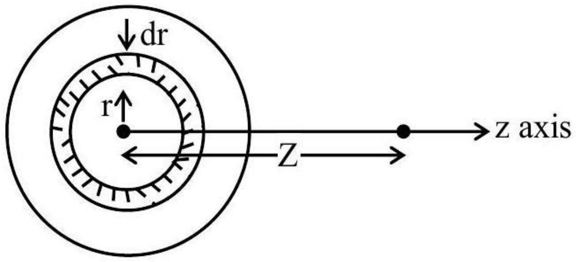
1. (a) Consider a small ring of radius r and thickness dr on disc.



area of elemental ring on disc

$dA = 2\pi r dr$

charge on this ring $dq = \sigma dA$

$$dEz = \frac{kdqz}{(z^2 + r^2)^{3/2}}$$

$$E = \int_0^R dE_z = \frac{\sigma}{2 \epsilon_0}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]$$

2. (b) $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$

$$\frac{N}{10^{10}} = \left(\frac{1}{2}\right)^{\frac{30}{60}}$$

$$\Rightarrow N = 10^{10} \times \left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{10^{10}}{\sqrt{2}} \approx 7 \times 10^9$$

3. (c) $[\mu_r] = 1$ as $\mu_r = \frac{\mu}{\mu_m}$

[power factor $(\cos\phi)$] $= 1$

$\mu_0 = \frac{B_0}{H}$ (unit $= NA^{-2}$) : Not dimensionless

$[\mu_0] = [MLT^{-2}\ A^{-2}]$

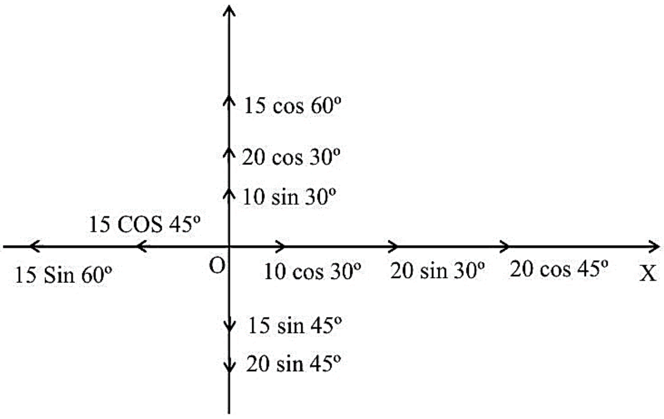
quality factor $(Q) = \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}}$

So Q is unitless & dimensionless.

4. (a)

Unit of $\frac{E}{H}$ is $\frac{\text{volt / metre}}{\text{Ampere / metre}} = \frac{\text{volt}}{\text{Ampere}} = \text{ohm}$

5. (a)



$$\vec{F}_x = \left(10 \times \frac{\sqrt{3}}{2} + 20\left(\frac{1}{2}\right) + 20\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{\sqrt{3}}{2}\right)\right)\hat{\imath}$$

$= 9.25\hat{\imath}$

$$\vec{F}_y = \left(15\left(\frac{1}{2}\right) + 20\left(\frac{\sqrt{3}}{2}\right) + 10\left(\frac{1}{2}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 20\left(\frac{1}{\sqrt{2}}\right)\right)\hat{\jmath} = 5\hat{\jmath}$$

6. **(d)** $P_m = \rho RT$

$\therefore \frac{P_1}{P_2} = \frac{\rho_1 T_1}{\rho_2 T_2}$

$\frac{\rho_1}{\rho_2} \Rightarrow \frac{P_1 T_2}{P_2 T_1} = \left(\frac{76}{45}\right) \times \frac{266}{300}$

$\frac{\rho_1}{\rho_2} \Rightarrow \frac{M_1}{M_2} = \frac{76 \times 266}{45 \times 300}$

$\therefore M_2 \Rightarrow \frac{45 \times 300 \times 185}{76 \times 266} = 123.54 \text{ kg}$

7. **(a)** Using Newton''s formula

$$(f + d_1)(f - d_2) = f^2$$
$$f^2 + fd_1 - fd_2 - d_1 d_2 = f^2$$
$$f = \frac{d_1 d_2}{d_1 - d_2}$$
$$\therefore R = \frac{2 d_1 d_2}{d_1 - d_2}$$

8. **(b)** $R = \frac{\ell}{\theta}$

$\text{Time} = \frac{4 \times 2\pi R}{v} = \frac{4 \times 2\pi}{v}\left(\frac{\ell}{\theta}\right)$

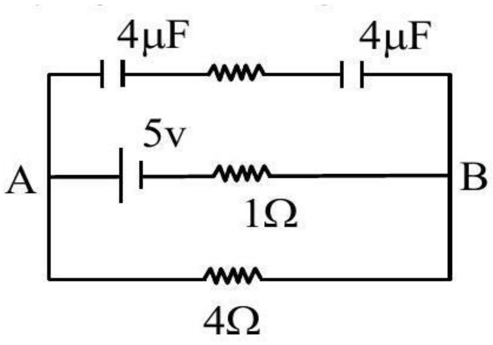
put $\ell = 4.4 \times 9.46 \times 10^{15}$

$v = 8 \times 1.5 \times 10^{11}$

$\theta = \frac{4}{3600} \times \frac{\pi}{180}$ rad.

we get time $= 4.5 \times 10^{10}$ sec

9. (a) On simplifying circuit we get



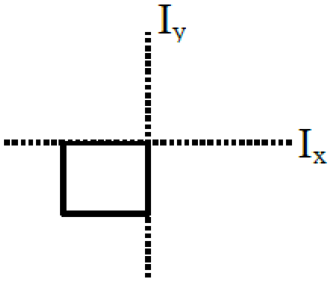
No current in upper wire.

$\therefore\ V_{AB} = \frac{5}{4+1} \times 4 = 4v.$

$\therefore \theta = (C_{eq})v$

$\Rightarrow 2 \times 4 = 8\mu C$

10. (d) According to perpendicular Axis theorem.



