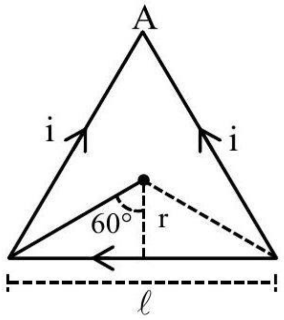
1. **(b)** Force on each column $= \frac{mg}{4}$

$$\text{Strain } = \frac{mg}{4AY}$$
$$= \frac{50 \times 10^3 \times 9.8}{4 \times \pi(1-0.25) \times 2 \times 10^{11}}$$
$$= 2.6 \times 10^{-7}$$

2. **(d)**

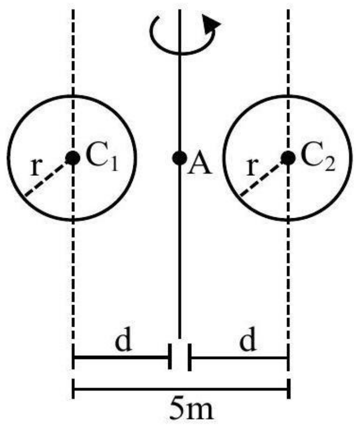


TEST PAPER WITH SOLUTION

$$B = 3\left[\frac{\mu_0 i}{4\pi r}(\sin 60^\circ + \sin 60^\circ)\right]$$
$$\tan 60^\circ = \frac{\ell/2}{r}$$
$$\text{Where } r = \frac{9 \times 10^{-2}}{2\sqrt{3}} \text{M}$$
$$\therefore B = 3 \times 10^{-5} \text{ T}$$

Current is flowing in clockwise direction so, $\vec{B}$ is inside plane of triangle by right hand rule.

3. **(c)**



$$M = 1.5 \text{ kg}, r = 0.5 \text{ m}, d = \frac{5}{2} \text{ m}$$

$$I = 2\left(\frac{2}{5}Mr^2 + Md^2\right)$$

$$= 19.05 \text{kgm}^2$$

4. **(b)** $\vec{A} = \vec{P} + \vec{Q}$

$\vec{B} = \vec{P} - \vec{Q}$

$|\vec{A}| = |\vec{B}| = \sqrt{P^2 + Q^2}$

$|\vec{A} + \vec{B}| = \sqrt{2(P^2 + Q^2)(1 + \cos\theta)}$

For $|\vec{A} + \vec{B}| = \sqrt{3(P^2 + Q^2)}$

$\theta_1 = 60^\circ$

For $|\vec{A} + \vec{B}| = \sqrt{2(P^2 + Q^2)}$

$\theta_2 = 90^\circ$

5. **(c)** For every large distance P.E. $= 0$

& total energy $= 2.6 + 0 = 2.6\text{eV}$

Finally in first excited state of H atom total energy $= -3.4\text{eV}$

Loss in total energy $= 2.6 - (-3.4)$

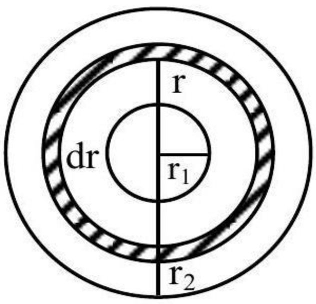
$= 6\text{eV}$

It is emitted as photon

$\lambda = \frac{1240}{6} = 206 \text{ nm}$

$$f = \frac{3 \times 10^8}{206 \times 10^{-9}} = 1.45 \times 10^{15} \text{ Hz}$$
$$= 1.45 \times 10^9 \text{ Hz}$$

6. **(a)**



Thermal resistance of spherical sheet of thickness dr and radius r is

$$dR = \frac{dr}{K(4\pi r^2)}$$

$$R = \int_{r_1}^{r_2} \frac{dr}{K(4\pi r^2)}$$

$$R = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{4\pi K}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

Thermal current $(i) = \frac{\theta_2 - \theta_1}{R}$

$$i = \frac{4\pi K r_1 r_2}{r_2 - r_1}(\theta_2 - \theta_1)$$

7. (b)

$V_A = 5\text{ V} \Rightarrow A = 1$

$V_A = 0\text{ V} \Rightarrow A = 0$

$V_B = 5\text{ V} \Rightarrow B = 1$

$V_B = 0\text{ V} \Rightarrow B = 0$

If $A = B = 0$, there is no potential anywhere here $V_0 = 0$

If $A = 1, B = 0$, Diode D_1 is forward biased, here $V_0 = 5\text{ V}$

If $A = 0, B = 1$, Diode D_2 is forward biased hence $V_0 = 5\text{ V}$

If $A = 1, B = 1$, Both diodes are forward biased hence $V_0 = 5\text{ V}$

Truth table for I st

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1

For II nd circuit

$V_B = 5\text{ V}, \ A = 1$

$V_B = 0\text{ V}, \ A = 0$

When $A = 0$, $E - B$ junction is unbiased there is no current through it

$\therefore V_0 = 1$

When $A = 1$, $E - B$ junction is forward biased $V_0 = 0$

8. (c) $\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$

$$\Delta x = \frac{h}{4\pi m \Delta v} \quad v = \sqrt{\frac{3KT}{m}}$$

$$\frac{\Delta x_e}{\Delta x_p}=\sqrt{\frac{m_p}{m_e}}$$

9. (d) $T=2\pi\sqrt{\ell/g}$

When bob is immersed in liquid $mg_{eff}=mg-$ Buoyant force $mg_{eff}=mg-v\sigma g$ ($\sigma=$ density of liquid) $=mg-v\frac{\rho}{4}g=mg-\frac{mg}{4}=\frac{3mg}{4}$

$\therefore g_{eff}=\frac{3g}{4}$

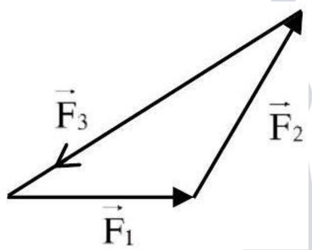
$T_1=2\pi\sqrt{\frac{\ell_1}{g_{eff}}}\ \ell_1=\ell+\frac{\ell}{3}=\frac{4\ell}{3},\ \ell_{eff}=\frac{3g}{4}$

By solving

$T_1=\frac{4}{3}2\pi\sqrt{\ell/g}$

$T_1=\frac{4T}{3}$

10. (d)



Here $\vec{F}_1+\vec{F}_2+\vec{F}_3=0$

$\vec{F}_1+\vec{F}_2=-\vec{F}_3$

Since $\vec{F}_{net}=0$ (equilibrium)

