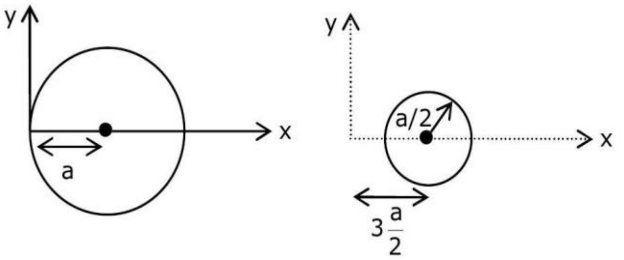
1. **(b)** The zener breakdown occurs in the heavily doped p-n junction diode. Heavily doped p-n junction diodes have narrow depletion region.
2. **(a)**
3. **(b)** $\lambda_{\min} = \frac{hc}{eV}$

$$\lambda_{\min} = \frac{1240 \text{ nm} - \text{eV}}{1.24 \times 10^{6}}$$

$$\lambda_{\min} = 10^{-3} \text{ nm}$$

4. **(a)** On the basis of kinetic theory of gases, the gas pressure is due to the molecules suffering change in momentum when impinge on the walls of container.
5. **(d)** Let σ is the surface mass density of disc.



$$X_{\text{com}} = \frac{(\sigma \times \pi a^2 \times a) - \left(\sigma \frac{\pi a^2}{4} \times \frac{3a}{2}\right)}{\sigma \pi a^2 - \frac{\sigma \pi a^2}{4}}$$

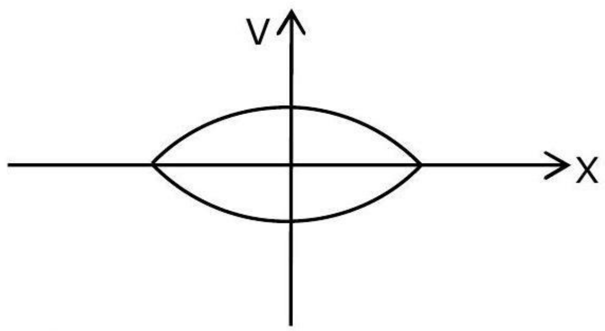
$$X_{\text{com}} = \frac{a - 3\frac{a}{8}}{1 - \frac{1}{4}}$$

$$X_{\text{com}} = \frac{\frac{5a}{8}}{\frac{3}{4}}$$

$$x_{\text{com}} = \frac{5a}{6}$$

6. **(a)**
7. **(a)** We know that is SHM;

$$V = \omega\sqrt{A^2 - x^2}$$



elliptical

8. (a)

(a) Source of microwave frequency - (ii) Magnetron

(b) Source of infra-red frequency - (iv) Vibration of atom and molecules

(c) Source of gamma ray - (i) Radioactive decay of nucleus

(d) Source of X-ray - (iii) inner shell electron

9. (b)
10. (b)

$$\beta = \frac{\lambda D}{d}$$

As $\lambda_v < \lambda_R$

$\Rightarrow \beta_{\mathrm{v}} < \beta_{\mathrm{R}}$

∴ Consecutive fringe line will come closer.b

11. (d) At north pole, weight

$Mg = 49$

Now, at equator

$g' = g - \underline{\underline{\omega}}^2 R$

$\Rightarrow Mg' = M(g - \omega^2 R)$

⇒ weight will be less than Mg at equator.

12. (c) AB → Isothermal process

$W_{AB} \to nRT\ln 2 = RT\ln 2$

BC → Isochoric process

$W_{BC} = 0$

CA → Adiabatic process

$$W_{CA} = \frac{P_1 V_1 - \frac{P_1}{4} X_2 V_1}{1-\gamma} = \frac{P_1 V_1}{2(1-\gamma)} = \frac{RT}{2(1-\gamma)}$$

$$W_{ABCA} = RT\ell n2 + \frac{RT}{2(1-\gamma)}$$

$$= RT\left[\ln 2 - \frac{1}{2(\gamma-1)}\right]$$

13. (c) $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T^2 = 4\pi^2\left[\frac{\ell}{g}\right]$

$g = 4\pi^2\left[\frac{\ell}{T^2}\right]$

$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T}{T}$

$= \left[\frac{1\text{ mm}}{1\text{ m}} + \frac{2(10\times10^{-3})}{1.95}\right]\times 100$

$= 1.13\%$

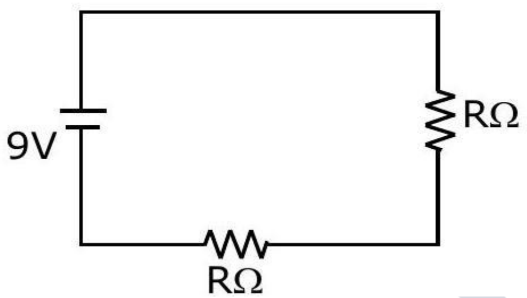
14. **(a)** Equivalent $K = K + K = 2K$

Now, $T = 2\pi\sqrt{\frac{m}{K_{eq}}}$

$\Rightarrow T = 2\pi\sqrt{\frac{m}{2k}}$

$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

15. **(c)** When switch S is closed-



Given : $v = 9v$

From $V = \text{IR}$

$I = \frac{V}{R}$

$R_{eq.} = 2 + 2 = 4\Omega$

$I = \frac{9}{4} = 2.25\text{ A}$

16. **(b)** From De-broglie's wavelength:-

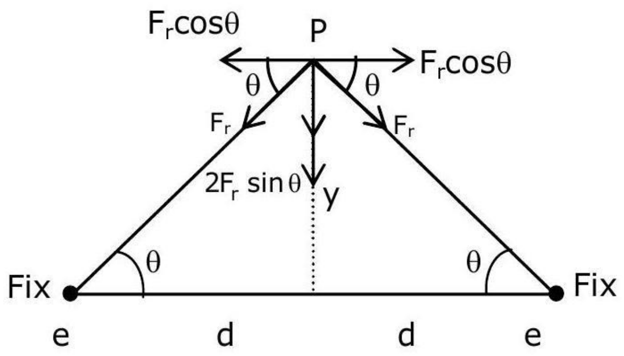
$\lambda = \frac{h}{mv}$

Given $\lambda_P = \lambda_\alpha$

$v \alpha \frac{1}{m}$

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p} = \frac{4}{1}$$

17. (a)



