1. (b)

D_{f_1}

$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R}\right)$

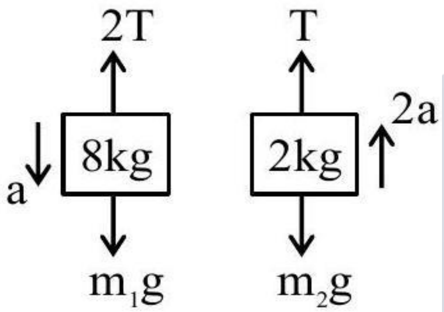
$\frac{1}{f_2} = (\mu_2 - 1)\left(-\frac{1}{R}\right)$

$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{eq}} = \frac{(\mu_1 - 1) - (\mu_2 - 1)}{R}$

$\frac{1}{f_{eq}} = \frac{(\mu_1 - \mu_2)}{R}$

$\frac{R}{f_{eq}} = (\mu_1 - \mu_2)$

2. (d)



$(m_1\ g - 2\ T) = m_1 a - (a)$

$T - m_2\ g = m_2(2a)$

$2\ T - 2\ m_2\ g = 4\ m_2 a - (b)$

$m_1\ g - 2\ m_2\ g = (m_1 + 4\ m_2)a$

$a = \frac{(8-4)g}{(8+8)} = \frac{4}{16}g = \frac{g}{4}$

$a = \frac{10}{4}\ m/s^2$

$S = \frac{1}{2}at^2$

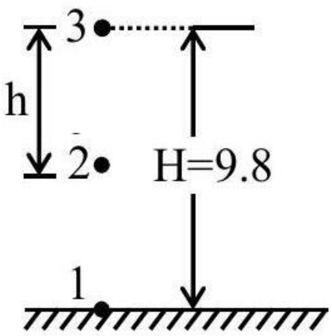
$\frac{0.2 \times 2 \times 4}{10} = t^2$

$t = 0.4sec$

3. (b)

$\alpha = \frac{I_C}{I_E}, \beta = \frac{I_C}{I_B}; I_E = I_C + I_B\ \alpha = \frac{I_C}{I_C + I_B} = \frac{I_C/I_B}{\frac{I_C}{I_B}+1} = \frac{\beta}{\beta+1} + 1 + \frac{1}{\beta} = \frac{1}{\alpha}\frac{1}{\beta} = \frac{1-\alpha}{\alpha}\ \beta = \frac{\alpha}{1-\alpha}$

4. (d)



$$H = \frac{1}{2}gt^2$$

$$\frac{9.8 \times 2}{9.8} = t^2$$

$$t = \sqrt{2}\text{sec}$$

Δt : time interval between drops

$$h = \frac{1}{2}\, g(\sqrt{2} - \Delta t)^2$$

$$0 = \frac{1}{2}\, g(\sqrt{2} - 2\Delta t)^2$$

$$\Delta t = \frac{1}{\sqrt{2}}$$

$$h = \frac{1}{2}\, g\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \times 9.8 \times \frac{1}{2} = \frac{9.8}{4} = 2.45 \text{ m}$$

$$H - h = 9.8 - 2.45$$

$$= 7.35 \text{ m}$$

5. **(c)** From conservation of angular momentum, we get

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

$$k_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

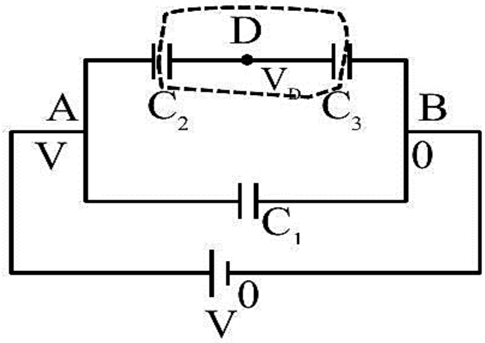
$$k_f = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$k_i - k_f = \frac{1}{2}\left[I_1\omega_1^2 + I_2\omega_2^2 - \frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}\right]$$

Solving above we get

$$k_i - k_f = \frac{1}{2}\left(\frac{I_1 I_2}{I_1 + I_2}\right)(\omega_1 - \omega_2)^2$$

6. (c)



$(V_D - V)C_2 + (V_D - 0)C_3 = 0$

$(V_D - V)6 + (V_D - 0)12 = 0$

$V_D - V + 2\,V_D = 0$

$V_D = \frac{V}{3}\; q_2 = (V - V_D)C_2 = \left(V - \frac{V}{3}\right)(6\mu F)$

$q_2 = (4\,V)\mu F$

$q_3 = (V_D - 0)C_3 = \frac{V}{3} \times 12\mu F = 4\,V\mu F$

$q_1 = (V - 0)C_1 = V(2\mu F)$

$q_1 : q_2 : q_3 = 2 : 4 : 4$

$q_1 : q_2 : q_3 = 1 : 2 : 2$

7. (d) $R = 75 \times 10^2 \pm 5\%$ of 7500

$R = (7500 \pm 375)\Omega$

8. (b) h : height of antenna

λ : wavelength of signal

$h < \lambda$

$\lambda > h$

$\lambda > 400$ m

9. (a) $V_{rms} = \sqrt{\frac{3KT}{M}}$

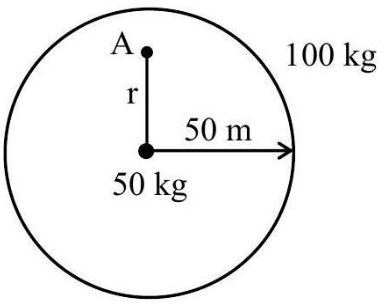
$$\frac{(V_{rms})_{O_2}}{(V_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} = \sqrt{\frac{2}{32}}$$

$$(V_{rms})_{H_2} = 4 \times (V_{rms})_{O_2}$$
$$= 4 \times 160$$
$$= 640 \text{ m/s}$$

10. (c) emf = BLV

$= 1.(2R).1$
$= 2$ V

11. (d)



$$V_A = \left[-\frac{GM_1}{r} - \frac{GM_2}{R}\right]$$
$$= \left[-\frac{50}{25}G - \frac{100}{50}G\right]$$
$$= -4G$$

12. (d) $I_{max} = \frac{50}{2} = 25$ mA

$$\mathbf{R} = \frac{V}{I} = \frac{50\text{mV}}{25\text{ mA}} = 2\Omega$$

13. (b) $kE_{max} = \frac{hc}{\lambda_i} + \phi$

or

$$eV_o = \frac{hc}{\lambda_i} + \phi$$

when $\lambda_i = 670.5$ nm; $V_o = 0.48$

when $\lambda_i = 474.6$ nm; $V_o = ?$

So, $e(0.48) = \frac{1240}{670.5} + \phi$

$$e(V_o) = \frac{1240}{474.6} + \phi$$

(b) $-$(a)

$$e(V_o - 0.48) = 1240\left(\frac{1}{474.6} - \frac{1}{670.5}\right) eV$$

$$V_o = 0.48 + 1240\left(\frac{670.5 - 474.6}{474.6 \times 670.5}\right) \text{ Volts}$$

$$V_o = 0.48 + 0.76$$

$$V_o = 1.24\text{ V} \simeq 1.25\text{ V}$$

14. (b) SI unit of Rydberg const. $= m^{-1}$

SI unit of Plank's const. $= kgm^2\ s^{-1}$

SI unit of Magnetic field energy density $= kgm^{-1}\ s^{-2}$

SI unit of coeff. of viscosity $= kgm^{-1}\ s^{-1}$

15. (a)

