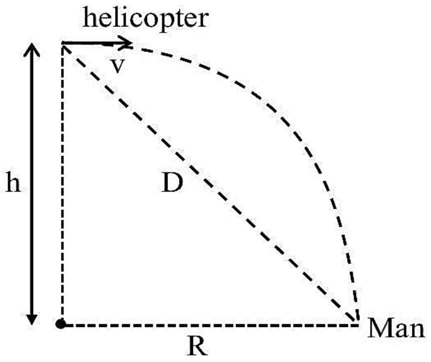
1. (c)



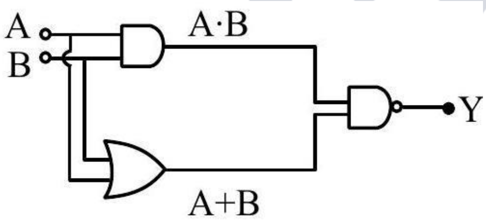
$$R = \sqrt{\frac{2\,h}{g}} \cdot v$$

$$D = \sqrt{R^2 + h^2}$$

$$= \sqrt{\left(\sqrt{\frac{2h}{g}} \cdot v\right)^2 + h^2}$$

$$D = \sqrt{\frac{2hvv^2}{g} + h^2}$$

2. (c)



$$Y = \overline{(A \cdot B) \cdot (A + B)}$$

$$Y)_{(0,0)} = 1$$

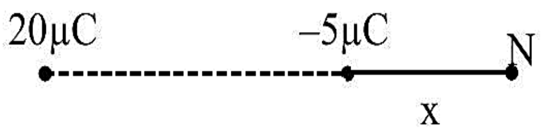
$$Y)_{(0,1)} = 1$$

$$Y)_{(1,0)} = 1$$

$$Y)_{(1,1)} = 0$$

3. (b)

Null point is possible only right side of $-5\mu C$



$$E_N = +\frac{k(-5\mu C)}{x^2} + \frac{k(20\mu C)}{(5+x)^2} = 0$$

$x = 5$ cm

4. (a) T_2 = sink temperature

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{1}{4} = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = \frac{3}{4}$$

$$\frac{1}{2} = 1 - \frac{T_2 - 58}{T_1}$$

$$\frac{T_2}{T_1} - \frac{58}{T_1} = \frac{1}{2}$$

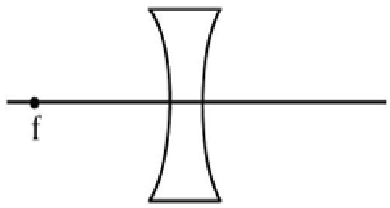
$$\frac{3}{4} = \frac{58}{T_1} + \frac{1}{2}$$

$$\frac{1}{4} = \frac{58}{T_1} \Rightarrow T_1 = 232$$

$$T_2 = \frac{3}{4} \times 232$$

$T_2 = 174$ K

5. (c)



$U = -f$

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{-f} \Rightarrow \frac{1}{V} = -\frac{2}{f}$$

$$V = \frac{-f}{2}$$

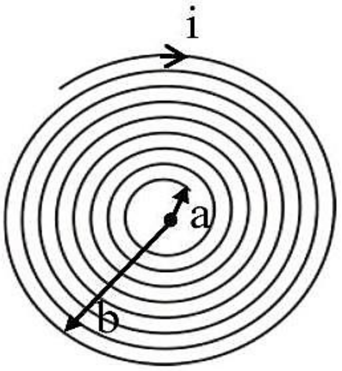
$$m = \frac{V}{U} = \frac{1}{2}$$

distance $= \frac{f}{2}$

6. (b) $A \longrightarrow B \longrightarrow C$ (stable)

Initially no. of atoms of $B = 0$ after $t = 0$, no. of atoms of B will starts increasing & reaches maximum value when rate of decay of $B =$ rate of formation of B. After that maximum value, no. of atoms will starts decreasing as growth & decay both are exponential functions, so best possible graph is (b)

7. (a)

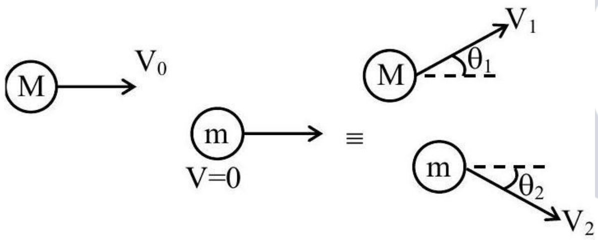


No. of turns in dx width $= \frac{N}{b-a} dx$

$$\int dB = \int_a^b \left(\frac{N}{b-a}\right) dx \frac{\mu_0 i}{2x}$$

$$B = \frac{N\mu_0 i}{2(b-a)} \ln\left(\frac{b}{a}\right)$$

8. (c)



given $\theta_1 = \theta_2 = \theta$

from momentum conservation

in x-direction $MV_0 = MV_1 \cos\theta + mV_2 \cos\theta$

in y-direction $0 = MV_1 \sin\theta - mV_2 \sin\theta$

Solving above equations $V_2 = \frac{MV_1}{m}$, $V_0 = 2V_1 \cos\theta$

From energy conservation

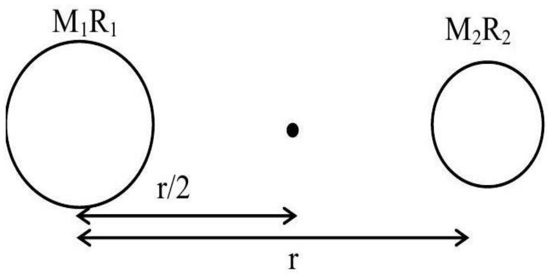
$$\frac{1}{2}MV_0^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}MV_2^2$$

Substituting value of $V_2 \& V_0$, we will get

$$\frac{M}{m} + 1 = 4\cos^2\theta \le 4$$

$$\frac{\mathrm{M}}{\mathrm{m}} \le 3$$

9. (b)