Both statements correct

11. (b) $[M]=K[F]^a[T]^b[V]^c$

$[M^1]=[M^1L^1T^{-2}]^a[T^1]^b[L^1T^{-1}]^c$

$a=1, b=1, c=-1$

$\therefore [M]=[FTV^{-1}]$

12. (c) $\vec{F}=q(\vec{V}\times\vec{B})$

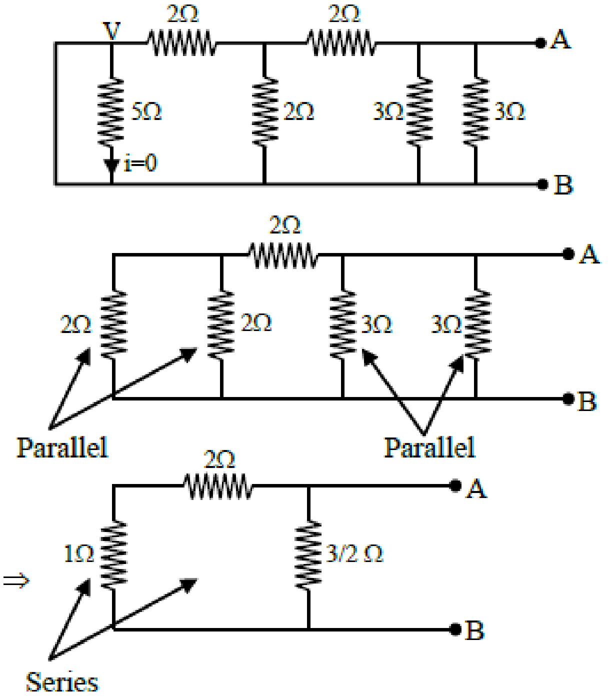
$$\vec{F}_1=4\pi\left[0.5c\hat{i}\times B_0\left(\frac{\hat{i}+\hat{j}}{2}\right)\cos\left(K\cdot\frac{\pi}{K}-0\right)\right]$$

$$\vec{F}_2 = 2\pi\left[0.5c\hat{i} \times B_0\left(\frac{\hat{i}+\hat{j}}{2}\right)\cos\left(K \cdot \frac{3\pi}{K} - 0\right)\right]$$

$\cos\pi = -1,\ \cos 3\pi = -1$

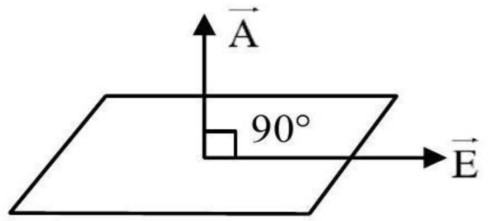
$\therefore \frac{F_1}{F_2} = 2$

13. (d)



$$R_{eq} = \frac{3 \times 3/2}{3 + 3/2} = \frac{9/2}{9/2} = 1\Omega$$

14. (c) Since $\phi = \vec{E} \cdot \vec{A} = EA\cos\theta$



$\theta = 90°$

$\therefore \phi = 0$

15. (c) PV = nRT

$$400 \times 10^3 \times 500 \times 10^{-6} = n\left(\frac{25}{3}\right)(300)$$

$$n = \frac{2}{25}$$

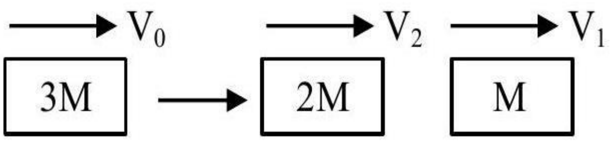
$n = n_1 + n_2$

$$\frac{2}{25}=\frac{M_1}{2}+\frac{M_2}{32}$$

Also $M_1+M_2=0.76\text{gm}$

$$\frac{M_2}{M_1}=\frac{16}{3}$$

16. (c)



$3MV_0=2MV_2+MV_1$

$3\ V_0=2\ V_2+V_1$

$120=2\ V_2+60\Rightarrow V_2=30\ \text{m/s}$

$$\frac{\Delta\ \text{K.E.}}{\text{K.E.}}=\frac{\frac{1}{2}MV_1^2+\frac{1}{2}2MV_2^2-\frac{1}{2}3MV_0^2}{\frac{1}{2}3MV_0^2}$$

$$=\frac{V_1^2+2\ V_2^2-3\ V_0^2}{3\ V_0^2}$$

$$=\frac{3600+1800-4800}{4800}$$

$$=\frac{1}{8}$$

17. (a) $\vec{B}$ must not be parallel to the plane of coil for non-zero flux and according to lenz law if B is outward it should be decreasing for anticlockwise induced current.

18. (a) In S.H.M. total mechanical energy remains constant and also $<\text{K.E.}>=<\text{P.E}>=\frac{1}{4}KA^2$

(for 1 time period)

19. (d) To convert pulsating dc into steady dc both of mentioned method are correct

20. (d) $g_{up}=\frac{g}{\left(1+\frac{r}{R}\right)^2}$

$$g_{down}=g\left(1-\frac{r}{R}\right)$$

$$\frac{g_{down}}{g_{up}}=\left(1-\frac{r}{R}\right)\left(1+\frac{r}{R}\right)^2$$

$$=\left(1-\frac{r}{R}\right)\left(1+\frac{2r}{R}+\frac{r^2}{R^2}\right)$$

$$=1+\frac{r}{R}-\frac{r^2}{R^2}-\frac{r^3}{R^3}$$

21. (500) Signal bandwidth $= 2fm$

$= 12kHz$

$$\therefore N = \frac{6MHZ}{12kHZ} = \frac{6 \times 10^6}{12 \times 10^3} = 500$$

22. (4) $\Delta U = \frac{1}{2}(\Delta C)V^2$

$$\Delta U = \frac{1}{2}(KC - C)V^2$$

$$\Delta U = \frac{1}{2}(2 - 1)CV^2$$

$$\Delta U = \frac{1}{2} \times 200 \times 10^{-6} \times 200 \times 200$$

$$\Delta U = 4\ J$$

23. (250) $\frac{\Delta M}{M} = \frac{\Delta \mu}{\mu} = \frac{250}{500} = \frac{1}{2}$

$$\frac{1}{2} = \frac{x}{499} \Rightarrow x \simeq 250$$

24. (1) $y = mx + C$

$$v^2 = \frac{20}{10}x + 20$$

$$v^2 = 2x + 20$$

$$2v\frac{dv}{dx} = 2$$

$$\therefore a = v\frac{dv}{dx} = 1$$

25. (5) $8\beta = 2.4$ cm

$$\frac{8\lambda\Delta}{d} = 2.4\ cm$$

$$\frac{8 \times 1.5 \times c}{0.3 \times 10^{-3} \times f} = 2.4 \times 10^{-2}$$

$$f = 5 \times 10^{14}\ Hz$$

26. (52) $9MSD = 10VSD$

9×1 mm $= 10VSD$

$\therefore 1VSD = 0.9$ mm

$LC = 1MSD - 1VSD = 0.1$ mm

Reading $= MSR + VSR \times LC$

$10 + 8 \times 0.1 = 10.8$ mm

Actual reading $= 10.8 - 0.4 = 10.4$ mm

radius $= \frac{d}{2} = \frac{10.4}{2} = 5.2$ mm

$= 52 \times 10^{-2}$ cm

27. (480) $v = 1.5$

$$p_1 v_1^v = p_2 v_2^v$$
$$(200)(1200)^{1.5} = P^2(300)^{1.5}$$
$$P_2 = 200[4]^{3/2} = 1600\text{kPa}$$
$$|\text{ W.D. }| = \frac{p_2 v_2 - p_1 v_1}{v - 1} = \left(\frac{480 - 240}{0.5}\right) = 480 \text{ J}$$

