1. (b) As electric field is in y-direction so electric flux is only due to top and bottom surface

Bottom surface y = 0

$\Rightarrow E = 0 \Rightarrow \phi = 0$

Top surface y = 0.5 m

$\Rightarrow E = 150(.5)^2 = \frac{150}{4}$

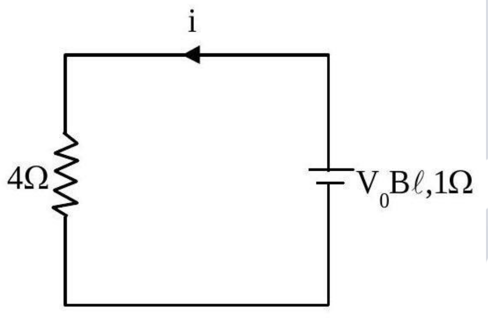
Now flux $\phi = EA = \frac{150}{4}(.5)^2 = \frac{150}{16}$

By Gauss's law $\phi = \frac{Q_{in}}{\epsilon_0}$

$$\frac{150}{16} = \frac{Q_{in}}{\epsilon_0}$$

$$Q_{in} = \frac{150}{16} \times 8.85 \times 10^{-12} = 8.3 \times 10^{-11} C$$

2. (b) Equivalent circuit



$$i = \frac{V_0 B\ell}{4+1} \Rightarrow V_0 = \frac{5(2\text{ mA})}{5 \times .2} = 10^{-2}\text{ m/s} = 1\text{ cm/s}$$

3. (a) De-Broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

Where E is kinetic energy

$E = \frac{3kT}{2}$ for gas

$$\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 9 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}}$$

$\lambda = 6.26 \times 10^{-9}$ m = 6.26 nm

4. (a) Work done = Change in kinetic energy

$$W_{mg} + W_{\text{air-friction}} = \frac{1}{2}m(.8\sqrt{gh})^2 - \frac{1}{2}m(0)^2$$

$W_{air-friction} = \frac{.64}{2}mgh - mgh = -0.68mgh$

5. **(b)** Range $R = \frac{u^2 \sin 2\theta}{g}$ and same for θ and $90 - \theta$

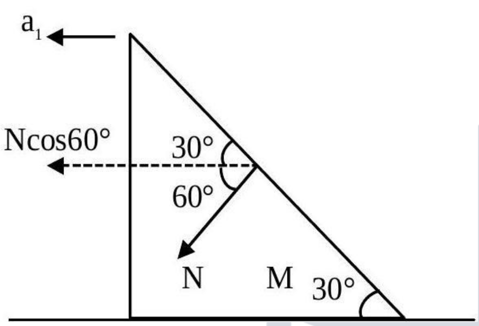
So same for 42° and 48°

Maximum height $H = \frac{u^2 \sin^2\theta}{2g}$

H is high for higher θ

So H for 48° is higher than H for 42°

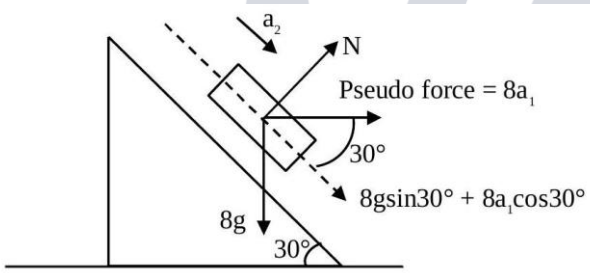
6. **(d)** Let acceleration of wedge is a_1 and acceleration of block w.r.t. wedge is a_2



$N\cos 60^\circ = Ma_1 = 16a_1$

$\Rightarrow N = 32a_1$

F.B.D. of block w.r.t wedge



$\perp$ to incline

$N = 8\,g\cos 30^\circ - 8a_1 \sin 30^\circ \Rightarrow 32a_1 = 4\sqrt{3}\,g - 4a_1$