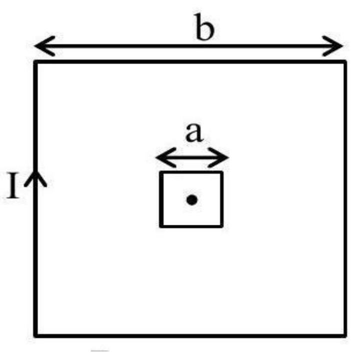


$$\frac{1}{2}\mathrm{mV}^2 - \frac{\mathrm{GM_1\,m}}{\mathrm{r}/2} - \frac{\mathrm{GM_2\,m}}{\mathrm{r}/2} = 0$$

$$\frac{1}{2}\mathrm{mV}^2 = \frac{2\mathrm{Gm}}{\mathrm{r}}(\mathrm{M_1} + \mathrm{M_2})$$

$$V = \sqrt{\frac{4G(M_1 + M_2)}{r}}$$

10. (a)



$$\mathrm{B} = \left[\frac{\mu_0}{4\pi}\frac{\mathrm{I}}{\mathrm{b}/2} \times 2\sin 45\right] \times 4$$

$$\phi = 2\sqrt{2}\frac{\mu_0}{\pi}\frac{I}{b} \times a^2$$

$$\therefore \mathrm{M} = \frac{\phi}{\mathrm{I}} = \frac{2\sqrt{2}\mu_0 \mathrm{a}^2}{\pi \mathrm{b}} = \frac{\mu_0}{4\pi}8\sqrt{2}\frac{\mathrm{a}^2}{\mathrm{b}}$$

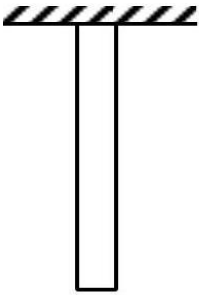
11. (c) When $\mathrm{V_i} > 3$ volt, $\mathrm{V_R} > 0$

Because diode will be in forward biased state

When $\mathrm{V_i} \le 3$ volt ; $\mathrm{V_R} = 0$

Because diode will be in reverse biased state.

12. (d)



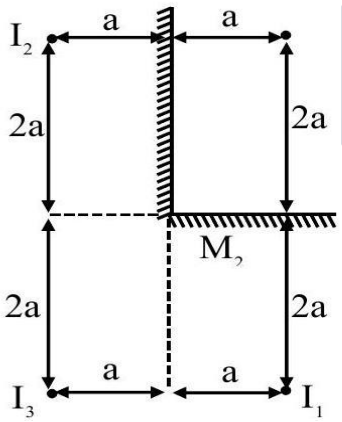
We know,

$\Delta \ell = \frac{WL}{2AY}$

$\Delta \ell = \frac{10 \times 1}{2 \times 5} \times 100 \times 10^{-4} \times 2 \times 10^{11}$

$\Delta \ell = \frac{1}{2} \times 10^{-9} = 5 \times 10^{-10}$ m

13. (b)



Shortest distance is 2a between I_1&I_3

But answer given is for I_1&I_2

$\sqrt{(4a)^2 + (2a)^2}$

$a\sqrt{20}$

4.47a

14. (a)

torque $\tau \rightarrow ML^2\ T^{-2}$ (III)

Impulse I $\Rightarrow MLT^{-1}$ (I)

Tension force $\Rightarrow MLT^{-2}$ (IV)

Surface tension $\Rightarrow MT^{-2}$ (II)

15. (a) $\int_{p_0}^{p} \frac{dp}{P} = -a\int_0^v dv$

$\ln\left(\frac{p}{p_0}\right) = -av$

$p = p_0 e^{-av}$

For temperature maximum $p - v$ product should be maximum

$T = \frac{pv}{nR} = \frac{p_0 v e^{-av}}{R}$

$\frac{dT}{dv} = 0 \Rightarrow \frac{p_0}{R}\{e^{-av} + v e^{-av}(-a)\}$

$\frac{p_0 e^{-av}}{R}\{1 - av\} = 0$

$v = \frac{1}{a}, \infty$

$T = \frac{p_0 1}{Rae} = \frac{p_0}{Rae}$

at $v = \infty$

$T = 0$

16. (a) (i) $\frac{\pi p a^4}{8\eta L} = \frac{dv}{dt}$ = Volumetric flow rate (poiseuille's law)

(ii) $h\rho g = \frac{2s}{r}\cos\theta$

(iii) RHS $\Rightarrow \varepsilon \times \frac{1}{4\pi\varepsilon_0}\frac{a}{r^2} \times \frac{1}{\varepsilon} = \frac{q}{t} \times \frac{1}{r^2}$

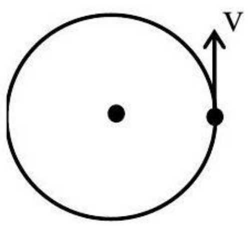
$= \frac{I}{L^2} = IL^{-2}$

LHS

$T = \frac{I}{A} = IL^{-2}$

(iv) $W = \tau\theta$

17. (b)



$|\vec{L}| = mvr$

And direction will be upward & remain constant

18. (c) $Z = \sqrt{(X_L - X_C)^2 + R^2} = R \because X_L = X_C$

19. (a) $\lambda_P = \frac{h}{P_P}\ \lambda_e = \frac{h}{P_e}$

$\because \lambda_P = \lambda_e$

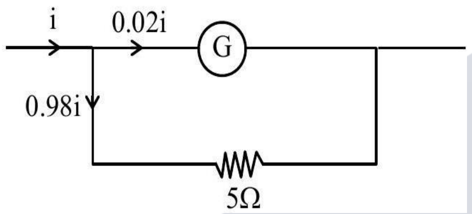
$\Rightarrow P_P = P_e$

$(K)_P = \frac{P_P^2}{2m_P}$

$(K)_e = \frac{P_e^2}{2\ m_e}$

$K_P < K_e$ as $m_P > m_e$

20. (c)



$0.02iRg = 0.98i \times 5$

$Rg = 245\Omega$

21. (500)

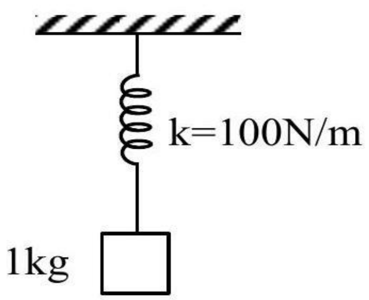
$B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = -\frac{\rho gh}{\left(\frac{\Delta V}{V}\right)}$

$-\frac{B\frac{\Delta V}{V}}{\rho g} = h$

$\frac{9.8 \times 10^8 \times 0.5}{100 \times 10^3 \times 9.8} = h$

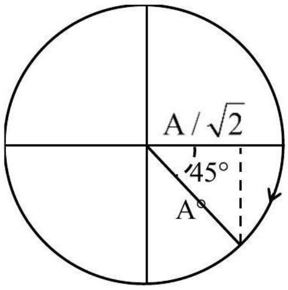
$h = 500$

22. (8)