Density $= \left[F^a L^b T^c\right]$

$[ML^{-3}] = \left[M^a L^a T^{-2a} L^b T^c\right]$

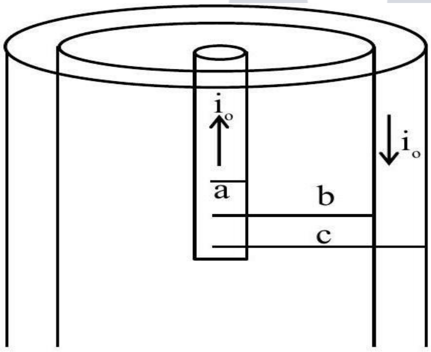
$[M^1\ L^{-3}] = \left[M^a L^{a+b} T^{-2a+c}\right]$

$a = 1;\ a + b = -3; -2a + c = 0$

$1 + b = -3 \quad c = 2a$
$b = -4 \quad c = 2$

So, density $= [F^1\ L^{-4}\ T^2]$

16. (a)



when $x < a$

$$B_1(2\pi x) = \mu_o\left(\frac{i_o}{\pi a^2}\right)\pi x^2$$

$$B(2\pi x) = \frac{\mu_0 i_0 x^2}{a^2}$$

$$B_1 = \frac{\mu_o i_o x}{2\pi a^2}$$

when $a < x < b$

$B_2(2\pi x) = \mu_0 i_o$

$$B_2 = \frac{\mu_o i_o}{2\pi x}$$

$$\frac{B_1}{B_2} = \frac{\mu_0 i_o \dfrac{x}{2\pi a^2}}{\dfrac{\mu_0 i_0}{2\pi x}} = \frac{x^2}{a^2}$$

17. (a) Change in P.E. = Heat energy

$mgh = mS\Delta T$

$$\Delta T = \frac{gh}{S}$$
$$= \frac{10 \times 63}{4200\ \text{J/kgC}}$$
$$= 0.147^\circ\text{C}$$

18. (c) $H = \frac{U^2 \sin^2\theta}{2\,g}$

$$= \frac{(25)^2 \cdot (\sin 45)^2}{2 \times 10}$$
$$= 15.625\ \text{m}$$

$$T = \frac{U \sin\theta}{g}$$

$$= \frac{25 \times \sin 45^\circ}{10}$$

$= 2.5 \times 0.7$

$= 1.77$ s

19. (d) $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$= 4I_o$

20. (b) $\varepsilon = \frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right)(-\hat{i})$

$$\lambda = \left(\frac{-Q}{R\theta}\right) = \left(\frac{-Q}{R \cdot \frac{2\pi}{3}}\right)$$

$$\lambda = \frac{-3Q}{2\pi R}$$

$$\varepsilon = \frac{2k}{R} \cdot \frac{-3Q}{2\pi R} \cdot \sin(60^\circ)(-\hat{i})$$

$$\varepsilon = \frac{3\sqrt{3}Q}{8\pi^2 \epsilon_o R^2}(+\hat{i})$$

21. (500)

$Q_{in} = 300$ J; $Q_{out} = 240$ J

Work done $= Q_{in} - Q_{out} = 300 - 240 = 60$ J

Efficiency $= \frac{W}{Q_{in}} = \frac{60}{300} = \frac{1}{5}$

efficiency $= 1 - \frac{T_2}{T_1}$

$$\frac{1}{5} = 1 - \frac{400}{T_1} \Rightarrow \frac{400}{T_1} = \frac{4}{5}$$

$T_1 = 500\text{k}$

22. (1) $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right) \Rightarrow$ Amplitude $= 10$

$y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

$$y_2 = 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right)$$

$$y_2 = 10\left(\cos\frac{\pi}{3}\sin 3\pi t + \sin\frac{\pi}{3}\cos 3\pi t\right)$$

$y_2 = 10\sin\left(3\pi t + \frac{\pi}{3}\right) \Rightarrow$ Amplitude $= 10$

So ratio of amplitudes $= \frac{10}{10} = 1$

23. (15) No. of different wavelengths $= \frac{n(n-1)}{2}$

$$= \frac{6 \times (6-1)}{2} = \frac{6 \times 5}{2} = 15$$

24. (20) When unregulated voltage is 14 V voltage across zener diode must be 10 V So potential difference across resistor $\Delta V_{Rs} = 4$ V

and $P_{zener} = 2$ W

$VI = 2$

$$I = \frac{2}{10} = 0.2 \text{ A}$$

$\Delta V_{Rs} = I_s$

$$4 \times 0.2R_s \Rightarrow R_s = \frac{40}{2} = 20\Omega$$

25. (40) B. $S_1 = \frac{T_{1max}}{8 \times 10^{-7}} \Rightarrow T_{1max} = 8 \times 1.25 \times 100$

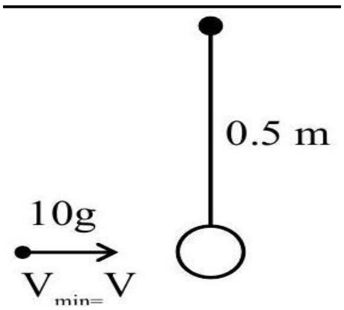
$= 1000$ N

B. $S_2 = \frac{T_{2max}}{4 \times 10^{-7}} \Rightarrow T_{2max} = 4 \times 1.25 \times 100$

$= 500 \text{ N}$

$$m = \frac{500 - 100}{10} = 40 \text{ kg}$$

26. (400)



$V' = \sqrt{5gR} = \sqrt{5 \times 10 \times 0.5}$

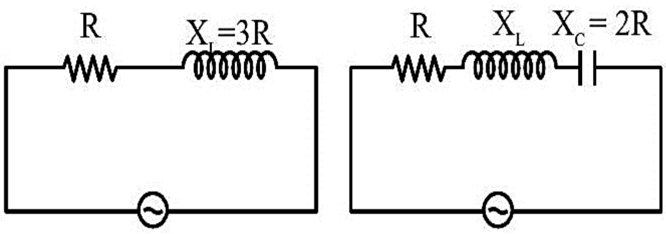
$V' = 5 \text{ m/s}$

$m_1 V = m_2 \times 5 - m_1 \times 100$

$$\frac{10}{1000} \times V = 5 - \frac{10}{1000} \times 100$$

$V = 400 \text{ m/s}$

27. (1)



$$\cos\phi = \frac{R}{\sqrt{R^2 + 3R^2}}$$

$$\cos\phi' = \frac{R}{\sqrt{R^2 + R^2}}$$

$$= \frac{1}{\sqrt{10}} = \frac{1}{\sqrt{2}}$$

$$\frac{\cos\phi'}{\cos\phi} = \frac{\sqrt{10}}{\sqrt{2}} = \frac{\sqrt{5}}{1}$$

$\therefore x = 1$

28. (9)

$$R_{\text{eq open}} = \frac{3R}{2}$$

$$R_{eq\ closed} = 2 \times \frac{R \times 2R}{3R} = \frac{4R}{3}$$

$$\frac{R_{eq\ open}}{R_{eq\ closed}} = \frac{3R}{2} \times \frac{3}{4R} = \frac{9}{8}$$

$\therefore x = 9$

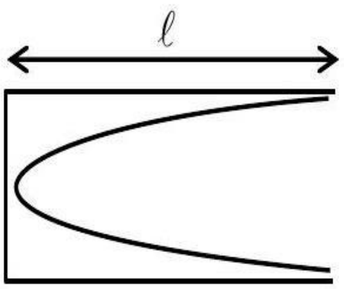
29. (2)

$$|B| = \frac{|E|}{C} = \frac{6}{3 \times 10^8}$$

$= 2 \times 10^{-8}$ T

$\therefore x = 2$

30. (34)



$$\frac{\lambda}{4} = \ell \Rightarrow \lambda = 4\ell$$

$$f = \frac{V}{\lambda} = \frac{V}{4\ell}$$

$$\Rightarrow 250 = \frac{340}{4\ell}$$

$$\Rightarrow \ell = \frac{34}{4 \times 25} = 0.34 \text{ m}$$

$\ell = 34$ cm

31. (c) Standard enthalpy of formation for alkali metal bromides becomes more negative on desending down the group.

