1. (d) $B_{axis} = \frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}$

$B_{centre} = \frac{\mu_0 i}{2R}$

$\therefore B_{centre} = \frac{\mu_0 i}{2a}$

$\therefore B_{axis} = \frac{\mu_0 i a^2}{2(a^2+r^2)^{3/2}}$

$\therefore$ fractional change in magnetic field =

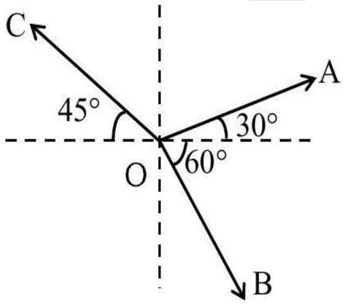
$$\frac{\frac{\mu_0 i}{2a} - \frac{\mu_0 i a^2}{2(a^2+r^2)^{3/2}}}{\frac{\mu_0 i}{2a}} = 1 - \frac{1}{\left[1+\left(\frac{r^2}{a^2}\right)\right]^{3/2}}$$

$$\approx 1 - \left[1 - \frac{3}{2}\frac{r^2}{a^2}\right] = \frac{3}{2}\frac{r^2}{a^2}$$

Note : $\left(1+\frac{r^2}{a^2}\right)^{-3/2} \approx \left(1 - \frac{3}{2}\frac{r^2}{a^2}\right)$

[True only if r << a]

2. (a)



Let magnitude be equal to λ.

$$\overrightarrow{OA} = \lambda[\cos 30°\hat{i} + \sin 30\hat{j}] = \lambda\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$$

$$\overrightarrow{OB} = \lambda[\cos 60°\hat{i} - \sin 60\hat{j}] = \lambda\left[\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$$

$$\overrightarrow{OC} = \lambda[\cos 45°(-\hat{i}) + \sin 45\hat{j}] = \lambda\left[-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]$$

$$\therefore \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$$

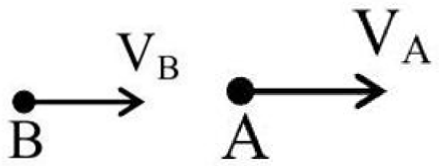
$$= \lambda\left[\left(\frac{\sqrt{3}+1}{2} + \frac{1}{\sqrt{2}}\right)\hat{i} + \left(\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)\hat{j}\right]$$

$\therefore$ Angle with x-axis

$$\tan^{-1}\left[\frac{\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}+\frac{1}{2}+\frac{1}{\sqrt{2}}}\right]=\tan^{-1}\left[\frac{\sqrt{2}-\sqrt{6}-2}{\sqrt{6}+\sqrt{2}+2}\right]$$

$$=\tan^{-1}\left[\frac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}\right]$$

3. (d)



Mirror used is convex mirror (rear-view mirror)

$\therefore V_{I/m}=-m^2 V_{O/m}$

Given,

$V_{O/m}=40\ \text{m/s}$

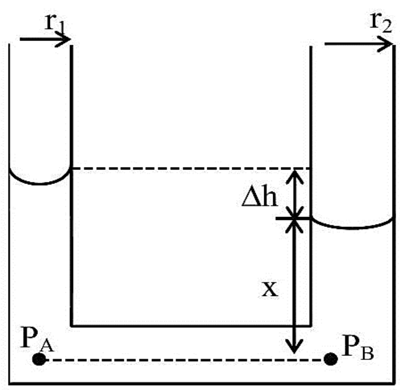
$$m=\frac{f}{f-u}=\frac{10}{10+190}=\frac{10}{200}$$

$$\therefore V_{I/m}=-\frac{1}{400}\times 40=-0.1\ \text{m/s}$$

$\therefore$ Car will appear to move with speed 0.1 m/s.

4. (a) Inside a spherical shell, gravitational field is zero and hence potential remains same everywhere

5. (b)



We have $P_A=P_B$. [Points A& B at same horizontal level]

$$\therefore P_{atm}-\frac{2T}{r_1}+\rho g(x+\Delta h)=P_{atm}-\frac{2T}{r_2}+\rho gx$$

$$\therefore \rho g\Delta h=2T\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

$= 2 \times 7.3 \times 10^{-2}\left[\frac{1}{2.5 \times 10^{-3}} - \frac{1}{4 \times 10^{-3}}\right]$

$\therefore \Delta h = \frac{2 \times 7.3 \times 10^{-2} \times 10^{3}}{10^{3} \times 10}\left[\frac{1}{2.5} - \frac{1}{4}\right]$

$= 2.19 \times 10^{-3}$ m $= 2.19$ mm

6. **(a)** $\Delta Q = \Delta U + \Delta W$

$\frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} + \frac{\Delta W}{\Delta t}$

$\frac{6000}{60}\frac{J}{sec} = \frac{2.5 \times 10^{3}}{\Delta t} + 90$

$\Delta t = 250 sec$

7. **(d)** $U = \frac{1}{2}Li^{2} = 64 \Rightarrow L = 2$

$i^{2}R = 640$

$R = \frac{640}{(8)^{2}} = 10$

$\tau = \frac{L}{R} = \frac{1}{5} = 0.2$

8. **(a)** For maximum average power

$X_L = X_C$

$250\pi = \frac{1}{2\pi(50)C}$

$C = 4 \times 10^{-6}$

9. **(c)**

A	B	X	Y	Z
1	1	0	0	0
1	0	0	1	0
0	1	1	0	0
0	0	1	1	1

10. **(d)** $nf_1 = k\left(\frac{1}{1} - \frac{1}{3^2}\right)$

$nf_2 = k\left(1 - \frac{1}{2^2}\right)$

$\frac{f_1}{f_2} = \frac{8/9}{3/4} \Rightarrow f_2 = 2.46 \times 10^{15}$

11. (a) $KE_{max} = eV_S = \frac{hc}{\lambda} - \phi$

$\Rightarrow eV_S = \frac{1240}{280} - 2.5 = 1.93eV$

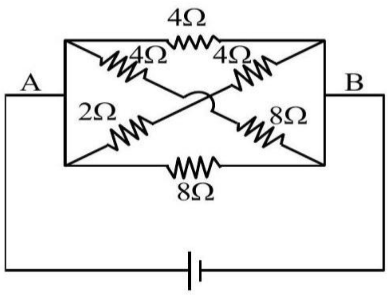
$\rightarrow V_{S_1} = 1.93\ V$

$\rightarrow eV_{S_2} = \frac{1240}{400} - 2.5 = 0.6eV$

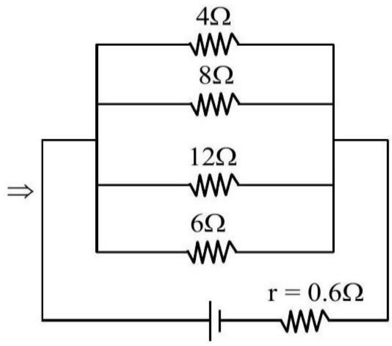
$\Rightarrow V_{S_2} = 0.6\ V$

$\Delta V = V_{S_1} - V_{S_2} = 1.93 - 0.6 = 1.33\ V$

12. (c)