$KE = PE$

$y=\frac{A}{\sqrt{2}}=A\sin\omega t$



$t=\frac{T}{8}=\frac{T}{x}$

$x=8$

23. (64) $h_T=h_R=160\ ...\ (i)$

$d=\sqrt{2Rh_T}+\sqrt{2Rh_R}$

$d=\sqrt{2R}[\sqrt{h_T}+\sqrt{h_R}]$

$d=\sqrt{2R}[\sqrt{x}+\sqrt{160-x}]$

$\frac{d(d)}{dx}=0$

$\frac{1}{2\sqrt{x}}+\frac{1(-1)}{2\sqrt{160-x}}=0$

$\frac{1}{\sqrt{x}}=\frac{1}{\sqrt{160-x}}$

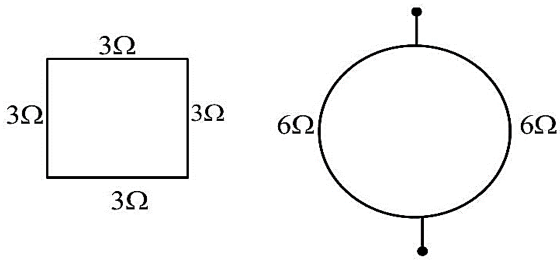
$x=80\text{ m}$

$d_{max}=\sqrt{2\times 6400}\left[\sqrt{\frac{80}{1000}}+\sqrt{\frac{20}{1000}}\right]$

$=\frac{80\sqrt{2}\times 2\sqrt{80}}{10\sqrt{10}}$

$=8\times 2\times\sqrt{2}\times 2\sqrt{2}=64\text{ km}$

24. (3)



$R_{eq} = 3\Omega$

25. (10) $\mu = 9.0 \times 10^{-4} \frac{kg}{m}$

$T = 900\ N$

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{900}{9 \times 10^{-4}}} = 1000\ m/s$$

$f_1 = 500\ Hz$

$f = 550$

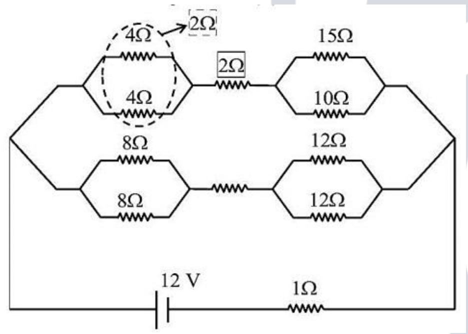
$$\frac{nV}{2\ell} = 500$$

$$\frac{(n+1)V}{2\ell} = 500$$

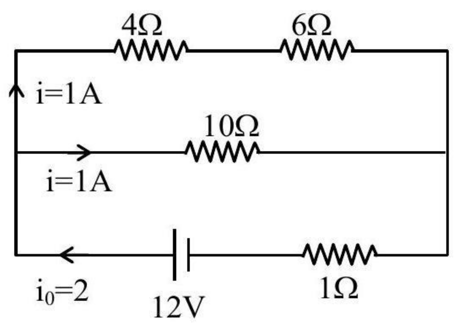
(ii) (i) $\frac{V}{2\ell} = 50$

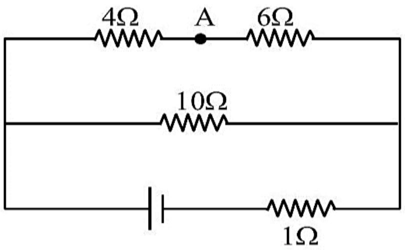
$$\ell = \frac{1000}{2 \times 50} = 10$$

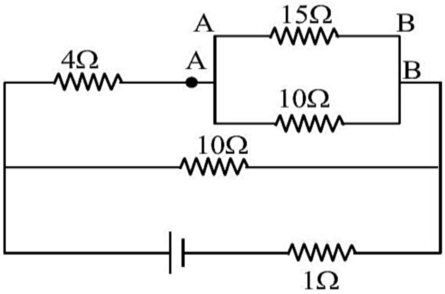
26. (6)



$\Rightarrow$ effective circuit diagram will be

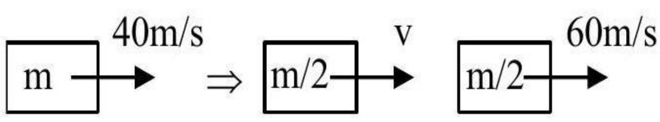




⇒ 

Point drop across $6\Omega = 1 \times 6 = 6 = V_{AB} \Rightarrow$ Hence point drop across $15\Omega = 6$ volt $= V_{AB}$

27. (1)



$P_i = P_f$

$$m \times 40 = \frac{m}{2} \times v + \frac{m}{2} \times 60$$

$$40 = \frac{v}{2} + 30$$

$$\Rightarrow v = 20$$

$$(\text{K.E.})_I = \frac{1}{2}\ m \times (40)^2 = 800\ m$$

$$(\text{K.E.})_f = \frac{1}{2}\frac{m}{2} \cdot (20)^2 + \frac{1}{2} \cdot \frac{m}{2}(60)^2 = 1000\ m$$

$$|\Delta \text{K.E.}| = |1000\ m - 800\ m| = 200\ m$$

$$\frac{\Delta \text{K.E}}{(\text{K.E.})_i} = \frac{200m}{800m} = \frac{1}{4} = \frac{x}{4}$$

$$x = 1$$

28. (500)

$$E = 50\sin\left(\omega t - \frac{\omega}{c} \cdot x\right)$$

Energy density $= \frac{1}{2}\epsilon_0 E_0^2$

Energy for volume $V = \frac{1}{2}\in_0 E_0^2 \cdot V = 5.5 \times 10^{-12}$

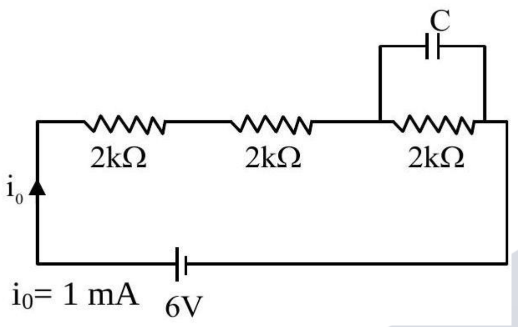
$$\frac{1}{2}8.8 \times 10^{-12} \times 2500\ \mathrm{V} = 5.5 \times 10^{-12}$$

$$\mathrm{V} = \frac{5.5 \times 2}{2500 \times 8.8} = .0005\ \mathrm{m}^3$$

$$= .0005 \times 10^6 (\mathrm{c.m})^3$$

$$= 500 (\mathrm{c.m})^3$$

29. (100)

