1. (c)

X	Y	Z
$m_1 = m$	$m_2 = m$	$m_3 = m$
$T_1 = 10°C$	$T_2 = 20°C$	$T_3 = 30°C$
s_1	s_2	s_3

when $x \& y$ are mixed, $T_{f_1} = 16°C$

$$m_1\, s_1\, T + m_2\, s_2\, T_2 = (m_1\, s_1 + m_2\, s_2)Tf_1$$

$$s_1 \times 10 + s_2 \times 20 = (s_1 + s_2) \times 16$$

$$s_1 = \frac{2}{3}\, s_2$$

when y & z are mixex, $T_{f_2} = 26°C$

$$m_2\, s_2\, T + m_3\, s_3\, T_3 = (m_3\, s_3 + m_3\, s_3)Tf_2$$

$$s_2 \times 20 + s_3 \times 30 = (s_2 + s_3) \times 26$$

$$s_3 = \frac{3}{2}\, s_2$$

when x&z are mixex

$$m_1\, s_1\, T_1 + m_3\, s_3\, T_3 = (m_1\, s_1 + m_3\, s_3)Tf$$

$$\frac{2}{3}\, s_2 \times 10 + \frac{2}{3}\, s_2 \times 20 = \left(\frac{2}{3}\, s_2 + \frac{3}{2}\, s_2\right) T_f$$

$$T_f = 23.84°C$$

2. (b) $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}, mv = \sqrt{2mE}$

$$\lambda \propto \frac{1}{\sqrt{E}}$$

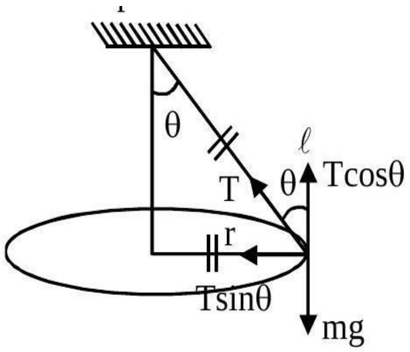
$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{E_1}{E_2}} = \frac{3}{4}, \lambda_2 = 0.75\lambda_1$$

$$\frac{E_1}{E_2} = \left(\frac{3}{4}\right)^2$$

$$E_2 = \frac{16}{9}E_1 = \frac{16}{9}E\ (E_1 = E)$$

Extra energy given $= \frac{16}{9}E - E = \frac{7}{9}E$

3. (a) Conical pendulum



$r = \frac{\ell}{\sqrt{2}}$

$\sin\theta = \frac{r}{\ell} = \frac{1}{\sqrt{2}}$

$\theta = 45°$

$$T\sin\theta = \frac{mv^2}{r}$$

$T\cos\theta = mg$

$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$

4. (c) $V = 4 \times 10^{-3}\ m^3$

$n = 3$ moles

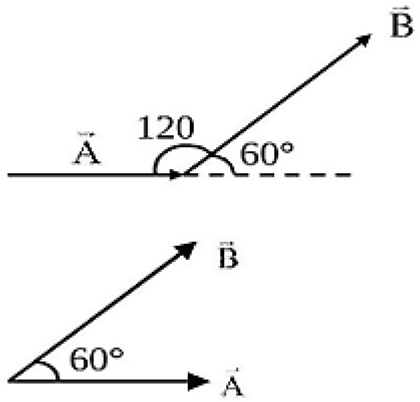
$T = 400$ K

$PV = nRT \Rightarrow P = \frac{nRT}{V}$

$P = \frac{3 \times 8.3 \times 400}{4 \times 10^{-3}}$

$= 24.9 \times 10^5$ Pa

5. (c)



Angle between $\vec{A}$ and $\vec{B}, \theta = 60°$

Angle betwenn $\vec{A}$ and $\vec{A} - \vec{B}$

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

$$= \frac{B\sqrt{\frac{3}{2}}}{A - B \times \frac{1}{2}2}$$

$$\tan\alpha = \frac{\sqrt{3}\,B}{2\,A - B}$$

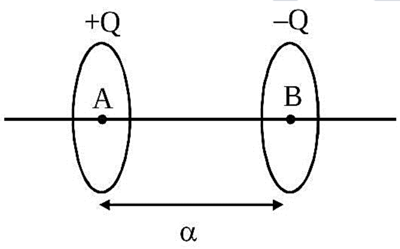
6. (d) $\frac{E_0}{C} = B_0$

$$F_{max} = eB_0\,V$$

$$= 1.6 \times 10^{-19} \times \frac{800}{3 \times 10^8} \times 3 \times 10^7$$

$$= 12.8 \times 10^{-18}\ N$$

7. (d)



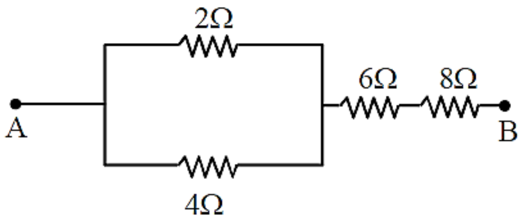
$$V_A = \frac{KQ}{a} - \frac{KQ}{\sqrt{a^2 + s^2}}$$

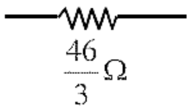
$$V_B = \frac{-KQ}{a} + \frac{KQ}{\sqrt{a^2 + s^2}}$$

$$V_A - V_B = \frac{2KQ}{a} - \frac{2KQ}{\sqrt{a^2 + s^2}}$$

$$= \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{s^2 + a^2}\right)$$

8. (d)