$\Rightarrow a_1 = \frac{\sqrt{3}}{9}g$

Along incline

$8g\sin 30^\circ + 8a_1 \cos 30^\circ = ma_2 = 8a_2$

$a_2 = g \times \frac{1}{2} + \frac{\sqrt{3}}{9}g \cdot \frac{\sqrt{3}}{2} = \frac{2g}{3}$

7. **(b)** mass of ice $m = \rho A\ell = 10^3 \times 10^{-4} \times 1 = 10^{-1}$ kg

Energy required to melt the ice

$Q = ms\Delta T + mL$

$= 10^{-1}(2 \times 10^3 \times 10 + 3.33 \times 10^5) = 3.53 \times 10^4\ \mathrm{J}$

$Q = i^2RT \Rightarrow 3.53 \times 10^4 = \left(\frac{1}{2}\right)^2 (4 \times 10^3)(t)$

Time = 35.3sec

8. (b) $y = \frac{MgL^3}{4bd^3\delta}$

$$\frac{\Delta y}{y} = \frac{\Delta M}{M} + \frac{3\Delta L}{L} + \frac{\Delta b}{b} + \frac{3\Delta d}{d} + \frac{\Delta\delta}{\delta}$$

$$\frac{\Delta y}{y} = \frac{10^{-3}}{2} + \frac{3 \times 10^{-3}}{1} + \frac{10^{-2}}{4} + \frac{3 \times 10^{-2}}{4} + \frac{10^{-2}}{5}$$

$= 10^{-3}[0.5 + 3 + 2.5 + 7.5 + 2] = 0.0155$

9. (c) $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{4} \Rightarrow R_{eq} = 2\Omega$$

Also $\frac{\Delta R_{eq}}{R_{eq}^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

$$\frac{\Delta R_{eq}}{4} = \frac{.8}{16} + \frac{.4}{16} = \frac{1.2}{16}$$

$\Delta R_{eq} = 0.3\Omega$

$R_{eq} = (2 \pm 0.3)\Omega$

10. (b) $\left(t_{1/2}\right)_x = (\tau)_y$

$$\Rightarrow \frac{\ell n2}{\lambda_x} = \frac{1}{\lambda_y} \Rightarrow \lambda_x = 0.693\lambda_y$$

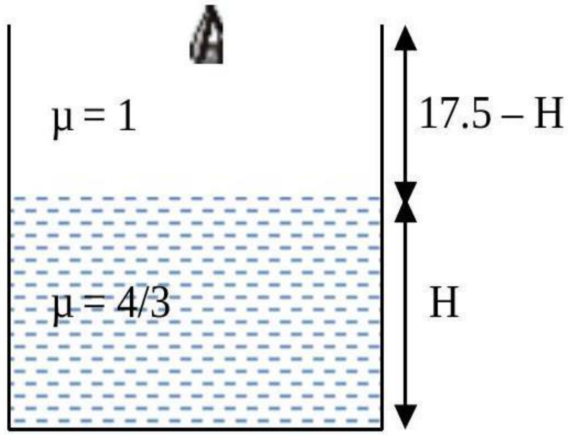
Also initially $N_x = N_y = N_0$

Activity $A = \lambda N$

As $\lambda_x < \lambda_y \Rightarrow A_x < A_y$

$\Rightarrow$ y will decay faster than x

11. (a)
12. (b)



Height of water observed by observer

$$= \frac{H}{\mu_w} = \frac{H}{(4/3)} = \frac{3H}{4}$$

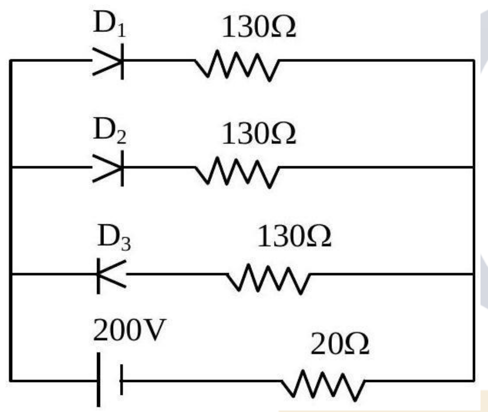
Height of air observed by observer $= 17.5 - H$