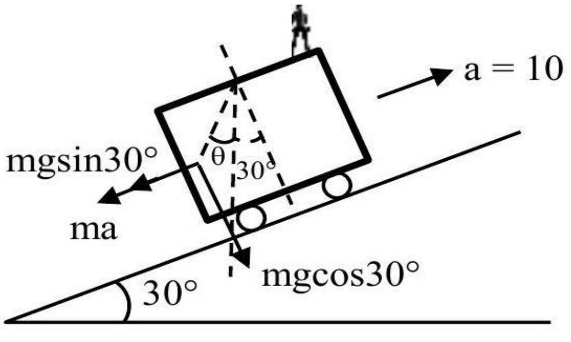


Pot. Diff. across each resistor = 2 V

$$q = CV$$

$$= 50 \times 10^{-6} \times 2 = 100 \times 10^{-6} = 100\mu C$$

30. (30)



$$\tan(30 + \theta) = \frac{\mathrm{mgsin}\,30^\circ + \mathrm{ma}}{\mathrm{mgcos}\,30^\circ}$$

$$\tan(30 + \theta) = \frac{5 + 10}{5\sqrt{3}} = \frac{1 + 2}{\sqrt{3}}$$

$$\frac{\tan\theta + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}\tan\theta} = \sqrt{3}$$

$$\sqrt{3}\tan\theta + 1 = 3 - \sqrt{3}\tan\theta$$

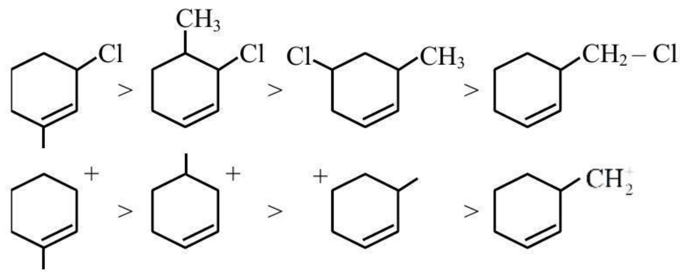
$$2\sqrt{3}\tan\theta = 2$$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30°$

31. **(a)** As it is example of SN^1.

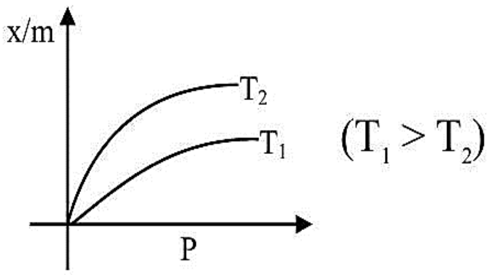
so carbocation stability ↑, reaction rate ↑



32. **(d)** $\frac{x}{m} \alpha P^{1/n} \left(0 < \frac{1}{n} < 1\right)$

On Increasing temperature $\frac{x}{m}$ decreases.

∵ adsorption is generally exothermic

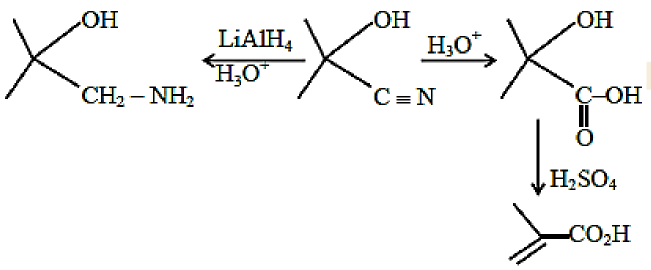


33. **(b)** Major component of portland cement is "Tricalcium silicate $(51\%, 3CaO^{-}SiO_2)$

34. **(b)** dichromate ion contain non-linear symmetrical $Cr - O - Cr$ Bond

35. **(a)** In Lactose it is $\beta C_1 - C_4$ glycosidic linkage. In Maltose, Amylose $\alpha C_1 - C_4$ glycosidic linkage is present

36. **(c)**



37. **(b)** Ytterbium shows +2 oxidation state with diamagnetic nature

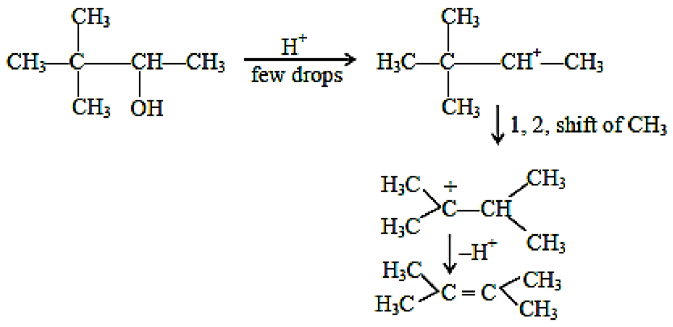
38. **(d)**

(A) Aluminium is reactive metal so Aluminium is extracted by electrolysis of Alumina with molten mixture of Cryolite

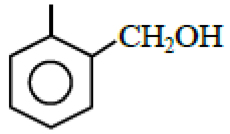
(B) Cryolite, Na_3AlF_6

Here Al is in +3 O.S.

39. (b)



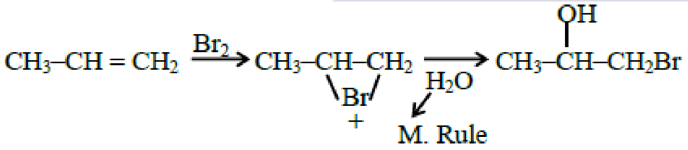
40. (c) Monomer of Novolac is



O-hydroxy methyl phenol

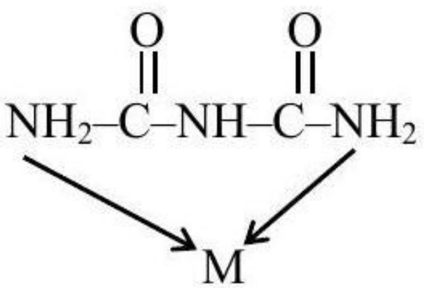
41. (b) The process of producing syn-gas from coal is called gasification of coal. Syn-gas having composition of $CO\&H_2$ in $1:1$

42. (c)



Its IUPAC name 1-bromopropan-2-ol *A* and *R* are true and (R) is the correct explanation of (A)

43. (a)



Biuret:- Bidentate ligand

The denticity of organic ligand is 2 .