In case of CsI, lattice energy is less, but $Cs^+$is having less hydration enthalpy due to which it is less soluble in water.

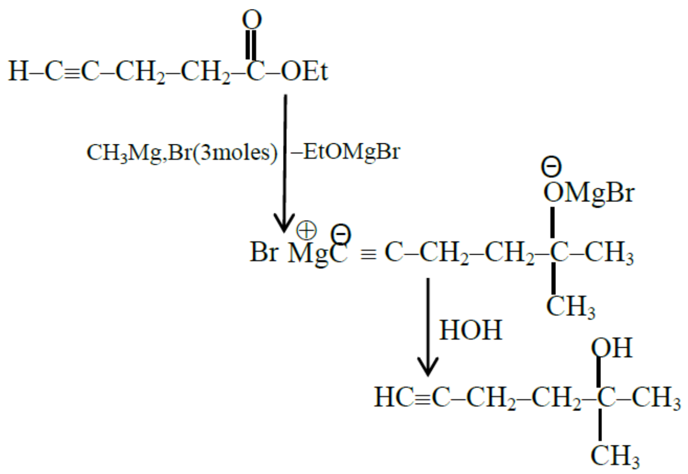
For alkali metal fluorides, the solubility in water increases from lithium to caesium. LiF is least soluble in water.

Standard enthalpy of formation for LiF is most negative among alkali metal fluorides.

32. (b)

$$Cr^{3+} + \underset{\text{dil.}}{NaOH} \longrightarrow \underset{\text{precipitate}}{Cr_2O_3.(H_2O)_n}$$

33. (c) Statement 1 is true
But it consume 3 moles of G R
So statement 2 is false.



34. (c) Ozone in the stratosphere is a product of UV radiations acting on dioxygen (O_2) molecules.

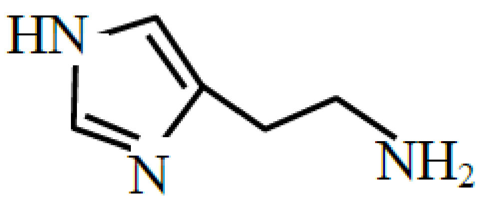
$$O_2(g) \xrightarrow{UV} O(g) + O(g)$$

$$O(g) + O_2(g) \xrightleftharpoons{UV} O_3(g)$$

35. (a)

(a) Shows intra molecular H-bonding
(b) Shows significant intermolecular H-bonding
(c) It do not show intermolecular H-bonding due to steric hindrance.

36. (b) Histamine stimulate the secretion of HCl



Histamine structure

37. (b)

$PbO_2 + 2H_2O \rightarrow Pb(OH)_4$

$Na_2O_2 + 2H_2O \rightarrow 2NaOH + H_2O_2$
this reaction is possible at room temperature

$SnO_2 + 2H_2O \rightarrow Sn(OH)_4$

Acidified $BaO_2 \cdot 8H_2O$ gives H_2O_2 after evaporation.

38. (d) All gives propanoic acid as product but option 4 gives butanoic as product

$$CH_3CH_2CH_2Br \xrightarrow[\text{Dry ether}]{Mg} CH_3CH_2CH_2MgBr$$

$$\xrightarrow{CO_2} CH_3-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-OMgBr$$

$$\xrightarrow{H_3O^+} CH_3-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-OH$$

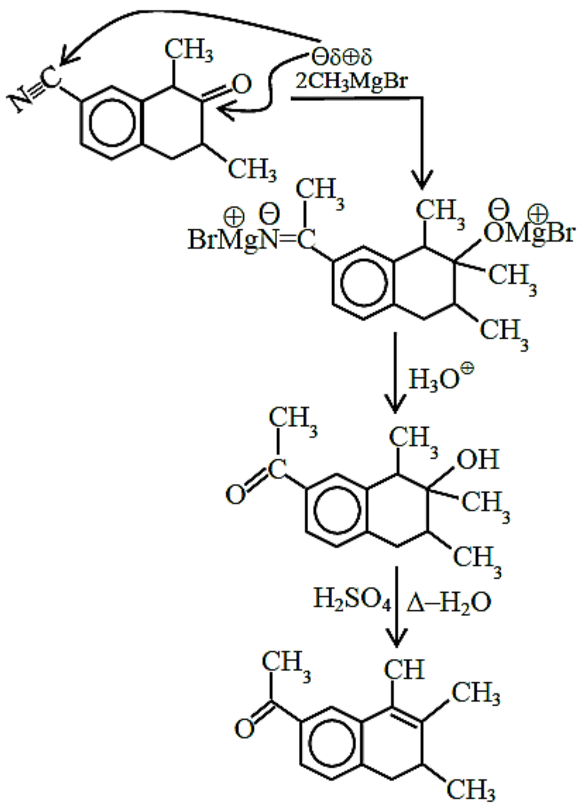
Butanoic acid

39. (a) $P^{3-} > S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$

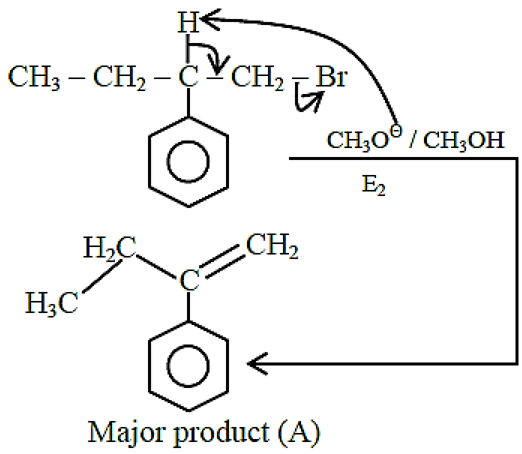
(Correct order of ionic radii) all the given species are isoelectronic species.

In isoelectronic species size increases with increase of negative charge and size decreases with increase in positive charge.

40. (a)



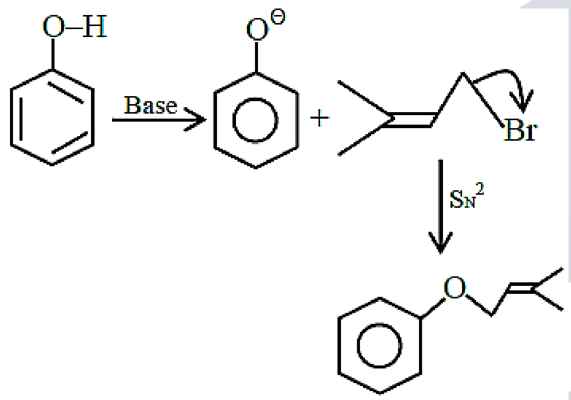
41. (b)



42. **(b)** O_2F_2 oxidises plutonium to PuF_6 and the reaction is used in removing plutonium as PuF_6 from spent nuclear fuel.