28. (2) $X_L = 2\pi fL$

f is very large

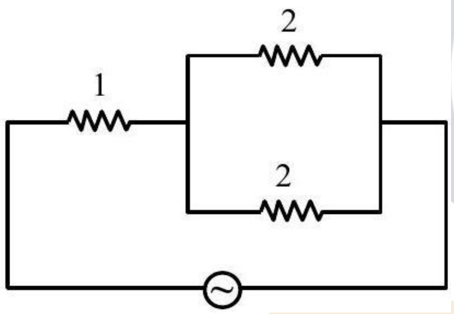
$\therefore X_L$ is very large hence open circuit.

$X_C = \frac{1}{2\pi fC}$

f is very large.

$\therefore X_C$ is very small, hence short circuit.

Final circuit



$Z_{eq} = 1 + \frac{2 \times 2}{2 + 2} = 2$

29. (5) $i = A = 60^\circ$

$\delta_{min} = 2i - A$

$= 2 \times 60^\circ - 60^\circ = 60^\circ$

$$\mu = \frac{\sin^{-1}\left(\frac{\delta_{min} + A}{2}\right)}{\sin^{-1}\left(\frac{A}{2}\right)}$$

$= \sqrt{3}$

$V_{prism} = \dfrac{3 \times 10^8}{\sqrt{3}}$

$AP = 10 \times 10^{-2} \times \dfrac{\sqrt{3}}{2}$

$\text{time} = \frac{5\times10^{-2}}{3\times10^8} \times \sqrt{3} \times \sqrt{3}$

$= 5 \times 10^{-10}\text{sec}$

30. (3840) $E = i^2Rt$

$192 = 16(R)(a)$

$R = 12\Omega$

$E^1 = (8)^2(12)(5)$

$= 3840$ J

31. (d) More stable less potential energy.

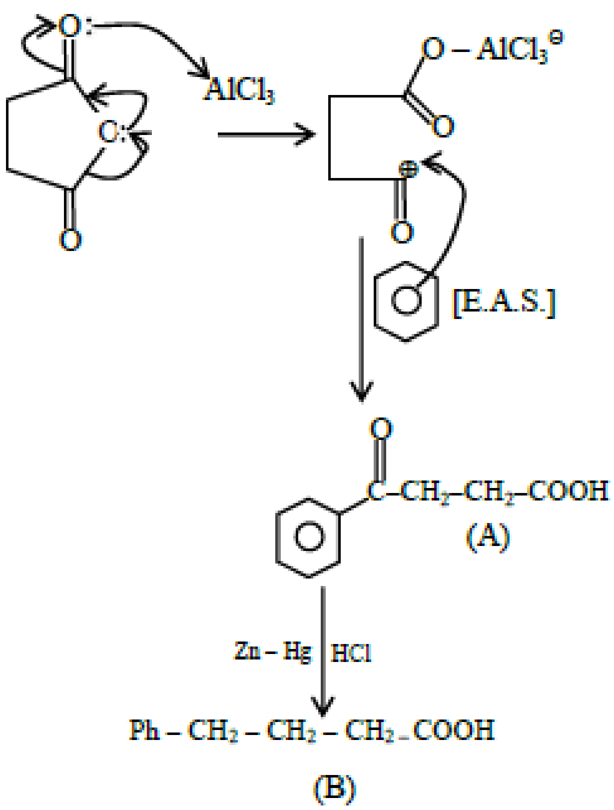
Stability order: I > III > IV > II

So Potential energy: II > IV > III > I

32. (c) Eu → $[Xe]4f^7 6s^2$

$Eu^{2+} \rightarrow [Xe]4f^7$

33. (a)



34. (c) No option contains all species that show disproportionation reaction. MnO_4^-
Mn is in +7 oxidation state (highest) hence cannot be simultaneously oxidized or reduced.

35. (a) Cell constant $= \left(\frac{\ell}{A}\right) \Rightarrow$ Units $= m^{-1}$

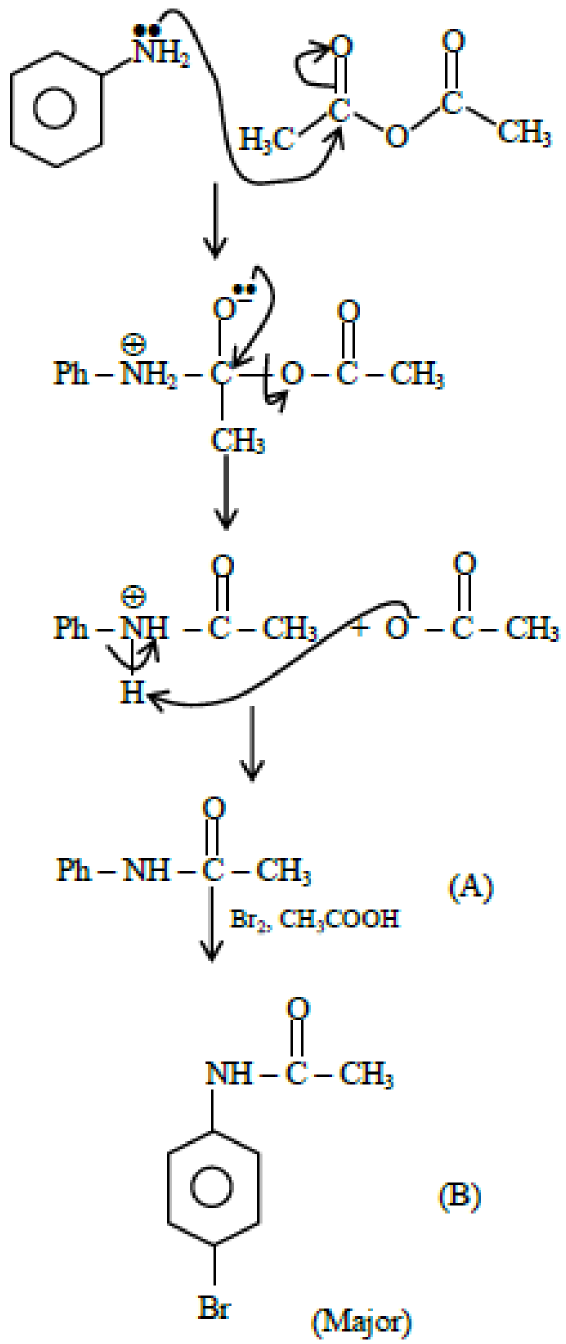
Molar conductivity $(\Lambda_m) \Rightarrow$ Units $= Sm^2$ mole $^{-1}$