$2.2\ V, r = 0.6\Omega$



2.2 V

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{6} = \frac{6+3+2+4}{24} = \frac{15}{24}$$

$$R_{eq} = \frac{24}{15} = 1.6 \Rightarrow R_T = 1.6 + 0.6 = 2.2\Omega$$

$$P = \frac{V^2}{R_T} = \frac{(2.2)^2}{2.2} = 2.2\ W$$

13. (a)

14. (a) $V_{RMS} = \sqrt{\frac{3RT}{M_W}}$

At the same temperature $V_{RMS} \propto \frac{1}{\sqrt{M_w}}$

$\Rightarrow V_H > V_O > V_C$

15. (c) Least count (L.C) $= \frac{0.5}{50}$

True reading $= 5 + \frac{0.5}{50} \times 20 - \frac{0.5}{50} \times 5$

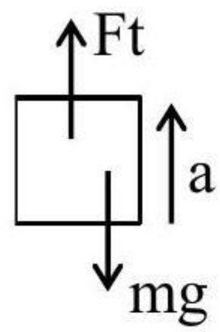
$= 5 + \frac{0.5}{50}(15) = 5.15 \text{ mm}$

16. (b) $\frac{R_1 R_2}{R_1 + R_2} = 3$

$$\frac{\frac{(12 \times 10^{-6} \times 10^{-2})\ell \times 4}{\pi(2)^2 \times 10^{-6}} \times \frac{(51 \times 10^{-6} \times 10^{-2})\ell \times 4}{\pi(2)^2 \times 10^{-6}}}{\frac{63 \times 10^{-6} \times 10^{-2} \times \ell \times 4}{\pi(2)^2 \times 10^{-6}}}$$

$\Rightarrow \ell = 97 \text{ m}$

17. (d)



$F_{thrust} = \left(\frac{dm}{dt} \cdot V_{rel}\right)$

$\left(\frac{dm}{dt} V_{rel} - mg\right) = ma$

$\Rightarrow \left(\frac{dm}{dt}\right) \times 500 - 10^3 \times 10 = 10^3 \times 20$

$\frac{dm}{dt} = (60 \text{ kg/s})$

18. (d) $E = ML^2 T^{-2}$

$L = ML^2 T^{-1}$

$m = M$

$G = M^{-1} L^{+3} T^{-2}$

$P = \frac{EL}{M^5 G^2}$

$$[P]=\frac{(ML^2\,T^{-2})(M^2\,L^4\,T^{-2})}{M^5(M^{-2}\,L^6\,T^{-4})}=M^0\,L^0\,T^0$$

19. (c) $\rho=200\Omega m$

$C=2\times10^{-12}$ F
$V=40$ V
$K=56$

$$i=\frac{q}{\rho k\varepsilon_0}=\frac{q_0}{\rho k\varepsilon_0}e^{-\frac{t}{\rho k\varepsilon_0}}$$

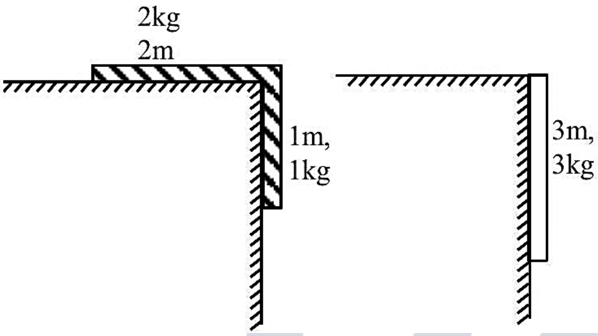
$$i_{max}=\frac{2\times10^{-12}\times40}{200\times50\times8.85\times10^{-12}}$$

$$=\frac{80}{10^4\times8.85}=903\mu A=0.9\text{ mA}$$

20. (a) Pentavalent activities have excess free e^-

So e^-density increases but overall semiconductor is neutral.

21. (40)



From energy conservation

$K_i+U_i=k_f+U_f$

$$0+\left(-1\times10\times\frac{1}{2}\right)=k_f+\left(-3\times10\times\frac{3}{2}\right)$$

$-5=k_f-45$

$k_f=40$ J

40.00

22. (354) $E_0=200$

$$I=\frac{1}{2}\varepsilon_0E_0^2\cdot C$$

Radiation pressure

$$P=\frac{2I}{C}$$

$$=\left(\frac{2}{C}\right)\left(\frac{1}{2}\varepsilon_0E_0^2C\right)$$

$= \varepsilon_0 E_0^2$

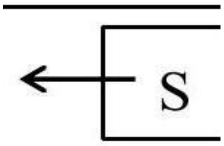
$= 8.85 \times 10^{-12} \times 200^2$

$= 8.85 \times 10^{-8} \times 4$

$= \dfrac{354}{10^9}$

354.0

23. (2025)



$V_S = 20\ \text{m/s}$

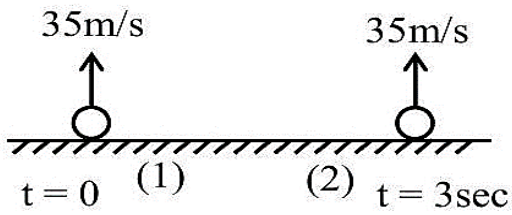
$f' = f\left(\dfrac{C - V_0}{C + V_s}\right)$

$1800 = f\left(\dfrac{340 - 20}{340 + 20}\right)$

$f = 2025\ \text{Hz}$

2025

24. (50)



When both balls will collied

$y_1 = y_2$

$35t - \dfrac{1}{2} \times 10 \times t^2 = 35(t-3) - \dfrac{1}{2} \times 10 \times (t-3)^2$

$35t - \dfrac{1}{2} \times 10 \times t^2 = 35t - 105 - \dfrac{1}{2} \times 10 \times t^2$

$-\dfrac{1}{2} \times 10 \times 3^2 + \dfrac{1}{2} \times 10 \times 6t$

$0 = 150 - 30t$

$t = 5\text{sec}$

$\therefore$ Height at which both balls will collied

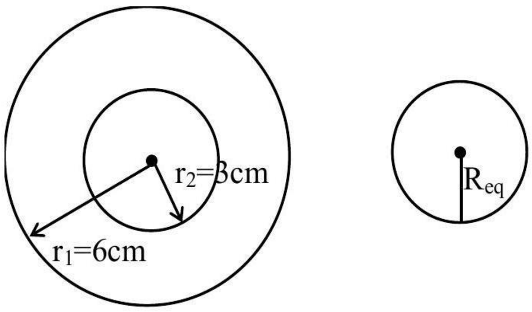
$h = 35t - \frac{1}{2} \times 10 \times t^2$

$= 35 \times 5 - \frac{1}{2} \times 10 \times 5^2$

$h = 50$ m

50.00

25. (2)



Excess pressure inside the smaller soap bubble

$\Delta P = \frac{4\,S}{r_1} + \frac{4\,S}{r_2}$

The excess pressure inside the equivalent soap bubble

$\Delta P = \frac{4\,S}{R_{eq}}$ ... (ii)