43. **(d)** In the lyophilic colloids, the colloidal particles are extensively solvated.

44. **(d)**



45. **(d)**

$$2KMnO_4 \xrightarrow[200^\circ C]{\Delta} \underset{\text{Green}}{K_2MnO_4} + \underset{\text{Black}}{MnO_2} + O_2$$

In K_2MnO_4, manganese oxidation state is +6 and hence it has one unpaired e^-.

46. **(b)** In Seliwanoff's reagent, Cu is not present. In Barfoed, Biuret and in Benediet reagent Cu is present.

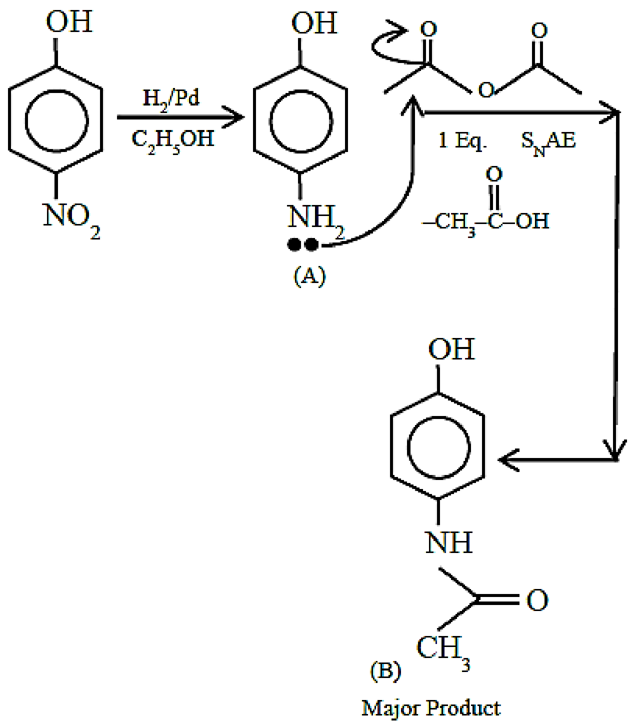
47. **(d)** Sucrose is formed by $\alpha - D(+)$. Glucose $+\beta - D(-)$ Fructose. we obtain these monomers on hydrolysis.

48. **(a)**

(Name of ore/mineral)
(A) Calamine $ZnCO_3$
(B) Malachite $CuCO_3 \cdot Cu(OH)_2$
(C) Siderite $FeCO_3$
(D) Sphalerite ZnS

49. **(d)** When red phosphorus is heated in a sealed tube at 803 K, α-black phosphorus is formed.

50. **(d)**



51. (16) $K_{700} = 6.36 \times 10^{-3}\ s^{-1}$;

$K_{600} = x \times 10^{-6}\ s^{-1}$

$E_a = 209$ kJ/mol

Applying;

$$\log\left(\frac{K_{T_2}}{K_{T_1}}\right) = \frac{-E_a}{2.303R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$\log\left(\frac{K_{700}}{K_{600}}\right) = \frac{-E_a}{2.303R}\left(\frac{1}{700} - \frac{1}{600}\right)$$

$$\log\left(\frac{6.36 \times 10^{-3}}{K_{600}}\right) = \frac{+209 \times 1000}{2.303 \times 8.31}\left(\frac{100}{700 \times 600}\right)$$

$$\log(6.36 \times 10^{-3}) - \log K_{600} = 2.6$$

$$\Rightarrow \log K_{600} = -2.19 - 2.6 = -4.79$$

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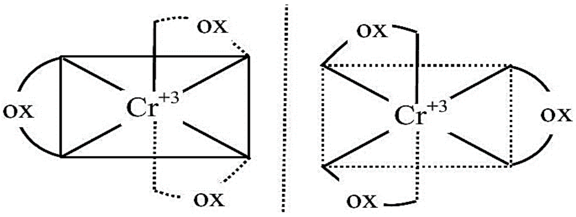
$$\Rightarrow K_{600} = 10^{-4.79} = 1.62 \times 10^{-5}$$
$$= 16.2 \times 10^{-6}$$
$$= x \times 10^{-6}$$

$\Rightarrow$ x = 16

52. **(2)** The number of optical isomers for $[Cr(C_2O_4)_3]^{3-}$ is two.



53. **(84)** Applying; $(n_I + n_{II})_{intital}$ $(n_I + n_{II})_{final}$

$\Rightarrow$ Assuming the system attains a final temperature of T (such that $300 < T < 60$)

$$\Rightarrow \begin{pmatrix}\text{Heat lost by} \\ N_2 \text{ of container} \\ \text{I}\end{pmatrix} = \begin{pmatrix}\text{Heat gained by} \\ N_2 \text{ of container} \\ \text{II}\end{pmatrix}$$

$$\Rightarrow n_I C_m (300 - T) = n_{II} C_m (T - 60)$$

$$\Rightarrow \left(\frac{2.8}{28}\right)(300 - T) = \frac{0.2}{28}(T - 60)$$

$$\Rightarrow 14(300 - T) = T - 60 \Rightarrow \frac{(14\times300+60)}{15} = T$$

$\Rightarrow T = 284$ K (final temperature)

$\Rightarrow$ If the final pressure $= P$

$$\Rightarrow (n_I + n_{II})_{final} = \left(\frac{3.0}{28}\right)$$

$$\Rightarrow \frac{P}{RT}(V_I + V_{II}) = \frac{3.0\text{gm}}{28\text{gm/mol}}$$

$$P = \left(\frac{3}{28}\text{ mol}\right) \times 8.31\frac{\text{J}}{\text{mol} - \text{K}} \times \frac{284\text{ K}}{3\times10^{-3}\text{ m}^3} \times 10^{-5}\frac{\text{bar}}{\text{Pa}}$$

$\Rightarrow 0.84287$bar

$\Rightarrow 84.28 \times 10^{-2}$bar

$\Rightarrow 84$

54. **(19)**

$$C_3H_{8(g)} + 5O_{2(g)} \longrightarrow 3CO_{2(g)} + 4H_2O_{(\ell)}$$

$t = 0$ 2.27 mole 31.25 mol

$t = \infty$ 0 19.9 mol 6.81 mol 9.08 mol mole fraction of CO_2 in the final reaction mixture (heterogenous)

$$X_{CO_2} = \frac{6.81}{19.9 + 6.81 + 9.08}$$

$= 0.1902 = 19.02 \times 10^{-2}$

$\Rightarrow 19$

55. **(271)**

molality $= \frac{\left(\frac{40}{180}\right)\text{mol}}{0.2\text{Kg}} = \left(\frac{10}{9}\right)$ molal

$\Rightarrow \Delta T_f = T_f - T_f{}' = 1.86 \times \frac{10}{9}$

$\Rightarrow T_f{}' = 273.15 - 1.86 \times \frac{10}{9}$

$= 271.08$ K

$\simeq 271$ K (nearest-integer)

56. (760) $K = \frac{1}{R} \times \ell/A = \left(\left(\frac{1}{1500}\right) \times 1.14\right) \text{Scm}^{-1}$

$\Rightarrow \Lambda_m = 1000 \times \frac{\left(\frac{1.14}{1500}\right)}{0.001}$ S cm^2 mol^{-1}

$= 760$ S cm^2 mol^{-1}

$\Rightarrow 760$

57. (50) Energy emitted in 0.1sec.