Restoring force on proton :-

$$F_r = \frac{2Kq^2y}{[d^2+y^2]^{\frac{3}{2}}}$$

Y <<< d

$$F_r = \frac{2kq^2y}{d^3} = \frac{q^2y}{2\pi\varepsilon_0 d^3} = ky$$

$$K = \frac{q^2}{2\pi\varepsilon_0 d^3}$$

Angular Frequency :-

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{q^2}{2\pi\varepsilon_0 m d^3}}$$

18. (b) Atoms of ferromagnetic material in unmagnetised state form domains inside the ferromagnetic material. These domains have large magnetic moment of atoms. In the absence of magnetic field, these domains have magnetic moment in different directions. But when the magnetic field is applied, domains aligned in the direction of the field grow in size and those aligned in the direction opposite to the field reduce in size and also its orientation changes.

19. (b) $Y = F(x,t)$

For travelling wave y should be linear function of x and t and they must exist as $(x \pm vt)$ $\mathrm{Y = Asin(15x - 2t)} \rightarrow$ linear function in x and t.

20. (c) $a = \frac{vdv}{dx}$

$\int_{v_i}^{v_f} V dv = \int_{x_i}^{x_f} a dx$

Given :- $v_i = v_0$

$V_f = 0$

$X_i = 0$

$X_f = x$

From Damping Force : $a = -\frac{\alpha x^2}{m}$

$\int_{V_0}^{0} VdV = -\int_0^x \frac{\alpha x^2}{m} dx$

$-\frac{v_0^2}{2} = \frac{-\alpha}{m}\left[\frac{x^3}{3}\right]$

$x = \left[\frac{3mv_0^2}{2\alpha}\right]^{\frac{1}{3}}$

21. (2)

$y = \frac{F/A}{\Delta\ell/\ell}$

$\Rightarrow \frac{F}{A} = y\frac{\Delta\ell}{\ell}$

$\Rightarrow \frac{F}{A} = y \times \frac{0.04}{\ell}$

When length & diameter is doubled.

$\Rightarrow \frac{F}{4\,A} = y \times \frac{\Delta\ell}{2\ell}$

(a) ÷ (b)

$\frac{F/A}{F/4\,A} = \frac{y \times \frac{0.04}{\ell}}{y \times \frac{\Delta\ell}{2\ell}}$

$4 = \frac{0.04 \times 2}{\Delta\ell}$

$\Delta\ell = 0.02$

$\Delta\ell = 2 \times 10^{-2}$

$\therefore x = 2$

22. (5) We know that

$J = \sigma E$

$\Rightarrow J = 5 \times 10^7 \times 10 \times 10^{-3}$

$\Rightarrow J = 50 \times 10^4 \ A/m^2$

Current flowing;

$I = J \times \pi R^2$

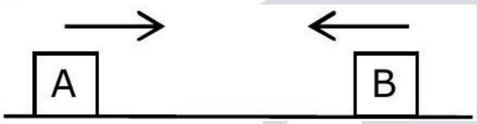
$I = 50 \times 10^4 \times \pi (0.5 \times 10^{-3})^2$

$I = 5 \times 10^4 \times \pi \times 0.25 \times 10^{-6}$

$I = 125 \times 10^{-3} \pi$

$X = 5$

23. (8)



Speed = 7.2 km/h = 2 m/s

Frequency as heard by A

$$f_A' = f_B \left(\frac{v + v_0}{v - v_s}\right)$$

$$f_A' = 676 \left(\frac{340 + 2}{340 - 2}\right)$$

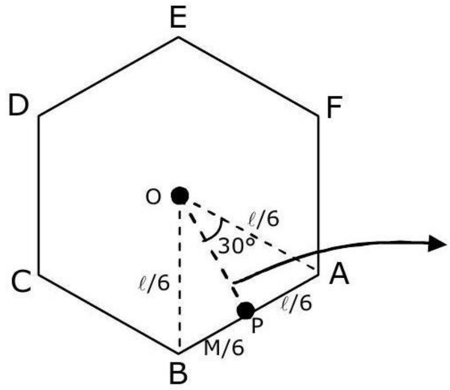
$f_A' = 684$ Hz

$\therefore f_{Beat} = f_A' - f_B$

$= 684 - 676$

$= 8$ Hz

24.(8)



MOI of AB about P: $I_{ABP} = \frac{\frac{M}{6}\left(\frac{\ell}{6}\right)^2}{12}$

MOI of AB about O,

$$I_{AB_O} = \left[\frac{\frac{M}{6}\left(\frac{\ell}{6}\right)^2}{12} + \frac{M}{6}\left(\frac{\ell}{6}\frac{\sqrt{3}}{2}\right)^2\right]$$

$$I_{Hexagon\ _o} = 6I_{AB_O} = M\left[\frac{\ell^2}{12\times 36} + \frac{\ell^2}{36}\times\frac{3}{4}\right]$$

$$= \frac{6}{100}\left[\frac{24\times 24}{12\times 36} + \frac{24\times 24}{36}\times\frac{3}{4}\right]$$

$= 0.8\text{kgm}^2$

$= 8\times 10^{-2}\ \text{kg/m}^2$

25. (226) Using Gauss law, it is a part of cube of side 12 cm and charge at centre so;

$$\phi = \frac{Q}{6\varepsilon_0} = \frac{12\mu C}{6\varepsilon_0} = 2\times 4\pi\times 9\times 10^9\times 10^{-6}$$

$= 226\times 10^3 \text{Nm}^2/\text{C}$

26. (2)