Conductivity (K) $\Rightarrow$ Units $= Sm^{-1}$

Degree of dissociation $(\alpha) \rightarrow$ Dimensionless

(A) - (iii) (B) - (i) (C) - (iv) (D) - (ii)

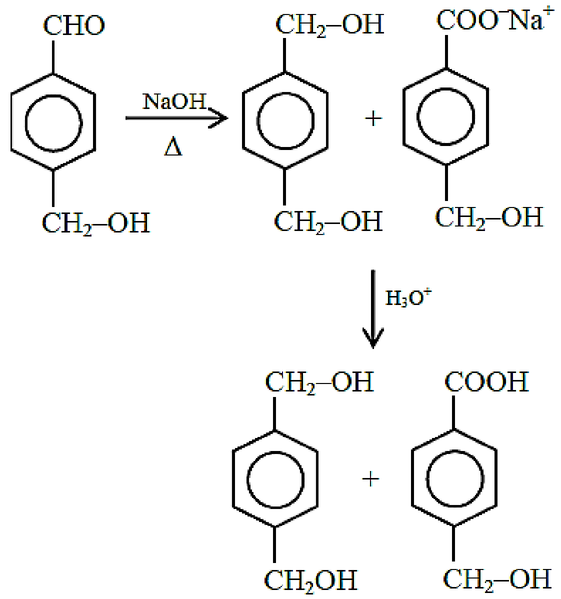
36. (b)



37. **(b)** Keratin, collagen and myosin are example of fibrous protein.

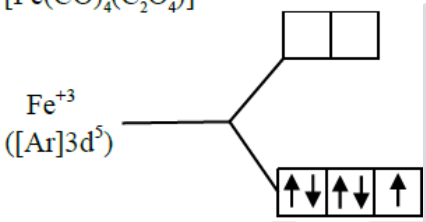
38. **(c)** Oxides of nitrogen and sulphur are acidic and settle down on ground as dry deposition. Ammonium salts in rain drops result in wet deposition

39. **(c)**



40. (d)

$[Fe(CO)_4(C_2O_4)]^+$



One unpaired electron Spin only magnetic moment $= \sqrt{3}$ B.M. $= 1.73$BM

41. (a) Lithium salts are hydrated due to high hydration energy of Li

Li^+due to smallest size in IA group has highest polarizing power.

42. (b) is incorrect

$\Delta G^\circ = -RT\ln K$

$\Delta H^\circ - T\Delta S^\circ = -RT\ln K$

$\ln K = -\left[\frac{\Delta H^\circ - \Delta S^\circ}{RT}\right]$

43. (b) Atomic hydrogen is produced at high temperature in an electric are or under ultraviolet radiations the dissociation of dihydrogen at 2000 K is only 0.081% $H-H$ bond dissociation enthalpy is highest for a single bond for any diatomic molecule. Dihydrogen can be produced on reacting Zn with dil. HCl as well as NaOH (aq.)

44. (a) Novalac is a linear polymer of $[Ph - OH + HCHO]$. So ester linkage not present. So novalac is not a polyester.

45. (d) Stability of oxides of Halogens is

$I > Cl > Br$

46. (b)

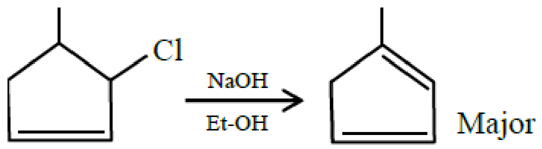
$Mn^{2+} \rightarrow$ III group

$As^{3+} \rightarrow$ II B group

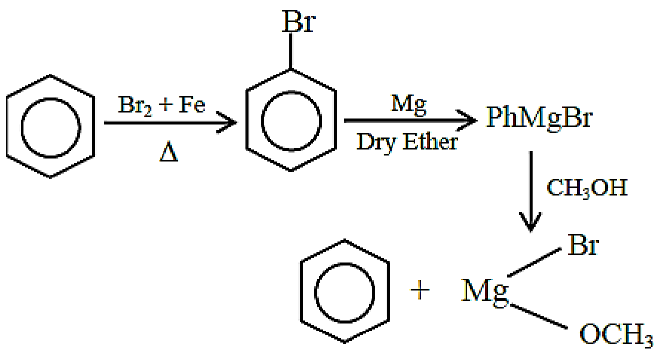
$Cu^{2+} \rightarrow$ II A group

$Al^{3+} \rightarrow$ IV group

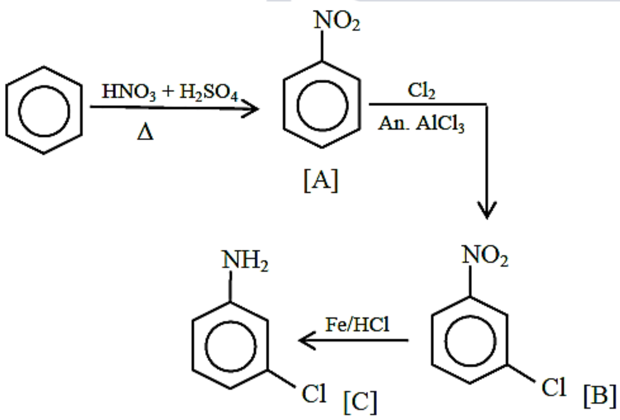
47. (c) NaOH + EtOH is known as alcoholic NaOH, so it give E^2 reaction with given alkyl halide.



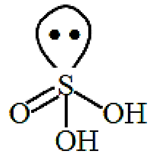
48. (b)



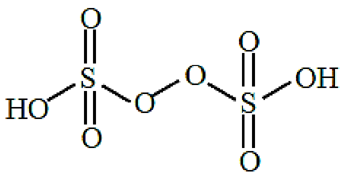
49. (a)



50. (d)



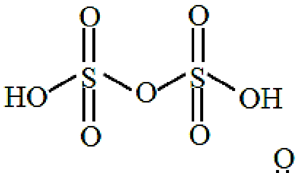
Sulphurous acid



Peroxodisulphuric acid

1 $S=O$ Bond.

4 $S=O$ Bond.



Pyrosulphuric acid

4 $S=O$ Bond.