9. **(d)** Parallel axis theorem

$$I = I_{CM} + Md^2$$

$$I = \frac{M^2}{2} + M\left(\frac{L}{2}\right)^2$$

$$2.7 = M\frac{(0.2)^2}{2} + M\left(\frac{0.8}{2}\right)^2$$

$$2.7 = M\left[\frac{2}{100} + \frac{16}{100}\right]$$

$$M = 15 \text{ kg}$$

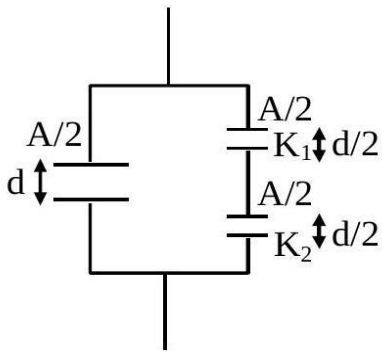
$$\Rightarrow \rho = \frac{M}{\pi r^2 L} = \frac{15}{\pi(0.2)^2 \times 0.8}$$

$$= 0.1492 \times 10^3$$

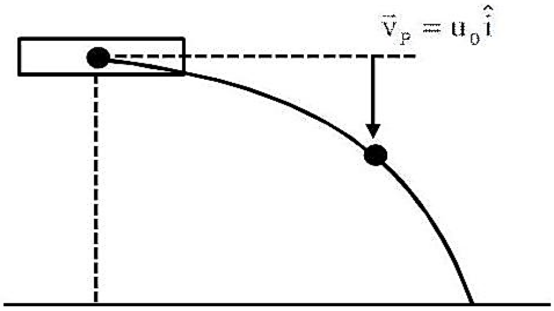
10. **(a)**

$$C_{eq} = \frac{\frac{A}{2}\varepsilon_0}{d} + \frac{A\varepsilon_0}{d}\frac{K_1 K_2}{K_1 + K_2}$$

$$= \frac{A\varepsilon_0}{d}\left(\frac{1}{2} + \frac{K_1 K_2}{K_1 + K_2}\right)$$



11. **(c)**



$v_B = u_0\hat{i} - gt\hat{j}$

$\vec{v}_{B/P} = \vec{v}_B - \vec{v}_P$

$\vec{v}_{B/P} = -8\hat{t}$

straight line vertically down

12. (c) $A \to B, B \to C$

$\frac{dN_B}{dt} = \lambda N_A - \lambda N_B$

$\frac{dN_B}{dt} = 2\lambda N_{B_0} e^{-\lambda t} - \lambda N_B$

$e^{-\lambda t}\left(\frac{dN_B}{dt} + \lambda N_B\right) = 2\lambda N_{B_0} e^{-\lambda t} \times e^{\lambda t}$

$\frac{d}{dt}\left(N_B e^{\lambda t}\right) = 2\lambda N_{B_0}$, on integrating

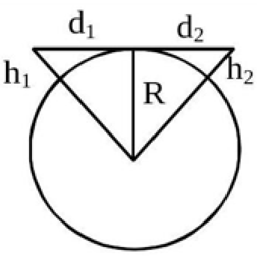
$N_B e^{\lambda t} = 2\lambda t N_{B_0} + N_{B_0}$

$N_B = N_{B_0}[1 + 2\lambda t]e^{-\lambda t}$

$\frac{dN_B}{dt} = 0$ at $-\lambda[1 + 2\lambda t)e^{-\lambda t} + 2\lambda e^{-\lambda t} = 0$

$N_{B_{max}}$ at $t = \frac{1}{2\lambda}$

13. (d)



$d_t = \sqrt{2Rh_1} + \sqrt{2Rh_2}$

$= \sqrt{2R}(\sqrt{h_1} + \sqrt{h_2})$

$= (2 \times 6400 \times 10^3)^{1/2}(\sqrt{50} + \sqrt{80})$

$= 3578(7.07 + 8.94)$

$= 57.28\text{Km}$

14. (a)

$$\frac{T_L}{T_H - T_L} = \text{C.O.P.} = \frac{\frac{dH}{dt}}{\frac{dW}{dt}}$$

$$\frac{263}{35} \times 35 = \frac{dH}{dt}$$

$$\frac{dH}{dt} = 263 \text{ watts}$$

15. (a) $P = Vi$

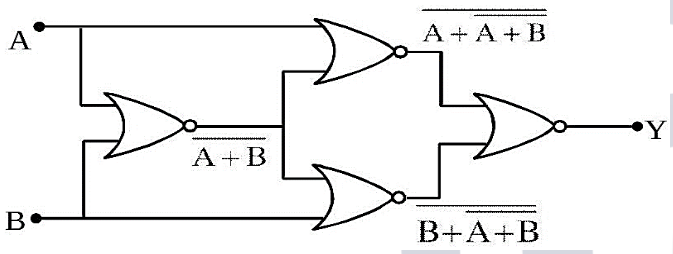
$500 = Vi$

$i = 5\text{Amp}$

$V = i \times R$

$R = 20$

16. (d)



$$y = \overline{(A + \overline{\overline{A + B}}) + (B + \overline{\overline{A + B}})}$$

$$y = (A + \overline{A + B}) \cdot (B + \overline{A + B})$$

A	B	y
0	0	1
0	1	0
1	0	0
1	1	1

17. (d)

(a) Magnetic Induction $= MT^{-2}\, A^{-1}$

(b) Magnetic Flux $= ML^2\, T^{-2}\, A^{-1}$

(c) Magnetic Permeability $= MLT^{-2}\ A^{-2}$

(d) Magnetization $= M^0\ L^{-1}\ A$

18. (b) $Z_C = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$

$$= \sqrt{\left(\frac{1}{100 \times 100 \times 10^{-6}}\right)^2 + 100^2}$$

$Z_C = \sqrt{(100)^2 + (100)^2}$

$= 100\sqrt{2}$

$Z_L = \sqrt{(\omega L)^2 + R^2}$

$\sqrt{(100 \times 0.5)^2 + 50^2}$

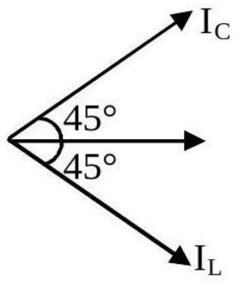
$= 50\sqrt{2}$

$$i_C = \frac{200}{z_C} = \frac{200}{100\sqrt{2}} = \sqrt{2}$$

$$i_L = \frac{200}{z_L} = \frac{200}{50\sqrt{2}} = 2\sqrt{2}$$

$$\cos\phi_1 = \frac{100}{10\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_1 = 45^\circ$$

$$\cos\phi_2 = \frac{50}{50\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_2 = 45^\circ$$



$I = \sqrt{I_C^2 + I_L^2}$

$= \sqrt{2 + 8}$

$= \sqrt{10}$

$I = 3.16$ A

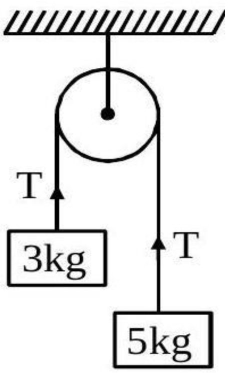
19. (c) $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$