Given that, $\frac{M_1}{M_2} = \frac{1}{2}$

Also, $p_1 = p_2 = p$

$\Rightarrow M_1\ V_1 = M_2\ V_2 = p$

Also, we know that

$$K = \frac{p^2}{2M} \Rightarrow K_1 = \frac{p^2}{2M_1} \ \&\Rightarrow K_2 = \frac{p^2}{2M_2}$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{p^2}{2M_1}\times\frac{2M_2}{p^2} \Rightarrow \frac{K_1}{K_2} = \frac{M_2}{M_1} = \frac{2}{1}$$

$$\Rightarrow \frac{A}{1} = \frac{2}{1} \Rightarrow \therefore A = 2$$

27. (400 m/s)

$V_{rms}\sqrt{\frac{3RT_1}{M_0}}$

$$200 = \sqrt{\frac{3R \times 300}{M_0}}$$

Also, $\frac{x}{\sqrt{3}} = \sqrt{\frac{3R\times400}{M_0}}$

(a) ÷ (b)

$$\frac{200}{x/\sqrt{3}} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$$

$\Rightarrow \mathrm{x} = 400\ \mathrm{m/s}$

28. (900)

$$P = \frac{V^2}{R}$$

$$16 = \frac{120^2}{R} \Rightarrow R = \frac{14400}{16}$$
$$\Rightarrow R = 900\Omega$$

29. (667)

$\mathrm{f} = 3\mathrm{GHz}, \varepsilon_{\mathrm{r}} = 2.25$

$$v = \lambda \mathrm{f} \Rightarrow \lambda = \frac{\mathrm{v}}{\mathrm{f}}$$

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$v = \frac{1}{\sqrt{\mu_0 \mu_{\mathrm{r}} \varepsilon_0 \varepsilon_{\mathrm{r}}}} \Rightarrow \lambda = \frac{1}{\mathrm{f} \cdot \sqrt{\mu_0 \varepsilon_0} \cdot \sqrt{\mu_{\mathrm{r}} \varepsilon_{\mathrm{r}}} \cdot \mathrm{f}}$$

$$\Rightarrow \lambda = \frac{C}{\mathrm{f} \cdot \sqrt{\mu_{\mathrm{r}}} \cdot \sqrt{\varepsilon_{\mathrm{r}}}} \Rightarrow \lambda = \frac{3 \times 10^8}{3 \times 10^9 \times \sqrt{1} \times \sqrt{2.25}}$$

$\Rightarrow \lambda = 667 \times 10^{-2}$ cm

30. (8) Power of signal transmitted: $P_i = 0.1\mathrm{Kw} = 100\mathrm{w}$

Rate of attenuation $= -5\ \mathrm{dB/Km}$

Total length of path $= 20$ km

Total loss suffered $= -5 \times 20 = -100$ dB

Gain in dB $= 10\log_{10}\frac{P_0}{P_i}$

$-100 = 10\log_{10}\frac{P_0}{P_i}$

$\Rightarrow \log_{10}\frac{P_i}{P_0} = 10$

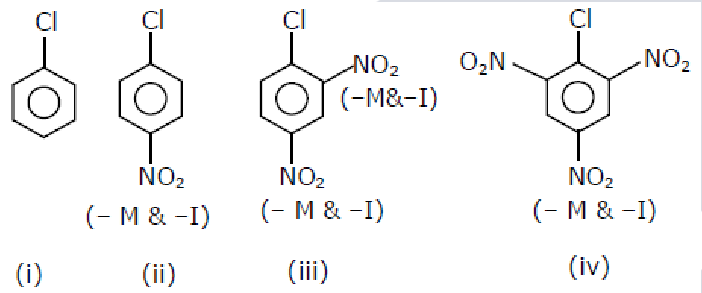
$\Rightarrow \log_{10}\frac{P_i}{P_0} = \log_{10} 10^{10}$

$\Rightarrow \frac{100}{P_0} = 10^{10}$

$\Rightarrow P_0 = \frac{1}{10^8} = 10^{-8}$

$\therefore x = 8$

31. (c)



Reactivity $\propto$ −m group present at O/P position.

32. (c)

Siderite	$FeCO_3$
Calamine	$ZnCO_3$
Kaolinite	$Si_2Al_2O_5(OH)_4$ or $Al_2O_3 \cdot 2SiO_2 \cdot 2H_2O$
Malachite	$CuCO_3 \cdot Cu(OH)_2$

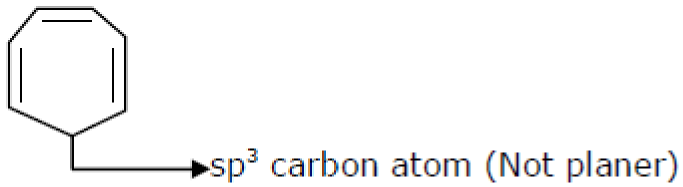
33. (b)

34. (b) Hydrogen is most abundant element in universe because all luminous body of universe i.e. stars & nebulae are made up of hydrogen which acts as nuclear fuel & fusion reaction is responsible for their light.

35. (c) For survival of aquatic life dissolved oxygen is responsible its optimum limit 6.5ppm and optimum limit of BOD ranges from 10-20 ppm & BOD stands for biochemical oxygen demand.

36. (a)

37. (b)



Hence It is non-aromatic.

38. (b) Red colour of ruby is due to presence of CrO_3 or Cr^{+6} not CO^{3+}

39. (b)

40. (d)

41. (b) Generally

$$\text{M.P.} \propto \text{ Lattice energy } = \frac{KQ_1Q_2}{r^+ + r^-}$$
$$\propto \text{ (packing efficiency)}$$

42. (c) $[FeCl_4]^{2-}Fe^{2+}3\ d^6 \rightarrow$ 4 unpaired electron. as Cl^-in a weak field liquid.

$\mu_{\text{spin}} = \sqrt{248}M$

$= 4.9BM$

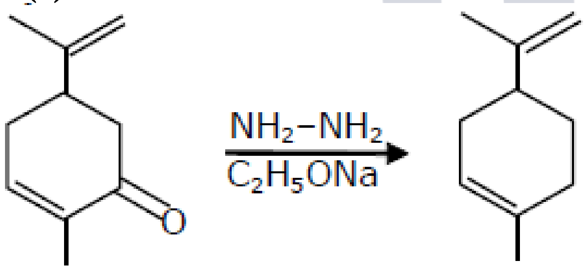
$[Co(C_2O_4)_3]^{3-}Co^{3+}3\ d^6 \rightarrow$ for Co^{3+} with coodination no. $6C_2O_4\ ^{2-}$ is strong field ligend & causes pairing & hence no. unpaired electron

$\mu_{\text{spin}} = 0$

$[MnO_4]^{2-}Mn^{+6}$ it has one unpaired electron.