From (i) & (ii)

$\frac{4\,S}{R_{eq}} = \frac{4\,S}{r_1} + \frac{4\,S}{r_2}$

$\frac{1}{R_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$

$= \frac{1}{6} + \frac{1}{3}$

$R_{eq} = 2$ cm

2.00

26. (2.00)

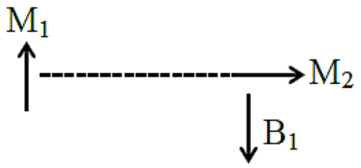
$W_m = 12560 = 2\pi f_m$

$f_m = \frac{12560}{2\pi}$

$= 2000$ Hz

2.00

27. (1)



$\vec{\tau} = \vec{M}_2 \times \vec{B}_1$

$\tau = M_2\ B_1 \sin 90^\circ$

$= 1 \times \frac{\mu_0}{4\pi} \frac{M_1}{(1)^3} 1$

$= 10^{-7}\ N \cdot m$

1.00

28. (1.00)

For node

$\cos(1.57\ cm^{-1})x = 0$

$(1.57\ cm^{-1})x = \frac{\pi}{2}$

$x = \frac{\pi}{2(1.57)} cm = 1\ cm$

1.00

29. (300) Position of bright fringe $y = n\frac{D\lambda}{d}$

y_1 of red $= \frac{D\lambda_r}{d} = 3.5\ mm$

$\lambda_r = 3.5 \times 10^{-3} \frac{d}{D}$

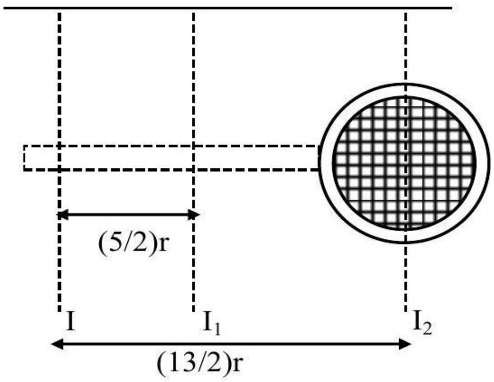
Similarly $\lambda_v = 2 \times 10^{-3} \frac{d}{D}$

$\lambda_r - \lambda_v = (1.5 \times 10^{-3}) \left(\frac{0.3 \times 10^{-3}}{1.5}\right)$

$= 3 \times 10^{-7} = 300\ nm$

300.0

30. (52.00)



$$I = \left[I_1 + M\left(\frac{5}{2}r\right)^2\right] + \left[I_2 + M\left(\frac{13r}{2}\right)^2\right]$$

$$= \left[\frac{M(36r^2)}{12} + \frac{M(25r^2)}{4}\right] + \left[\frac{M^2}{2} + \frac{169\mathrm{Mr}^2}{4}\right]$$

$$= 52\mathrm{Mr}^2$$

52.00

31. (d)

(a) $[Fe(CN)_6]^4 \rightarrow$ Pale yellow solution

(b) $[Fe(SCN)_6]^4 \rightarrow$ Blood red colour

(c) $Fe_4[Fe(CN_6)]_3 \cdot H_2O \rightarrow$ Prussian blue

(d) $[Fe(CN)_5NOS]^4 \rightarrow$ Violet colour

32. (c) (i) Adsorption of gas at metal surface is an exothermic process so $\Delta H < 0$

(ii) As the adsorption of gas on metal surface reduces the free movement of gas molecules thus restricting its randomness hences $\Delta S < 0$

33. (a)

(a) $CuCl_2 + nH_2O \rightarrow Cu^{+2}_{(aq.)}$

blue colour

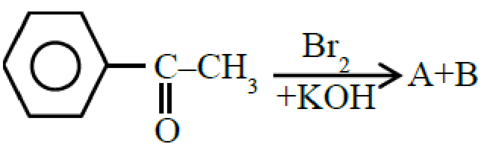
(b) $AgCl + nH_2O \rightarrow$ Insoluble

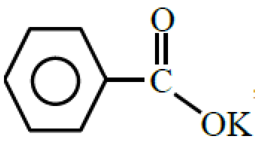
(c) $ZnCl_2 + nH_2O \rightarrow Zn^{+2}_{(aq.)}$

Colourless

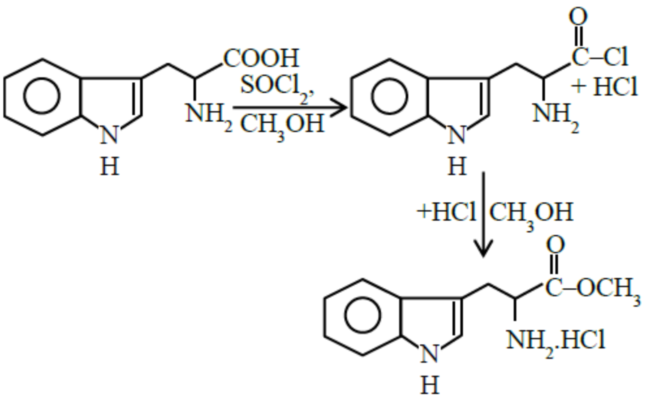
(d) $Cu_2Cl_2 + nH_2O \rightarrow$ Insoluble

34. (a)

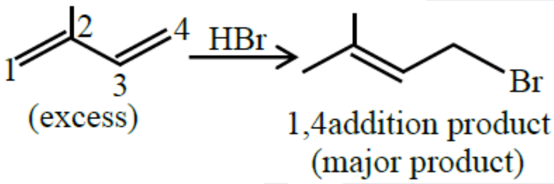


(a)  , B=$CHBr_3$

35. (c)



36. (a)



37. (a) Novolac + formaldehyde → Bakelite

38. (d)

Ion	H^+	K^+	Cl^-	CH_3COO^-
$\Lambda^{\infty}_{mScm/mole}$	349.8	73.5	76.3	40.9

$$= 349.8 + 40.9$$
$$= 390.7 Scm^2/n$$
$$\Lambda^{\infty}_{mKCl} = \Lambda^{\infty}_{m(K^+)} + \Lambda^{\infty}_{m(Cl^-)}$$
$$= 73.5 + 76.3$$
$$= 149.3 Scm^2/mole$$

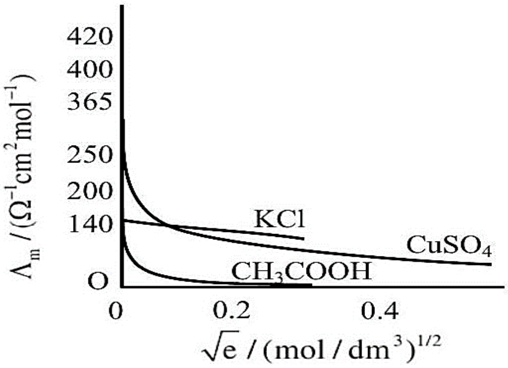
So $\Lambda^{3}_{mCH}COOH^{\infty} = \Lambda^{\infty}_{m(H^+)} + \Lambda^{\infty}_{mCH_3COO^-}$

$= 390.7 Scm^2/mole$

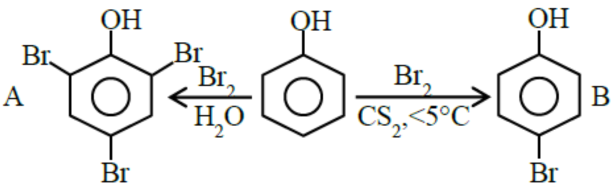
So statement-I is wrong or False.