51. (128) We know

$$\frac{X}{m} = KP^{1/n};\ \text{using } (x \propto V)$$

$$\Rightarrow \frac{10}{1} = K \times (100)^{1/n}$$

$$\frac{15}{1} = K \times (200)^{1/n}$$

$$\frac{V}{1} = K \times (300)^{1/n}$$

Divide

(2) / (1)

$$\frac{15}{10} = 2^{1/n}$$

$$\log\left(\frac{3}{2}\right) = \frac{1}{n}\log 2$$

$$\frac{1}{n} = \frac{\log 3 - \log 2}{\log 2} = \frac{0.4771 - 0.3010}{0.3010}$$

$$\frac{1}{n} = 0.585$$

Divide

(3) / (1)

$$\frac{V}{10} = 3^{1/n}$$

$$\log\left(\frac{V}{10}\right) = \frac{1}{n}\log 3$$

$$\log\left(\frac{V}{10}\right) = 0.585 \times 0.4771 = 0.2791$$

$$\frac{V}{10} = 10^{0.279} \Rightarrow V = 10 \times 10^{0.279}$$

$$\Rightarrow V = 10^{1.279} = 10^{x}$$

$$\Rightarrow x = 1.279$$

$$\Rightarrow x = 128 \times 10^{-2} \text{ (Nearest integer)}$$

52. (13) With benzene as solvent

$$\Delta T_b = iKK_b m$$

$$\Delta T_b = \frac{1}{2} \times 2.6 \times \frac{1.22/M_w}{100/1000}$$

With Acetone as solvent

$$\Delta T_b = iK_b m$$

$$0.17 = 1 \times 1.7 \times \frac{1.22/M_w}{100/1000}$$

$(1)/(2)$

$$\frac{\Delta T_b}{0.17} = \frac{\frac{1}{2} \times 2.6 + \frac{1.22/M_w}{100/1000}}{1 \times 1.7 \times \frac{1.22/M_w}{100/1000}}$$

$$\Delta T_b = \frac{0.26}{2}$$

$$\Delta T_b = 13 \times 10^{-2}$$

$$\Rightarrow x = 13$$

53. (0) $Zn^{+} \rightarrow 1\,s^2 2\,s^2 2p^6 3\,s^2 3p^6 3\,d^{10} 4\,s^1$

Outermost electron is in 4 s subshell

$m = 0$

54. (1) Anions froms CCP or FCC $(A^{-}) = 4\,A^{-}$per unit cell Cations occupy all octahedral voids $(B^{+}) = 4\,B^{+}$per unit cell

cell formula $\rightarrow A_4\,B_4$

Empirical formula $\rightarrow$ AB

$\rightarrow (x = 1)$

55. (6) Anode mud contains Sb, Se, Te, Ag, Au and Pt

56. (6021)

HCl (aq.) +NaOH (aq.) $\rightarrow$ NaCl(aq.) + $H_2O(\ell)$

50ml, 1M 30ml, 1M

$t = 0$ 50 mm 30 mm

$t = \infty$ 20 mm

$[HCl] = \frac{20}{80} = \frac{1}{4} M = 2.5 \times 10^{-1} M$

$pH = -\log 2.15 \times 10^{-1} = 1 - 0.3979 = 0.6021$

$pH = 6021 \times 10^{-4}$

57. (47) Given

$\log K = 20.35 - \frac{2.47 \times 10^3}{T}$

We know $\log K = \log A - \frac{E_a}{2.303RT}$

$\Rightarrow \frac{E_a}{2.303RT} = 2.47 \times 10^3$

$E_a = 2.47 \times 10^3 \times 2.303 \times \frac{8.314}{1000} KJ/mole$

$= 47.29 = 47$ (Nearest integer)

58. (13) $Na_2O + H_2O \rightarrow 2NaOH$

$\frac{20}{62}$ moles

Moles of NaOH formed $= \frac{20}{62} \times 2$

$[NaOH] = \frac{\frac{40}{62}}{\frac{500}{1000}} = 1.29M = 13 \times 10^{-1} M$

(Nearest integer)

59. (0) Molecular orbital configuration of O_2^{2-} is

$\sigma_{1s}^2 \sigma_{1s}^{*2} \sigma_{2s}^2 \sigma_{2s}^{*2} (\pi 2p_x^2 = \pi 2p_y^2)(\pi_{2px}^{*2} = \pi_{2py}^{*2})$

Zero unpaired electron

60. (7)

$C_2H_7 N + \left(2x + \frac{y}{2}\right) CuO \rightarrow xCO_2 + \frac{y}{2} H_2O + \frac{z}{2} N_2 + \left(2x + \frac{y}{2}\right) Cu$

On balancing

$$C_2H_7\ N + \frac{15}{2}CuO \rightarrow 2CO_2 + \frac{7}{2}H_2O + \frac{1}{2}\ N_2 + \frac{15}{2}Cu$$

On comparing

$y = 7$

61. (b) $\alpha + \beta + \gamma = 2\pi$

$$\begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$= 1 + 2\cos\alpha \cdot \cos\beta \cdot \cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma$

$= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + (\cos(\alpha+\beta) + \cos(\alpha-\beta)\cos\gamma$

$= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + \cos^2\gamma + \cos(\alpha-\beta)\cos\gamma$

$= \sin^2\alpha - \cos^2\beta + \cos(\alpha-\beta)\cos(\alpha+\beta)$

$= \sin^2\alpha - \cos^2\beta + \cos^2\alpha - \sin^2\beta = 0$

62. (c) Suppose $\vec{r} = x\vec{a} + y\vec{b} + 2\vec{c}$

and $|\vec{a}| = |\vec{b}| = |\vec{c}| = k$

$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0}$

$\Rightarrow k^2(\vec{r} - \vec{b}) - k^2 xa + k^2(\vec{r} - \vec{c}) - k^2 y\vec{b} +$

$k^2(\vec{r} - \vec{a}) - k^2 z\vec{c} = \vec{0}$

$\Rightarrow 3\vec{r} - (\vec{a} + \vec{b} + \vec{c}) - \vec{r} = \vec{0}$

$$\Rightarrow \vec{r} = \frac{\vec{a} + \vec{b} + \vec{c}}{2}$$

63. (c) $f(x) = \sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$

$$-1 \le \frac{x-1}{x+1} \le 1 \Rightarrow 0 \le x < \infty$$

$$-1 \le \frac{3x^2+x-1}{(x-1)^2} \le 1 \Rightarrow x \in \left[\frac{-1}{4}, \frac{1}{2}\right] \cup \{0\}$$

(1) & (2)

$\Rightarrow$ Domain $= \left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$

64. (a) g(c) = 2g(a) can be defined in 3 ways number of onto functions in this condition $= 3 \times 4$! Total number of onto functions = 6 ! Required probability $= \frac{3\times 4!}{6!} = \frac{1}{10}$

65. (b) $f(\mathrm{m}+\mathrm{n}) = f(\mathrm{m}) + f(\mathrm{n})$

Put $\mathrm{m}=1, \mathrm{n}=1$

$f(\mathrm{b}) = 2f(\mathrm{a})$

Put $\mathrm{m}=2, \mathrm{n}=1$

$f(\mathrm{c}) = f(\mathrm{b}) + f(\mathrm{a}) = 3f(\mathrm{a})$

Put $\mathrm{m}=3, \mathrm{n}=3$

$f(6) = 2f(\mathrm{c}) \Rightarrow f(\mathrm{c}) = 9$

$\Rightarrow f(\mathrm{a}) = 3, f(\mathrm{b}) = 6$

$f(\mathrm{b}).f(\mathrm{c}) = 6 \times 9 = 54$

66. (d) $\mathrm{P}_1: 2\mathrm{x}+3\mathrm{y}+2\mathrm{z}=0$

$\Rightarrow \vec{n}_1 = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

$\mathrm{P}_2: \mathrm{x}-2\mathrm{y}+\mathrm{z}=0$

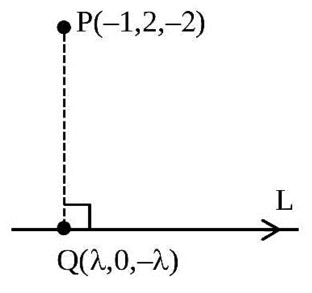
$\Rightarrow \vec{\mathrm{n}}_2 = \hat{\imath} - 2\hat{\jmath} + \mathrm{k}$