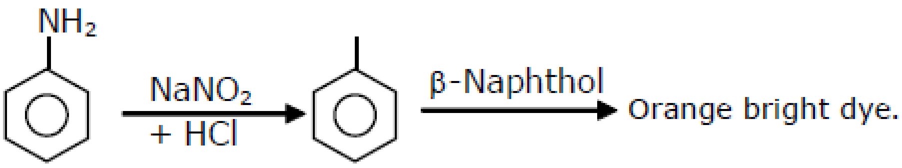
$\mu_{\text{spin}} = \sqrt{3}BM$

43. (b)

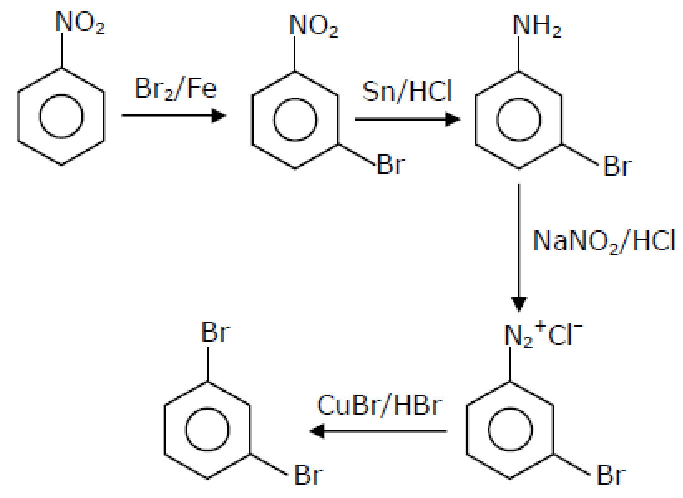


It is wolf-kishner reduction of carbonyl compounds.

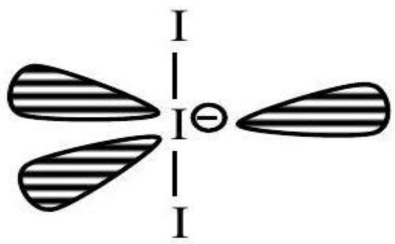
44. (c)



45. (d)



46. (c) I_3^- sp^3 d hybridisation (2BP +3 L.P.) Linear geometry



47. (d)
48. (d)
49. (a) Buna-S is the co-polymer of buta- 1, 3 diene & styrene.
50. (d) Blood is a negative sol. According to hardy-Schulz's rule, the cation with high charge has high coagulation power. Hence, $FeCl_3$ can be used for clotting blood.

51. (3776)

$$5e^- + MnO_4^- + 8H^+ \longrightarrow Mn^{+2} + 4H_2O$$

$$Q = \frac{[Mn^{+2}]}{[H^+]^8[MnO_4^-]} \quad \Rightarrow \quad E_1 = E^\circ - \frac{0.059}{5}\log(Q_1)$$

$$E_2 = E^\circ - \frac{0.059}{5}\log(Q_2) \quad \Rightarrow \quad E_2 - E_1 = \frac{0.059}{5}\log\left(\frac{Q_1}{Q_2}\right)$$

$$= \frac{0.059}{5}\log\left\{\frac{[H^+]_{II}}{[H^+]_I}\right\}^8 \quad \Rightarrow \quad = \frac{0.059}{5}\log\left(\frac{10^{-4}}{1}\right)^8$$

$$(E_2 - E_1) = \frac{0.059}{5} \times (-32) \quad \Rightarrow \quad |(E_2 - E_1)| = 32 \times \frac{0.059}{5} = x \times 10^{-4}$$

$$= \frac{32 \times 590}{5} \times 10^{-4} = x \times 10^{-4} \quad \Rightarrow \quad = 3776 \times 10^{-4}\ x = 3776$$

52. (a) S_2 is like O_2 i; e paramagnetic as per molecular orbital theory.
53. (a) $\Delta T_f = i \times K_f \times m$

$$= (a) \times 5.12 \times \frac{10/58}{200} \times 1000 \Rightarrow \Delta T_f = \frac{5.12 \times 50}{58} = 4.414$$

$$T_{f(\text{ solution })} = T_{K(\text{ solvent })} - \Delta T_f$$

$= 5.5 - 4.414$

$= 1.086^\circ C$

$\approx 1.09^\circ C = 1$ (nearest integer)

54. (5) $T = 50^\circ C = 323.15\ K$

$P = 740$ mm of Hg $= \frac{740}{760}$ atm

V= ?

moles $(n) = \frac{4.75}{26}$

$$V = \frac{4.75}{26} \times \frac{0.0821 \times 323.15}{740} \times 760$$

$V = 4.97 \simeq 5$ Lit

55. (141)

$K_{SP}(PbI_2) = 8 \times 10^{-9}$

$PbI_2(s) \rightleftharpoons Pb^{+2}(aq) + 2I^-(aq)$
$S + 0.1\ 2\ S$
$K_{SP} = [Pb^{+2}][I^-]^2$
$8 \times 10^{-9} = (S + 0.1)(2\ S)^2 \Rightarrow 8 \times 10^{-9} \simeq 0.1 \times 4\ S^2$
$\Rightarrow S^2 = 2 \times 10^{-8}$
$S = 1.414 \times 10^{-4}$ mol/Lit
$= x \times 10^{-6}$ mol/Lit $\therefore x = 141.4 \simeq 141$

56. (c) Only aliphatic amines can be prepared by Gabriel synthesis.