According to question, both height observed by observer is same.

$$\frac{3H}{4} = 17.5 - H$$

$\Rightarrow H = 10$ cm

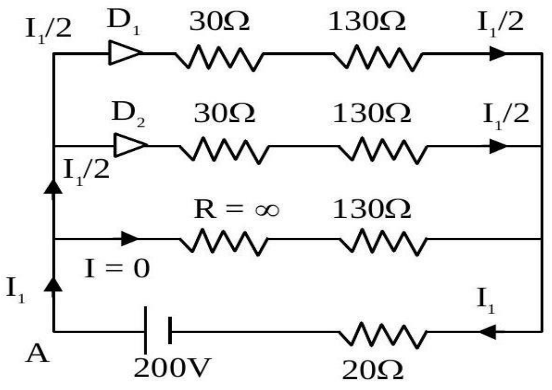
13. (c)



As per diagram,

Diode $D_1 \& D_2$ are in forward bias i.e. $R = 30\Omega$ whereas diode D_3 is in reverse bias i.e. $R =$ infinite $\Rightarrow$ Equivalent circuit will be

Applying KVL starting from point A



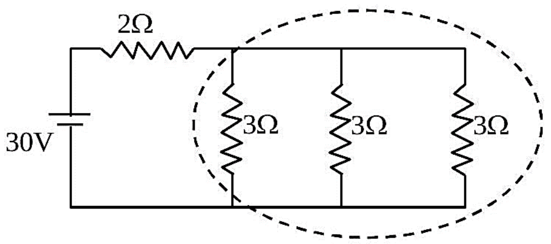
$$-\left(\frac{I_1}{2}\right) \times 30 - \left(\frac{I_1}{2}\right) \times 130 - I_1 \times 20 + 200 = 0$$

$$\Rightarrow -100I_1 + 200 = 0$$

$$I_1 = 2$$

14. (c) In steady state, inductor behaves as a conducting wire.

So, equivalent circuit becomes

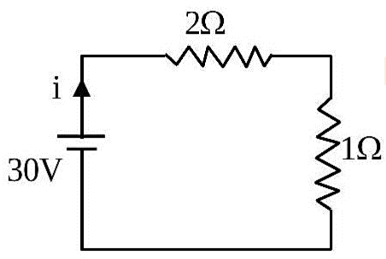


Parallel (R_{eq})

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

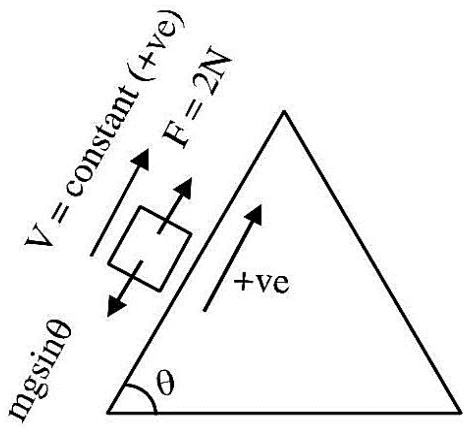
$$\Rightarrow R_{eq} = 1\Omega$$

$\Rightarrow$ Circuit becomes



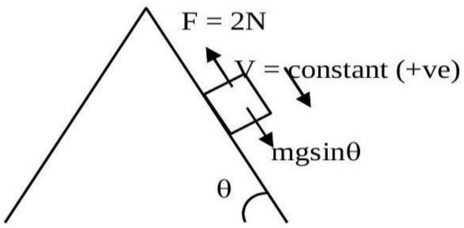
$$\Rightarrow i = \frac{30}{3} = 10 \text{ A}$$