$I_x + I_y = I_z$

$I_z \Rightarrow \frac{m\ell^2}{3} + \frac{m\ell^2}{3}$

$= \frac{2\ m\ell^2}{3}$

11. (d) Active region of the CE transistor is linear region and is best suited for its use as an amplifier

12. (c) PT^3 = constant

$\left(\frac{nRT}{v}\right) T^3$ = constant

$T^4\ V^{-1}$ = constant

$T^4 = kV$

$\Rightarrow 4\frac{\Delta T}{T} = \frac{\Delta V}{V}$

$\Delta V = V\gamma\Delta T$

comparing (a) and (b)

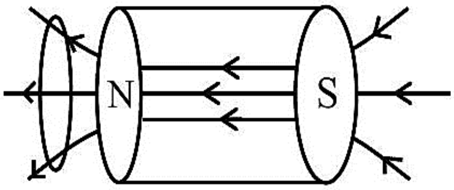
we get

$\gamma = \frac{4}{T}$

13. (d) Increasing intensity means number of incident photons are increased.

→ Kinetic energy of ejected electrons depend on the frequency of incident photons, not the intensity.

14. (c)



→ When magnet passes through centre region of solenoid, no current / Emf is induced in loop.

→ While entering flux increases so negative induced emf

→ While leaving flux decreases so positive induced emf.

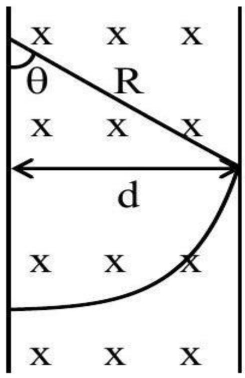
15. (b) $r = \frac{P}{qB} = \frac{\sqrt{2mk}}{qB}$

Given they have same kinetic energy

$r \propto \frac{\sqrt{m}}{q}$

$\frac{r_1}{r_2} = \frac{\sqrt{4}}{2} \times \frac{3}{\sqrt{16}} = \frac{3}{4}$

$r_2 = \frac{4r_1}{3}$ (r_2 is for hearier ion and r_1 is for lighter ion)



$\sin\theta = \frac{d}{R}$

$\theta \rightarrow$ Deflection

$\theta \propto \frac{1}{R}$

(R → Radius of path)

$\because R_2 > R_1 \Rightarrow \theta_2 < \theta_1$

16. (a)

$\frac{1}{V_1} + \frac{1}{30} = \frac{1}{10}$

$\frac{1}{V_1} = \frac{2}{30} \Rightarrow V_1 = 15 \text{ cm}$

$\frac{1}{V_2} - \frac{1}{10} = -\frac{1}{10}$

$\frac{1}{V_2} = 0$

$V_2 = \infty$

$V_3 = 30 \text{ cm}$

$OV_3 = 75 \text{ cm}$

17. (b) Viscous force = Weight

$= \rho \times \left(\frac{4}{3}\pi r^3\right) g$

$= 3.9 \times 10^{-10}$

18. (c) $V = \frac{\omega}{K} = \frac{10 \times 10^{10}}{500} = 2 \times 10^8$

V $= \frac{2C}{3}$.

19. (a) $i_1 = \frac{25}{5+R}$

$i_2 = \frac{5}{R + \frac{1}{5}}$

$i_1 = i_2 \Rightarrow 5\left(R + \frac{1}{5}\right) = 5 + R$

$4R = 4$

$R = 1\Omega$

20. (d) Potential energy is maximum at maximum distance from mean.

21. (2) Energy is maximum when mass is split equally so $\frac{M}{m} = 2$

22. (11) $f_{rms}^2 = f_{1rms}^2 + f_{2rms}^2$

$$= \left(\frac{\sqrt{42}}{\sqrt{2}}\right)^2 + 10^2$$

$$= 121 \Rightarrow f_{rms} = 11\text{ A}$$

23. (3) In triangle shape $N_t = \frac{24a}{3a} = 8$

In square $N_s = \frac{24a}{4a} = 6$

$\frac{M_t}{M_3} = \frac{N_t I A_t}{N_s I A_s}$ [I will be same in both]

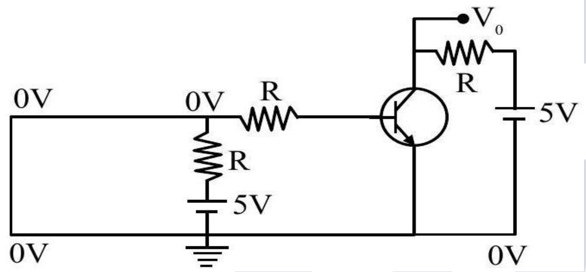
$$= \frac{8 \times \frac{\sqrt{3}}{4} \times a^2}{6 \times a^2}$$

$$\frac{M_t}{M_s} = \frac{1}{\sqrt{3}}$$

$y = 3$

24. (5) As diodes D_1 and D_2 are in forward bias, so they acted as neligible resistances

$\Rightarrow$ Input voltage become zero



$\Rightarrow$ Input current is zero

$\Rightarrow$ Output current is zero

$\Rightarrow V_0 = 5$ volt

25. (20) In series

$R_{eq} = nR = 10n$

$$i_s = \frac{20}{10 + 10n} = \frac{2}{1+n}$$

in parallel

$$R_{eq} = \frac{10}{n}$$

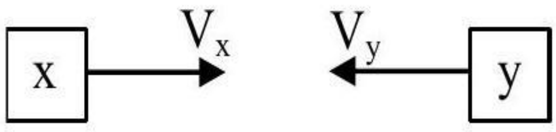
$$i_p = \frac{20}{\frac{10}{n} + 10} = \frac{2n}{1+n}$$

$$\frac{i_p}{i_s} = 20$$

$$\frac{\left(\frac{2n}{1+n}\right)}{\left(\frac{2}{1+n}\right)} = 20$$

$n = 20$

26. (1210)



$V_x = 36\ \text{km/hr} = 10\ \text{m/s}$

$V_y = 72\ \text{km/hr} = 20\ \text{m/s}$

by doppler's effect

$$F' = F_0\left(\frac{V \pm V_0}{V \pm V_s}\right)$$

$$1320 = F_0\left(\frac{340+20}{340-10}\right) \Rightarrow F_0 = 1210\ \text{Hz}$$

27. (12) $V = \sqrt{5000+24x}$

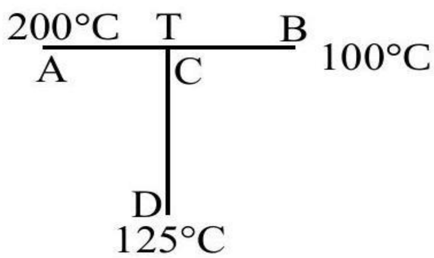
$$\frac{dV}{dx} = \frac{1}{2\sqrt{5000+24x}} \times 24 = \frac{12}{\sqrt{5000+24x}}$$

now $a = V\frac{dV}{dx}$

$$= \sqrt{5000+24x} \times \frac{12}{\sqrt{5000+24x}}$$

$a = 12\ \text{m/s}^2$

28. (2)