44. (b)

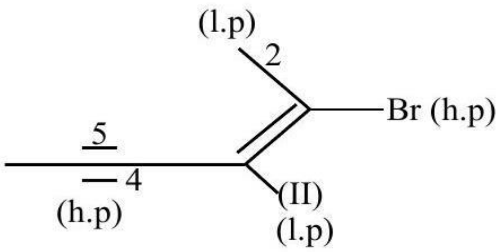
From left to right in periodic table: -

Metallic character decreases

Non-metallic character increases

⇒ It is due to increase in ionization enthalpy and increase in electron gain enthalpy.

45. (c)



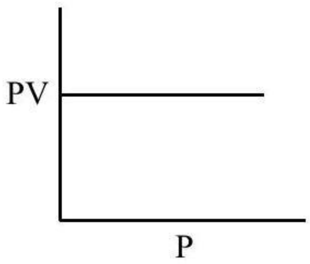
h.p. ⇒ higher priority

l.p. ⇒ lower priority

2E -2- bromo hex -2- en-4-yne

46. (a) $PV = nRT(n, T \text{ constant })$

$PV = \text{constant}$



47. (d) Both assertion & reason are correct & (R) is the correct explanation of (A)

48. (d) Van't Hoff factor is highest for $KHSO_4$

∴ colligative property (ΔT_f) will be highest for $KHSO_4$

49. (c)

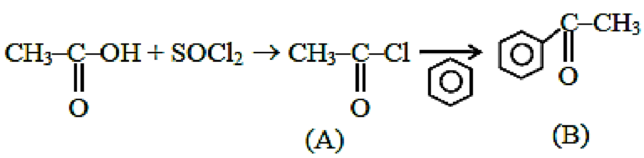
BOD values of clean water (A) is less than 5ppm

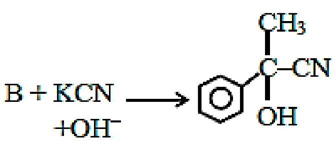
So $A < 5$

BOD values of polluted water (B is greater than 17 ppm

So $B > 17$

50. (a)





51. (25) $\Delta G^\circ = -nFE^\circ = \Delta H^\circ - T\Delta S^\circ$

$= \frac{\Delta H^o + nFE}{T}$

$= \frac{(-825.2 \times 10^3) + (2 \times 96487 \times 4.315)}{298}$

$= \frac{-825.2 \times 10^3 + 832.682 \times 10^3}{298}$

$= \frac{7.483 \times 10^3}{298} = 25.11 JK^{-1}\ mol^{-1}$

$\therefore$ Nearest integer answer is 25

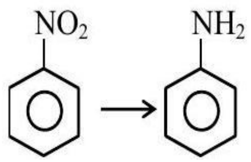
52. (20) $[H_2C_2O_4 \cdot 2H_2O] = \frac{\text{weight }/M_W}{V(L)}$

$\Rightarrow x \times 10^{-2} = \frac{6.3/126}{250/1000}$

$x = 20$

53. (4) Pbs, CuS, $As_2\ S_3$, CdS are soluble in 50%HNO_3 HgS, $Sb_2\ S_3$ are insoluble in 50%HNO_3

54. (5)



Reagents used can be

(i) Sn + HCl

(ii) Fe + HCl

(iii) Zn + HCl

(iv) $H_2 - Pd$

(v) H_2 (Raney Ni)

55. (3) The number of halogen forming halic (V) acid

$HClO_3$

$HBrO_3$

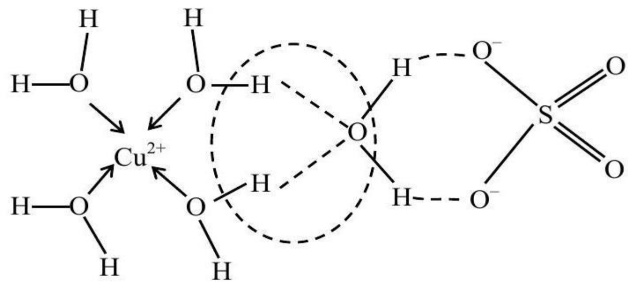
HIO_3

56. (2) $k = \frac{2.303}{t} \log \frac{a}{a-x}$

$\frac{2.303}{t_{50\%}} \log \frac{100}{100-50} = \frac{2.303}{t_{75\%}} \log \frac{100}{100-75}$

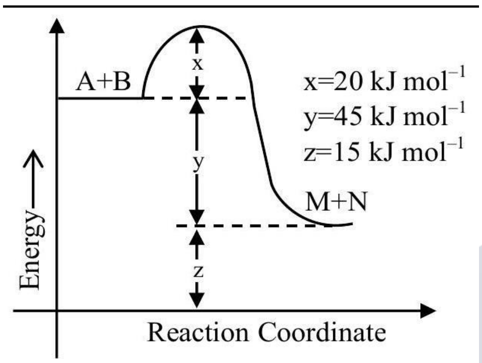
$t_{75\%} = 2t_{50\%}$

57. (1)



One hydrogen bonded H_2O molecule

58. (45)



$\Delta H = E_{a_f} - E_{a_b}$

$= 20 - 65$

$= -45 KJ/mol$

$|\Delta H| = 45 KJ/mol$

59. (7)
60. (108)

$A_3\ B_2(\ s) \rightleftharpoons 3\ A^{+2}_{(aq)} + 2\ B^{-3}_{(aq)}$

3 s 2 s

$K_{SP} = (3\ s)^3(2\ s)^2$

$K_{SP} = 108\ S^5 \&\ s = (X/M)$

$K_{SP} = 108\left(\frac{x}{m}\right)^5$

given $K_{SP} = a\left(\frac{x}{m}\right)^5$

comparing $a = 108$

61. (c) $(p \wedge \sim q) \rightarrow (p \vee q)$ is tautology

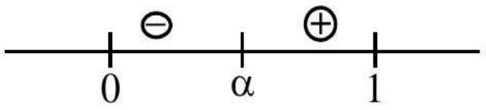
p	q	~ q	p ∧~ q	p ∨ q	(p ∧~ q) → (p ∨ q)