15. (b) During upward motion



F = 2 N = (+ve) constant

During downward motion



⇒ F = 2 N = (−ve) constant

16. (c) From potential energy curve

$U_{max} = \frac{1}{2}kA^2 \Rightarrow 10 = \frac{1}{2}k(2)^2$

$\Rightarrow k = 5$

Now $T_{spring} = T_{pendulum}$

$2\pi\sqrt{\frac{5}{5}} = 2\pi\sqrt{\frac{4}{g}}$

$\Rightarrow 1 = \sqrt{\frac{4}{g}} \Rightarrow g = 4$ on planet

17. (b) $V = V_0\left(1 - e^{-t/RC}\right)$

$2 = 20\left(1 - e^{-t/RC}\right)$

$\frac{1}{10} = 1 - e^{-t/RC}$

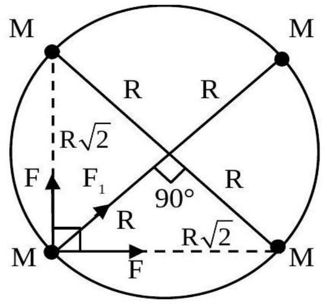
$e^{-t/RC} = \frac{9}{10}$

$e^{t/RC} = \frac{10}{9}$

$$\frac{t}{RC} = \ell n\left(\frac{10}{9}\right) \Rightarrow C = \frac{t}{R\ell n\left(\frac{10}{9}\right)}$$

$$C = \frac{10^{-6}}{10 \times .105} = .95\mu F$$

18. (b)



$$F_{net} = \frac{MV^2}{R}$$

$$\sqrt{2}\,F + F_1 = \frac{MV^2}{R}$$

$$\sqrt{2}\frac{GMM}{(\sqrt{2}R)^2} + \frac{GMM}{(2R)^2} = \frac{MV^2}{R}$$

$$\frac{GM}{R}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = V^2$$

$$\frac{GM}{R}\left(\frac{4+\sqrt{2}}{4\sqrt{2}}\right) = V^2$$

$$V = \sqrt{\frac{GM(4+\sqrt{2})}{R4\sqrt{2}}}$$

$$V = \frac{1}{2}\sqrt{\frac{GM(2\sqrt{2}+1)}{R}}$$

19. (a) Speed of wave $= \frac{2\times10^{10}}{200} = 10^8\ m/s$

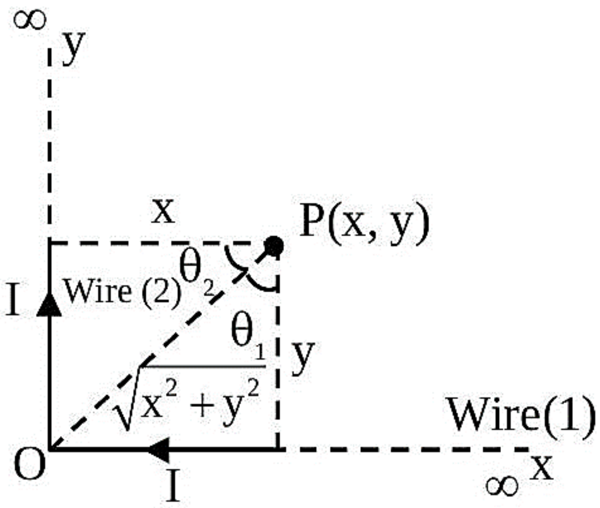
Refractive index $= \frac{3\times10^8}{10^8} = 3$

Now refractive index $= \sqrt{\varepsilon_r \mu_r}$

$3 = \sqrt{\varepsilon_r (a)}$

$\Rightarrow \varepsilon_r = 9$

20. (a)



$B_{\text{due to wire (a)}} = \frac{\mu_0 I}{4\pi y}[\sin 90 + \sin\theta_1]$

$= \frac{\mu_0}{4\pi}\frac{I}{y}\left(1 + \frac{x}{\sqrt{x^2+y^2}}\right)$

$B_{\text{due to wire (b)}} = \frac{\mu_0}{4\pi}\frac{I}{X}(\sin 90^\circ + \sin\theta_2)$

$= \frac{\mu_0}{4\pi}\frac{I}{x}\left(1 + \frac{y}{\sqrt{x^2+y^2}}\right)$.