Rods are identical so

$R_{AB} = R_{CD} = 10\text{Kw}^{-1}$

C is mid-point of AB, so

$R_{AC} = R_{CB} = 5Kw^{-1}$

at point C

$\frac{200-T}{5} = \frac{T-125}{10} + \frac{T-100}{5}$

$2(200-T) = T-125+2(T-100)$

$400-2T = T-125+2T-200$

$T = \frac{725}{5} = 145°C$

$I_h = \frac{145-125}{10} w = \frac{20}{10} w$

$I_h = 2w$

29. (2) Given $W_A = W_B$

$F_A d\cos 45° = F_B d\cos 60°$

$F_A \times \frac{1}{\sqrt{2}} = F_B \times \frac{1}{2}$

$\frac{F_A}{F_B} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$x = 2$

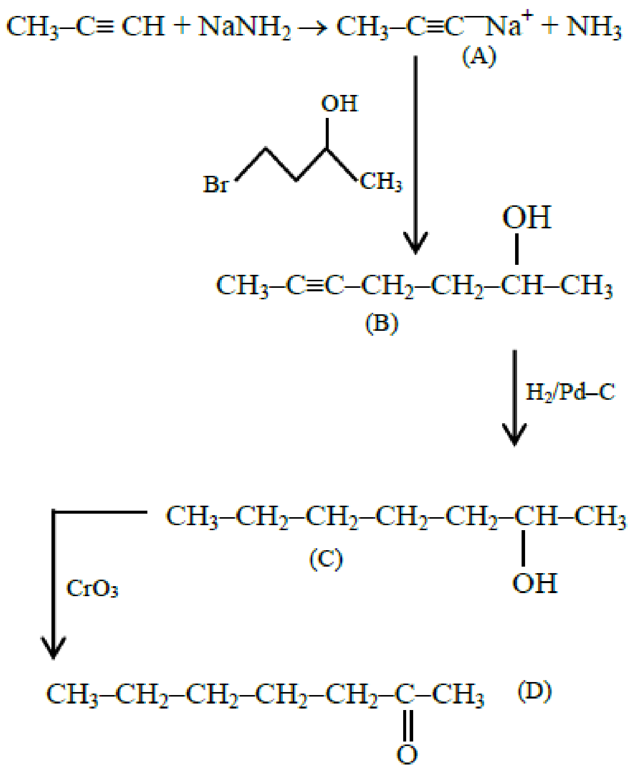
30. (224)

$d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

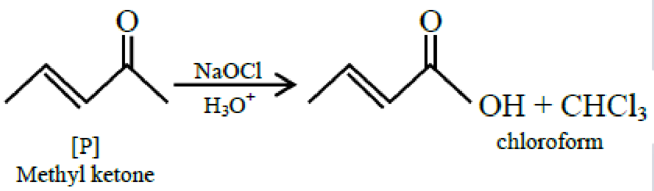
$d_m = \left(\sqrt{2 \times 6400 \times 10^3 \times 320} + \sqrt{2 \times 6400 \times 10^3 \times 2000}\right) m$

$d_m = 224$ km

31. (d)



32. (a)

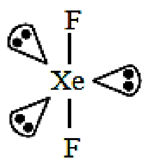


NaOCl is used in haloform reaction as reagent.

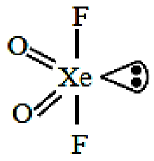
33. (d)

Species (Number of lone pairs of electrons on the central atom)

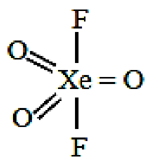
XeF_2 3



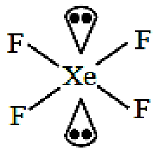
XeO_2F_2 1



XeO_3F_2 0



XeF_4 2



34. (b) Type of back bonding

BF_3	BCl	BBr_3	BI_3
$(2p\pi - 2p\pi)$	$(2p\pi - 3p\pi)$	$(2p\pi - 4p\pi)$	$(2p\pi - 5p\pi)$

Therefore, back bonding strength is as follows

$BF_3 > BCl > BBr_3 > BI_3$

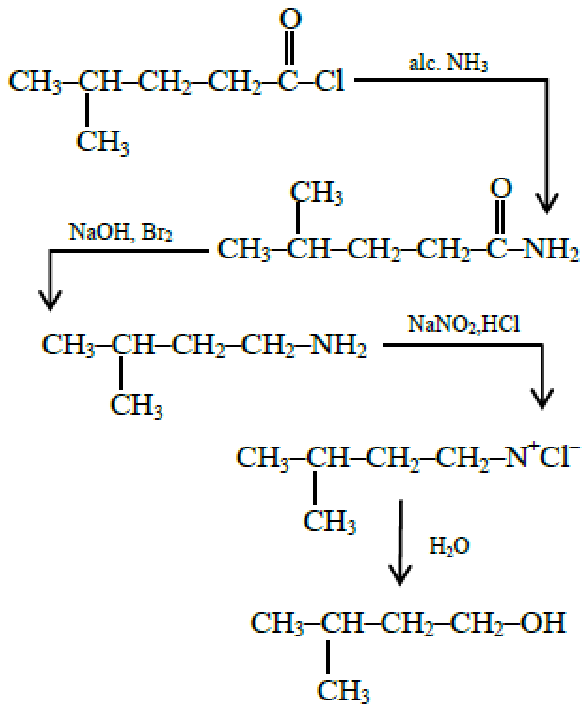
35. (a) The bond dissociation energy of D_2 is greater than H_2 and therefore D_2 reacts slower than H_2.

36. (b) Liquation method is used to purify those impure metals which has lower melting point than the melting point of impurities associated. This method is used for metal having low melting point

37. (c)

38. (d)

39. (c)



40. (a) The intermolecular association is more prominent in case of primary amines as compared to secondary, due to the availability of two hydrogen atom.