$\Delta T = \frac{1}{2} \times \frac{0.1}{100} \times 24 \times 3600$

$\Delta T = 43.2$

20. (c)



$T = \frac{2\,m_1\,m_2\,g}{m_1 + m_2} = \frac{2 \times 3 \times 5 \times 10}{8}$

$= \frac{75}{2}$

$\text{Stress} = \frac{T}{A}$

$\frac{24}{\pi} \times 10^2 = \frac{75}{2 \times \pi R^2}$

$R^2 = \frac{75}{2 \times 24 \times 100} = \frac{3}{8 \times 24}$

$\Rightarrow R = 0.125 \text{ m}$

$R = 12.5 \text{ cm}$

21. (7) $y_1 = A_1 \sin k(x - vt)$

$y_1 = 12 \sin 6.28(x - vt)$

$y_2 = 5 \sin 6.28(x - vt + 3.5)$

$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$

$= K(\Delta x)$

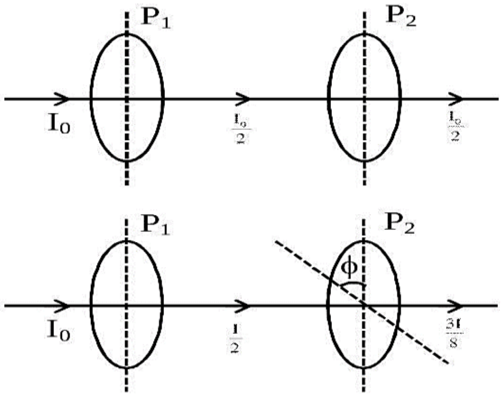
$= 6.28 \times 3.5 = \frac{7}{2} \times 2\pi = 7\pi$

$A_{\text{net}} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$A_{\text{net}} = \sqrt{(12)^2 + (5)^2 + 2(12)(5)\cos(7\pi)}$

$= \sqrt{144 + 25 - 120}$

22. (30) $I = \frac{I_0}{2}\cos^2\phi$



$\frac{\text{I}}{2}\cos^2\phi = \frac{3\text{I}}{8}$

$\cos^2\phi = \frac{3}{4}$

$\cos^2\phi = \frac{\sqrt{3}}{2}$

$\Rightarrow \phi = 30$

23. (543) $\text{V} = 12\text{kV}$

Number of revolution = n

$\text{n}[2 \times \text{q}_\text{p} \times \text{V}] = \frac{1}{2}\ \text{m}_\text{P} \times \text{v}_\text{P}^2$

$\text{n}[2 \times 1.6 \times 10^{-19} \times 12 \times 10^3$

$= \frac{1}{2} \times 1.67 \times 10^{-27} \times \left[\frac{3 \times 10^8}{6}\right]^2$

$\text{n}(38.4 \times 10^{-16}) = 0.2087 \times 10^{-11}$

$\text{n} = 543.4$

24. (14)

$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \ell = \frac{T^2 g}{4\pi^2}$

$\text{E} = \text{mg}\ell\frac{\theta^2}{2} = \text{mg}^2\frac{\text{T}^2\theta^2}{8\pi^2}$

$$\frac{dE}{E} = 2\left(\frac{dg}{g} + \frac{dT}{T}\right)$$

$= (4 + 3) = 14\%$

25. (60) Maximum emf $\varepsilon = N\omega AB$

$N = 20, \omega = 50, B = 3 \times 10^{-2}$ T

$\varepsilon = 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2} = 60.28 \times 10^{-2}$

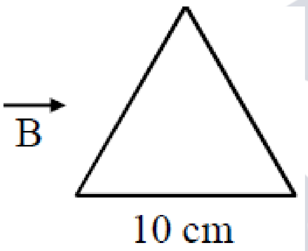
Rounded off to nearest integer $= 60$

26. (2) $x_2 = 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 2\pi t + \frac{1}{\sqrt{2}}\cos 2\pi t\right)\sqrt{2}$

$= 10\sin\left(2\pi t + \frac{\pi}{4}\right)$

$\therefore \frac{A_2}{A_1} = \frac{10}{5} = 2$

27. (3)

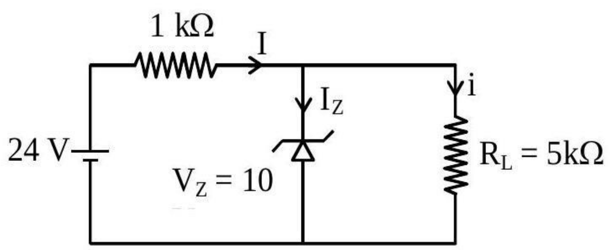


$\vec{\tau} = \vec{M} \times \vec{B} = MB\sin 90°$

$= MB = \frac{i\sqrt{3}\ell^2}{4}B$

$= \sqrt{3} \times 10^{-5}$ N − m

28. (120)



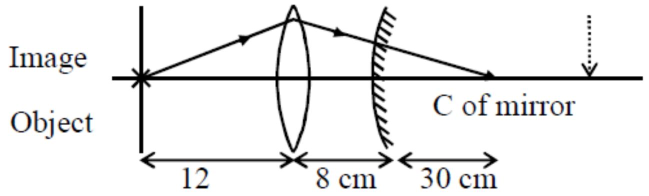
$i = \frac{10\text{ V}}{5k\Omega} = 2$ mA

$I = \frac{14\text{ V}}{1k\Omega} = 14$ mA

$\therefore I_z = 12$ mA

$\therefore P = I_z V_z = 120 \text{ mW}$

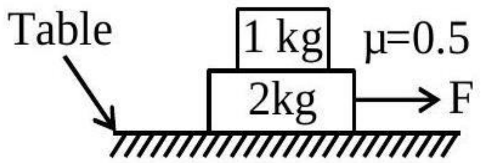
29. (50)



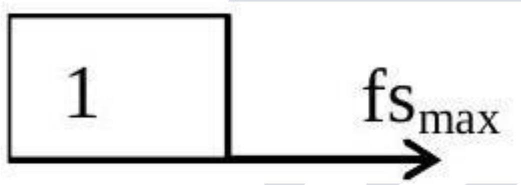
For the object to coincide with image, the light must fall perpendicularly to mirror. Which means that the light will have to converge at C of mirror. Without the mirror also, the light would coverage at C.