Total magnetic field

$B = B_1 + B_2$

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{1}{y} + \frac{x}{y\sqrt{x^2+y^2}} + \frac{1}{x} + \frac{y}{x\sqrt{x^2+y^2}}\right]$$

$$B = \frac{\mu_0}{4\pi}\left[\frac{x+y}{xy} + \frac{x^2+y^2}{xy\sqrt{x^2+y^2}}\right]$$

$$B = \frac{\mu_0 0}{4\pi}\left[\frac{x+y}{xy} + \frac{\sqrt{x^2+y^2}}{xy}\right]$$

$$B = \frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2} + (x+y)\right]$$

21. (25) Pressure is not changing ⇒ isobaric process

$\Rightarrow \Delta U = nC_v\Delta T = \frac{5nR\Delta T}{2}$

and $W = nR\Delta T$

$\frac{\Delta U}{W} = \frac{5}{2} = \frac{x}{10} \Rightarrow x = 25.00$

22. (1) Given amplitude ∝ slit width

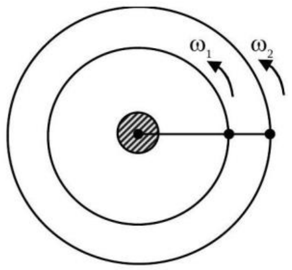
Also, intensity $\propto$ (Amplitude ^{2} $\propto$ (Slit width ^{2}

$$\frac{I_1}{I_2} = \left(\frac{3}{1}\right)^2 = 9 \Rightarrow I_1 = 9I_2$$

$$\frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2 = \left(\frac{3-1}{3+1}\right)^2 = \frac{1}{4} = \frac{x}{4}$$

$\Rightarrow x = 1.00$

23. (3)



$T_1 = 1$ hour

$\Rightarrow \omega_1 = 2\pi \text{rad/ hour}$

$T_2 = 8$ hours

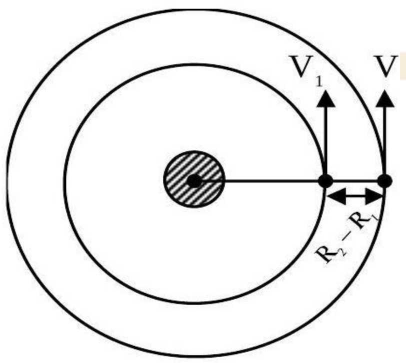
$\Rightarrow \omega_2 = \frac{\pi}{4} \text{rad/ hour}$

$R_1 = 2 \times 10^3$ km

As $T^2 \propto R^3$

$$\Rightarrow \left(\frac{R_2}{R_1}\right)^3 = \left(\frac{T_2}{T_1}\right)^2$$

$$\Rightarrow \frac{R_2}{R_1} = \left(\frac{8}{1}\right)^{2/3} = 4 \Rightarrow R_2 = 8 \times 10^3 \text{ km}$$



$V_1 = \omega_1 R_1 = 4\pi \times 10^3$ km/h

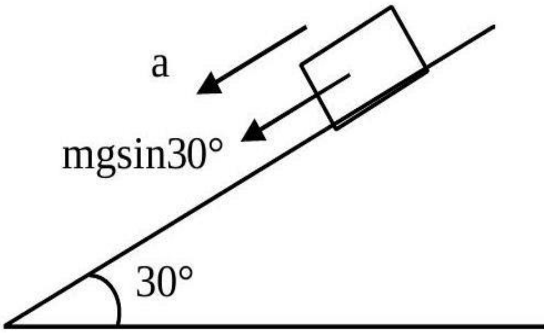
$V_2 = \omega_2 R_2 = 2\pi \times 10^3$ km/h

Relative $\omega = \frac{V_1 - V_2}{R_2 - R_1} = \frac{2\pi \times 10^3}{6 \times 10^3}$