As the concentration decreases, the dilution increases which increases the degree of dissociation, thus increasing the no. of ions, which increases the molar conductance.

So statement-II is false.

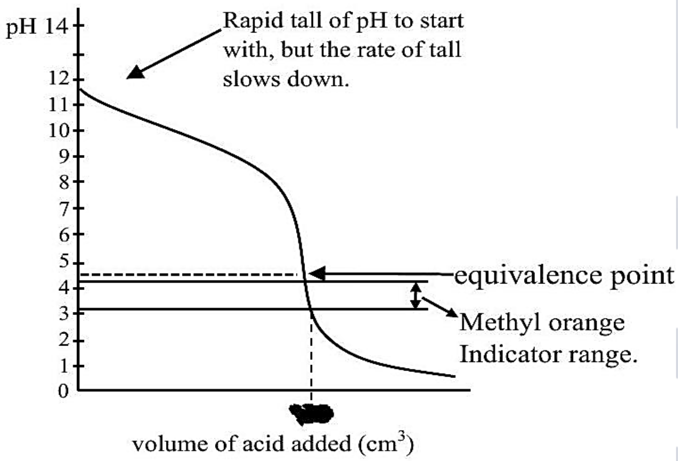


39. (b)

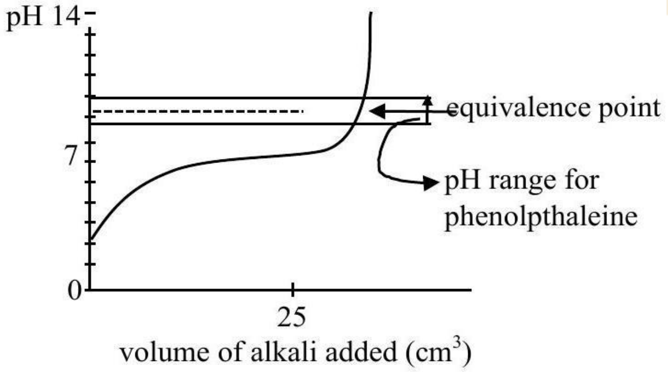


40. (a) The $F^{\ominus}$ ions make the enamel on teeth much harder by converting hydroxyapatite, $[3(Ca_3(PO_4)_2] \cdot Ca(OH)_2]$, the enamel on the surface of the teeth into much harder fluroappatite. $[3Ca_3(PO_4)_2 \cdot CaF_2]$

41. (b) Titration curve for strong acid and weak base initially a buffer of weak base and conjugate acid is :



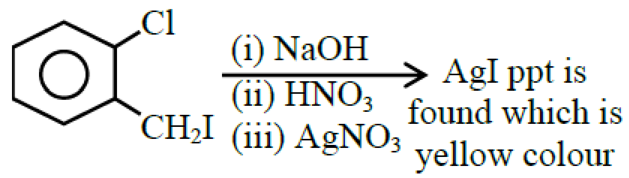
Formed, thus pH falls slowly and after equivalence point, so the pH falls sharply so methyl arrange, having pH range of 3.2 to 4.4 will weak as indicator. So statement-I is correct.



Titration curve for weak acid and strong base (NaOH)

Initially weak acid will form a buffer so pH increases slowly but after equivalence point. it rises sharply covering range of phenolphthalein so it will be suitable indicator so statement-II is false.

42. (b)



Other compounds halide can't be removed because corresponding C^+ is highly unstable.

43. (a) Pure demineralised (de-ionized) water free from all soluble mineral salts is obtained by passing water successively through a cation exchange (in the H^+ form) and an anion exchange (in the OH^- form) resins.

44. (d) Given statement-I is true as in a number of processes, one element is used to reduce the oxide of another metal. Any element will reduce the oxide of other metal which lie above it in the Ellingham diagram because the free energy change will become more negative.

Given statement-II is false as the value of ΔS is decreases from left to right in Ellingham diagram.

45. (b) $Ca(OH)_2 + CO_2 \longrightarrow CaCO_3 \downarrow + H_2O$

$CaCO_3 \downarrow + CO_2 + H_2O \rightarrow Ca(HCO_3)_2$

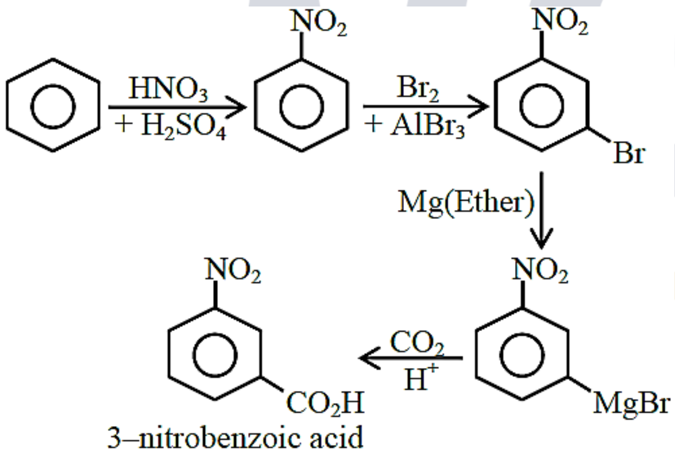
46. (c) Velocity of electron in Bohr's atom is given by $V \propto \frac{Z}{n}$

Z = atomic number of atom, corresponds to +ve

charge so as Z increase velocity increases so statement-I is wrong.

and as 'n' decreases velocity increases so statement-II is correct.

47. (d)



48. (c)

49. (a)

(i) Reactivity order:

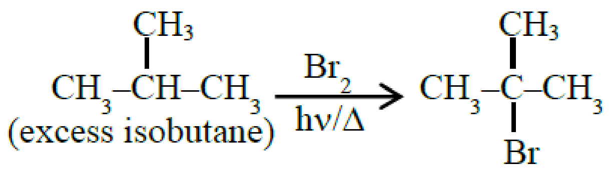
$F_2 > ClF$ (inter halogen) $> Cl_2$

(ii) $ClF + H_2O \rightarrow HOCl + HF$

(iii) Oxidizing power in aqueous solution

$F_2 > Cl_2 > Br_2 > I_2$

50. (d)



51. (2) T-shaped molecule means 3 sigma bond and 2 lone pairs of electron on central atom.

52. (3) Sublimation enthalpy, Ionisation enthalpy and hydration enthalpy affect the reduction potential.

53. (4) As $0.1 M C_2H_5OH$ is non-dissociative and rest all salt given are electrolyte so in each case effective molarity > 0.1 so each will have lower freezing point.