So the distance is : $12 + 8 + 30 = 50 \text{ cm}$

30. (15)



$F = 3a$ (For system)



$fs_{max} = 1a$ (for 1 kg block)

$\mu \times 1 \times g = a$

$\Rightarrow 5 = a$

$F = 15 \text{ N}$

31. (b) Only p-methyl, phenol does not give any colour with phthalic anhydroxide with cons. H_2SO_4.

32. (d) Photochemical smog causes cracking of rubber, the common component of photochemical smog are ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyle nitrate (PAN).

33. (c) For london dispersion forces.

$E \propto \frac{1}{r^6}$

Hence $x = -6$

34. (a)

35. (a) $O_2^- = (\sigma_{1s})^2(\sigma^*_{1s})^2(\sigma_{2s})^2(\sigma^*_{2s})^2(\sigma_{2p_z})^2$

$(\pi^2_{2p_x} = \pi^2_{2p_y})(\pi^{*2}_{2p_x} = \pi^{*1}_{2p_y})$

Bond order $= \frac{10-7}{2} = 1.5$ and paramagnetic

36. (c) Surcrose is example of disaccharide & non reducing sugar

Assertion: correct

Sucrose involves glycosidic linkage between C_1 of α-D-glucose C_2 of β-D-fructose

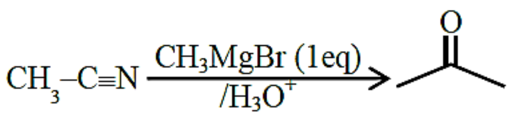
Reason: Incorrect

37. (c)

$$CH_3-\underset{\underset{O}{\|}}{C}-O-CH_2CH_3 \xrightarrow{H_3O^+} CH_3CO_2H + CH_3CH_2OH$$

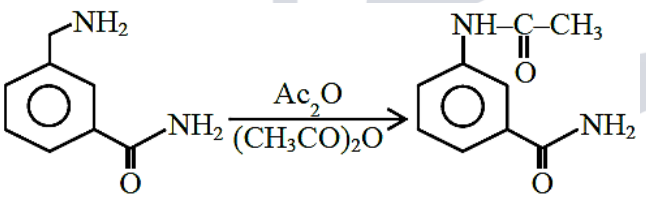
$$CH_3-\underset{\underset{O}{\|}}{C}-O-CH_3 \xrightarrow[-78^\circ C]{DIBALH/H_2O} CH_3CHO$$

$$CH_3-CN \xrightarrow[/H_2O]{SnCl_2+HCl} CH_3CH=O$$



38. (a) In IIA group on moving down the group size of cation increases and show thermal stability of carbonate increases.

39. (d)



40. (b)

(a) $[CoCl_2(en)_2]$ show Cis-trans isomerism

(b) $[Co(CN)_5(NC)]^{-3}$ can't Show G.I.

(c) $[Co(NH_3)_3(NO_2)_3]$ Show fac & mer isomerism

(d) $[Co(NH_3)_4Cl_2]^{\oplus}$ show cis & trans isomerism

41. (c)

42. (b) Sphalerite-ZnS, copper glance - Cu_2 S two sulphide ores can be separated by adjusting proportions of oil to water or by using ' Depressants '

43. (d) D_2O in used for the study of reaction mechanism. Rate of reaction for the cleavage of $O-H$ bond > O-D bond.

44. (b)

(i) CFSE $\propto$ charge or oxidation no. of central metal ion.
(ii) CFSE $\propto$ strength of ligand

$en > NH_3 > H_2O > F^-$

∴ order of CFSE

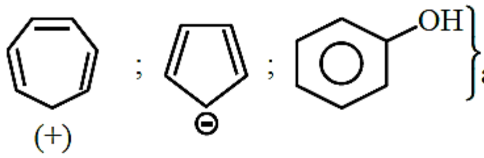
$$\left[\overset{III}{Co}(en_3)_3\right]^{+3} > \left[\overset{III}{Co}(NH_3)_6\right]^{+3} > \left[\overset{III}{Co}F_6\right]^{-3} > \left[\overset{II}{Co}(H_2O)_6\right]^{+2}$$

45. (c) The drug named chlordiate poxide is example of tranquilizer.

46. (b) Group 16/oxygen family is known as Chalcogens the members are O, S, Se, Te, Po

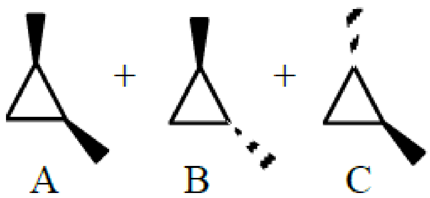
47. (c)


: Non aromatic

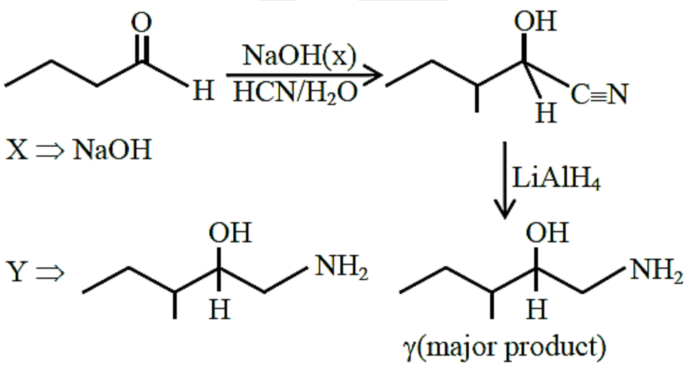


All are aromatic

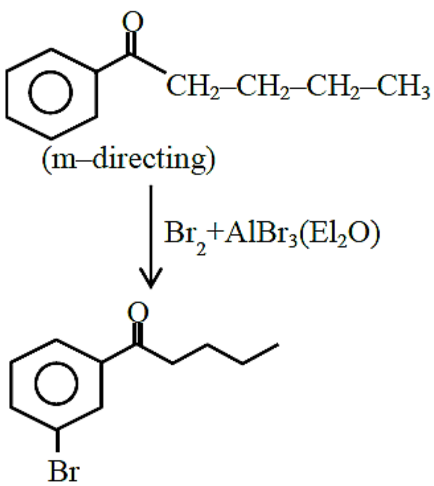
48. (d)