T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T

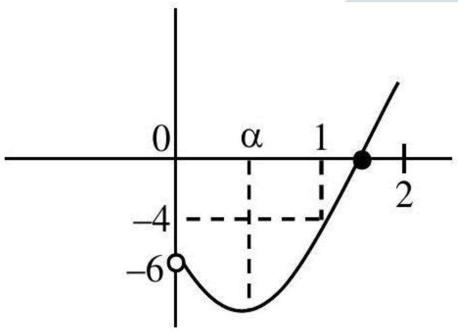
62. (c) Let $e^x = t > 0$

$f(t) = t^4 + 2t^3 - t - 6 = 0$

$f'(t) = 4t^3 + 6t^2 - 1$



$f''(t) = 12t^2 + 12t > 0$



$f(0) = -6, f(a) = -4, f(b) = 24$

$\Rightarrow$ Number of real roots $= 1$

63. (b) $S = \frac{2^2-1^2}{1^2\times 2^2} + \frac{3^2-2^2}{2^2\times 3^2} + \frac{4^2-3^2}{3^2\times 4^2} + \cdots$

$= \left[\frac{1}{1^2} - \frac{1}{2^2}\right] + \left[\frac{1}{2^2} - \frac{1}{3^2}\right] + \left[\frac{1}{3^2} - \frac{1}{4^2}\right] + \cdots + \left[\frac{1}{10^2} - \frac{1}{11^2}\right]$

$= 1 - \frac{1}{121}$

$= \frac{120}{121}$

64. (a) Equation of plane is

$3x - 2y + 4z - 7 + \lambda(x + 5y - 2z + 9) = 0$
$(3 + \lambda)x + (5\lambda - 2)y + (4 - 2\lambda)z + 9\lambda - 7 = 0$

passing through $(1,4,-3)$

$\Rightarrow 3 + \lambda + 20\lambda - 8 - 12 + 6\lambda + 9\lambda - 7 = 0$

$\Rightarrow \lambda = \frac{2}{3}$

$\Rightarrow$ equation of plane is

$-11x - 4y - 8z + 3 = 0$

$\Rightarrow \alpha + \beta + \gamma = -23$

65. (d) $\int_0^x \sqrt{1-(f'(t))^2}dt = \int_0^x f(t)dt \; 0 \le x \le 1$

differentiating both the sides

$\sqrt{1-(f'(x))^2} = f(x)$

$\Rightarrow 1-(f'(x))^2 = f^2(x)$

$\frac{f'(x)}{\sqrt{1-f^2(x)}} = 1$

$\sin^{-1} f(x) = x + C$

$\because f(0) = 0 \Rightarrow C = 0 \Rightarrow f(x) = \sin x$

Now $\lim_{x\to 0} \frac{\int_0^x \sin t dt}{x^2}\left(\frac{0}{0}\right) = \frac{1}{2}$

66. (c) $|3\vec{a}+\vec{b}|^2 = |2\vec{a}+3\vec{b}|^2$

$(3\vec{a}+\vec{b})\cdot(3\vec{a}+\vec{b}) = (2\vec{a}+3\vec{b})\cdot(2\vec{a}+3\vec{b})$

$9\vec{a}\cdot\vec{a} + 6\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b} = 4\vec{a}\cdot\vec{a} + 12\vec{a}\cdot\vec{b} + 9\cdot\vec{b}\cdot\vec{b}$

$5|\vec{a}|^2 - 6\vec{a}\cdot\vec{b} = 8|\vec{b}|^2$

$5(8)^2 - 6.8.|\vec{b}|\cos 60^\circ = 8|\vec{b}|^2 \begin{pmatrix} \because \frac{1}{8}|\vec{a}| = 1 \\ \Rightarrow |\vec{a}| = 8 \end{pmatrix} 40 - 3|\vec{b}| = |\vec{b}|^2$

$\Rightarrow |\vec{b}|^2 + 3|\vec{b}| - 40 = 0$

$|\vec{b}| = -8, \; |\vec{b}| = 5$

67. (c) $f(x) = |(x-3)(x+1)|\cdot e^{(3x-2)^2}$

$$f(x) = \begin{cases} (x-3)(x+1)\cdot e^{(3x-2)^2} & ; x \in (3,\infty) \\ -(x-3)(x+1)\cdot e^{(3x-2)^2} & ; x \in [-1,3] \\ (x-3)\cdot(x+1)\cdot e^{(3x-2)^2} & ; x \in (-\infty,-1) \end{cases}$$

Clearly, non-differentiable at $x = -1 \,\&\, x = 3$.

68. (b) Let numbers be $\frac{a}{r}, a, ar \to$ G.P

$\frac{a}{r}, 2a, ar \rightarrow A \cdot P \Rightarrow 4a = \frac{a}{r} + ar \Rightarrow r + \frac{1}{r} = 4$

$r = 2 \pm \sqrt{3}$

4^{th} form of G.P $= 3r^2 \Rightarrow a^2 = 3r^2 \Rightarrow a = 3$

$r = 2 + \sqrt{3}, a = 3, d = 2a - \frac{a}{r} = 3\sqrt{3}$

$r^2 - d = (2 + \sqrt{3})^2 - 3\sqrt{3}$

$= 7 + 4\sqrt{3} - 3\sqrt{3}$

$= 7 + \sqrt{3}$

69. (b) Note that (1,2) and (2,3) satisfy $0 < |x - y| \leq 1$

but (1,3) does not satisfy it so

$0 \leq |x - y| \leq 1$ is symmetric but not transitive

70. (c)

$$\int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}}$$
$$= \int \frac{dx}{\left(\frac{x+2}{x-1}\right)^{5/4} \cdot (x-1)^2}$$
put $\frac{x+2}{x-1} = t$
$$= -\frac{1}{3}\int \frac{dt}{t^{5/4}}$$
$$= \frac{4}{3} \cdot \frac{1}{t^{1/4}} + C$$
$$= \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$$

71. (a)

First line is $\frac{x}{\sin\alpha} - \frac{y}{\cos\alpha} = \frac{k\cos 2\alpha}{\sin 2\alpha}$

$\Rightarrow x\cos\alpha - y\sin\alpha = \frac{k}{2}\cos 2\alpha$

$\Rightarrow p = \left|\frac{k}{2}\cos\alpha\right| \Rightarrow 2p = |k\cos 2\alpha|$

second line is $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$

$\Rightarrow q = |k\sin 2\alpha|$

Hence $4p^2 + q^2 = k^2$ (From (i) & (ii))

72. (d) $\operatorname{cosec} 18^\circ = \frac{1}{\sin 18^\circ} = \frac{4}{\sqrt{5}-1} = \sqrt{5} + 1$

Let $\operatorname{cosec} 18^\circ = x = \sqrt{5}+1$

$\Rightarrow x-1=\sqrt{5}$

Squaring both sides, we get

$x^2-2x+1=5$

$\Rightarrow x^2-2x-4=0$

73. (d) Here $D=\begin{vmatrix}2 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & a\end{vmatrix}=2(-a-1)-1(a-1)+1+1$ $D_3=\begin{vmatrix}2 & 1 & 5\\ 1 & -1 & 3\\ 1 & 1 & b\end{vmatrix}=2(-b-3)-1(b-3)+5(1+1)$ for $a=\frac{1}{3}, b\neq\frac{7}{3}$, system has no solutions