57. (243)

$Ph - NH_2 \longrightarrow Ph - NH - C - CH_3$

$(C_6H_7\ N)$ (Ace tanilide) (C_8H_9NO)

Molar mass = 93 Molarmass = 135

93 g Aniline produce 135 g acetanilide

1.86 g produce $\frac{135 \times 1.86}{93} = 2.70$ g

At 10% loss, 90% product will be formed after purification.

$\therefore$ Amount of product obtained $= \frac{2.70 \times 90}{100} = 2.43\ g = 243 \times 10^{-2}$ g

58. (8)

$C_xH_y + 6O_2 \longrightarrow 4CO_2 + \frac{y}{2}H_2O$

Applying POAC on 'O' atoms

$6 \times 2 = 4 \times 2 + y/2 \times 1$

$y/2 = 4 \Rightarrow y = 8$

59. (81)

Sucose $\xrightarrow{\text{Hydrolysis}}$ Glucose + Fructose

$t_{1/2} = 3.33\ \text{h} = \frac{10}{3}\ \text{h} \Rightarrow C_t = \frac{C_\circ}{2^{t/t_{1/2}}}$

Fraction of sucrose remaining $= f = \frac{C_t}{C_0} = \frac{1}{2^{t/t_{1/2}}}$

$\frac{1}{f} = 2^{t/t_{1/2}}$

$\log(1/f) = \log\left(2^{t/t_{1/2}}\right) = \frac{t}{t_{1/2}}\log(\text{b})$

$= \frac{9}{10/3} \times 0.3 = \frac{8.1}{10} = 0.81 = x \times 10^{-2}\ x = 81$

60. (855)

$3\text{HC} \equiv \text{CH(g)} \rightleftharpoons \text{C}_6\text{H}_6(\ell)$

$\Delta G_r^\circ = \Delta G_f^\circ[\text{C}_6\text{H}_8(\ell)] - 3 \times \Delta G_f^0[\text{HC} \equiv \text{CH}]$

$= [-1.24 \times 10^5 - 3 \times (-2.04 \times 10^5)]$

$= 4.88 \times 10^5\ \text{J/mol}$

$\Delta G_r^0 = -\text{RTln}\left(\text{K}_{\text{eq}}\right)$

$\log\left(\text{K}_{\text{eq}}\right) = \frac{-\Delta G^\circ}{2.303\text{RT}}$

$= \frac{-4.88 \times 10^5}{2.303 \times 8.314 \times 298}$

$= -8.55 \times 10^1 = 855 \times 10^{-1}$

61. (b) $P(a, 6,9), Q(20, b, -a - 9)$

mid point of PQ $= \left(\frac{\text{a}+20}{2}, \frac{\text{b}+6}{2}, -\frac{\text{a}}{2}\right)$

lie on line

$$\frac{\frac{\text{a}+20}{2} - 3}{7} = \frac{\frac{\text{b}+6}{2} - 2}{5} = \frac{-\frac{\text{a}}{2} - 1}{-9}$$

$$\frac{\text{a} + 20 - 6}{14} = \frac{\text{b} + 6 - 4}{10} = \frac{-\text{a} - 2}{-18}$$

$$\frac{a+14}{14}=\frac{a+2}{18}$$

$18a+252=14a+28$

$4a=-224$

$a=-56$

$$\frac{b+2}{10}=\frac{a+2}{18}$$

$$\frac{b+2}{10}=\frac{-54}{18}$$

$$\frac{b+2}{10}=-3 \Rightarrow b=-32$$

$|a+b|=|-56-32|=88$

62. (b) Given $f(x)f''(X)-(f'(x))^2=0$

Let $h(x)=\frac{f(x)}{f'(x)}$

$\Rightarrow h'(x)=0 \ \Rightarrow h(x)=k$

$$\Rightarrow \frac{f(x)}{f'(x)}=k \ \Rightarrow f(x)=kf'(x)$$

$$\Rightarrow f(0)=kff'(0) \ \Rightarrow 1=k(b) \Rightarrow k=\frac{1}{2}$$

Now $f(x)=\frac{1}{2}f'(x) \Rightarrow \int 2dx=\int \frac{f'(x)}{f(x)}dx$

$\Rightarrow 2x=\ln|f(x)|+C$

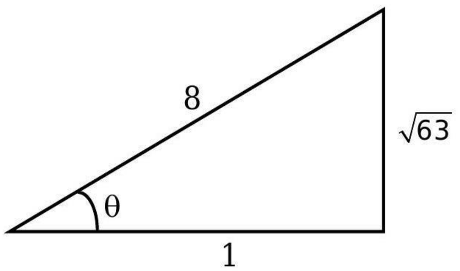
As $f(0)=1 \Rightarrow C=0$

$\Rightarrow 2x=\ln|f(X)| \Rightarrow f(x)=\pm e^{2x}$

As $f(0)=1 \Rightarrow f(x)=e^{2x} \Rightarrow f(a)=e^2$

63. (b) $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)=\theta \ \sin\theta=\frac{\sqrt{63}}{8}$



$\cos\theta = \frac{1}{8}$

$2\cos^2\frac{\theta}{2} - 1 = \frac{1}{8}$

$\cos^2\frac{\theta}{2} = \frac{9}{16}$

$\cos\frac{\theta}{2} = \frac{3}{4}$

$\frac{1-\tan^2\frac{\theta}{4}}{1+\tan^2\frac{\theta}{4}} = \frac{3}{4}$

$\tan\frac{\theta}{4} = \frac{1}{\sqrt{7}}$

64. **(b)** Required probability

$= \frac{{}^5C_2 \times 3^3}{4^5}$

$= \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$

65. **(b)** Plane passing through intersection of plane is

$\{\vec{r}\cdot(\hat{\imath}+\hat{\jmath}+\hat{k})-1\} + \lambda\{\vec{r}\cdot(\hat{\imath}-2\hat{\jmath})+2\} = 0$