49.(c)



50. (c)



51. (42) Molecular mass of $BaSO_4 = 233$ g

$\because$ $233 BaSO_4$ contain $\rightarrow$ 32 g sulphur

$\therefore$ 1.44 g$BaSO_4$ contain $\rightarrow \frac{32}{233} \times 1.44$ g sulphur given : 0.471 g of organic compound

% of $S = \frac{32 \times 1.44}{233 \times 0.471} \times 100 = 41.98\% \approx 42\%$

OR

$\underset{w_{OC}=0.471\text{ g}}{\text{O.C.}} \longrightarrow \underset{1.44\text{ g}}{BaSO_4}$

$\Rightarrow n_s = n_{BaSO_4} = \frac{1.44}{233}$

$\Rightarrow w_s = \frac{1.44}{233} \times 32$ g

therefore % S $= \frac{W_s}{W_{o.c.}} \times 100 = \frac{1.44 \times 32}{233 \times 0.471} \times 100$

$= \frac{46.08}{109.743} \times 100 = 41.98 \simeq 42$

52. (182) $A + B \rightleftharpoons C + D : K_{eq} = 100$

1M 1M 1M 1M

First check direction of reversible reaction.

Since $Q_C = \frac{[C][D]}{[A][B]} = 1 < K_{eq.} \Rightarrow$ reaction will move in forward direction to attain equilibrium state.

$\Rightarrow A + B \rightleftharpoons C + D : K_{eq} = 100$

to 1 1 1

teq. $1 - x$ $1 - x$ $1 + x$ $1 + x$

Now : $K_{eq} = 100 = \frac{(1+x)(1+x)}{(1-x)(1-x)}$

$\Rightarrow 100 = \left(\frac{1+x}{1-x}\right)^2$

(i) $10 = \left(\frac{1+x}{1-x}\right)$

$\Rightarrow 10 - 10x = 1 + x$

$\Rightarrow 11x = 9$

$\Rightarrow x = \frac{9}{11}$

(ii) $-10 = \frac{1+x}{1-x}$

$\Rightarrow -10 + 10x = 1 + x$

$\Rightarrow -9x = -11$

$\Rightarrow x = \frac{11}{9}$

→ ' x ' cannot be more than one, therefore not valid. therefore, equation concretion of $(D) = 1 + x$

$= 1 + \frac{9}{11} = \frac{20}{11}$
$= 1.8181 = 181.81 \times 10^{-2}$
$\simeq 182 \times 10^{-2}$

53. (38) Given equation is

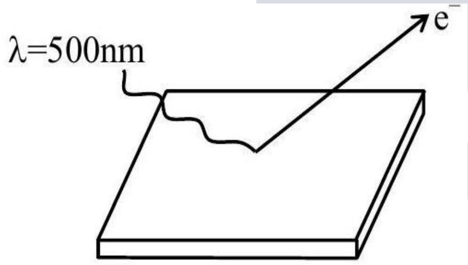
$H_2O(\ell) \longrightarrow H_2O(g): \Delta H = 41 \frac{kJ}{mol}$

$\Rightarrow$ From the relation: $\Delta H = \Delta U + \Delta n_g RT$

$\Rightarrow 41 \frac{kJ}{mol} = \Delta U + (a) \times \frac{8.3}{1000} \times 373$
$\Rightarrow DU = 41 - 3.0959$

$= 38$ kJ/mol

54. (5)



v : speed of electron having max. K.E.

$\Rightarrow$ from Einstein equation: $E = \phi + K \cdot E_{.max}$

$$\Rightarrow \frac{hc}{\lambda} = hv_0 + \frac{1}{2} mv^2$$

$$\Rightarrow \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9}} = 6.63 \times 10^{-34} \times 4.3 \times 10^{14} + \frac{1}{2} mv^2$$

$$\Rightarrow \frac{6.63 \times 30 \times 10^{-20}}{5} = 6.63 \times 4.3 \times 10^{-20} + \frac{1}{2} mv^2$$

$$\Rightarrow 11.271 \times 10^{-20}\ J = \frac{1}{2} \times 9 \times 10^{-31} \times v^2$$

$$\Rightarrow v = 5 \times 10^5\ m/sec.$$

55. (109) Galvanic cell:

$Zn_{(s)} + \underset{0.02M}{Cu^{+2}_{(aq.)}} \rightarrow \underset{0.04M}{Zn^{+2}} + Cu(s)$

Nernst equation $= F_{cell} = E^o_{cell} - \frac{0.059}{2} \log \frac{[2n^{+2}]}{[Cu^{+2}]}$

$\Rightarrow E_{cell}\left[E^{o}_{cell} - E^{o}_{Zn^{+2}/Zn}\right] - \frac{0.059}{2}\log\frac{0.04}{0.02}$

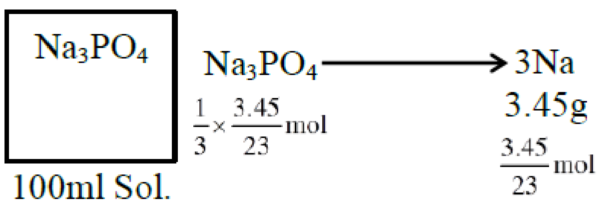
$\Rightarrow E_{cell}\,[0.34 - (-0.76)] - \frac{0.059}{2}\log^{2}$

$\Rightarrow E_{cell}\ 1 - 1 - \frac{0.059}{2} \times 0.3010$

$= 1.0911 = 109.11 \times 10^{-2}$

$= 109$

56. (50)



therefore molarity of Na_3PO_4 Solution $=$

$$\frac{n_{Na_3PO_4}}{\text{volume of solution in L}}$$

$$= \frac{\frac{1}{3}\times\frac{3.45}{23}\ \text{mol}}{0.1\ \text{L}}$$

$= 0.5 = 50 \times 10^{-2}$

57. (5) Given $k_f = 2.1 \times 10^{13}$

$K_d = \frac{1}{k_f} = 4.7 \times 10^{-14}$

$\therefore y = 4.7 \approx 5$

58. (269)