$= \frac{\pi}{3}$ rad/ hour

$x = 3$

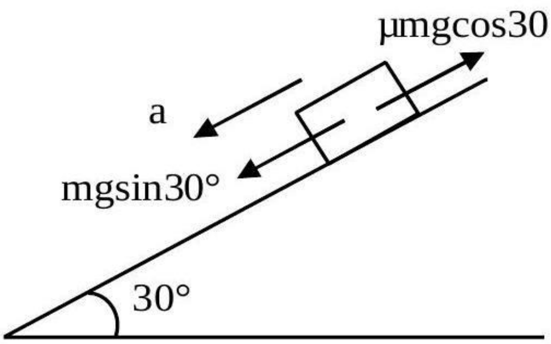
24. (3)



On smooth incline $a = g\sin 30°$

by $S = ut + \frac{1}{2}at^2$

$S = \frac{1}{2}\frac{g}{2}T^2 = \frac{g}{4}T^2$



On rough incline

$a = g\sin 30° - \mu g\cos 30°$

by $S = ut + \frac{1}{2}at^2$

$S = \frac{1}{4}g(1 - \sqrt{3}\mu)(\alpha T)^2$

By (i) and (ii)

$\frac{1}{4}gT^2 = \frac{1}{4}g(1 - \sqrt{3}\mu)\alpha^2 T^2$

$\Rightarrow 1 - \sqrt{3}g = \frac{1}{\alpha^2} \Rightarrow g = \left(\frac{\alpha^2 - 1}{\alpha^2}\right) \cdot \frac{1}{\sqrt{3}}$

$\Rightarrow x = 3.00$

25. (500) Given

Translation K.E. of N_2 = K.E. of electron

$$\frac{3}{2}kT = eV$$

$$\frac{3}{2} \times 1.38 \times 10^{-23}\ T = 1.6 \times 10^{-19} \times 0.1$$

$$\Rightarrow T = 773k$$

$$T = 773 - 273 = 500^\circ C$$

26. (4) First case $P_1 = \frac{V^2}{R} = \frac{(240)^2}{36}$

Second case Resistance of each half = 18Ω

$$P_2 = \frac{(240)^2}{18} + \frac{(240)^2}{18} = \frac{(240)^2}{9}$$

$$\frac{P_1}{P_2} = \frac{1}{4}$$

$$x = 4.00$$

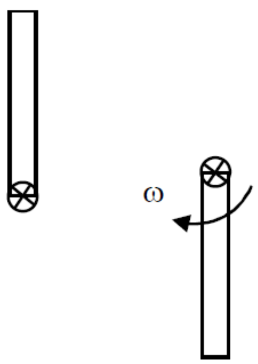
27. (8) Thermal force $F = Ay \propto \Delta T$

$$F = (10 \times 10^{-4})(2 \times 10^{11})(10^{-5})(400)$$

$$F = 8 \times 10^5\ N$$

$$\Rightarrow x = 8$$

28. (6)



by energy conservation $mg\ell = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{m\ell^2\omega^2}{3}$

$$\Rightarrow \omega = \sqrt{\frac{6g}{\ell}}$$

Speed $v = \omega r = \omega\ell = \sqrt{6\ g\ell}$

$$v = \sqrt{6 \times 10 \times .6} = 6\ m/s$$

29. (200) $A_{max} = A_C + A_m = 250 + 150 = 400$

$A_{min} = A_C - A_m = 250 - 150 = 100$

$$\frac{A_{min}}{A_{max}} = \frac{100}{400} = \frac{1}{4} = \frac{50}{200}$$

$x = 200$

30. (16)

Work = Δ K.E.

$$W_{friction} + W_{spring} = 0 - \frac{1}{2}mv^2$$

$$-\frac{90}{100}\left(\frac{1}{2}mv^2\right) + W_{Spring} = -\frac{1}{2}mv^2$$

$$W_{spring} = -\frac{10}{100} \times \frac{1}{2}mv^2$$

$$-\frac{1}{2}kx^2 = -\frac{1}{20}mv^2$$

$$\Rightarrow k = \frac{40000 \times (20)^2}{10 \times (1)^2} = 16 \times 10^5$$

31. (c) Clean water could have BOD value of less than 5ppm whereas highly polluted water could have a BOD value of 17ppm or more.