Passes through $\hat{\imath}+2\hat{k}$, we get

$(3-1)+\lambda(1+2) = 0 \Rightarrow \lambda = -\frac{2}{3}$

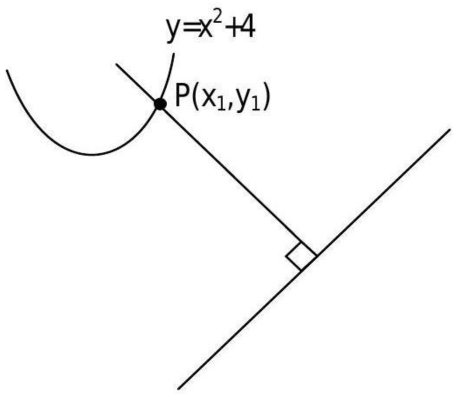
Hence, equation of plane is $3\{\vec{r}\cdot(\hat{\imath}+\hat{\jmath}+\hat{k})-1\} - 2\{\vec{r}\cdot(\hat{\imath}-2\hat{\jmath})+2\} = 0$

$\Rightarrow \vec{r}\cdot(\hat{\imath}+7\hat{\jmath}+3\hat{k}) = 7$

66. **(d)**

$\left.\frac{dy}{dx}\right|_p = 4$

$\therefore 2x_1 = 4$



$\Rightarrow x_1 = 2$

$\therefore$ Point will be (2,8)

67. (d) $2b = a + c$

$\frac{2a+2}{3} = \frac{10}{3}$ and $\frac{2b+c}{3} = \frac{7}{3}$

$a = 4, \left.\begin{matrix} 2b + c = 7 \\ 2b - c = 4 \end{matrix}\right\}$, solving

$b = \frac{11}{4}$

$c = \frac{3}{2}$

$\therefore$ Quadratic Equation is $4x^2 + \frac{11}{4}x + 1 = 0$

$\therefore$ The value of $(\alpha + \beta)^2 - 3\alpha\beta = \frac{121}{256} - \frac{3}{4} = -\frac{71}{256}$

68. (c)

$I = \int_1^3 -3dx + \int_1^3 [(x-1)^2]dx$

Put $x - 1 = t; dx = dt$

$I = (-6) + \int_0^2 [t^2]dt$

$I = -6 + \int_0^1 0dt + \int_1^{\sqrt{2}} 1dt + \int_{\sqrt{2}}^{\sqrt{3}} 2dt + \int_{\sqrt{3}}^{2} 3dt$

$I = -6 + (\sqrt{2} - 1) + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

$I = -1 - \sqrt{2} - \sqrt{3}$

69. (a)

$$f(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x^2 - x - 20) & ; \quad -5 < x < 4 \\ 6(x^2 - x - 6) & ; \quad x > 4 \end{cases}$$

$$f(x)=\begin{cases}-55 & ; \quad x<-5\\ 6(x-5)(x+4) & ; \quad -5<x<4\\ 6(x-3)(x+2) & ; \quad x>4\end{cases}$$

Hence, $f(x)$ is monotonically increasing in interval $(-5,-4)\cup(4,\infty)$

70. (a) $2=a+b+c$

$$\frac{dy}{dx}=2ax+b\Rightarrow\left.\frac{dy}{dx}\right|_{(0,0)}=1$$

$\Rightarrow \mathrm{b}=1\Rightarrow \mathrm{a}+\mathrm{c}=1$

$(0,0)$ lie on curve

$\therefore \mathrm{c}=0, \mathrm{a}=1$

71. (d)

72. (c) $x-2y+0.z=1$

$\mathrm{x}-\mathrm{y}+\mathrm{kz}=-2$

$0.x+ky+4z=6$

$$\Delta=\begin{vmatrix}1 & -2 & 0\\ 1 & -1 & \mathrm{k}\\ 0 & \mathrm{k} & 4\end{vmatrix}=4-\mathrm{k}^2$$

For unique solution $4-\mathrm{k}^2\neq 0$

$\mathrm{k}\neq\pm 2$

For $\mathrm{k}=2$

$x-2y+0.z=1$

$x-y+2z=-2$

$0.x+2y+4z=6$

$$\Delta\mathrm{x}=\begin{vmatrix}1 & -2 & 0\\ -2 & -1 & 2\\ 6 & 2 & 4\end{vmatrix}=(-8)+2[-20]$$

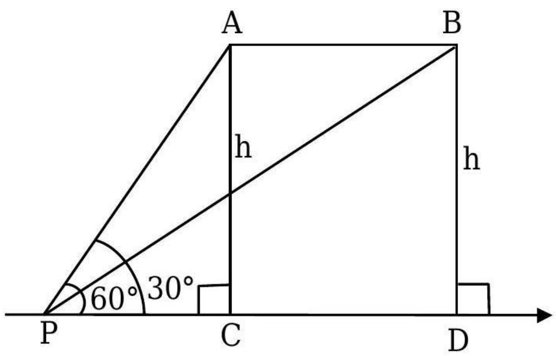
$\Delta\mathrm{x}=-48\neq 0$

For $\mathrm{k}=2 \Delta\mathrm{x}\neq 0$

For $\mathrm{K}=2$; The system has no solution

73. (a)

74. (a)



$v = 432 \times \frac{1000}{60 \times 60}$ m/sec $= 120$ m/sec

Distance $AB = v \times 20 = 2400$ meter

In $\triangle$ PAC

$\tan 60^{\circ} = \frac{\mathrm{h}}{\mathrm{PC}} \Rightarrow \mathrm{PC} = \frac{\mathrm{h}}{\sqrt{3}}$

In $\triangle$ PBD

$\tan 30^{\circ} = \frac{h}{PD} \Rightarrow PD = \sqrt{3h}$

$PD = PC + CD$

$\sqrt{3}\ \mathrm{h} = \frac{\mathrm{h}}{\sqrt{3}} + 2400 \Rightarrow \frac{2\ \mathrm{h}}{\sqrt{3}} = 2400$

$\mathrm{h} = 1200\sqrt{3}$ meter

75. (c)
76. (c) $A^{\top} = A, B^{\top} = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^{\top} = (A^2B^2 - B^2A^2)^{\top} = (A^2B^2)^{\top} - (B^2A^2)^{\top}$