74. (c)

75. (a) If $f(x)$ is continuous at $x=0$, RHL = LHL = $f(0)$

$\lim_{x\to0^+} f(x)=\lim_{x\to0^+}\frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1}\cdot\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1}$ (Rationalisation)

$$\lim_{x\to0^+}\frac{2\sin^2 x}{x^2}\cdot\left(\sqrt{x^2+1}+1\right)=-4$$

$$\lim_{x\to0^-} f(x)=\lim_{x\to0^-}\frac{1}{x}\ell n\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right)$$

$$\lim_{x\to0^-}\frac{\ln\left(1+\frac{x}{a}\right)}{\left(\frac{x}{a}\right)\cdot a}+\frac{\ln\left(1-\frac{x}{b}\right)}{\left(-\frac{x}{b}\right)\cdot b}$$

$$=\frac{1}{a}+\frac{1}{b}$$

So $\frac{1}{a}+\frac{1}{b}=-4=k$

$$\Rightarrow\frac{1}{a}+\frac{1}{b}+\frac{4}{k}=-4-1=-5$$

76. (b) $\frac{dy}{dx}=\frac{2^x 2^y-2^x}{2^y}$

$$2^y\frac{dy}{dx}=2^x(2^y-1)$$

$$\int\frac{2^y}{2^y-1}dy=\int 2^x dx$$

$$\frac{\ln(2^y-1)}{\ln 2}=\frac{2^x}{\ln 2}+C$$

$\Rightarrow \log_2(2^y-1)=2^x\log_2 e+C$

$\because y(0)=1\Rightarrow 0=\log_2 e+C$

$C=-\log_2 e$

$\Rightarrow \log_2(2^y - 1) = (2^x - 1)\log_2 e$

put $x = 1, \log_2(2^y - 1) = \log_2 e$

$2^y = e + 1$

$y = \log_2(e + 1)$

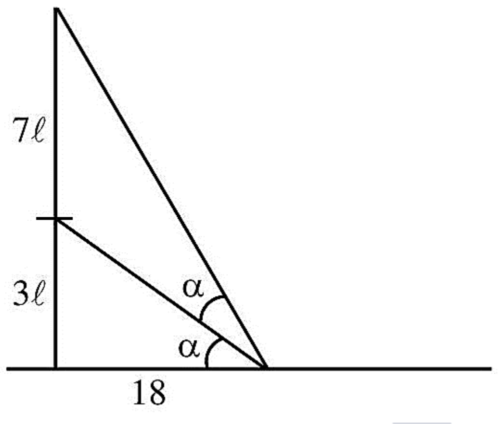
77. (c) $\lim_{x\to 0} \frac{\sin^2(\pi\cos^4 x)}{x^4}$

$$\lim_{x\to 0} \frac{1 - \cos(2\pi\cos^4 x)}{2x^4}$$

$$\lim_{x\to 0} \frac{1 - \cos(2\pi - 2\pi\cos^4 x)}{[2\pi(1 - \cos^4 x)]^2} 4\pi^2 \cdot \frac{\sin^4 x}{2x^4}(1 + \cos^2 x)^2$$

$$= \frac{1}{2} \cdot 4\pi^2 \cdot \frac{1}{2}(2)^2 = 4\pi^2$$

78. (b)



Let height of pole $= 10\ell$

$$\tan\alpha = \frac{3\ell}{18} = \frac{\ell}{6}$$

$$\tan 2\alpha = \frac{10\ell}{18}$$

$$\frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{10\ell}{18}$$

use $\tan\alpha = \frac{\ell}{6} \Rightarrow \ell = \sqrt{\frac{72}{5}}$

height of pole $= 10\ell = 12\sqrt{10}$

79. (c) $a_r = e^{\frac{i2\pi r}{9}}, r = 1,2,3, \ldots a_1, a_2, a_3, \ldots$ are in G.P.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_n & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_2^2 & a_1^3 \\ a_1^4 & a_1^5 & a_1^6 \\ a_1^7 & a_1^8 & a_1^9 \end{vmatrix} = a_1 \cdot a_1^4 \cdot a_1^7 \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \end{vmatrix} = 0$$

Now $a_1a_9 - a_3a_7 = a_1{}^{10} - a_1{}^{10} = 0$

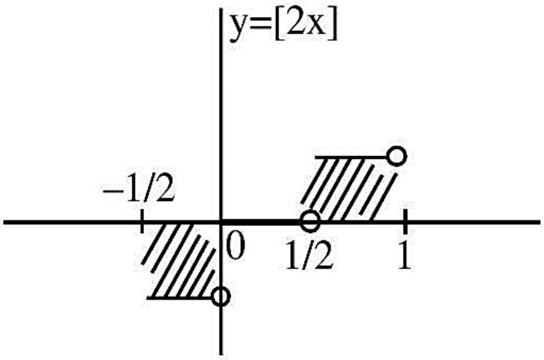
80. (b) $12x\cos\theta + 5y\sin\theta = 60$

$$\frac{\mathrm{x}\cos\theta}{5} + \frac{\mathrm{y}\sin\theta}{12} = 1$$

is tangent to $\frac{x^2}{25} + \frac{y^2}{144} = 1$

$144x^2 + 25y^2 = 3600$

81. (5) $I = \int_{-1/2}^{1}([2x] + |x|)dx$



$$= \int_{-1/2}^{1}[2\mathrm{x}]\mathrm{dx} + \int_{-1/2}^{1}|\mathrm{x}|\mathrm{dx}$$

$$= 0 + \int_{-1/2}^{0}(-x)dx + \int_{0}^{1}xdx$$

$$= \left(-\frac{\mathrm{x}^2}{2}\right)_{-1/2}^{0} + \left(\frac{\mathrm{x}^2}{2}\right)_{0}^{1}$$

$$= \left(0 + \frac{1}{8}\right) + \frac{1}{2}$$

$$= \frac{5}{8}$$

$8\mathrm{I} = 5$

82. (98) Let $\mathrm{z} = \mathrm{x} + \mathrm{iy}$

$$\arg\left(\frac{x-2+iy}{x+2+iy}\right) = \frac{\pi}{4}$$

$$\arg(\mathrm{x}-2+\mathrm{iy}) - \arg(\mathrm{x}+2+\mathrm{iy}) = \frac{\pi}{4}$$