$k_f = 1.86k.\,kg/mol$

$T^{o}_{f} = 273k$

solvent : $H_2O(625\ g)$

Solute : 83 g $\begin{pmatrix} CH_2 & -CH_2 \\ | & | \\ OH & OH \end{pmatrix}$ $\Rightarrow$ Non dissociative

solute

$\Rightarrow \Delta T_f = k_f \times m$

$\Rightarrow \left(T^{o}_{f} - T^{1}_{f}\right) = 1.86 \times \frac{83/62}{624/1000}$

$\Rightarrow 273 - T_f^1 = \dfrac{1.86 \times 83 \times 1000}{62 \times 625} = \dfrac{154380}{38750}$

$\Rightarrow 273 - T_f^1 = 4$

$\Rightarrow T_f^1 = 259\ K$

59. (50) $[PtCl_4]^{-2} + H_2O \rightleftharpoons [Pt(H_2O)Cl_3]^- + Cl^-$

$\dfrac{-d[PtCl_4]^{-2}}{dt} = 4.8 \times 10^{-5}[PtCl_4^{-2}] - 2.4 \times 10^3$

$[Pt(H_2O)Cl_3][\mathring{u}]$

$\Rightarrow K_{eq} = \dfrac{k_f}{k_b} = \dfrac{4.8 \times 10^{-5}}{2.4 \times 10^{-3}} = 0.02$

60. (3) 448ml of A $\Rightarrow$ 1.53gmA

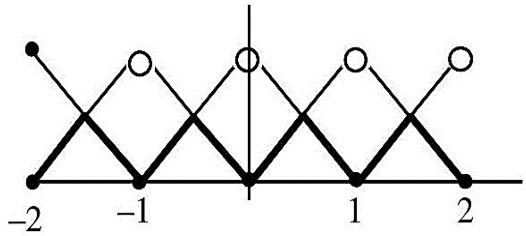
22400ml of A $\Rightarrow \frac{1.53}{445} \times 22400$gmA $= 7650$

$\underset{\text{has 3 carbon atoms}}{H_3CHC - CH - Cl/H_2O} \xrightarrow{O_3} \underset{\text{Aldehyde}}{CH_3 - CH = O}$

& mm is $36 + 5 + 35.5 = 76.5$

61. (a) $\min\{x - [x], 1 - x + [x]\}$

$h(x) = \min\{x - [x], 1 - [x - [x]]\}$



$\Rightarrow$ always continuous in $[-2,2]$

but non differentiable at 7 Points

62. (a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$$A^{2025} - A^{2020} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^6 - A = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

63. (c) $f(x) = \left(\frac{2}{x}\right)^{x^2}$; $x > 0$

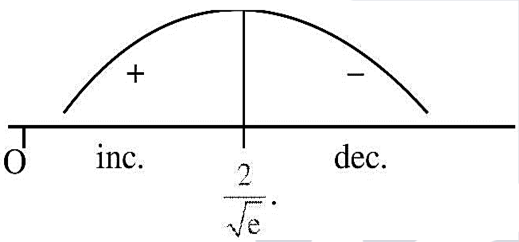
$\ln f(x) = x^2(\ln 2 - \ln x)$

$f'(x) = f(x)\{-x + (\ln 2 - \ln x)2x\}$

$f'(x) = \underbrace{f(x)}_{+} \cdot \underbrace{x}_{g(x)} \underbrace{(2\ell n2 - 2\ell nx - 1)}$

$g(x) = 2\ell n^2 - 2\ell nx - 1$

$= \ln\frac{4}{x^2} - 1 = 0 \Rightarrow x = \frac{2}{\sqrt{e}}$



$\text{LM} = \frac{2}{\sqrt{e}}$

Local maximum value $= \left(\frac{2}{2/\sqrt{e}}\right)^{\frac{4}{e}} \Rightarrow e^{\frac{2}{e}}$

64. (b) $I = \int_0^5 \frac{x+[x]}{e^{x-[x]}} dx$

$\int_0^1 \frac{x}{e^x} dx + \int_1^2 \frac{x+1}{e^{x-1}} dx + \int_2^3 \frac{x+2}{e^{x-2}} dx + \cdots .. \int_4^5 \frac{x+4}{e^{x-4}} dx$

$\Downarrow \quad \Downarrow$

$x = t + 1 \; x = z + 2 \; x = y + 4$

$\int_0^1 \frac{t+2}{e^t} dt + \int_0^1 \frac{z+4}{e^z} dz + \cdots .. + \int_0^1 \frac{y+8}{e^y} dx$

$\Rightarrow \int_0^5 \frac{5x+20}{e^x} dt = 5\int_0^1 \frac{x+4}{e^x} dx$

$\Rightarrow 5\int_0^1 (x+4)e^{-x} dx$

$\Rightarrow 5e^{-x}(-x-5)|_0^1 \Rightarrow -\frac{30}{e} + 25$

$\alpha = -30$

$\beta = 25 \Rightarrow 5\alpha + 6\beta = 0$

$(\alpha + \beta)^2 = 5^2 = 25$

65. (c) $P(-2\sqrt{6}, \sqrt{3})$ lies on hyperbola

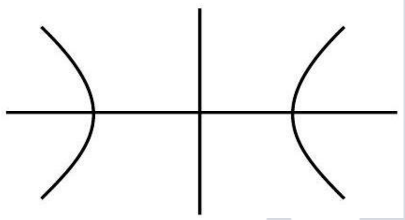
$$\Rightarrow \frac{24}{a^2} - \frac{3}{b^2} = 1$$

$$e = \frac{\sqrt{5}}{2} \Rightarrow b^2 = a^2\left(\frac{5}{4} - 1\right) \Rightarrow 4\,b^2 = a^2$$

Put in (i) $\Rightarrow \frac{6}{b^2} - \frac{3}{b^2} = 1 \Rightarrow b = \sqrt{3}$

$\Rightarrow a = \sqrt{12}$

$$\frac{x^2}{12} - \frac{y^2}{3} = 1$$



Tangent at P :

$$\frac{-x}{\sqrt{6}} - \frac{y}{\sqrt{3}} = 1 \Rightarrow Q(0, \sqrt{3})$$

Slope of T $= -\frac{1}{\sqrt{2}}$

Normal at P :

$$y - \sqrt{3} = \sqrt{2}(x + 2\sqrt{6})$$
$$\Rightarrow R = (0, 5\sqrt{3})$$
$$QR = 6\sqrt{3}$$