54. (3) $[NH_4^+] = 0.0504 \& [NH_3] = 0.0210$

So $K_b = \frac{[NH_4^+][HO^-]}{[NH_3]}$

$$[HO^-] = \frac{K_b \times [NH_3]}{[NH_4^+]} = 1.8 \times 10^{-5} \times \frac{2}{5} \times \frac{210}{504}$$

$= 3 \times 10^{-6}$

55. (7) The electronic configuration of

${}_{64}Gd: [Xe]4f^7 5\,d^1 6\,s^2$

So, the electronic configuration of

${}_{64}Gd^{2+}: [Xe]4f^7 5\,d^1 6\,s^0$

i.e. the number of 4f electrons in the ground state electronic configuration of Gd^{2+} is 7

56. (5) Mohr's salt: $(NH_4)_2Fe(SO_4)_2 \cdot 6H_2O$

The number of water molecules in Mohr's salt = 6

Potash alum : $KAl(SO_4)_2 \cdot 12H_2O$

The number of water molecules in potash alum = 12

So ratio of number of water molecules in Mohr's

salt and potash alum $= \frac{6}{12}$

$$= \frac{1}{2}$$
$$= 0.5$$
$$= 5 \times 10^{-1}$$

57. (1) $7 \times 10^{-9} = K \times (8 \times 10^{-5})^x (8 \times 10^{-5})^y$

$2.1 \times 10^{-8} = K \times (24 \times 10^{-5})^x (8 \times 10^{-5})^y$

$\frac{1}{3} = \left(\frac{1}{3}\right)^x \Rightarrow x = 1$

58. (718)

$$\Delta_f H^{\ominus}_{KCl} = \Delta_{sub} H^{\ominus}_{(K)} + \Delta_{ionization} H^{\ominus}_{(K)} + \frac{1}{2}\Delta_{bond} H^{\ominus}_{(Cl_2)}$$

$$+\Delta_{electron\ gain} H^{\ominus}_{(Cl)} + \Delta_{lattice} H^{\ominus}_{(KCl)}$$

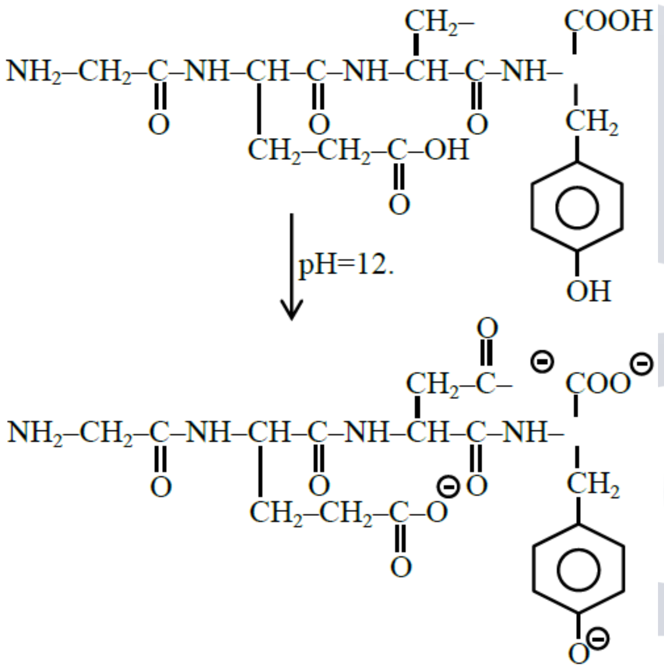
$$\Rightarrow -436.7 = 89.2 + 419.0 + \frac{1}{2}(243.0) + \{-348.6\}$$

$$+\Delta_{lattice} H^{\ominus}_{(KCl)}$$

$$\Rightarrow \Delta_{lattice} H^{\ominus}_{(KCl)} = -717.8 \text{ kJ mol}^{-1}$$

The magnitude of lattice enthalpy of KCl in kJmol^{-1} is 718 (Nearest integer).

59. (4)



Total negative charge produced = 4.

60. (3) 1000 kg solvent has 3.3 moles of KCl

1000 kg solvent $\longrightarrow 3.3 \times 74.5$gmKCl

$\longrightarrow 245.85$

Weight of solution $= 1245.85$gm

Volume of solution $= \frac{1245.85}{1.2}$ml

So molarity $= \frac{3.3 \times 1.2}{1245.85} \times 1000 = 3.17$

61. (a) $\frac{\cos x}{1+\sin x} = |\tan 2x|$

$$\Rightarrow \frac{\cos^2 x/2 - \sin^2 x/2}{(\cos x/2 + \sin x/2)} = |\tan 2x|$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \tan^2 2x$$

$$\Rightarrow 2x = n\pi \pm \left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\Rightarrow x = \frac{-3\pi}{10}, \frac{-\pi}{6}, \frac{\pi}{10}$$

or sum $= \frac{-11\pi}{6}$.

62. (d) Given:

Mean $(\bar{x}) = \frac{\Sigma x_i}{20} = 10$

or $\Sigma x_i = 200$ (incorrect)

or $200 - 25 + 35 = 210 = \Sigma x_i$ (Correct)
TEST PAPER WITH SOLUTION

Now correct $\bar{x} = \frac{210}{20} = 10.5$

again given $\mathrm{S.D} = 2.5(\sigma)$

$$\sigma^2 = \frac{\Sigma x_i^2}{20} - (10)^2 = (2.5)^2$$

or $\Sigma x_i^2 = 2125$ (incorrect)

or $\Sigma x_i^2 = 2125 - 25^2 + 35^2$

$= 2725$ (Correct)

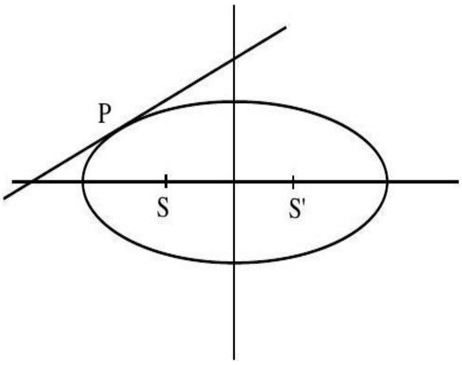
$\therefore$ correct $\sigma^2 = \frac{2725}{20} - (10.5)^2$

$\underline{\underline{\sigma}}^2 = 26$

or $\sigma = 26$

$\therefore \underline{\alpha} = 10.5, \beta = 26$

63. (a)