41. (d)

$$FeCl_3 + K_4[Fe(CN)_6] \text{ (excess)} \xrightarrow{} \underset{\text{Colloidal species}}{KFe[Fe(CN)_6]}$$

42. (d) In stratosphere CFCs get broken down by powerful UV radiations releasing Cl^* $CF_2Cl_2(g) \xrightarrow{U.V.} Cl^{\cdot}(g) + {}^{\cdot}CF_2Cl(g)$

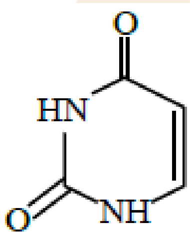
43. (d)

44. (d) V_2O_3 basic
CrO basic

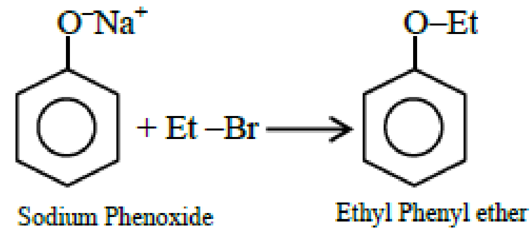
45. (d)

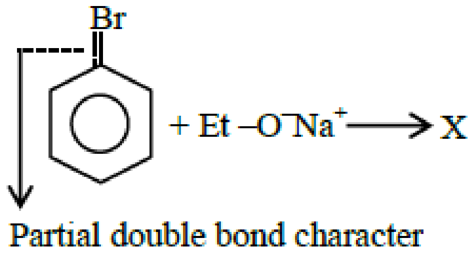
Gypsum	$CaSO_4 \cdot 2H_2O$	
Plaster of Paris	$CaSO_4 \cdot \frac{1}{2}H_2O$	
Dead burnt plaster	$CaSO_4$	

Isomeric form of uracil present in RNA

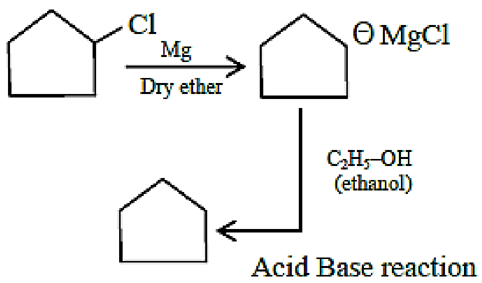


46. (b)





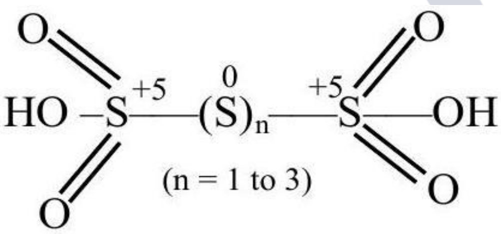
47. (a)



48. (d)

$$\frac{an^2}{V^2} = \text{atm} \Rightarrow a = \text{atm} \times \frac{\text{dm}^6}{\text{mol}^2}$$

49. (d)



50. (c) Tyndall effect is observed in lyophobic colloids

51. (40)

$$n_{AgBr} = \frac{0.188\ \text{g}}{188\ \text{g/mol}} = 10^{-3}\ \text{mol}$$

$$\Rightarrow n_{Br} = n_{AgBr} = 0.001\ \text{mol}$$

$$\Rightarrow \text{mass}_{Br} = (0.001 \times 80)\text{gm} = 0.08\text{gm}$$

$$\Rightarrow \text{mass}\ \% = \frac{0.08 \times 100}{0.2} = 40\%$$

52. (4.00)

$$\left(\frac{\text{Rate of disappearance of } C_2H_6O}{3}\right)$$

$$= \left(\frac{\text{Rate of appearance of } Cr_2(SO_4)_3}{2}\right)$$

$$\Rightarrow \left(\frac{2.67\ \text{mol/min}\times 3}{2}\right) = \text{rate of disappearance of}$$

C_2H_6O.

$\Rightarrow$ Rate of disappearance of $C_2H_6O = 4.005\ \text{mol/min}$

53. (3155)

$$\text{mvr} = \frac{nh}{2\pi}$$

$$\text{K.E.} = \frac{n^2 h^2}{8\pi^2 mr^2} = \frac{4\,h^2}{8\pi^2\, m(4a_0)^2} = \left(\frac{4}{8\pi^2\times 16}\right)\frac{h^2}{ma_0^2}$$

$\Rightarrow x = 315.507$

$\Rightarrow 10x = 3155$ (nearest integer)

54. (518) Let mass of water initially present = x gm

$\Rightarrow$ Mass of sucrose $= (1000 - x)$gm

$\Rightarrow$ moles of sucrose $= \left(\frac{1000-x}{342}\right)$

$$\Rightarrow 0.75 = \frac{\left(\frac{1000-x}{342}\right)}{\left(\frac{x}{1000}\right)} \Rightarrow \frac{x}{1000} = \frac{1000-x}{342\times 0.75}$$

$\Rightarrow 256.5x = 10^6 - 1000x$

$\Rightarrow x = 795.86$gm

$\Rightarrow$ moles of sucrose $= 0.5969$

New mass of H_2O = akg

$$\Rightarrow 4 = \frac{0.5969}{a}\times 1.86 \Rightarrow a = 0.2775\ \text{kg}$$

$\Rightarrow$ ice separated $= (795.86 - 277.5) = 518.3$gm

55. (2)
56. (1125)

Moles of N in N,N - dimethylaminopentane

$= \left(\frac{57.5}{115}\right) = 0.5\ \text{mol}$

$\Rightarrow C_7H_{17}\ N + \frac{45}{2}CuO \rightarrow 7CO_2 + \frac{17}{2}H_2O + \frac{1}{2}\ N_2 + \frac{45}{2}Cu$

$\frac{n_{CuO}\ \text{reacted}}{\left(\frac{45}{2}\right)} = \frac{n_{C_7H_{17}\ N}\ \text{reacted}}{1}$

$\Rightarrow n_{CuO}\ \text{reacted} = \left(\frac{45}{2}\right) \times 0.5 = 11.25$

57. (4)

58. (82) Millimoles of HCl $= 200 \times 0.2 = 40$

$\Rightarrow$ Millimoles of NaOH $= 300 \times 0.1 = 30$

$\Rightarrow$ Heat released $= \left(\frac{30}{1000} \times 57.1 \times 1000\right) = 1713\ \text{J}$

$\Rightarrow$ Mass of solution $= 500\text{ml} \times 1\text{gm/ml} = 500\text{gm}$

$\Rightarrow \Delta T = \frac{q}{m \times C} = \frac{1713\ \text{J}}{500\ \text{g} \times 4.18 \frac{\text{J}}{\text{g}-\text{K}}} = 0.8196\ \text{K}$