66. (c) $2x^2 dy + (e^y - 2x)dx = 0$

$$\frac{dy}{dx} + \frac{e^y - 2x}{2x^2} = 0 \Rightarrow \frac{dy}{dx} + \frac{e^y}{2x^2} - \frac{1}{x} = 0$$
$$e^{-y}\frac{dy}{dx} - \frac{e^{-y}}{x} = -\frac{1}{2x^2} \Rightarrow Put\, e^{-y} = z$$
$$\frac{-dz}{dx} - \frac{z}{x} = -\frac{1}{2x^2} \Rightarrow xdz + zdx = \frac{dx}{2x}$$
$$d(xz) = \frac{dx}{2x} \Rightarrow xz = \frac{1}{2}\log_e x + c$$

$xe^{-y} = \frac{1}{2}\log_e x + c$, passes through $(e, 1)$

$\Rightarrow C = \frac{1}{2}$

$xe^{-y} = \frac{\log_e ex}{2}$

$e^{-y} = \frac{1}{2} \Rightarrow y = \log_e 2$

67. (c) $S_1: (\sim p \vee q) \vee (q \vee p) = (q \vee \sim p) \vee (q \vee p)$

$S_1 = q \vee (\sim p \vee p) = qvt = t = \text{tautology}$

$S_2: (p \wedge \sim q) \wedge (\sim p \vee q) = (p \wedge \sim q) \wedge \sim (p \wedge \sim q) = C$

$= \text{fallacy}$

68. (d) $\frac{1+x}{x} \in (-\infty, -1] \cup [1, \infty)$

$\frac{1}{x} \in (-\infty, -2] \cup [0, \infty)$

$x \in \left[-\frac{1}{2}, 0\right) \cup (0, \infty)$

$x \in \left[-\frac{1}{2}, \infty\right) - \{0\}$

69. (d) $P(x \geq 5 \mid x > 2) = \frac{P(x \geq 5)}{P(x > 2)}$

$$\frac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \cdots \ldots.. + \infty}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \cdots \ldots + \infty}$$

$$\frac{\dfrac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}}{1 - \frac{5}{6}}}{\dfrac{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}}{1 - \frac{5}{6}}} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

70. (b)

$\sum_{r=1}^{50} \tan^{-1}\left(\frac{2}{4r^2}\right) = \sum_{r=1}^{50} \tan^{-1}\left(\frac{(2r+1) - (2r-1)}{1 + (2r+1)(2r-1)}\right)$

$\sum_{r=1}^{50} \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$

$\tan^{-1}(101) - \tan^{-1} 1 \Rightarrow \tan^{-1}\frac{50}{51}$

71. (b) $D \neq 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} \neq 0 \Rightarrow \lambda \neq 5$

For no solution $D = 0 \Rightarrow \lambda = 5$

$$D_1 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix} \neq 0 \Rightarrow \mu \neq 3$$

$$p = \frac{5}{6}$$

$$q = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$$

72. (c) $T = S_1$

$$xh - yk = h^2 - k^2$$

$$y = \frac{xh}{k} - \frac{(h^2 - k^2)}{k}$$

this touches $y^2 = 8x$ then $c = \frac{a}{m}$

$$\left(\frac{k^2 - h^2}{k}\right) = \frac{2k}{h}$$

$$2y^2 = x(y^2 - x^2)$$

$$y^2(x - 2) = x^3$$

73. (c)

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

$$2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}$$

$$\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$$

$$\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$$

$$\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right) = \frac{1}{8}$$

74. (a) $\left(2e^{i\pi/6}\right)^{100} = 2^{99}(p + iq)$

$$2^{100}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right) = 2^{99}(p + iq)$$

$$p + iq = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$p = -1, q = \sqrt{3}$$

$$x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0.$$

75. (d)

$A(\hat{j}) \cdot B(10\hat{i})$

H$(\hat{j} + 10\hat{k})$

G $(10\hat{i} + h\hat{j} + 10\hat{k})$

$\overrightarrow{AG} = 10\hat{i} + h\hat{j} + 10\hat{k}$
$\overrightarrow{BH} = -10\hat{i} + h\hat{j} + 10\hat{k}$
$\cos\theta = \frac{\overrightarrow{AG}\overrightarrow{BH}}{|\overrightarrow{AG}||\overrightarrow{BH}|}$
$\frac{1}{5} = \frac{h^2}{h^2 + 200}$
$4\,h^2 = 200 \Rightarrow h = 5\sqrt{2}$

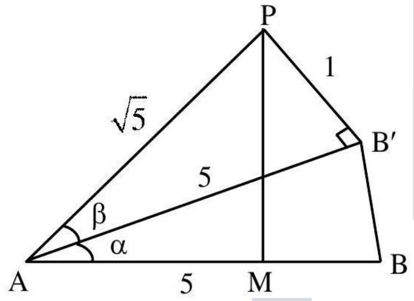
76. (c)

$(x + y + 4z - 16) + \lambda(-x + y + z - 6) = 0$

Passes through (1,2,3)

$-1 + \lambda(-2) \Rightarrow \lambda = -\frac{1}{2}$
$2(x + y + 4z - 16) - (-x + y + z - 6) = 0$
$3x + y + 7z - 26 = 0$

77. (c)



From figure.

$\sin\beta = \frac{1}{\sqrt{5}}$

$\therefore \tan\beta = \frac{1}{2}$

$\tan(\alpha + \beta) = 2$

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = 2$

$\frac{\tan\alpha + \frac{1}{2}}{1 - \tan\alpha\left(\frac{1}{2}\right)} = 2$

$\tan\alpha = \frac{3}{4}$

$\alpha = \tan^{1}\left(\frac{3}{4}\right)$

78. (c) $I=\int_0^{\pi/2}\frac{(1+\sin^2 x)}{(1+\pi^{\sin x})}+\frac{\pi^{\sin x}(1+\sin^2 x)}{(1+\pi^{\sin x})}dx$

$I=\int_0^{\pi/2}(1+\sin^2 x)dx$

$\mathrm{I}=\frac{\pi}{2}+\frac{\pi}{2}\cdot\frac{1}{2}=\frac{3\pi}{4}$

79. (a) $(x-2)^2+(y-1)^2+\lambda(x-2y)=0$

$C: x^2+y^2+x(\lambda-4)+y(-2-2\lambda)+5=0$

$C_1: x^2+y^2+2y-5=0$

$\mathrm{S}_1-\mathrm{S}_2=0$ (Equation of PQ)

