 0$$
$$\Rightarrow \beta = \frac{-1}{6}, \alpha = \frac{5}{6} \Rightarrow 5\alpha - \beta = \frac{27}{6} = \frac{9}{2}$$

57. $(2\ln 2 - \sqrt{2}\ln 2 + 3)$



Area $== \int_{\frac{1}{2}}^{1} 2^x dx + \int_{1}^{2} (2^x - \ln^x) dx$

$= 2^x \ln 2|_{\frac{1}{2}}^{1} + 2^x \ln 2|_1^2 - (x\ln^x - x)|_1^2$

$= \left(2\ln 2 - 2^{\frac{1}{2}}\ln 2\right) - (2^2\ln 2 - 2^1\ln 2) - [(2\ln^2 - 2) - (1\ln 1 - 1)]$

$= 2\ln 2 - \sqrt{2}\ln 2 + 4\ln 2 - 2\ln 2 - 2\ln 2 + 2 = 1$

$= 2\ln 2 - \sqrt{2}\ln 2 + 3$

58. (13[$\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$]

$$
\begin{aligned}
&(\bar{a}+\bar{b}) \times (\bar{a} \times ((\bar{a}-\bar{b}) \times b)) \\
&= (\bar{a}+\bar{b}) \times (\bar{a} \times (\bar{a} \times \bar{b} - \bar{b} \times \bar{b})) \\
&= (\bar{a}+\bar{b}) \times (\bar{a} \times (\bar{a} \times \bar{b})) \\
&= (\bar{a}+\bar{b}) \times ((\bar{a}\cdot\bar{b})\bar{a} - (\bar{a}\cdot\bar{a})\bar{b}) \\
&= (-(\bar{a}\cdot\bar{a})(\bar{a}\times\bar{b}) + (\bar{a}\cdot\bar{b})(\bar{b}\times\bar{a})) \\
&= (-6(\bar{a}\times\bar{b}) + (7)(\bar{b}\times\bar{a})) \\
&= (6(\bar{b}\times\bar{a}) + 7(\bar{b}\times\bar{a})) \\
&= 13(\bar{b}\times\bar{a}) = 13\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix} \\
&= 13[\hat{\imath}(4-3) - \hat{\jmath}(-2-3) + \hat{k}(-1-2)] \\
&= 13[\hat{\imath} + 3\hat{\jmath} - 3\hat{k}]
\end{aligned}
$$

59. ($-\frac{1}{4}\ln 2$)

$\ln\left(\frac{dy}{dx}\right) = 3x + 4y$

$\frac{dy}{dx} = e^{3x+4y}$

$\frac{dy}{dx} = e^{3x+4y}$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$

$y(0) = 0$

$\Rightarrow \frac{-1}{4} = \frac{1}{3} + c$

$\Rightarrow c = \frac{-1}{4} - \frac{1}{3} = \frac{-7}{12}$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$

$\Rightarrow 3e^{-4y} = -4e^{3x} + 7$

$x = \frac{-2}{3}\ln 2$

$\Rightarrow 3e^{-4y} = -4e^{-2\ln 2} + 7$

$\Rightarrow \frac{-1}{4} = \frac{1}{3} + c$

$\Rightarrow c = \frac{-1}{4} - \frac{1}{3} = \frac{-7}{12}$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$$

$$\Rightarrow 3e^{-4y} = -4e^{3x} + 7$$

$$x = \frac{-2}{3}\ln 2$$

$$\Rightarrow 3e^{-4y} = -4e^{-2\ln 2} + 7$$

$$\Rightarrow 3e^{-4y} = -4 \times \frac{1}{4} + 7$$

$$\Rightarrow 3e^{-4y} = 6$$

$$\Rightarrow e^{-4y} = 2$$

$$\Rightarrow -4y = \ln 2$$

$$\Rightarrow y = -\frac{1}{4}\ln 2$$

60. $\left(\frac{578}{115}\right)$

Let plane be $ax + by + cz = d$

It passes through $A(3,7,-9)$

It contains $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$

$\Rightarrow$ Plane passes through B$(2,3,-2)$

$$\overrightarrow{AB} = -\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$$

Normal to plane is $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -4 & 7 \\ -3 & 2 & 1 \end{vmatrix}$

$$= \hat{\imath}(-4 - 14) - \hat{\jmath}(-1 + 21) + \hat{k}(-2 - 12)$$

$$= -18\hat{\imath} - 20\hat{\jmath} - 14\hat{k}$$

$$\vec{n} = 9\hat{\imath} + 19\hat{\jmath} + 7\hat{k}$$

Equation of plane is $9x + 10y + 7z = d$

It passes through $(3,7,-9)$

$$\Rightarrow 9 \times 3 + 10 \times 7 + 7 \times (-9) = d$$

$$\Rightarrow 27 + 70 - 63 = d$$

$$\Rightarrow 97 - 63 = d$$

$$\Rightarrow 34 = d$$

Plane is $9x + 10y + 7z = 34$

$$d^2 = \frac{34^2}{9^2 + 10^2 + 7^2} = \frac{34 \times 34}{230} = \frac{578}{115}$$

61. (0.00)

$$\sec x\left(\frac{dy}{dx}\right) - \sin(x+y) - \sin(x-y) = 2\sin x \cos y$$

$$\int \sec y dy = \int \sin 2x \cdot dx \Rightarrow \log(\sec y + \tan y) = \frac{-\cos 2x}{2} + c$$

At $x = 0, y = 0$

$$\therefore \sec y + \tan y = e^{\frac{1-\cos 2x}{2}} = e^{\sin^2 x}$$
$$\Rightarrow (\sec y \tan y + \sec^2 y)y' = e^{\sin^2 x}\sin 2x = 0$$
$$\Rightarrow y' = 0 \Rightarrow 5\left[y'\left(\frac{\pi}{2}\right)\right] = 0$$