Direction vector of line L which is line of intersection of $\mathrm{P}_1 \& \mathrm{P}_2$

$\vec{\mathrm{r}} = \vec{\mathrm{n}}_1 \times \vec{\mathrm{n}}_2 = 7\hat{\imath} - 7\hat{\mathrm{k}}$

DR's of L are $(1,0,-1)$

$\Rightarrow$ Equation of L: $\frac{\mathrm{x}}{1} = \frac{\mathrm{y}}{0} = \frac{\mathrm{z}}{-1} = \lambda$



DR's of $\overrightarrow{\mathrm{PQ}} = (\lambda+1, -2, 2-\lambda)$

$\because \overrightarrow{\mathrm{PQ}} \perp \vec{\mathrm{r}}$

$\Rightarrow (\lambda+1)(\mathrm{a}) + (-2)(0) + (2-\lambda)(-1) = 0$

$\Rightarrow \lambda = \frac{1}{2} \Rightarrow \mathrm{Q}\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$

$\Rightarrow PQ = \frac{\sqrt{34}}{2}$

67. (a)

. $\because \sim (A \Rightarrow B) = A \wedge \sim B$

$\therefore \sim ((p \vee r) \Rightarrow (q \vee r))$

$= (p \vee r) \wedge (\sim q \wedge \sim r)$

$= ((p \vee r) \wedge (\sim r)) \wedge (\sim q)$

$= p \wedge (\sim r) \wedge (\sim q)$

68. (d) $\alpha = \lim_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}; \frac{0}{0}$ form

Using L Hopital rule

$$\alpha = \lim_{x \to \frac{\pi}{4}} \frac{3\tan^2 x \sec^2 x - \sec^2 x}{-\sin\left(x + \frac{\pi}{4}\right)}$$

$\Rightarrow \alpha = -4$

$$\beta = \lim_{x \to 0} (\cos x)^{\cot x} = e^{\lim_{x \to 0} \frac{(\cos x - 1)}{\tan x}}$$
$$\beta = e^{\lim_{x \to 0} \frac{-(1-\cos x)}{x^2} \cdot \frac{x^2}{\left(\frac{\tan x}{x}\right)^x}}$$
$$\beta = e^{\lim_{x \to 0} \left(\frac{-1}{2}\right) \cdot \frac{x}{1}} = e^0 \Rightarrow \beta = 1$$
$\alpha = -4; \beta = 1$

If $ax^2 + bx - 4 = 0$ are the roots then

$16a - 4b - 4 = 0 \,\&\, a + b - 4 = 0$

$\Rightarrow a = 1 \,\&\, b = 3$

69. (c) General point on $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is $A(2\cos\theta, 3\sin\theta)$ given $B(-3, -5)$

midpoint $C\left(\frac{2\cos\theta - 3}{2}, \frac{3\sin\theta - 5}{2}\right)$

$$h = \frac{2\cos\theta - 3}{2}; k = \frac{3\sin\theta - 5}{2}$$

$$\Rightarrow \left(\frac{2h + 3}{2}\right)^2 + \left(\frac{2k + 5}{3}\right)^2 = 1$$

$\Rightarrow 36x^2 + 16y^2 + 108x + 80y + 145 = 0$

70. (a) $\frac{dy}{dx} = \frac{2^x(y + 2^y)}{2^x(1 + 2^y \ln 2)}$

$$\Rightarrow \int \frac{(1 + 2^y)\ell n2}{(y + 2^y)} dy = \int dx$$

$\Rightarrow \ell n|y+2^y| = x + c$

$x = 0; y = 0 \Rightarrow c = 0$

$\Rightarrow x = \ell n y + 2^y$ |

$\Rightarrow$ at $y = 1, x = \ln 3$

$\because 3 \in (e, e^2) \Rightarrow x \in (1,2)$

71. (c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, x^2+y^2=ab$

$$\frac{2x_1}{a^2}+\frac{2y_1y'}{b^2}=0$$

$$\Rightarrow y_1' = \frac{-x_1}{a^2}\frac{b^2}{y_1}$$

$\therefore 2x_1 + 2y_1y' = 0$

$$\Rightarrow y_2' = \frac{-x_1}{y_1}$$

Here (x_1y_1) is point of intersection of both curves

$$\therefore x_1^2 = \frac{a^2\,b}{a+b}, y_1^2 = \frac{ab^2}{a+b}$$

$$\therefore \tan\theta = \left|\frac{y_1{}' - y_2{}'}{1+y_1{}'y_2{}'}\right| = \left|\frac{\frac{-x_1\,b^2}{a^2y_1}+\frac{x_1}{y_1}}{1+\frac{x_1^2\,b^2}{a^2y_1^2}}\right|$$

$$\tan\theta = \left|\frac{-b^2x_1y_1 + a^2x_1y_1}{a^2y_1^2+b^2x_1^2}\right|$$

$$\tan\theta = \left|\frac{a-b}{\sqrt{ab}}\right|$$

72. (b) Let, $y = tx$

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$\therefore tx\left(t + x\frac{dt}{dx}\right) = x\left(t^2 + \frac{\varphi(t^2)}{\varphi'(t^2)}\right)$$

$$t^2 + xt\frac{dt}{dx} = t^2 + \frac{\varphi(t^2)}{\varphi'(t^2)}$$

$$\int \frac{t\varphi'(t^2)}{\varphi(t^2)}dt = \int \frac{dx}{x}$$

Let $\varphi(t^2) = p$

$\therefore \varphi'(t^2)2tdt = dp$

$\Rightarrow \int \frac{dy}{2p} = \int \frac{dx}{x}$

$\frac{1}{2}\ln \varphi(t^2) = \ell nx + \ell nc$

$\varphi(t^2) = x^2k$

$\varphi\left(\frac{y^2}{x^2}\right) = kx^2, \varphi(a) = k$

$\varphi\left(\frac{y^2}{4}\right) = 4\varphi(a)$

73. (b)