$= (B^2)^{\top}(A^2)^{\top} - (A^2)^{\top}(B^2)^{\top}$

$= B^2A^2 - A^2B^2$

$\Rightarrow P$ is skew-symmetric matrix

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore\ ay + bz = 0$

$-ax + cz = 0$

$-bx - cy = 0$

From equation 1,2,3

$\Delta = 0 \& \Delta_1 = \Delta_2 = \Delta_3 = 0$

$\therefore$ equation have infinite number of solution

77. (c) ${}^2C_2 = {}^3C_3$

$S = {}^3C_3 + {}^3C_2 + \cdots \ldots + {}^nC_2 = {}^{n+1}C_3$

$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$$= \frac{(n+1)!}{3!\,(n-1)!} + \frac{(n+1)!}{3!\,(n-2)!}$$

$$= \frac{(n+2)(n+1)n}{6} + \frac{(n+1)(n)(n-1)}{6} = \frac{n(n+1)(2n+1)}{6}$$

78. (d) $\frac{dy}{dx} + \frac{y}{x} = bx^3$, I.F. $= e^{\int \frac{dx}{x}} = x$

$\therefore yx = \int bx^4 dx = \frac{bx^5}{5} + c$

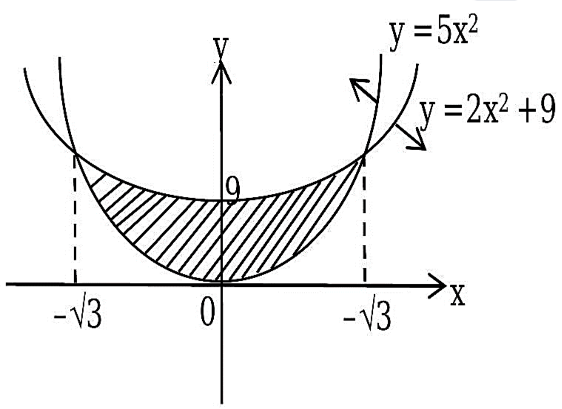
Passes through $(1,2)$, we get

$2 = \frac{b}{5} + C$

Also, $\int_1^2 \left(\frac{bx^4}{5} + \frac{c}{x}\right) dx = \frac{62}{5}$

$\Rightarrow \frac{b}{25} \times 32 + C\ln 2 - \frac{b}{25} = \frac{62}{5} \Rightarrow C = 0 \& b = 10$

79. (b)



Required area

$= 2\int_0^{\sqrt{3}} (2x^2 + 9 - 5x^2) dx$

$= 2\int_0^{\sqrt{3}} (9 - 3x^2) dx$

$= 2|9x - x^3|_0^{\sqrt{3}} = 12\sqrt{3}$

80. (a) $f'(x) = f'(2-x)$

On integrating both side $f(x) = -f(2-x) + c$

put $x = 0$

$\mathrm{f(0) + f(b) = c \Rightarrow c = 1 + e^2}$

$\mathrm{\Rightarrow f(x) + f(2-x) = 1 + e^2}$

$I = \int_0^2 f(x)dx = \int_0^1 \{f(x) + f(2-x)\}dx = (1 + e^2)$

81. (2)

$x \geq 5$

$$(x+1)^2 + (x-5) = \frac{27}{4}$$
$$\Rightarrow x^2 + 3x - 4 = \frac{27}{4}$$
$$\Rightarrow x^2 + 3x - \frac{43}{4} = 0$$
$$\Rightarrow 4x^2 + 12x - 43 = 0$$
$$x = \frac{-12 \pm \sqrt{144 + 688}}{8}$$
$$x = \frac{-12 \pm \sqrt{832}}{8} = \frac{-12 \pm 28.8}{8}$$
$$= \frac{-3 \pm 7.2}{2}$$
$$= \frac{-3 + 7.2}{2}, \frac{-3 - 7.2}{2} \text{ (Therefore no solution)}$$

For $x \leq 5$

$$(x+1)^2 - (x-5) = \frac{27}{4}$$

$$x^2 + x + 6 - \frac{27}{4} = 0$$

$4x^2 + 4x - 3 = 0$

$$x = \frac{-4 \pm \sqrt{16 + 48}}{8}$$

$$x = \frac{-4 \pm 8}{8} \Rightarrow x = -\frac{12}{8}, \frac{4}{8}$$

$\therefore$ 2 Real Root's

82. (31650)

$C \to 1 \quad 9L_{B}^{A}$

$C \to 2 \quad 8\left[\begin{matrix} A \\ C \to 3 \end{matrix}\right.$

$= {}^{10}C_1[2^9 - 2] + {}^{10}C_2[2^8 - 2] + {}^{10}C_3[2^7 - 2]$

$= 2^7[{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3] - 20 - 90 - 240$
$= 128[40 + 90 + 120] - 350$
$= (128 \times 250) - 350$
$= 10[3165] = 31650$

83. (2) $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$

$x \to \frac{1}{x}$

$\text{af}\left(\frac{1}{x}\right) + \text{af}(x) = \frac{b}{x} + \beta x$

(i) + (ii)

$(a + \alpha)\left[f(x) + f\left(\frac{1}{x}\right)\right] = \left(x + \frac{1}{x}\right)(b + \beta)$

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{2}{1} = 2$$

84. (11) $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$$\sigma^2 = \frac{(9 + k^2)}{10} - \left(\frac{9 + k}{10}\right)^2 < 10$$

$(90 + k^2)10 - (81 + k^2 + 8k) < 1000$

$90 + 10k^2 - k^2 - 18k - 81 < 1000$

$9k^2 - 18k + 9 < 1000$

$$(k - 1)^2 < \frac{1000}{9} \Rightarrow k - 1 < \frac{10\sqrt{10}}{3}$$

$$k < \frac{10\sqrt{10}}{3} + 1$$

Maximum integral value of $k = 11$

85. (1)

$$\frac{x - \lambda}{1} = \frac{y - \frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$$

$$\frac{x-\lambda}{2}=\frac{y-\frac{1}{2}}{1}=\frac{2}{-1}$$
$$\frac{x}{1}=\frac{y+2\lambda}{1}=\frac{z-\lambda}{1}$$