Equation of tangent : $y = 2x + 6$

at P

$\therefore$ P$(-8/3, 2/3)$

$e = \frac{1}{\sqrt{2}}$

$S \& S' = (-2,0) \& (2,0)$

Area of Δ SPS $' = \frac{1}{2} \times 4 \times \frac{2}{3}$

$A = \frac{4}{3}$

$\therefore (5 - e^2)A = \left(5 - \frac{1}{2}\right)\frac{4}{3} = 6$

64. (d) $(y+1)\tan^2 x dx + \tan x dy + y dx = 0$

or $\frac{dy}{dx} + \frac{\sec^2 x}{\tan x} \cdot y = -\tan x$

$\text{IF} = e^{\int \frac{\sec^2 x}{\tan x} dx} = e^{\ln \tan x} = \tan x$

$\therefore y \tan x = -\int \tan^2 x dx$

or $y \tan x = -\tan x + x + C$

or $y = -1 + \frac{x}{\tan x} + \frac{C}{\tan x}$

or $\lim_{x \to 0} xy = -x + \frac{x^2}{\tan x} + \frac{Cx}{\tan x} = 1$

or $C = 1$

$y(x) = \cot x + x\cot x - 1$

$y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

65. (d) P (Exactly one of A or B)

$= P(A \cap \overline{B}) + P(\overline{A} \cap B) = \frac{5}{9}$

$= P(A)P(\overline{B}) + P(\overline{A})P(B) = \frac{5}{9}$

$\Rightarrow P(A)(1 - P(B)) + (1 - P(A))P(B) = \frac{5}{9}$

$\Rightarrow p(1 - 2p) + (1 - p)2p = \frac{5}{9}$

$\Rightarrow 36p^2 - 27p + 5 = 0$

$\Rightarrow p = \frac{1}{3}$ or $\frac{5}{12}$

$p_{max} = \frac{5}{12}$

66. (b) Case-I

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 2 & 1+\sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & 1+4\sin^3\theta \end{vmatrix} = 0$$

or $4\sin 3\theta = -2$

$\sin 3\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{18}$

67. (c) $f(x) = \cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$

$\cot^{-1}\sqrt{\frac{1-x}{x}} = \sin^{-1}\sqrt{x}$

or $f(x) = \cos\left(2\tan^{-1}\sqrt{x}\right)$

$= \cos \tan^{-1}\left(\frac{2\sqrt{x}}{1-x}\right)$

$f(x) = \frac{1-x}{1+x}$

Now $f'(x) = \frac{-2}{(1+x)^2}$

or $f'(x)(1-x)^2 = -2\left(\frac{1-x}{1+x}\right)^2$

or $(1-x)^2 f'(x) + 2(f(x))^2 = 0.$

68. (d)

69. (d) $\sum_{r=0}^{20} r^2 \cdot {}^{20}C_r$

$\sum(4(r-1)+r) \cdot {}^{20}C_r$

$\sum r(r-1) \cdot \frac{20 \times 19}{r(r-1)} \cdot {}^{18}C_r + r \cdot \frac{20}{r} \cdot \sum {}^{19}C_{r-1}$

$\Rightarrow 20 \times 19.2^{18} + 20.2^{19}$

$\Rightarrow 420 \times 2^{18}$

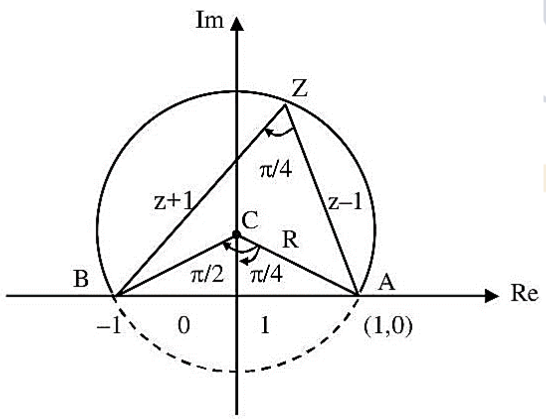
70. (c) $n(A \cup B) \geq n(A) + n(B) - n(A \cap B)$

$100 \geq 89 + 98 - n(A \cup B)$

$n(A \cup B) \geq 87$

$87 \leq n(A \cup B) \leq 89$

71. (b)



In $\triangle$ OAC

$\sin\left(\frac{\pi}{4}\right) = \frac{1}{AC}$

$\Rightarrow AC = \sqrt{2}$

Also, $\tan\frac{\pi}{4} = \frac{OA}{OC} = \frac{1}{OC}$

$\Rightarrow OC = 1$

$\therefore$ centre (0,1); Radius $= \sqrt{2}$

72. (a)

$|\vec{a}| = \sqrt{3}; \vec{a}.\vec{c} = 3; \vec{a}\times\vec{b} = -2\hat{\imath} + \hat{\jmath} + \hat{k}, \vec{a}\times\vec{c} = \vec{b}$

Cross with $\vec{a}$.

$\vec{a}\times(\vec{a}\times\vec{c}) = \vec{a}\times\vec{b}$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{a} - a^2\vec{c} = \vec{a}\times\vec{b}$

$\Rightarrow 3\vec{a} - 3\vec{c} = -2\hat{\imath} + \hat{\jmath} + \hat{k}$

$\Rightarrow 3\hat{\imath} + 3\hat{\jmath} + 3\hat{k} - 3\vec{c} = -2\hat{\imath} + \hat{\jmath} + \hat{k}$

$$\Rightarrow \vec{c} = \frac{5\hat{\imath}}{3} + \frac{2\hat{\jmath}}{3} + \frac{2\hat{k}}{3}$$

$$\therefore\ \vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{c} = \frac{-10}{3} + \frac{2}{3} + \frac{2}{3} = -2$$

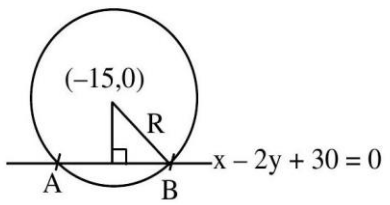
73. (d) Equation of tangent to $y^2 = 30x$

$$y = mx + \frac{30}{4m}$$

Pass thru $(-30,0)$: $a = -30m + \frac{30}{4m} \Rightarrow m^2 = 1/4$

$\Rightarrow m = \frac{1}{2}$ or $m = -\frac{1}{2}$

At $m = \frac{1}{2}$: $y = \frac{x}{2} + 15 \Rightarrow x - 2y + 30 = 0$



$P = \frac{15}{\sqrt{5}}$

$$\ell_{AB} = 2\sqrt{R^2 - P^2} = 2\sqrt{\frac{225}{4} - \frac{225}{5}}$$

$$\Rightarrow \ell_{AB} = 30\cdot\sqrt{\frac{1}{20}} = \frac{15}{\sqrt{5}} = 3\sqrt{5}$$

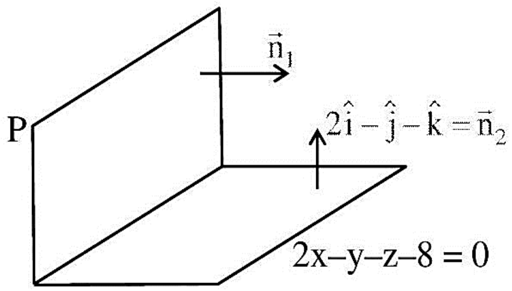
74. (b) $I = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1} - \frac{x-1}{x+1}\right)^2\right)^{1/2} dx$

$I=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left|\frac{4x}{x^2-1}\right|dx \Rightarrow I=2.4\int_0^{1/\sqrt{2}}\left|\frac{x}{x^2-1}\right|dx$