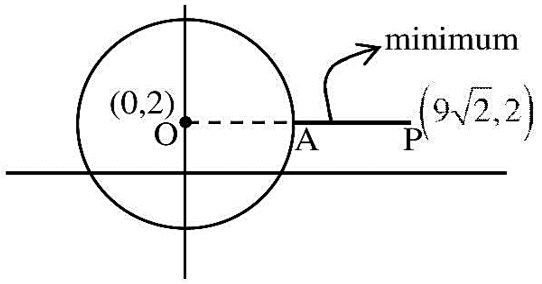
$$\tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$$

$$\frac{\frac{y}{x-2} - \frac{y}{x+2}}{1+\left(\frac{y}{x-2}\right)\cdot\left(\frac{y}{x+2}\right)} = \tan\frac{\pi}{4} = 1 \quad \frac{xy+2y-xy+2y}{x^2-4+y^2} = 1$$

$4y = x^2 - 4 + y^2$

$x^2 + y^2 - 4y - 4 = 0$

locus is a circle with center $(0,2)$& radius $= 2\sqrt{2}$



min. value $= (AP)^2 = (OP - OA)^2$

$= (9\sqrt{2} - 2\sqrt{2})^2$

$= (7\sqrt{2})^2 = 98$

83. (61) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6} = \lambda$

$\mathrm{x} = 2\lambda + 1, \mathrm{y} = 3\lambda + 2, \mathrm{z} = 6\lambda - 1$

for point of intersection of line & plane

$2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$

$7\lambda = 7 \Rightarrow \lambda = 1$

point: $(3,5,5)$

$(\text{ distance })^2 = (3+1)^2 + (5+1)^2 + (5-2)^2$

$= 16 + 36 + 9 = 61$

84. (2) $f(\mathrm{x}) = \mathrm{x}^2 + \mathrm{ax} + 1$

$f'(\mathrm{x}) = 2\mathrm{x} + \mathrm{a}$

when $f(\mathrm{x})$ is increasing on $[1,2]$

$2\mathrm{x} + \mathrm{a} \geq 0\ \forall \mathrm{x} \in [1,2]$

$a \geq -2x \forall x \in [1,2]$

$\mathrm{R} = -4$

when $f(\mathrm{x})$ is decreasing on $[1,2]$

$2\mathrm{x} + \mathrm{a} \leq 0\ \forall \mathrm{x} \in [1,2]$

$\mathrm{a} \leq -2\ \forall \mathrm{x} \in [1,2]$

$S = -2$

$|R - S| = |-4 + 2| = 2$

85. (398)

$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14$

$T_n = (3n + 4)(2n + 6) = 2(3n + 4)(n + 3)$

$= 2(3n^2 + 13n + 12) = 6n^2 + 26n + 24$

$S_{10} = \sum_{n=1}^{10} T_n = 6\sum_{n=1}^{10} n^2 + 26\sum_{n=1}^{10} n + 24\sum_{n=1}^{10} 1$

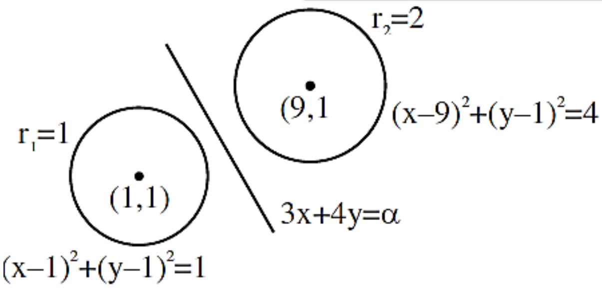
$= \dfrac{6(10 \times 11 \times 21)}{6} + 26 \times \dfrac{10 \times 11}{2} + 24 \times 10$

$= 10 \times 11(21 + 13) + 240$

$= 3980$

Mean $= \frac{S_{10}}{10} = \frac{3980}{10} = 398$

86. (165)



Both centres should lie on either side of the line as well as line can be tangent to circle.

$(3 + 4 - \alpha) \cdot (27 + 4 - \alpha) < 0$

$(7 - \alpha).(31 - \alpha) < 0 \Rightarrow \alpha \in (7,31)$

d_1 = distance of (1,1) from line

d_2 = distance of (9,1) from line

$d_1 \geq r_1 \Rightarrow \dfrac{|7 - \alpha|}{5} 1 \Rightarrow \alpha \in (-\infty, 2] \cup [12, \infty)$

$d_2 \geq r_2 \Rightarrow \dfrac{|31 - \alpha|}{5} \geq 2 \Rightarrow \alpha \in (-\infty, 21] \cup [41, \infty)$

$(1) \cap (2) \cap (3) \Rightarrow \alpha \in [12,21]$

Sum of integers $= 165$

87. (576)

VOWELS → 2 Vowels

VOWELS → 4 Consonants

All Consonants should not be together

= Total - All consonants together,

$= 6! - 3!\,4! = 576$

88. (4)

$x\phi(x) = \int_5^x 3t^2 - 2\phi'(t)dt$

$x\phi(x) = x^3 - 125 - 2[\phi(x) - \phi(5)]$

$x\phi(x) = x^3 - 125 - 2\phi(x) - 2\phi(5)$

$\phi(0) = 4 \Rightarrow \phi(5) = -\frac{133}{2}$

$\phi(x) = \frac{x^3+8}{x+2}$

$\phi(2) = 4$

89. (55) $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$

$T_{r+1} = (-1)^r \cdot {}^{12}C_r \left(\frac{x}{4}\right)^{12-r} \left(\frac{12}{x^2}\right)^r$

$T_{r+1} = (-1)^r \cdot {}^{12}C_r \left(\frac{1}{4}\right)^{12-r} (12)^r \cdot (x)^{12-3r}$

Term independent of $x \Rightarrow 12 - 3r = 0 \Rightarrow r = 4$

$T_5 = (-1)^4 \cdot {}^{12}C_4 \left(\frac{1}{4}\right)^8 (12)^4 = \frac{3^6}{4^4} \cdot k$

$\Rightarrow k = 55$

90. (28) I_1 = first unit is functioning

I_2 = second unit is functioning

$P(I_1) = 0.9, P(I_2) = 0.8$

$P(\bar{I}_1) = 0.1, P(\bar{I}_2) = 0.2$

$P = \frac{0.8 \times 0.1}{0.1 \times 0.2 + 0.9 \times 0.2 + 0.1 \times 0.8} = \frac{8}{28}$

$98P = \frac{8}{28} \times 98 = 28$