Point on line $=\left(\lambda,\frac{1}{2},0\right)$

Point on line $=(0,-2\lambda,\lambda)$

Distance between skew lines = $\dfrac{\left[\vec{a}_2-\vec{a}_1\ \vec{b}_1\ \vec{b}_2\right]}{\left|\vec{b}_1\times\vec{b}_2\right|}$

$$\frac{\begin{vmatrix}\lambda & \frac{1}{2}+2\lambda & -\lambda\\ 2 & 1 & -1\\ 1 & 1 & 1\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 1 & 1 & 1\end{vmatrix}}$$

$=\frac{\left|-5\lambda-\frac{3}{2}\right|}{\sqrt{14}}=\frac{\sqrt{7}}{2\sqrt{2}}$ (given)

$=|10\lambda+3|=7\Rightarrow\lambda=-1$

$\Rightarrow|\lambda|=1$

86. (310)

$$\frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^{24}}+\frac{\left(2e^{i\frac{\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{24}}$$

$\Rightarrow\frac{2^{21}\cdot e^{i14\pi}}{2^{12}\cdot e^{-i6\pi}}+\frac{2^{21}(e^{i7\pi})}{2^{12}(e^{i6\pi})}$

$\Rightarrow 2^9e^{i(20\pi)}+2^9e^{i\pi}$

$\Rightarrow 2^9+2^9(-1)=0$

$n=0$

$\sum_{j=0}^{5}(j+5)^2-\sum_{j=0}^{5}(j+5)$

$\Rightarrow[5^2+6^2+7^2+8^2+9^2+10^2]-[5+6+7+8+9+10]$
$\Rightarrow[(1^2+2^2+\cdots.+10^2)-(1^2+2^2+3^2+4^2)]-[(1+2+3+\cdots..+10)-(1+2+3+4)]$
$\Rightarrow(385-30)-[55-10]$
$\Rightarrow 355-45\Rightarrow 310$ ans.

87. (56.25)

Let $P(h,k)$

Given

PA = 3 PB

$PA^2 = 9\ PB^2$

$\Rightarrow (h-5)^2 + k^2 = 9[(h+5)^2 + k^2]$

$\Rightarrow 8\ h^2 + 8k^2 + 100\ h + 200 = 0$

$\therefore$ Locus

$$x^2 + y^2 + \left(\frac{25}{2}\right)x + 25 = 0$$

$$\therefore c \equiv \left(\frac{-25}{4}, 0\right)$$

$$\therefore r^2 = \left(\frac{-25}{4}\right)^2 - 25$$

$$= \frac{625}{16} - 25$$

$$= \frac{225}{16}$$

$$\therefore 4r^2 = 4 \times \frac{225}{16} = \frac{225}{4} = 56.25$$

88. (12)

$$(1+x))^{10} = {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x + \cdots .. + {}^{10}C_{10}x^{10}$$

$$(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1x + \cdots .\ {}^{15}C_{k-1}x^{k-1} + {}^{15}C_kx^k + {}^{15}C_{k+1}x^{k+1} + \cdots ..\ {}^{15}C_{15}x^{15}$$

$$\sum_{i=0}^{k}(10C_i)(15C_{k-i}) = {}^{10}C_0 \cdot {}^{15}C_k + {}^{10}C_1 \cdot {}^{15}C_{k-1} + \cdots .. + {}^{10}C_k \cdot {}^{15}C_0$$

Coefficient of x_k in $(1+x)^{25}$

$= {}^{25}C_k$

$$\sum_{i=0}^{k+1}({}^{12}C_i)({}^{13}C_{k+1-i}) = {}^{12}C_0 \cdot {}^{13}C_{k+1} + {}^{12}C_1 \cdot {}^{13}C_k + \cdots .. + {}^{12}C_{k+1} \cdot {}^{13}C_0$$

Coefficient of x^{k+1} in $(1+x)^{25}$

$= {}^{25}C_{k+1}$

$${}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$$

For maximum value

$k + 1 = 13$

K = 12

89. (3)

a, ar, ar^2, ar^3

$$a + ar + ar^2 + ar^3 = \frac{65}{12}$$

$$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$$

$$\frac{1}{a}\left(\frac{r^3 + r^2 + r + 1}{r^3}\right) = \frac{65}{18}$$

$$\frac{(i)}{(ii)}, a^2r^3 = \frac{18}{12} = \frac{3}{2}$$

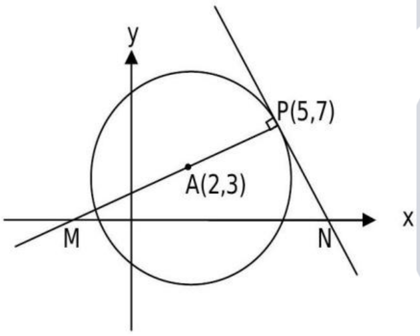
$$a^3r^3 = 1 \Rightarrow a\left(\frac{3}{2}\right) = 1 \Rightarrow a = \frac{2}{3}$$

$$\frac{4}{9}r^3 = \frac{3}{2} \Rightarrow r^3 = \frac{3^3}{2^3} \Rightarrow r = \frac{3}{2}$$

$$\alpha = ar^2 = \frac{2}{3} \cdot \left(\frac{3}{2}\right)^2 = \frac{3}{2}$$

$$2\alpha = 3$$

90. (1225)



Equation of normal at P

$$(y - 7) = \left(\frac{7-3}{5-2}\right)(x - 5)$$

$$3y - 21 = 4x - 20$$

$$\Rightarrow 4x - 3y + 1 = 0$$

$$\Rightarrow M\left(-\frac{1}{4}, 0\right)$$

Equation of tangent at P

$$(y-7)=-\frac{3}{4}(x-5)$$

$$4y-28=-3x+15$$

$$\Rightarrow 3x+4y=43$$

$$\Rightarrow N\left(\frac{43}{3},0\right)$$

Hence $\operatorname{ar}(\triangle \mathrm{PMN})=\frac{1}{2}\times \mathrm{MN}\times 7$

$$\lambda=\frac{1}{2}\times\frac{175}{12}\times 7$$

$$\Rightarrow 24\lambda=1225$$