$\Rightarrow \mathrm{I}=-4\int_0^{1/\sqrt{2}}\frac{2\mathrm{x}}{\mathrm{x}^2-1}\mathrm{dx} \Rightarrow \mathrm{I}=-4\ln|\mathrm{x}^2-1|_0^{1/\sqrt{2}}$

$\Rightarrow \mathrm{I}=4\ln 2 \Rightarrow \mathrm{I}=\ln 16$

75. (b) Equation of plane P can be assumed as



$P: x+2y+3z+1+\lambda(x-y-z-6)=0$
$\Rightarrow P: (1+\lambda)x+(2-\lambda)y+(3-\lambda)z+1-6\lambda=0$
$\Rightarrow \vec{n}_1=(1+\lambda)\hat{\imath}+(2-\lambda)\hat{\jmath}+(3-\lambda)\hat{k}$
$\therefore \vec{n}_1\cdot\vec{n}_2=0$
$\Rightarrow 2(1+\lambda)-(2-\lambda)-(3-\lambda)=0$
$\Rightarrow 2+2\lambda-2+\lambda-3+\lambda=0 \Rightarrow \lambda=\frac{3}{4}$
$\Rightarrow P: \frac{7x}{4}+\frac{5}{4}y+\frac{9z}{4}-\frac{14}{4}=0$
$\Rightarrow 7x+5y+9z=14$
$(0,1,1)$ lies on P

76. (b) $\mathrm{AA^T}=\begin{pmatrix}\frac{1}{5} & \frac{2}{\sqrt{5}}\\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}}\\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\end{pmatrix}$

$\mathrm{AA^T}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\mathrm{I}$

$\mathrm{Q}^2=\mathrm{A^TBAA^TBA}=\mathrm{A^TBIBA}$

$\Rightarrow Q^2=A^TB^2A$

$Q^3=A^TB^2AA^TBA \Rightarrow Q^3=A^TB^3A$

Similarly : $\mathrm{Q}^{2021}=\mathrm{A^TB^{2021}\,A}$

Now $B^2=\begin{pmatrix}1 & 0\\ i & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ i & 1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 2\mathrm{i} & 1\end{pmatrix}$

$B^3=\begin{pmatrix}1 & 0\\ 2\mathrm{i} & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \mathrm{i} & 1\end{pmatrix} \Rightarrow \mathrm{B}^3=\begin{pmatrix}1 & 0\\ 3\mathrm{i} & 1\end{pmatrix}$

Similarly, $B^{2021}=\begin{pmatrix}1 & 0\\ 2021\mathrm{i} & 1\end{pmatrix}$

$\therefore \mathrm{AQ^{2021}\,A^T}=\mathrm{AA^TB^{2021}AA^T}=\mathrm{IB^{2021}I}$

$\Rightarrow AQ^{2021}A^T = B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

$\therefore \left(AQ^{2021}A^T\right)^{-1} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

77. (b) Sum of infinite terms:

$\dfrac{a}{1-r} = 15$

Series formed by square of terms:

$a^2, a^2r^2, a^2r^4, a^2r^6$ .....

$\text{Sum} = \frac{a^2}{1-r^2} = 150$

$\Rightarrow \dfrac{a}{1-r} \cdot \dfrac{a}{1+r} = 150 \Rightarrow 15 \cdot \dfrac{a}{1+r} = 150$

$\Rightarrow \dfrac{a}{1+r} = 10$

by (i) and (ii) $a = 12; r = \frac{1}{5}$

Now series : ar^2, ar^4, ar^6

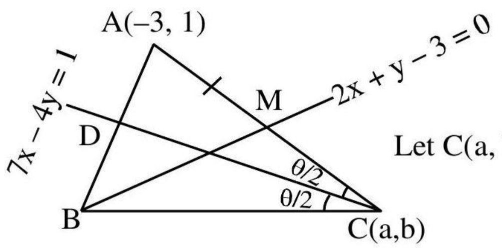
$\text{Sum} = \frac{ar^2}{1-r^2} = \frac{12\cdot(1/25)}{1-1/25} = \frac{1}{2}$

78. (b) $L = \lim_{n\to\infty} \frac{1}{n} \cdot \sum_{r=0}^{2n-1} \frac{1}{1+4\left(\frac{r}{n}\right)^2}$

$\Rightarrow L = \int_0^2 \dfrac{1}{1+4x^2}dx$

$\Rightarrow L = \dfrac{1}{2}\tan^{-1}(2x)\Big|_0^2 \Rightarrow L = \dfrac{1}{2}\tan^{-1}4$

79. (c)



Let C(a, b)

$\therefore\ M\left(\frac{a-3}{2}, \frac{b+1}{2}\right)$ lies on $2x + y - 3 = 0$

$\Rightarrow 2a + b = 11$

$\because$ C lies on $7x - 4y = 1$

$\Rightarrow 7a - 4b = 1$

$\therefore$ by (i) and (ii) : $a = 3, b = 5$

$\Rightarrow C(3,5)$

$\therefore m_{AC} = 2/3$

Also, $m_{CD} = 7/4$

$$\Rightarrow \tan\frac{\theta}{2} = \left|\frac{\frac{2}{3}-\frac{4}{4}}{1+\frac{14}{12}}\right| \Rightarrow \tan\frac{\theta}{2} = \frac{1}{2}$$

$$\Rightarrow \tan\theta = \frac{2\cdot\frac{1}{2}}{1-\frac{1}{4}} = \frac{4}{3}$$

80. (c)

p	q	r	$\underbrace{p \vee q}_{a}$	$\underbrace{q \to r}_{b}$	$a \wedge b$	$\sim r$	$\underbrace{a \wedge b \wedge (\sim r)}_{c}$	$\underbrace{p \wedge q}_{d}$	$c \to d$
T	F	T	T	T	T	F	F	F	T
F	F	T	F	T	F	F	F	F	T
T	F	F	T	T	T	T	T	F	F
F	T	F	T	F	F	T	F	F	T

81. (13) $Z = \frac{1-\sqrt{3}i}{2} = e^{-i\frac{\pi}{3}}$

$$z^r + \frac{1}{z^r} = 2\cos\left(-\frac{\pi}{3}\right) r = 2\cos\frac{r\pi}{3}$$

$$\Rightarrow 21 + \sum_{r=1}^{21}\left(z^r+\frac{1}{z^r}\right)^3 = 8\left(\cos^3\frac{r\pi}{3}\right) = 2\left(\cos r\pi + 3\cos\frac{r\pi}{3}\right)$$

$$\Rightarrow 21 + \left(z+\frac{1}{2}\right)^3 + \left(z^2+\frac{1}{z^2}\right)^3 + \cdots \ldots \left(z^{21}+\frac{1}{z^{21}}\right)^3$$