$= 0.1\text{sec.} \times 10^{-3} \frac{\text{J}}{\text{s}}$

$= 10^{-4}$ J

If ' n ' photons of $\lambda = 1000$ nm are emitted,

then; $10^{-4} = n \times \frac{hc}{\lambda}$

$\Rightarrow 10^{-4} = \frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^8}{1000 \times 10^{-9}}$

$\Rightarrow n = 5.02 \times 10^{14} = 50.2 \times 10^{13}$

$\Rightarrow 50$ (nearest integer)

58. (6) moles of NH_4HS initially taken $= \frac{5.1\text{ g}}{51\text{ g/mol}}$

$= 0.1$ mol

volume of vessel $= 2\ell$

$NH_4HS_{(s)} \rightleftharpoons NH_{3(g)} + H_2S_{(g)}$

t = 0 0.1 mol

t = ∞ 0.1(1 − 0.2) 0.1 × 0.2 0.1 × 0.2

$\Rightarrow$ partial pressure of each component

$P = \frac{nRT}{V} = \frac{0.1 \times 0.2 \times 0.082 \times 300}{2}$

$= 0.246$ atm

$\Rightarrow k_P = P_{NH_3} \times P_{H_2S} = (0.246)^2 = 0.060516$

$= 6.05 \times 10^{-2}$

$\Rightarrow 6$

59. (3)
60. (964)

$$T_{min} = \left(\frac{\Delta^0 H}{\Delta^0 S}\right)$$

$\Delta^0 H_{rxn} = [\Delta_f^0 H(Fe) + \Delta_f^0 H(CO)] -$
$= [\Delta_f^0 H(FeO) + \Delta_f^0 H(C_{(graphite)})]$
$= [0 - 110.5] - [-266.3 + 0]$
$= 155.8$ kJ/mol
$\Delta^0 S_{rxn} = [\Delta^0 S(Fe) + \Delta^0 S(CO)] -$
$[\Delta^0 S(FeO) + \Delta^0 S(C_{(graphite)})]$
$= [27.28 + 197.6] - [57.49 + 5.74]$
$= 161.65$ J/mol – K

$$T_{min} = \frac{155.8 \times 10^3 \text{ J/mol}}{161.65 \text{ J/mol} - \text{K}} = 963.8 \text{ K}$$

$\simeq 964$k (nearest integer)

61. (a)

$n = 2(\ell + m)$

$\ell m + n(\ell + m) = 0$

$\ell m + 2(\ell + m)^2 = 0$

$2\ell^2 + 2m^2 + 5m\ell = 0$

$2\left(\frac{\ell}{m}\right)^2 + 2 + 5\left(\frac{\ell}{m}\right) = 0.$

$2t^2 + 5t + 2 = 0$

$(t + 2)(2t + 1) = 0$

$\Rightarrow t = -2; -\frac{1}{2}$

(i) $\frac{\ell}{m} = -2$

$\frac{n}{m} = -2$	$n = -2\ell$
$(-2m, m, -2m)$	$(\ell, -2\ell, -2\ell)$

$(-2,1,-2) \quad\Big|\quad (1,-2,-2)$

$$\cos\theta = \frac{-2-2+4}{\sqrt{9}\sqrt{9}} = 0 \Rightarrow 0 = \frac{\pi}{2}$$

62. (b)

$$\begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix} = 192$$

$R_1 \to R_1 - R_3 \& R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix} = 192$$

$2[x] + 6 + [x] = 192 \Rightarrow [x] = 62$

63. (d) Let $g(x) = \sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

$g(x) \in [1, \sqrt{2}]$ for $x \in [0, \pi/2]$

$$f(x) = \tan^{-1}(\sin x + \cos x) \in \left[\frac{\pi}{4}, \tan^{-1}\sqrt{2}\right]$$

$$\tan\left(\tan^{-1}\sqrt{2} - \frac{\pi}{4}\right) = \frac{\sqrt{2}-1}{1+\sqrt{2}} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = 3 - 2\sqrt{2}$$

64. (c)

C - I ' 0 ' Head

T T T $\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 = \frac{1}{64}$

C - II '1' head

H T T $\left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = \frac{9}{64}$

C - III '2' Head

H H T $\left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = \frac{9}{64}$

C-IV '3' Heads

H H H $\left(\frac{1}{8}\right)\left(\frac{1}{8}\right) = \frac{1}{64}$

Total probability $= \frac{5}{16}$.

65. (d) $\alpha \cdot R = \frac{|3(b)+4(-3)-5|}{5} = \frac{11}{5}$

$(x-h)^2=\frac{11}{5}(y-k)$

differentiate w.r.t 'x' :-

$2(x-h)=\frac{11}{5}\frac{dy}{dx}$

again differentiate

$2=\frac{11}{5}\frac{d^2y}{dx^2}$

$\frac{11d^2y}{dx^2}=10$

66. (b) Locus is directrix of parabola

$x-3+4=0 \Rightarrow x+1=0.$

67. (a) Equation of planes are

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0 \Rightarrow x+y+z-1=0$

and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0 \Rightarrow 2x+3y-z+4=0$

equation of planes through line of intersection of these planes is:-

$(x+y+z-1)+\lambda(2x+3y-z+4)=0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-1+4\lambda=0$

But this plane is parallel to x-axis whose direction are (1,0,0)

$\therefore (1+2\lambda)1+(1+3\lambda)0+(1-\lambda)0=0$

$\lambda=-\frac{1}{2}$

$\therefore$ Required plane is

$0x+\left(1-\frac{3}{2}\right)y+\left(1+\frac{1}{2}\right)z-1+4\left(\frac{-1}{2}\right)=0$

$\Rightarrow \frac{-y}{2}+\frac{3}{2}z-3=0$

$\Rightarrow y-3z+6=0$

$\Rightarrow \vec{r}\cdot(\hat{j}-3\hat{k})+6=0$

68. (d) $(2x-10y^3)dy+ydx=0$

$\Rightarrow \frac{dx}{dy}+\left(\frac{2}{y}\right)x=10y^2$

$\text{I. F.}=e^{\int\frac{2}{y}dy}=e^{2\ln(y)}=y^2$

Solution of D.E. is

$\therefore\ x.y = \int (10y^2)y^2 \cdot dy$

$xy^2 = \frac{10y^5}{5} + C \Rightarrow xy^2 = 2y^5 + C$

It passes through $(0,1) \rightarrow 0 = 2 + \mathrm{C} \Rightarrow \mathrm{C} = -2$

$\therefore$ Curve is $xy^2 = 2y^5 - 2$

Now, it passes through $(2, \beta)$

$2\beta^2 = 2\beta^5 - 2 \Rightarrow \beta^5 - \beta^2 - 1 = 0$

$\therefore\ \beta$ is root of an equation $y^5 - y^2 - 1 = 0$

69. (d) $\left|\frac{1}{2}\right| \begin{matrix} \mathrm{a} & 0 & 1 \\ \mathrm{b} & 2\,\mathrm{b}+1 & 1 \\ 0 & \mathrm{b} & 1 \end{matrix} | = 1$

$\Rightarrow \begin{vmatrix} \mathrm{a} & 0 & 1 \\ \mathrm{b} & 2\,\mathrm{b}+1 & 1 \\ 0 & \mathrm{b} & 1 \end{vmatrix} = \pm 2$

$\Rightarrow \mathrm{a}(2\,\mathrm{b} + 1 - \mathrm{b}) - 0 + 1(\mathrm{b}^2 - 0) = \pm 2$

$\Rightarrow \mathrm{a} = \frac{\pm 2 - \mathrm{b}^2}{\mathrm{b} + 1}$

$\therefore\ a = \frac{2-\mathrm{b}^2}{\mathrm{b}+1}$ and $\mathrm{a} = \frac{-2-\mathrm{b}^2}{\mathrm{b}+1}$

sum of possible values of ' a ' is

$= \frac{-2b^2}{a+1}$

70. (a)

$\mathrm{D} = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28 + [\lambda] \end{vmatrix} = -24 - [\lambda] + 15 = -[\lambda] - 9$

if $[\lambda] + 9 \neq 0$ then unique solution

if $[\lambda] + 9 = 0$ then $\mathrm{D}_1 = \mathrm{D}_2 = \mathrm{D}_3 = 0$

so infinite solutions

Hence λ can be any red number.