$= 81.96 \times 10^{-2}\ \text{K}$

59. (4) Let moles added $=$ a

$Ag^+_{(aq.)} + 2NH_{3(aq.)} \rightleftharpoons Ag(NH_3)^+_{2(aq.)}$

$t = 0\ 0.8\ \left(\frac{a}{2}\right)$

$t = \infty\ 5 \times 10^{-8}\ \left(\frac{a}{2} - 1.6\right)\ 0.8$

$\frac{0.8}{(5 \times 10^{-8})\left(\frac{a}{2} - 1.6\right)^2} = 10^8$

$\Rightarrow \frac{a}{2} - 1.6 = 0.4 \Rightarrow a = 4$

60. (316) Let molarity of $KMnO_4 = x$

$KMnO_4 + FeSO_4 \rightarrow Fe_2(SO_4)_3 + Mn^{2+}$

$n = 5\ n = 1$

(Equivalents of $KMnO_4$ reacted) $=$ (Equivalents of $FeSO_4$ reacted)

$\Rightarrow (5 \times x \times 10\text{ml}) = 1 \times 0.1 \times 10\text{ml}$

$\Rightarrow x = 0.02\text{M}$

Molar mass of $KMnO_4 = 158\text{gm/mol}$

$\Rightarrow$ Strength $= (x \times 158) = 3.16\ \text{g}/\ell$

61. (a) Let $t = \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \cdots \infty$

$$= \left(2 - \frac{1}{2}\right)x^2 + \left(2 - \frac{1}{3}\right)x^3 + \left(2 - \frac{1}{4}\right)x^4 + \cdots \infty$$

$$= 2(x^2 + x^3 + x^4 + \cdots \infty) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots \infty\right)$$

$$= \frac{2x^2}{1-x} - (\ln(1-x) - x)$$

$$\Rightarrow t = \frac{2x^2}{1-x} + x - \ln(1-x)$$

$$\Rightarrow t = \frac{x(1+x)}{1-x} - \ln(1-x)$$

62. (d) $\frac{2(1+2+3+\cdots+y)}{3(1+2+3+\cdots+y)} = \frac{4}{\log_{10} x}$

$$\Rightarrow \log_{10} x = 6 \Rightarrow x = 10^6$$

Now,

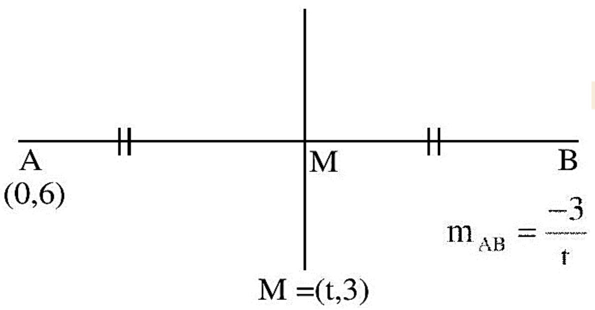
$$y = (\log_{10} x) + \left(\log_{10} x^{\frac{1}{3}}\right) + \left(\log_{10} x^{\frac{1}{9}}\right) + .. \infty$$

$$= \left(1 + \frac{1}{3} + \frac{1}{9} + \cdots \infty\right)\log_{10} x$$

$$= \left(\frac{1}{1-\frac{1}{3}}\right)\log_{10} x = 9$$

So, $(x, y) = (10^6, 9)$

63. (c) $A(0,6)$ and $B(2t, 0)$



Perpendicular bisector of AB is

$$(y-3) = \frac{t}{3}(x-t)$$

So, $C = \left(0, 3 - \frac{t^2}{3}\right)$

Let P be (h, k)

$h = \frac{t}{2}; k = \left(3 - \frac{t^2}{6}\right)$

$\Rightarrow k = 3 - \frac{4\,h^2}{6} \Rightarrow 2x^2 + 3y - 9 = 0$

64. (b) Given $a = (\sin^{-1} x)^2 - (\cos^{-1} x)^2$

$= (\sin^{-1} x + \cos^{-1} x)(\sin^{-1} x - \cos^{-1} x)$

$= \frac{\pi}{2}\left(\frac{\pi}{2} - 2\cos^{-1} x\right)$

$\Rightarrow 2\cos^{-1} x = \frac{\pi}{2} - \frac{2a}{\pi}$

$\Rightarrow \cos^{-1}(2x^2 - 1) = \frac{\pi}{2} - \frac{2a}{\pi}$

$\Rightarrow 2x^2 - 1 = \cos\left(\frac{\pi}{2} - \frac{2a}{\pi}\right)$

65. (a) Given matrix $A = \begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix}$

$A^4 + 3IA = 2I$

$\Rightarrow A^4 = 2I - 3\,A$

Also characteristic equation of A is

$|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 0 - \lambda & 2 \\ k & -1 - \lambda \end{vmatrix} = 0$

$\Rightarrow \lambda + \lambda^2 - 2k = 0$

$\Rightarrow A + A^2 = 2\,K.I$

$\Rightarrow A^2 = 2KI - A$

$\Rightarrow A^4 = 4\,K^2I + A^2 - 4AK$

Put $A^2 = 2KI - A$

and $A^4 = 2I - 3\,A$

$2I - 3\,A = 4\,K^2I + 2KI - A - 4AK$

$\Rightarrow I(2 - 2\,K - 4\,K^2) = A(2 - 4\,K)$

$\Rightarrow -2I(2\,K^2 + K - 1) = 2\,A(1 - 2\,K)$

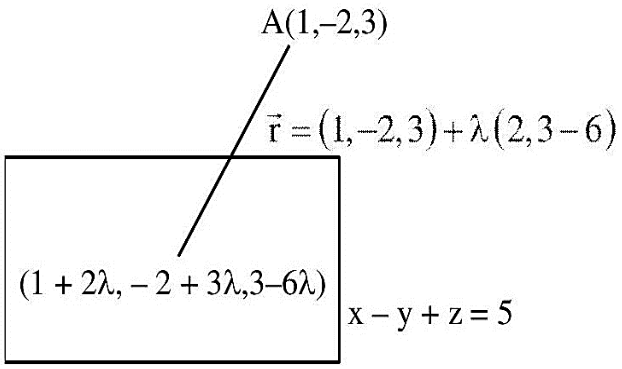
$\Rightarrow -2I(2\,K - 1)(K + 1) = 2\,A(1 - 2\,K)$

$\Rightarrow (2\,K - 1)(2\,A) - 2I(2\,K - 1)(K + 1) = 0$

$\Rightarrow (2\,\mathrm{K}-1)[2\,\mathrm{A}-2\mathrm{I}(\mathrm{K}+1)]=0$

$\Rightarrow \mathrm{K}=\dfrac{1}{2}$

66. (d)