$$= 21 + \sum_{r=1}^{21}\left(z^r+\frac{1}{z^r}\right)^3$$

$$= 21 + \sum_{r=1}^{21}\left(2\cos r\pi + 6\cos\frac{r\pi}{3}\right)$$

$$= 21 - 2 - 6$$
$$= 13$$

82. (66) $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$

$x \in R - \{1,2\}$

$\Rightarrow k(2x - 4 - x + 1) = 2(x^2 - 3x + 2)$

$\Rightarrow k(x - 3) = 2(x^2 - 3x + 2)$

for $x \neq 3,\ k = 2\left(x - 3 + \frac{2}{x-3} + 3\right)$

$$x - 3 + \frac{2}{x-3} \geq 2\sqrt{2}, \forall x > 3$$

& $x - 3 + \frac{2}{x-3} \leq -2\sqrt{2}, \forall x < -3$

$$\Rightarrow 2\left(x - 3 + \frac{2}{x-3} + 3\right) \in (-\infty, 6 - 4\sqrt{2}] \cup [6 + 4\sqrt{2}, \infty)$$

for no real roots

$k \in (6 - 4\sqrt{2}, 6 + 4\sqrt{2}) - \{0\}$

Integral $k \in \{1,2 \ldots .11\}$

Sum of $k = 66$

83. (26) $L_1: \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$

for foot of $\perp$ r of (1,3,4) on $x - 2y - z - 3 = 0$

$(1 + t) - 2(3 - 2t) - (4 - t) - 3 = 0$

$\Rightarrow t = 2$

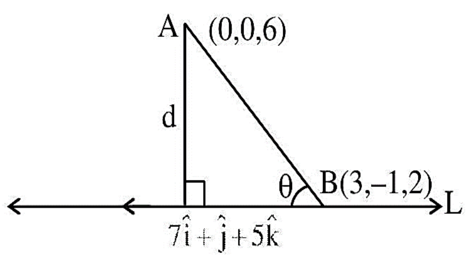
So foot of $\perp$ r $\triangleq (3, -1, 2)$

& point of intersection of L_1 with plane

is $(-11, -3, -8)$

dr's of L is $< 14,2,10 >$

$\cong \langle 7,1,5 \rangle$



$$d = AB\sin\theta = \left| \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -4 \\ 7 & 1 & 5 \end{vmatrix}}{\sqrt{7^2 + 1^2 + 5^2}} \right|$$

$\Rightarrow d^2 = \dfrac{1^2 + (43)^2 + (10)^2}{49 + 1 + 25} = 26$

84. (136) ${}^{1}P_1 + 2 \cdot {}^{2}P_2 + 3 \cdot {}^{3}P_3 + \cdots + 15 \cdot {}^{15}P_{15}$

$= 1! + 2.2! + 3.3! + \cdots .15 \times 15!$

$= \sum_{r=1}^{15} (r + 1 - 1)r\,!$

$= \sum_{r=1}^{15} (r + 1)! - (r)!$

$= 16! - 1$

$= {}^{16}P_{16} - 1$

$\Rightarrow q = r = 16, s = 1$

${}^{q+5}C_{r-s} = {}^{17}C_{15} = 136$

85. (36) Let $x + y = 36$

x is perimeter of square and y is perimeter of circle side of square $= x/4$

radius of circle $= \frac{y}{2\pi}$

Sum Areas $= \left(\frac{x}{4}\right)^2 + \pi\left(\frac{y}{2\pi}\right)^2$

$= \dfrac{x^2}{16} + \dfrac{(36 - x)^2}{4\pi}$

For min Area :

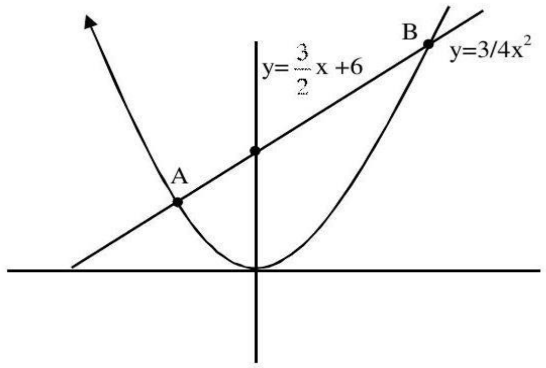
$x = \dfrac{144}{\pi + 4}$

$\Rightarrow$ Radius $= y = 36 - \frac{144}{\pi+4}$

$\Rightarrow k = \dfrac{36\pi}{\pi + 4}$

$\left(\dfrac{4}{\pi} + 1\right) k = 36$

86. (27)



For A & B

$3x^2 = 6x + 24 \Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2,4$

Area $= \int_{-2}^{4} \left(\frac{3}{2}x + 6 - \frac{3}{4}x^2\right) dx$

$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4} = 27$

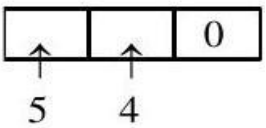
87. (16) Let $P(x, y)$

$$x^2 + y^2 + x^2 + (y-1)^2 + (x-1)^2 + y^2 + (x-1)^2 + (y-1)^2$$
$$\Rightarrow 4(x^2 + y^2) - 4y - 4x = 14$$
$$\Rightarrow x^2 + y^2 - x - y - \frac{7}{2} = 0$$
$$d = 2\sqrt{\frac{1}{4} + \frac{1}{4} + \frac{7}{2}}$$
$$\Rightarrow d^2 = 16$$

88. (40)

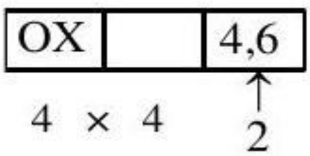
$$\overline{\qquad \ln(x+y) = 4xy \quad (\text{At } x = 0, y = 1)}$$
$$x + y = e^{4xy}$$
$$\Rightarrow 1 + \frac{dy}{dx} = e^{4xy}\left(4x\frac{dy}{dx} + 4y\right)$$
$$\text{At } x = 0 \ \frac{dy}{dx} = 3$$
$$\frac{d^2y}{dx^2} = e^{4xy}\left(4x\frac{dy}{dx} + 4y\right)^2 + e^{4xy}\left(4x\frac{d^2y}{dx^2} + 4y\right)$$
$$\text{At } x = 0, \frac{d^2y}{dx^2} = e^0(4)^2 + e^0(24)$$
$$\Rightarrow \frac{d^2y}{dx^2} = 40$$

89. (52) (i) When ' 0 ' is at unit place



Number of numbers = 20

(ii) When 4 or 6 are at unit place



Number of numbers $= 32$

So number of numbers $= 52$

90. (14)

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), & x\leq 0 \\ \frac{\tan 2x-\sin 2x}{bx^3}, & x>0 \end{cases}$$

For continuity at ' 0 '

$$\lim_{x\to 0^+} f(x)=f(0)$$

$$\Rightarrow \lim_{x\to 0^+}\frac{\tan 2x-\sin 2x}{bx^3}=-a$$

$$\Rightarrow \lim_{x\to 0^+}\frac{\frac{8x^3}{3}+\frac{8x^3}{3!}}{bx^3}=-a$$

$$\Rightarrow 8\left(\frac{1}{3}+\frac{1}{3!}\right)=-\mathrm{ab}$$

$$\Rightarrow 4=-\mathrm{ab}$$

$$\Rightarrow 10-\mathrm{ab}=14$$