71. (a)

$(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2)$

$+\mathrm{k}(3\mathrm{x}^2 + 4\mathrm{x} + 2)^2 = 0$

Let $3x^2 + 4x + 3 = a$

and $3x^2 + 4x + 2 = b \Rightarrow b = a - 1$

Given equation becomes

$\Rightarrow a^2 - (k+1)ab + kb^2 = 0$

$\Rightarrow a(a - kb) - b(a - kb) = 0$

$\Rightarrow (a - kb)(a - b) = 0 \Rightarrow a = kb$ or $a = b$ (reject)

$\because a = kb$

$\Rightarrow 3x^2 + 4x + 3 = k(3x^2 + 4x + 2)$

$\Rightarrow 3(k-1)x^2 + 4(k-1)x + (2k-3) = 0$ for real roots

$D \geq 0$

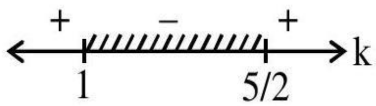
$\Rightarrow 16(k-1)^2 - 4(3(k-1))(2k-3) \geq 0$

$\Rightarrow 4(k-1)\{4(k-1) - 3(2k-3)\} \geq 0$

$\Rightarrow 4(k-1)\{-2k+5\} \geq 0$

$\Rightarrow -4(k-1)\{2k-5\} \geq 0$

$\Rightarrow (k-1)(2k-5) \leq 0$

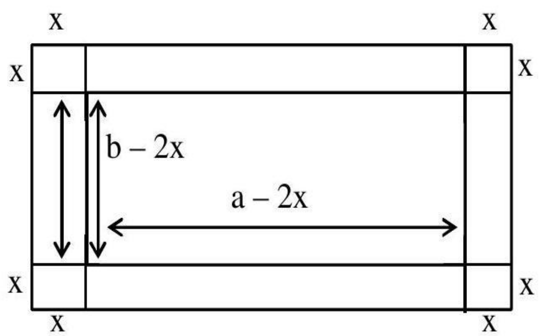


$\therefore k \in \left[1, \frac{5}{2}\right]$

$\because k \neq 1$

$\therefore k \in \left(1, \frac{5}{2}\right]$

72. (c)



$V = \ell.b.h = (a - 2x)(b - 2x)x$

$\Rightarrow V(x) = (2x - a)(2x - b)x$

$\Rightarrow V(x) = 4x^3 - 2(a+b)x^2 + abx$

$\Rightarrow \frac{d}{dx}v(x) = 12x^2 - 4(a+b)x + ab$

$\frac{d}{dx}(v(x)) = 0 \Rightarrow 12x^2 - 4(a+b)x + ab = 0 <_{\beta}^{\alpha}$

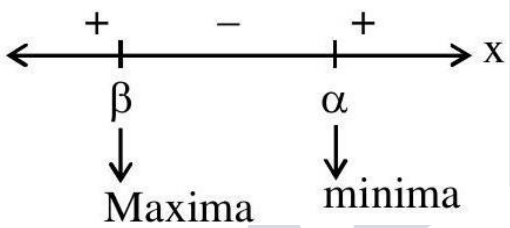
$\Rightarrow x = \frac{4(a+b) \pm \sqrt{16(a+b)^2 - 48ab}}{2(12)}$

$= \frac{(a+b) \pm \sqrt{a^2+b^2-ab}}{6}$

Let $x = \alpha = \frac{(a+b)+\sqrt{a^2+b^2-ab}}{6}$

$\beta = \frac{(a+b)-\sqrt{a^2+b^2-ab}}{6}$

Now, $12(x-\alpha)(x-\beta) = 0$



$\therefore x = \beta$

$= \frac{a+b-\sqrt{a^2+b^2-ab}}{b}$

73. (a)

$(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge p) \wedge (p \wedge q)$

$\Rightarrow [\sim (p \wedge q) \vee (p \wedge q)] \wedge (\sim (p \wedge q) \vee (r \wedge p))$

$\Rightarrow t \wedge [\sim (p \wedge q) \vee (r \wedge p)]$

$\Rightarrow \sim (p \wedge q) \vee (r \wedge p)$

$\Rightarrow (p \wedge q) \Rightarrow (r \wedge p)$

Aliter:

given statement says

" if p and q both happen then

p and q and r will happen"

it Simply implies

" If p and q both happen then

'r' too will happen "

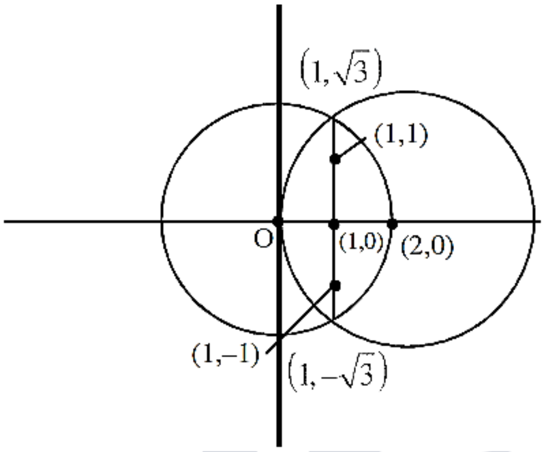
i.e.

" if p and q both happen then r and p too will happen

i.e.

$(p \wedge q) \Rightarrow (r \wedge p)$

74. (b)



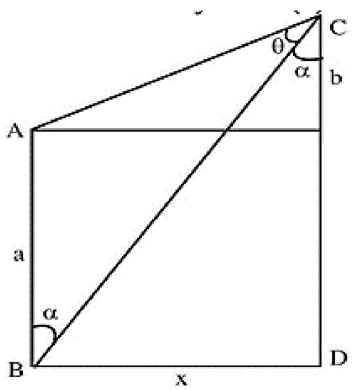
$$(x-2)^2 + y^2 \leq 4$$
$$x^2 + y^2 \leq 4$$

No. of points common in C_1&C_2 is 5 .

$(0,0), (1,0), (2,0), (1,1), (1,-1)$

Similarly in C_2&C_3 is 5 .
No. of relations $= 2^{5\times5} = 2^{25}$.