$(\lambda-4)\mathrm{x}-(2\lambda+4)\mathrm{y}+10=0$ Passes through $(0,-1)$

$\Rightarrow \lambda=-7$

$C: x^2+y^2-11x+12y+5=0$

$=\frac{\sqrt{245}}{4}$

Diometer $=7\sqrt{5}$

80. (a)

$S=\lim_{x\to 2}\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}$

$S=\sum_{n=1}^{9}\frac{2}{4(n^2+3n+2)}=\frac{1}{2}\sum_{n=1}^{9}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

$S=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{11}\right)=\frac{9}{44}$

81. (7744) $209,220,231,\ldots,495$

Sum $=\frac{27}{2}(209+495)=9504$

$\underline{2}\ \underline{3}\ 1$

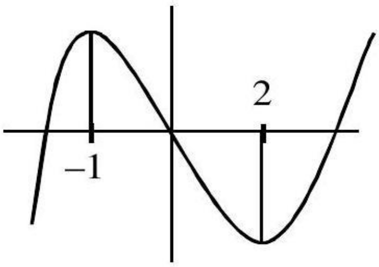
Number containing 1 at unit place $\underline{3}\ \underline{4}\ \underline{1}$

Number containing 1 at 10^{th} place $\begin{matrix}\underline{3} & \underline{1} & \underline{9}\\ \underline{4} & \underline{1} & \underline{8}\end{matrix}$

Required $=9501-(231+341+451+319+418)$
7744

82. (114) $\mathrm{f}'(\mathrm{x})=6\mathrm{x}^2-6\mathrm{x}-12=6(\mathrm{x}-2)(\mathrm{x}+1)$

Point $=(2,-20)\&(-1,7)$



$A=\int_{-1}^{0}(2x^3-3x^2-12x)dx+\int_{0}^{2}(12x+3x^2-2x^3)dx$

$A=\left(\frac{x^4}{2}-x^3-6x^2\right)_{-1}^{0}+\left(6x^2+x^3-\frac{x^4}{2}\right)_{0}^{2}$

$4\,A=114$

83. (5)

$\vec{a}=\hat{\imath}+2\hat{\jmath}+\hat{k}$

$\vec{b}=(2-\lambda)\hat{\imath}+6\hat{\jmath}-2\hat{k}$

$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=1, \vec{a}\cdot\vec{b}=12-\lambda$

$(\vec{a}\cdot\vec{b})=|\vec{b}|^2$

$\lambda^2-24\lambda+144=\lambda^2-4\lambda+4+40$

$20\lambda=100\Rightarrow\lambda=5.$

84. (2021)

$c_2=a_2+b_2=a_1-3+2\,b_1=12$
$a_1+2\,b_1=15$

$c_3=a_3+b_3=a_1-6+4\,b_1=13$

$a_1+4\,b_1=19$

from (a) & (b) $b_1=2, a_1=11$

$\sum_{k=1}^{10}c_k=\sum_{k=1}^{10}(a_k+b_k)=\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}b_k$

$=\frac{10}{2}(2\times11+9\times(-3))+\frac{2(2^{10}-1)}{2-1}$

$=5(22-27)+2(1023)$

$=2046-25=2021$

85. (96) Containing the line $\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}=0$

$9(x+1)-18(y-1)+9(z-3)=0$

$x-2y+z=0$

$PQ=\left|\frac{7+4+13}{\sqrt{6}}\right|=4\sqrt{6}$

$PQ^2=96$

86. (49)

$A_k=\sum_{i=0}^{9}{}^{9}C_i\,{}^{12}C_{k-i}+\sum_{i=0}^{8}{}^{8}C_i\,{}^{13}C_{k-i}$

$A_k={}^{21}C_k+{}^{21}C_k=2\cdot{}^{21}C_k$

$A_4-A_3=2({}^{21}C_4-{}^{21}C_3)=2(5985-1330)$

$190p=2(5985-1330)\Rightarrow p=49.$

87. (18) $3\alpha^2-10\alpha+27\lambda=0$

$\alpha^2-\alpha+2\lambda=0$

(a) -3(b) gives

$-7\alpha+21\lambda=0\Rightarrow\alpha=3\lambda$

Put $\alpha=3\lambda$ in equation (a) we get

$9\lambda^2-3\lambda+2\lambda-0$

$9\lambda^2=\lambda\Rightarrow\lambda=\frac{1}{9}$ as $\lambda\neq 0$

Now $\alpha=3\lambda\Rightarrow\lambda=\frac{1}{3}$

$\alpha+\beta=1\Rightarrow\beta=2/3$

$\alpha+\gamma=\frac{10}{3}\Rightarrow\gamma=3$

$\frac{\beta\gamma}{\lambda}=\frac{\frac{2}{3}\times 3}{\frac{1}{9}}=18$

88. (12) $5=\frac{3+7+x+y}{4}\Rightarrow x+y=10$

$\mathrm{Var}(x)=10=\frac{3^2+7^2+x^2+y^2}{4}-25$

$140=49+9+x^2+y^2$

$x^2+y^2=82$

$x+y=10$

$\Rightarrow (x, y) = (9,1)$

Four numbers are 21,9,10,8

Mean $= \frac{48}{4} = 12$

89. (4) $\operatorname{adj}(2\,A) = 2^2 \operatorname{adj} A$

$\Rightarrow \operatorname{adj}(\operatorname{adj}(2\,A)) = \operatorname{adj}(4 \operatorname{adj} A) = 16 \operatorname{adj}(\operatorname{adj} A)$

$= 16|\,A|A$

$\Rightarrow \operatorname{adj}(32|\,A|A) = (32|\,A|)^2 \operatorname{adj} A$

$12(32\mid Al)^2 \text{ladj}\,Al = 2^3(32\mid Al)^6 \text{ladj}\,Al$

$2^3.2^{30}|\,A|^6.|A|^2 = 2^{41}$

$|A|^8 = 2^8 \Rightarrow |A| = \pm 2$

$|A|^2 = |A|^2 = 4$

90. (6) $\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(-2i)^{n-2}}$

$$= \frac{(2i)^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}} = \frac{2^{\frac{n+2}{2}} ; i^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}}$$

This is positive integer for $n = 6$