$(1+2\lambda)+2-3\lambda+3-6\lambda=5$

$\Rightarrow 6-7\lambda=5 \Rightarrow \lambda=\dfrac{1}{7}$

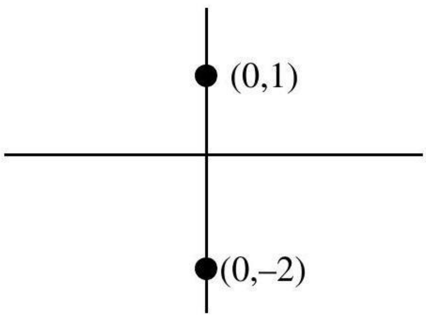
so, $\mathrm{P}=\left(\frac{9}{7},-\frac{11}{7},\frac{15}{7}\right)$

$$\mathrm{AP}=\sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$$

$$\mathrm{AP}=\sqrt{\left(\frac{4}{49}\right)+\frac{9}{49}+\frac{36}{49}}=1$$

67. (d) Given $\frac{z-i}{z+2i}\in R$

Then arg $\left(\frac{z-i}{z+2i}\right)$ is 0 or Π



$\Rightarrow$ S is straight line in complex

68. (a) $\frac{dy}{dx}-2xy=2(2\sin x-5)x-2\cos x$

$\mathrm{IF}=e^{-x^2}$

so, $y\cdot e^{-x^2}=\int e^{-x^2}(2x(2\sin x-5)-2\cos x)dx$

$\Rightarrow y \cdot e^{-x^2} = e^{-x^2}(5 - 2\sin x) + c$

$\Rightarrow y = 5 - 2\sin x + c \cdot e^{x^2}$

Given at $x = 0, y = 7$

$\Rightarrow 7 = 5 + c \Rightarrow c = 2$

So, $y = 5 - 2\sin x + 2e^{x^2}$

Now at $x = \pi$,

$y = 5 + 2e^{\pi^2}$

69. **(d)** Required equation of plane

$P_1 + \lambda P_2 = 0$

$(x - y - z - 1) + \lambda(2x + y - 3z + 4) = 0$

Given that its dist. From origin is $\frac{2}{\sqrt{21}}$

Thus $\frac{|4\lambda - 1|}{\sqrt{(2\lambda+1)^2 + (\lambda-1)^2 + (-3\lambda-1)^2}} = \frac{\sqrt{2}}{\sqrt{21}}$

$\Rightarrow 21(4\lambda - 1)^2 = 2(14\lambda^2 + 8\lambda + 3)$

$\Rightarrow 336\lambda^2 - 168\lambda + 21 = 28\lambda^2 + 16\lambda + 6$

$\Rightarrow 308\lambda^2 - 184\lambda + 15 = 0$

$\Rightarrow 308\lambda^2 - 154\lambda - 30\lambda + 15 = 0$

$\Rightarrow (2\lambda - 1)(154\lambda - 15) = 0$

$\Rightarrow \lambda = \frac{1}{2}$ or $\frac{15}{154}$

for $\lambda = \frac{1}{2}$ reqd. plane is

$4x - y - 5z + 2 = 0$

70. **(a)** $U_n = \prod_{r=1}^{n}\left(1 + \frac{r^2}{n^2}\right)^r$

$L = \lim_{n\to\infty}(U_n)^{-4/n^2}$

$\log L = \lim_{n\to\infty} \frac{-4}{n^2}\sum_{r=1}^{n} \log\left(1 + \frac{r^2}{n^2}\right)^r$

$\Rightarrow \log L = \lim_{n\to\infty}\sum_{r=1}^{n} -\frac{4r}{n} \cdot \frac{1}{n} \log\left(1 + \frac{r^2}{n^2}\right)$

$\Rightarrow \log L \Rightarrow -4\int_0^1 x\log(1 + x^2)dx$

put $1+x^2=t$

Now, $2xdx=dt$

$=-2\int_1^2 \log(t)dt=-2[t\log t-t]_1^2$

$\Rightarrow \log L=-2(2\log 2-1)$

$\therefore L=e^{-2(2\log 2-1)}$

$=e^{-2\left(\log\left(\frac{4}{e}\right)\right)}$

$=e^{\log\left(\frac{4}{e}\right)^{-2}}$

$=\left(\frac{e}{4}\right)^2=\frac{e^2}{16}$

71. (a)

$(p\wedge(p\to q)\wedge(q\to r))\to r$

$\equiv(p\wedge(\sim p\vee q)\vee(\sim q\vee r))\to r$

$\equiv((p\wedge q)\wedge(\sim p\vee r))\to r$

$\equiv(p\wedge q\wedge r)\to r$

$\equiv\sim(p\wedge q\wedge r)\vee r$

$\equiv(\sim p)\vee(\sim q)\vee(\sim r)\vee r$

$\Rightarrow$ tautology

72. (c) $y+\frac{xdy}{dx}=x^2$ (given)

$\Rightarrow \frac{dy}{dx}+\frac{y}{x}=x$

$\text{If}=e^{\int\frac{1}{x}dx}=x$

Solution of DE

$\Rightarrow y\cdot x=\int x\cdot xdx$

$\Rightarrow xy=\frac{x^3}{3}+\frac{c}{3}$

Passes through $(-2,2)$, so

$-12=-8+c\Rightarrow c=-4$

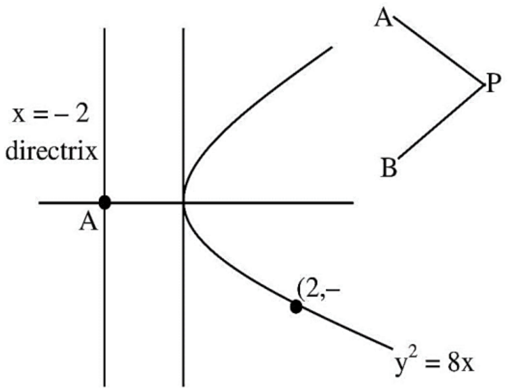
$\therefore 3xy=x^3-4$

ie. $3x.f(x)=x^3-4$

73. (c) $\sum_{k=0}^{20} {}^{20}C_k \cdot {}^{20}C_{20-k}$

sum of suffix is const. so summation will be ${}^{40}C_{20}$

74. (a)