$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$

$\log_2(2^{x+1}) - \log_2(3 + 2^x)^2 + \log_2(10 - 2^{-x}) = 0$

$\log_2\left(\frac{2^{x+1} \cdot (10 - 2^{-x})}{(3 + 2^x)^2}\right) = 0$

$\frac{2(10 \cdot 2^x - 1)}{(3 + 2^x)^2} = 1$

$\Rightarrow 20.2^x - 2 = 9 + 2^{2x} + 6.2^x$

$\therefore (2^x)^2 - 14(2^x) + 11 = 0$

Roots are 2^{x_1}&2^{x_2}

$\therefore 2^{x_1} \cdot 2^{x_2} = 11$

$x_1 + x_2 = \log_2(11)$

74. (d) $\frac{z-i}{z-1}$ is purely Imaginary number

Let $z = x + iy$

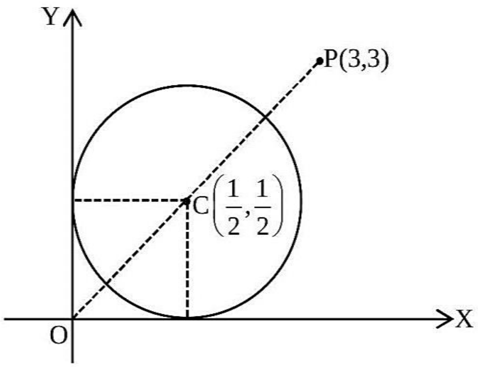
$\therefore \frac{x + i(y-1)}{(x-1) + i(y)} \times \frac{(x-1) - iy}{(x-1) - iy}$

$\Rightarrow \frac{x(x-1)+y(y-1)+i(-y-x+1)}{(x-1)^2+y^2}$ is purely

Imaginary number

$\Rightarrow x(x-1) + y(y-1) = 0$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$



$$\therefore |z-(3+3i)|_{min} = |PC| - \frac{1}{\sqrt{2}}$$

$$= \frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2\sqrt{2}$$

75. (c) $\frac{\frac{10}{2}(2a_1+9d)}{\frac{p}{2}(2a_1+(p-1)d)} = \frac{100}{p^2}$

$(2a_1 + 9\,d)p = 10(2a_1 + (p-1)d)$

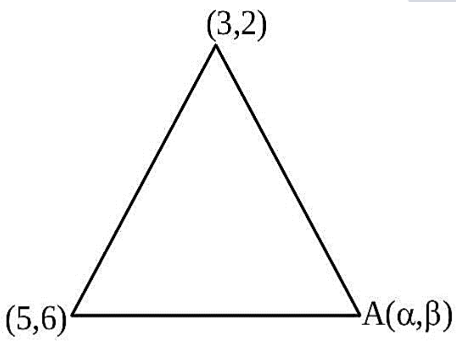
$9dp = 20a_1 - 2pa_1 + 10d(p-1)$

$9p = (20-2p)\frac{a_1}{d} + 10(p-1)$

$\frac{a_1}{d} = \frac{(10-p)}{2(10-p)} = \frac{1}{2}$

$\therefore \frac{a_{11}}{a_{10}} = \frac{a_1 + 10\,d}{a_1 + 9\,d} = \frac{\frac{1}{2}+10}{\frac{1}{2}+9} = \frac{21}{19}$

76. (c)



$\left|\frac{1}{2}\right|\begin{matrix} 5 & 6 & 1 \\ 3 & 2 & 1 \\ \alpha & \beta & 1 \end{matrix}| = 12$

$4\alpha - 2\beta = \pm 24 + 8$

$\Rightarrow 4\alpha - 2\beta = +24 + 8 \Rightarrow 2\alpha - \beta = 16$

$2x - y - 16 = 0$

$\Rightarrow 4\alpha - 2\beta = -24 + 8 \Rightarrow 2\alpha - \beta = -8$

$2x - y + 8 = 0$

perpendicular distance of (1) from (0,0)

$$\left|\frac{0-0-16}{\sqrt{5}}\right| = \frac{16}{\sqrt{5}}$$

perpendicular distance of (2) from (0,0) is $\left|\frac{0-0+8}{\sqrt{5}}\right| = \frac{8}{\sqrt{5}}$

77. (b) $(32)^{\tan^2 x} + (32)^{\sec^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} + (32)^{1+\tan^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} = \frac{81}{33}$

In interval $\left[0, \frac{\pi}{4}\right]$ only one solution

78. (b) $f(0) = 0 \; f(1) = 1$ and $f(2) = 2$

Let $h(x) = f(x) - x$ has three roots

By Rolle's theorem $h'(x) = f'(x) - 1$ has at least two roots

$h''(x) = f''(x) = 0$ has at least one roots

79. (b) $\pi^2 \left[\int_0^1 \sin\frac{\pi x}{2} dx + \int_1^2 \sin\frac{\pi x}{2}(x-1)dx\right]$

$$= \pi^2 \left[-\frac{2}{\pi}\left(\cos\frac{\pi x}{2}\right) + \left((x-1)\left(-\frac{2}{\pi}\cos\frac{\pi x}{2}\right)\right)_1^2 - \int_1^2 -\frac{2}{\pi}\cos\frac{\pi x}{2}dx\right]$$

$$= \pi^2 \left[0 + \frac{2}{\pi} + \frac{2}{\pi} + \frac{2}{\pi}\cdot\frac{2}{\pi}\left(\sin\frac{\pi x}{2}\right)_1^2\right]$$

$= 4\pi - 4 = 4(\pi - 1)$

80. (c) Let 8, 16, x_1, x_2, x_3, x_4, x_5 be the observations.

Now $\frac{x_1+x_2+\cdots+x_5+14}{7} = 8$

$\Rightarrow \sum_{i=1}^{5} x_i = 42$

Also $\frac{x_1^2+x_2^2+\cdots x_5^2+8^2+6^2}{7} - 64 = 16$

$\Rightarrow \sum_{i=1}^{5} x_i^2 = 560 - 100 = 460$

So variance of $x_1, x_2, \ldots, x_5$

$$= \frac{460}{5} - \left(\frac{42}{5}\right)^2 = \frac{2300 - 1764}{25} = \frac{536}{25}$$

81. (315) $\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha}(2b)^{\beta}\cdot(4ab)^{\gamma}$

$$\frac{10!}{\alpha!\,\beta!\,\gamma!}a^{\alpha+\gamma}\cdot b^{\beta+\gamma}\cdot 2^{\beta}\cdot 4^{\gamma}$$

$\alpha+\beta+\gamma=10$

$\alpha+\gamma=7$

$\beta+\gamma=8$

(b) $+$(c) $-$ (a) $\Rightarrow \gamma=5$

$\alpha=2$

$\beta=3$

so coefficients $=\frac{10!}{2!3!5!}2^{3}\cdot 2^{10}$

$$=\frac{10\times 9\times 8\times 7\times 6\times 5}{2\times 3\times 2\times 5!}\times 2^{13}$$

$=315\times 2^{16}\Rightarrow \mathrm{k}=315$

82. (7) Point $(2,2,-2)$ also lies on given plane

So $2+3\times 2-2(-2)+\beta=0$

$\Rightarrow 2+6+4+\beta=0\Rightarrow \beta=-12$

Also $\alpha\times 1-5\times 3+2\times -2=0$

$\Rightarrow \alpha-15-4=0\Rightarrow \alpha=19$

$\therefore \alpha+\beta=19-12=7$

83. (5143) A = 4 - digit numbers divisible by 3

$\mathrm{A}=1002,1005,\ldots,9999.$

$9999=1002+(\mathrm{n}-1)3$

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$\Rightarrow (\mathrm{n}-1)3=8997\Rightarrow \mathrm{n}=3000$