75. (a)
76. (a)
77. (c)



$\tan\theta = \frac{1}{2}$

$\tan(\theta+\alpha) = \frac{x}{b}, \tan\alpha = \frac{x}{a+b}$

$\Rightarrow \frac{1}{2} + \frac{x}{a+b}$

$\Rightarrow \frac{\frac{1}{2}+\frac{x}{a+b}}{1-\frac{1}{2}\times\frac{x}{a+b}} = \frac{x}{b}.$

$\Rightarrow x^2 - 2ax + ab + b^2 = 0$

78. (a) $y = \left(1-\frac{1}{2}\right)x^2 + \left(1-\frac{1}{3}\right)x^3 + \cdots \ldots$

$= (x^2 + x^3 + x^4 + \cdots \ldots) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots.\right)$

$= \frac{x^2}{1-x} + x - \left(x + \frac{x^2}{2} + \frac{x^2}{3} + \cdots\right)$

$= \frac{x}{1-x} + \ln(1-x)$

$x = \frac{1}{2} \Rightarrow y = 1 - \ln 2$

$e^{1+y} = e^{1+1-\ln 2}$

$= e^{2-\ln 2} = \frac{e^2}{2}$

79. (a) $I = \int_0^1 \frac{\sqrt{x}}{(1+x)(1+3x)(3+x)} dx$

Let $x = t^2 \Rightarrow dx = 2t.dt$

$I = \int_0^1 \frac{t(2t)}{(t^2+1)(1+3t^2)(3+t^2)} dt$

$I = \int_0^1 \frac{(3t^2+1)-(t^2+1)}{(3t^2+1)(t^2+1)(3+t^2)} dt$

$I = \int_0^1 \frac{dt}{(t^2+1)(3+t^2)} - \int_0^1 \frac{dt}{(1+3t^2)(3+t^2)}$

$= \frac{1}{2}\int_0^1 \frac{(3+t^2)-(t^2+1)}{(t^2+1)(3+t^2)} dt + \frac{1}{8}\int_0^1 \frac{(1+3t^2)-3(3+t^2)}{(1+3t^2)(3+t^2)} dt$

$= \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} - \frac{1}{2}\int_0^1 \frac{dt}{t^2+3} + \frac{1}{8}\int_0^1 \frac{dt}{t^2+3} - \frac{3}{8}\int_0^1 \frac{dt}{(1+3t^2)}$

$= \frac{1}{2}\int_0^1 \frac{dt}{t^2+1} - \frac{3}{8}\int_0^1 \frac{dt}{t^2+3} - \frac{3}{8}\int_0^1 \frac{dt}{1+3t^2}$

$$=\frac{1}{2}(\tan^{-1}(t))_0^1-\frac{3}{8\sqrt{3}}\left(\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right)_0^1$$

$$-\frac{3}{8\sqrt{3}}\left(\tan^{-1}(\sqrt{3}t)\right)_0^1$$

$$=\frac{1}{2}\left(\frac{\pi}{4}\right)-\frac{\sqrt{3}}{8}\left(\frac{\pi}{6}\right)-\frac{\sqrt{3}}{8}\left(\frac{\pi}{3}\right)$$

$$=\frac{\pi}{8}-\frac{\sqrt{3}}{16}\pi$$

$$=\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$$

80. (b)

$$\lim_{x\to\infty}\left(\sqrt{x^2-x+1}\right)-ax=b\ (\infty-\infty)$$
$$\Rightarrow a>0$$

Now, $\lim_{x\to\infty}\frac{(x^2-x+1-a^2x^2)}{\sqrt{x^2-x+1}+ax}=b$

$$\Rightarrow \lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{\sqrt{x^2-x+1}+ax}=b$$

$$\Rightarrow \lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$$

$$\Rightarrow 1-a^2=0\Rightarrow a=1$$

Now, $\lim_{x\to\infty}\frac{-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$$\Rightarrow \frac{-1}{1+a}=b\Rightarrow b=-\frac{1}{2}$$

$$(a,b)=\left(1,-\frac{1}{2}\right)$$

81. (56) Given equation

$\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$
$\Rightarrow 1 - \sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$
$\Rightarrow 2 - (\sin 2\theta)^2 - \sin 2\theta = 0$
$\Rightarrow (\sin 2\theta)^2 + (\sin 2\theta) - 2 = 0$
$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$
$\Rightarrow \sin 2\theta = 1$ or $\frac{\sin 2\theta = -2}{(\text{ not possible })}$
$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$
$\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$
$\Rightarrow S = \frac{\pi}{4} + \frac{5\pi}{4} + \frac{9\pi}{4} + \frac{13\pi}{4} = 7\pi$
$\Rightarrow \frac{8S}{\pi} = \frac{8 \times 7\pi}{\pi} = 56.00$

82. (72) Since $R(3,5,\gamma)$ lies on the plane $2x - y + z + 3 = 0$.

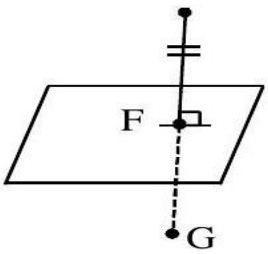
Therefore, $6 - 5 + \gamma + 3 = 0$

$\Rightarrow \gamma = -4$

Now,

dr's of line QS

are $2, -1, 1$



equation of line QS is

$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$ (say)

$\Rightarrow F(2\lambda + 1, -\lambda + 3, \lambda + 4)$

F lies in the plane

$\Rightarrow 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$

$\Rightarrow 4\lambda + 2 + \lambda - 3 + \lambda + 7 = 0$

$\Rightarrow 6\lambda + 6 = 0 \Rightarrow \lambda = -1.$

$\Rightarrow F(-1,4,3)$

Since, F is mid-point of QS.

Therefore, co- ordinated of S are $(-3,5,2)$.

So, $SR = \sqrt{36+0+36} = \sqrt{72}$

$SR^2 = 72.$

83. (30) $\sum P(X) = 1 \Rightarrow k + 2k + 2k + 3k + k = 1$

$\Rightarrow k = \frac{1}{9}$

Now, $p = P\left(\frac{kX<4}{X<3}\right) = \frac{P(X=2)}{P(X<3)} = \frac{\frac{2k}{9k}}{\frac{k}{9k}+\frac{2k}{9k}} = \frac{2}{3}$

$\Rightarrow p = \frac{2}{3}$

Now, $5p = \lambda k$

$\Rightarrow (5)\left(\frac{2}{3}\right) = \lambda(1/9)$

$\Rightarrow \lambda = 30$

84. (6) $|z-3| = \mathrm{Re}(z)$

let $Z = x = iy$

$\Rightarrow (x-3)^2 + y^2 = x^2$

$\Rightarrow x^2 + 9 - 6x + y^2 = x^2$

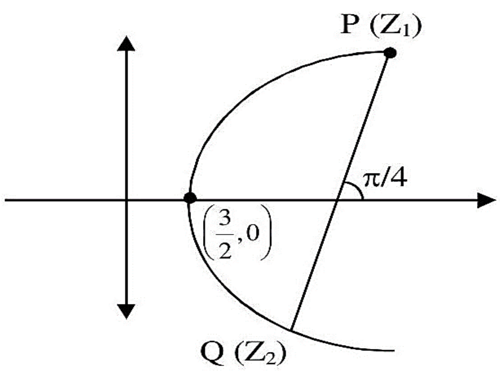
$\Rightarrow y^2 = 6x - 9$

$\Rightarrow y^2 = 6\left(x - \frac{3}{2}\right)$

$\Rightarrow z_1$ and z_2 lie on the parabola mentioned in eq.(a)

$\arg(z_1 - z_2) = \frac{\pi}{4}$

$\Rightarrow$ Slope of PQ = 1.