Equation of tangent at $(2,-4)(\mathrm{T}=0)$

$-4y = 4(x+2)$

$\mathrm{x}+\mathrm{y}+2=0$

equation of normal

$\mathrm{x}-\mathrm{y}+\lambda=0$

$\downarrow (2,-4)$

$\lambda=-6$

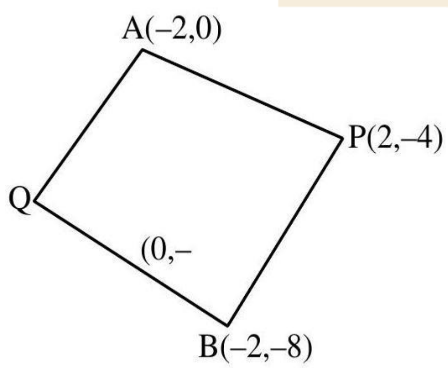
thus $\mathrm{x}-\mathrm{y}=6$ ... (b) equation of normal

POI of (a) & $\mathrm{x}=-2$ is $\mathrm{A}(-2,0)$

POI of (b) & $x=-2$ is $\mathrm{A}(-2,8)$

Given AQBP is a sq.

(a,b)



$\Rightarrow \mathrm{m}_{\mathrm{AQ}} \cdot \mathrm{m}_{\mathrm{AP}} = -1$

$\Rightarrow \left(\frac{b}{a+2}\right)\left(\frac{4}{-4}\right) = -1 \Rightarrow a+2=b$

Also PQ must be parallel to x-axis thus

$\Rightarrow \mathrm{b} = -4$

$\therefore a = -6$

Thus $2\mathrm{a} + \mathrm{b} = -16$

75. (b) $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$

As A, B, C are angles of triangle

$\mathrm{A} + \mathrm{B} + \mathrm{C} = \pi$

$\mathrm{A} = \pi - (\mathrm{B} + \mathrm{C})$

So, $\sin \mathrm{A} = \sin(\mathrm{B} + \mathrm{C})$

Similarly $\sin B = \sin(A + C)$ ... (b)

From (a) and (b)

$$\frac{\sin(B+C)}{\sin(A+C)} = \frac{\sin(A-C)}{\sin(C-B)}$$

$\sin(\mathrm{C}+\mathrm{B}) \cdot \sin(\mathrm{C}-\mathrm{B}) = \sin(\mathrm{A}-\mathrm{C})\sin(\mathrm{A}+\mathrm{C})$

$\sin^2 \mathrm{C} - \sin^2 \mathrm{B} = \sin^2 \mathrm{A} - \sin^2 \mathrm{C}$

$\{\because \sin(\mathrm{x}+\mathrm{y})\sin(\mathrm{x}-\mathrm{y}) = \sin^2 \mathrm{x} - \sin^2 \mathrm{y}\}$

$2\sin^2 \mathrm{C} = \sin^2 \mathrm{A} + \sin^2 \mathrm{B}$

By sine rule

$2\mathrm{c}^2 = \mathrm{a}^2 + \mathrm{b}^2$

$\Rightarrow \mathrm{b}^2, \mathrm{c}^2$ and a^2 are in A.P.

76. (c) $\lim_{x\to\beta} \frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$

$$\Rightarrow \lim_{x\to\beta} \frac{1\left(1+\frac{2(x^2+bx+c)}{1!}+\frac{2^2(x^2+bx+c)^2}{2!}+\cdots\right)-1-2(x^2+bx+c)}{(x-\beta)^2}$$

$$\Rightarrow \lim_{x\to\beta} \frac{2(x^2+bx+1)^2}{(x-\beta)^2}$$

$$\Rightarrow \lim_{x\to\beta} \frac{2(x-\alpha)^2(x-\beta)^2}{(x-\beta)^2}$$

$\Rightarrow 2(\beta-\alpha)^2 = 2(b^2-4c)$

77. (b) Probability of obtaining total sum 7 = probability of getting opposite faces.

Probability of getting opposite faces

$$= 2\left[\left(\frac{1}{6}-x\right)\left(\frac{1}{6}+x\right)+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\right]$$

$$\Rightarrow 2\left[\left(\frac{1}{6}-x\right)\left(\frac{1}{6}+x\right)+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\right]=\frac{13}{96}$$

(given)

$$x = \frac{1}{8}$$

78. (d) $x^2 + 9y^2 - 4x + 3 = 0$

$(x^2 - 4x) + (9y^2) + 3 = 0$

$(x^2 - 4x + 4) + (9y^2) + 3 - 4 = 0$

$(x-2)^2 + (3y)^2 = 1$

$\frac{(x-2)^2}{(1)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$ (equation of an ellipse).

As it is equation of an ellipse, x & y can vary inside the ellipse.

So, $x - 2 \in [-1,1]$ and $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

$x \in [1,3] y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

79. (c) Let $I = \int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$

$$I = \int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x-22)^2} dx$$

We know

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$(king)

So $I = \int_6^{16} \frac{\log_e (22-x)^2}{\log_e (22-x)^2 + \log_e (22-(22-x))^2}$

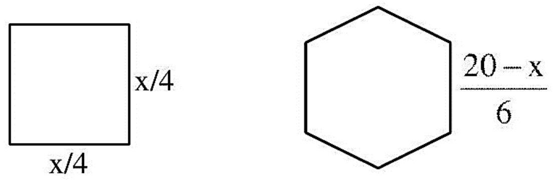
$$I = \int_0^{16} \frac{\log_e (22-x)^2}{\log_e x^2 + \log_e (22-x)^2} dx$$

(a) +(b)

$2I = \int_6^{16} 1 \cdot dx = 10$

$I = 5$

80. (d) Let the wire is cut into two pieces of length x and 20 − x.