B = 4 - digit numbers divisible by 7

$\mathrm{B}=1001,1008,\ldots,9996$

$\Rightarrow 9996=1001+(\mathrm{n}-1)7$

$\Rightarrow \mathrm{n}=1286$

$\mathrm{A}\cap\mathrm{B}=1008,1029,\ldots,9996$

$9996 = 1008 + (n-1)21$

$\Rightarrow n = 429$

So, no divisible by either 3 or 7

$= 3000 + 1286 - 429 = 3857$

total 4-digits numbers $= 9000$

required numbers $= 9000 - 3857 = 5143$

84. (3)

$$= \int \frac{\frac{\sin x}{\cos^3 x}}{1+\tan^3 x} dx = \int \frac{\tan x \cdot \sec^2 x}{(\tan x+1)(1+\tan^2 x-\tan x)} dx$$

Let $\tan x = t \Rightarrow \sec^2 x \cdot dx = dt$

$$= \int \frac{t}{(t+1)(t^2-t+1)} dt$$

$$= \int \left(\frac{A}{t+1} + \frac{B(2t-1)}{t^2-t+1} + \frac{C}{t^2-t+1}\right) dx$$

$\Rightarrow A(t^2-t+1) + B(2t-1)(t^2-t+1) + C(t+1) = t$

$\Rightarrow t^2(A+2B) + t(-A+B+C) + A - B + C = 1$

$\therefore A + 2B = 0$

$-A + B + C = 1$

$A - B + C = 0$

$\Rightarrow C = \frac{1}{2} \Rightarrow A - B = -\frac{1}{2}$

$A + 2B = 0$

$A - B = -\frac{1}{2} \Rightarrow 3B = \frac{1}{2} \Rightarrow B = \frac{1}{6}$

$A = -\frac{1}{3}$

$$I = -\frac{1}{3}\int \frac{dt}{1+t} + \frac{1}{6}\int \frac{2t-1}{t^2-t+1} dt + \frac{1}{2}\int \frac{dt}{t^2-t+1}$$

$$= -\frac{1}{3}\ln|(1+\tan x)| + \frac{1}{6}\ln|\tan^2 x - \tan x + 1|$$

$$+\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\left(\tan x - \frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right)$$

$= -\frac{1}{3}\ln|(1+\tan x)| + \frac{1}{6}\ell\ln \tan^2 x - \tan x + 1\,|$

$+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$

$\alpha = -\frac{1}{3}, \beta = \frac{1}{6}, \gamma = \frac{1}{\sqrt{3}}$

$18(\alpha+\beta+\gamma^2) = 18\left(-\frac{1}{3}+\frac{1}{6}+\frac{1}{3}\right) = 3$

85. (2) tangent of $y^2 = 8x$ is $y = mx + \frac{2}{m}$

$P(2,-4) \Rightarrow -4 = 2\,m + \frac{2}{m}$

$\Rightarrow m + \frac{1}{m} = -2 \Rightarrow m = -1$

$\therefore$ tangent is $y = -x - 2$

$\Rightarrow x + y + 2 = 0$

(1) is also tangent to $x^2 + y^2 = a$

So $\frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow \sqrt{a} = \sqrt{2}$

$\Rightarrow a = 2$

86. (305)

$$S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \cdots$$
$$\frac{1}{5}S = \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + \cdots$$

On subtracting

$\frac{4}{5}S = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \cdots$

$S = \frac{7}{4} + \frac{1}{10}\left(1 + \frac{2}{5} + \frac{3}{5^2} + \cdots\right)$

$S = \frac{7}{4} + \frac{1}{10}\left(1 - \frac{1}{5}\right)^{-2}$

$= \frac{7}{4} + \frac{1}{10} \times \frac{25}{16} = \frac{61}{32}$

$\Rightarrow 160\,S = 5 \times 61 = 305$

87. (8) $(I-A)^3 = I^3 - A^3 - 3A(I-A) = I - A^3$

$\Rightarrow 3\,A(I-A) = 0$ or $A^2 = A$

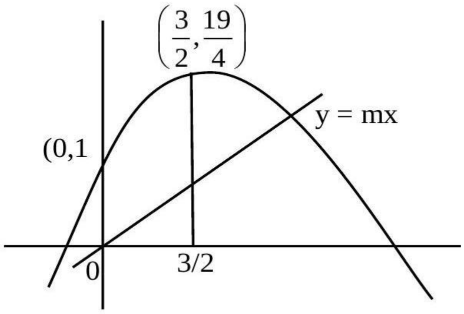
$\Rightarrow \begin{bmatrix} a^2 & ab+bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$

$\Rightarrow a^2 = a, b(a+d-1) = 0, d^2 = d$

If $b \neq 0, a+d = 1 \Rightarrow 4$ ways
If $b = 0, a = 0,1 \& d = 0,1 \Rightarrow 4$ ways

$\Rightarrow$ Total 8 matrices

88. (26)



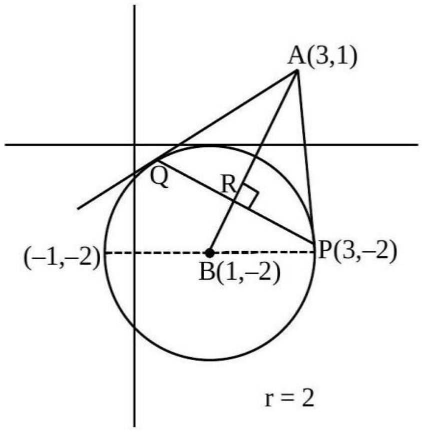
Total area $= \int_0^{3/2} (1 + 4x - x^2)dx$

$= x + 2x^2 - \frac{x^3}{3}\Big|_0^{3/2} = \frac{39}{8}$

$\& \frac{39}{16} = \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3}{2}$ m

$\Rightarrow 3\text{ m} = \frac{13}{2} \Rightarrow 12\text{ m} = 26$

89. (18)



$\tan\theta = \frac{3}{2}$

$\frac{\text{Area } \triangle APQ}{\text{Area } \triangle BPQ} = \frac{AR}{RB} = \frac{3\sin\theta}{2\cos\theta} = \frac{9}{4}$

$8\left(\frac{\text{Area } \triangle APQ}{\text{Area } \Delta BPQ}\right) = 18$

90. (22) $F'(x) = a(x-1)(x+3)$

$F''(x) = 6a(x+1)$

$F'(x) = 3a(x+1)^2 + b$

$F'(a) = 0 \Rightarrow b = -12a$

$F(x) = a(x+1)^3 - 12ax + c$

$= (x+1)^3 - 12x - 6$

$F(c) = 64 - 36 - 6 = 22$