Let $P\left(\frac{3}{2}+\frac{3}{2}t_1^2, 3t_1\right)$ and $Q\left(\frac{3}{2}+\frac{3}{2}t_2^2, 3t_2\right)$

Slope of PQ $= \frac{3(t_2-t_1)}{\frac{3}{2}(t_2^2-t_1^2)} = 1$

$$\Rightarrow \frac{2}{t_2+t_2} = 1$$

$$\Rightarrow t_2 + t_1 = 2$$

$$\mathrm{Im}(z_1+z_2) = 3t_1 + 3t_2 = 3(t_1+t_2) = 3(b)$$

Ans. 6.00
Aliter :

Let $z_1 = x_1 + iy_1; z_2 = x_2 + iy_2$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$\therefore \arg(z_1 - z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{y_1-y_2}{x_1-x_2}\right) = \frac{\pi}{4}$$

$$y_1 - y_2 = x_1 - x_2$$

$$|z_1 - 3| = \mathrm{Re}(z_1) \Rightarrow (x_1-3)^2 + y_1^2 = x_1^2$$

$$|z_2 - 3| = \mathrm{Re}(z_2) \Rightarrow (x_2-3)^2 + {y_2}^2 = {x_2}^2$$

sub (b) & (c)

$$(x_1-3)^2 - (x_2-3)^2 + y_1^2 - y_2^2 = x_1^2 - x_2^2$$

$$(x_1-x_2)(x_1+x_2-6) + (y_1-y_2)(y_1+y_2)$$

$$= (x_1-x_2)(x_1+x_2)$$

$$x_1 + x_2 - 6 + y_1 + y_2 = x_1 + x_2 \Rightarrow y_1 + y_2 = 6.$$

85. (80) 3n type → 3,6,9 = P

3n – 1 type → 2,5 = Q

3n – 2 type → 1,4 = R

number of subset of S containing one element

which are not divisible by 3 = ${}^2C_1 + {}^2C_1 = 4$

number of subset of S containing two numbers

whose some is not divisible by 3

$$= {}^3C_1 \times {}^2C_1 + {}^3C_1 \times {}^2C_1 + {}^2C_2 + {}^2C_2 = 14$$

number of subsets containing 3 elements whose sum is not divisible by 3

$$= {}^3C_2 \times {}^4C_1 + ({}^2C_2 \times {}^2C_1)2 + {}^3C_1({}^2C_2 + {}^2C_2) = 22$$

number of subsets containing 4 elements whose sum is not divisible by 3

$= {}^3C_3 \times {}^4C_1 + {}^3C_2({}^2C_2 + {}^2C_2) + ({}^3C_1\, {}^2C_1 \times {}^2C_2)2$

$= 4 + 6 + 12 = 22.$

number of subsets of S containing 5 elements

whose sum is not divisible by 3 .

$= {}^3C_3({}^2C_2 + {}^2C_2) + ({}^3C_2\, {}^2C_1 \times {}^2C_2) \times 2 = 2 + 12 = 14$

number of subsets of S containing 6 elements

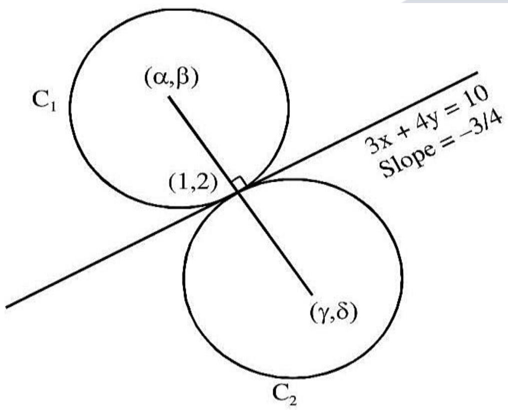
whose sum is not divisible by $3 = 4$

$\Rightarrow$ Total subsets of Set A whose sum of digits is

not divisible by $3 = 4 + 14 + 22 + 22 + 14 + 4 = 80.$

86. (36)

87. (40) Slope of line joining centres of circles $= \frac{4}{3} = \tan\theta$



$\Rightarrow \cos\theta = \dfrac{3}{5}, \sin\theta = \dfrac{4}{5}$

Now using parametric form

$\dfrac{x-1}{\cos\theta} = \dfrac{y-2}{\sin\theta} = \pm 5$

$\oplus$ $(x, y) = (1 + 5\cos\theta, 2 + 5\sin\theta)$

$(\alpha, \beta) = (4,6)$

$\Theta(x, y) = (\gamma, \delta) = (1 - 5\cos\theta, 2 - 5\sin\theta)$

$(\gamma, s) = (-2, -2)$

$\Rightarrow |(\alpha + \beta)(\gamma + \delta)| = |10x - 4| = 40$

88. (15)

$3(1+6)^{22} + 2 \cdot (1+9)^{22} - 44 = (3 + 2 - 44) = 18.I$

$= -39 + 18.I$
$= (54 - 39) + 18(I - 3)$
$= 15 + 18I_1$

$\Rightarrow$ Remainder $= 15$.

89. (25)

$\sigma_b^2 = 2$ (variance of boys) $n_1 =$ no. of boys

$\bar{x}_b = 12$ $n_2 =$ no. of girls
$\sigma_g^2 = 2$

$$\bar{x}_g = \frac{50 \times 15 - 12 \times \sigma_b}{30} = \frac{750 - 12 \times 20}{30} = 17 = \mu$$

variance of combined series

$$\sigma^2 = \frac{n_1\sigma_b^2 + n_2\sigma_g^2}{n_1 + n_2} + \frac{n_1 \cdot n_2}{(n_1 + n_2)^2}(\bar{x}_b - \bar{x}_g)^2$$
$$\sigma^2 = \frac{20 \times 2 + 30 \times 2}{20 + 30} + \frac{20 \times 30}{(20 + 30)^2}(12 - 17)^2$$
$$\sigma^2 = 8.$$
$$\Rightarrow \mu + \sigma^2 = 17 + 8 = 25$$

90. (7)

$$\int \frac{2e^x}{4e^x + 7e^{-x}} dx + 3\int \frac{e^{-x}}{4e^x + 7e^{-x}} dx$$

$$= \int \frac{2e^{2x}}{4e^{2x} + 7} dx + 3\int \frac{e^{-2x}}{4 + 7e^{-2x}} dx$$

Let $4e^{2x} + 7 = T$ Let $4 + 7e^{-2x} = t$

$8e^{2x}dx = dT$ $-14e^{-2x}dx = dt$

$2e^{2x}dx = \frac{dT}{4}$ $e^{-2x}dx = -\frac{dt}{14}$

$$\int \frac{dT}{4\,T} - \frac{3}{14}\int \frac{dt}{t}$$

$$= \frac{1}{4}\log T - \frac{3}{14}\log t + C$$

$$= \frac{1}{4}\log(4e^{2x} + 7) - \frac{3}{14}\log(4 + 7e^{-2x}) + C$$

$$= \frac{1}{14}\left[\frac{1}{2}\log(4e^x + 7e^{-x}) + \frac{13}{2}x\right] + C$$

$$u = \frac{13}{2}, v = \frac{1}{2} \Rightarrow u + v = 7$$

Aliter :

$2e^{x}+3e^{-x}=A(4e^{x}+7e^{-x})+B(4e^{x}-7e^{-x})+\lambda$

$2=4A+4B; 3=7A-7B; \lambda=0$

$A+B=\frac{1}{2}$

$A-B=\frac{3}{7}$

$A=\frac{1}{2}\left(\frac{1}{2}+\frac{3}{7}\right)=\frac{7+6}{28}=\frac{13}{28}$

$B=A-\frac{3}{7}=\frac{13}{28}-\frac{3}{7}=\frac{13-12}{28}=\frac{1}{28}$

$\int \frac{13}{28}dx+\frac{1}{28}\int \frac{4e^{x}-7e^{-x}}{4e^{x}+7e^{-x}}dx$

$\frac{13}{28}x+\frac{1}{28}\ln|4e^{x}+7e^{-x}|+C$

$u=\frac{13}{2}; v=\frac{1}{2}$

$\Rightarrow u+v=7$