Area of square $= \left(\frac{x}{4}\right)^2$ Area of regular hexagon

$= 6 \times \frac{\sqrt{3}}{4}\left(\frac{20-x}{6}\right)^2$

Total area $= A(x) = \frac{x^2}{16} + \frac{3\sqrt{3}}{2}\frac{(20-x)^2}{36}$

$A'(x) = \frac{2x}{16} + \frac{3\sqrt{3} \times 2}{2 \times 36}(20-x)(-1)$

$A'(x) = 0$ at $x = \frac{40\sqrt{3}}{3+2\sqrt{3}}$

Length of side of regular Hexagon $= \frac{1}{6}(20-x)$

$= \frac{1}{6}\left(20 - \frac{4 \cdot \sqrt{3}}{3+2\sqrt{3}}\right)$

$= \frac{10}{2+2\sqrt{3}}$

81. (90) since, $\vec{a} \cdot \vec{b} = 0$

$1 + 15 + \alpha\beta = 0 \Rightarrow \alpha\beta = -16$

Also,

$|\vec{b} \times \vec{c}|^2 = 75 \Rightarrow (10 + \beta^2)14 - (5 - 3\beta)^2 = 75$

$\Rightarrow 5\beta^2 + 30\beta + 40 = 0$

$\Rightarrow \beta = -4, -2$

$\Rightarrow \alpha = 4, 8$

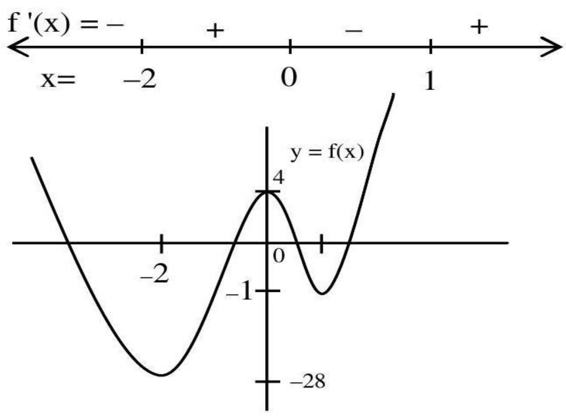
$\Rightarrow |\vec{a}|^2_{\max} = (26 + \alpha^2)_{\max} = 90$

82. (4) $3x^4 + 4x^3 - 12x^2 + 4 = 0$

So, Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$

$\therefore f'(x) = 12x(x^2 + x - 2)$

$= 12x(x+2)(x-1)$



83. (61)

$$r=\sqrt{\frac{p^2}{4}+\frac{(1-p)^2}{4}-5}=\frac{\sqrt{2p^2-2p-19}}{2}$$

Since, $r\in(0,5]$

So, $0<2p^2-2p-19\le 100$

$\Rightarrow p\in\left[\frac{1-\sqrt{239}}{2},\frac{1-\sqrt{39}}{2}\right)\cup\left(\frac{1+\sqrt{39}}{2},\frac{1+\sqrt{239}}{2}\right]$ so, number

of integral values of p^2 is 61

84. (256) $A=(-\infty,1)\cup(3,\infty)$

$B=(-\infty,-2)\cup(2,\infty)$

$C=(-\infty,2]\cup[6,\infty)$

So, $A\cap B\cap C=(-\infty,-2)\cup[6,\infty)$

$z\cap(A\cap B\cap C)'=\{-2,-1,0,-1,2,3,4,5\}$

$2^8=256.$

85. (15) $I=\int\frac{dx}{\left[\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right]^2}$

$\int\frac{dt}{\left(t^2+\frac{3}{4}\right)^2}\left(\quad \text{Put } x+\frac{1}{2}=t\right)$

$=\frac{\sqrt{3}}{2}\int\frac{\sec^2\theta d\theta}{\frac{9}{16}\sec^4\theta}\left(\quad \text{Put } t=\frac{\sqrt{3}}{2}\tan\theta\right)$

$$=\frac{4\sqrt{3}}{9}\int(1+\cos 2\theta)d\theta$$

$$=\frac{4\sqrt{3}}{9}\left[\theta+\frac{\sin 2\theta}{2}\right]+c$$

$$=\frac{4\sqrt{3}}{9}\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{\sqrt{3}(2x+1)}{3+(2x+1)^2}\right]+c$$

$$=\frac{4\sqrt{3}}{9}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{3}\left(\frac{2x+1}{x^2+x+1}\right)+c$$

$9(\sqrt{3}a+b)=15$

86. (5)

$2\times$ (i) - (ii) - (iii) gives:

$-(1+\beta)z=3-\alpha$

For infinitely many solution

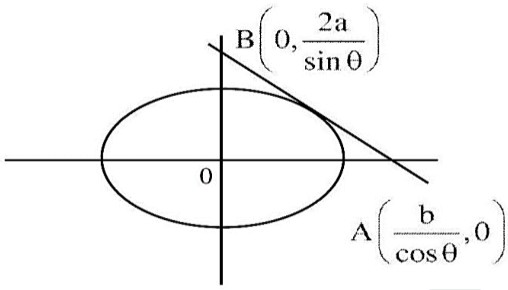
$\beta+1=0=3-\alpha\Rightarrow(\alpha,\beta)=(3,-1)$

Hence, $\alpha+\beta-\alpha\beta=5$

87. (13)

$\frac{n^2-1}{12}=14\Rightarrow n=13$

88. (2) $\frac{x\cos\theta}{b}+\frac{y\sin\theta}{2a}=1$



So, $\text{area}(\Delta OAB)=\frac{1}{2}\times\frac{b}{\cos\theta}\times\frac{2a}{\sin\theta}$

$=\frac{2ab}{\sin 2\theta}\geq 2ab$

$\Rightarrow k=2$

89. (100)

90. (17) $y^{\frac{1}{4}}+\frac{1}{y^{\frac{1}{4}}}=2x$

$\Rightarrow\left(y^{\frac{1}{4}}\right)^2-2xy^{\left(\frac{1}{4}\right)}+1=0$

$\Rightarrow y^{\frac{1}{4}}=x+\sqrt{x^2-1}$ or $x-\sqrt{x^2-1}$

So, $\frac{1}{4}\frac{1}{y^{\frac{3}{4}}}\frac{dy}{dx}=1+\frac{x}{\sqrt{x^2-1}}$

$$\Rightarrow \frac{1}{4}\frac{1}{y^{3/4}}\frac{dy}{dx} = \frac{y^{\frac{1}{4}}}{\sqrt{x^2-1}}$$

$$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{4\mathrm{y}}{\sqrt{\mathrm{x}^2-1}}$$

Hence, $\frac{d^2y}{dx^2} = 4\frac{(\sqrt{x^2-1})y' - \frac{yx}{\sqrt{x^2-1}}}{x^2-1}$

$$\Rightarrow (x^2-1)y'' = 4\frac{(x^2-1)y' - xy}{\sqrt{x^2-1}}$$

$$\Rightarrow (x^2-1)y'' = 4\left(\sqrt{x^2-1}y' - \frac{xy}{\sqrt{x^2-1}}\right)$$

$$\Rightarrow (x^2-1)y'' = 4\left(4y - \frac{xy'}{4}\right) \text{ (from I)}$$

$$\Rightarrow (x^2-1)y'' + xy' - 16y = 0$$

So, $|\alpha - \beta| = 17$