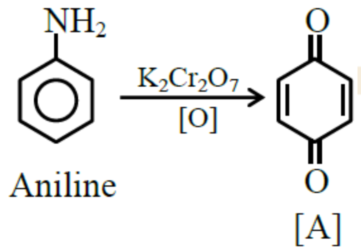
32. (b) Calamine $\Rightarrow ZnCO_3$

Malachite $\Rightarrow Cu(OH)_2 \cdot CuCO_3$

33. (**b**) NaH_2 is not reducing agent

34. (c) By observation we get this plot during measurable temperatures

35. (a)



36. (c)

$$\underset{\text{(A) } [C_6H_{12}O_2]}{CH_3-CH_2-CH_2-\underset{\underset{O}{\|}}{C}-O-CH_2-CH_3}$$

$$\Big\downarrow \begin{matrix}(1)\ LiAlH_4 \\ (2)\ H_3O^+\end{matrix}$$

$$\underset{\text{(B)}}{CH_3-CH_2-CH_2-CH_2-OH} + CH_3-CH_2-$$

$$\Big\downarrow [O]$$

$$CH_3-CH_2-CH_2-\underset{\underset{O}{\|}}{C}-OH \quad [C_4 \text{ carboxylic acid}]$$

37. (b) According to type of reactions for preparation, colloids have been classified

38. (d)

$V^{3+} \Rightarrow \underbrace{\boxed{\ }\boxed{\ }}_{e_g} = 2 \times 0.4\ \Delta_0$

$\underbrace{\boxed{1}\boxed{1}\boxed{\ }}_{t_{2g}} = -0.8\ \Delta_0$

$= 2$ unpaired e^-

$\mu = 2.89$ Bm

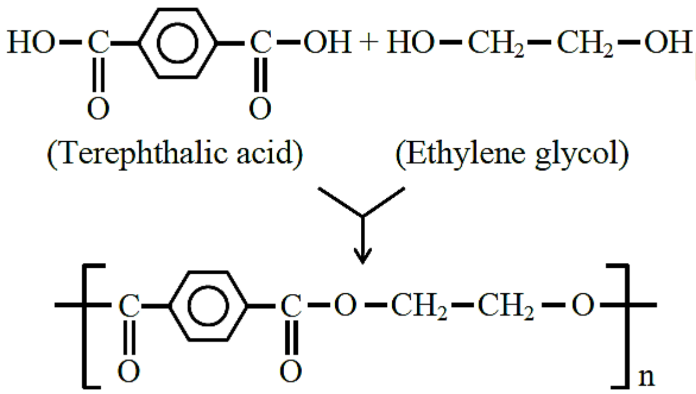
$Co^{2+} \Rightarrow \underbrace{\boxed{1}\boxed{1}}_{e_g} \ [2 \times 0.6\ \Delta_0 - 5 \times 0.4\ \Delta_0]$

$= -0.8\ \Delta_0$

$\underbrace{\boxed{1\!\downarrow}\boxed{1\!\downarrow}\boxed{1}}_{t_{2g}}$ 3 unpaired $e^- \Rightarrow \mu = 3.87$ BM

hence d^7 configuration is of Co^{2+} Ans.

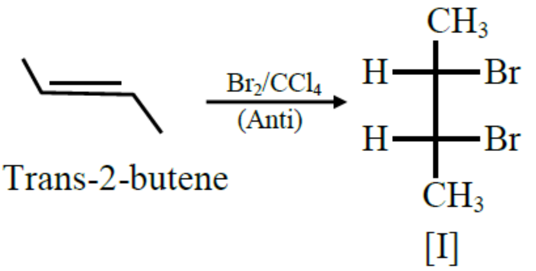
39.(b)



40. (d)

41. (d) Colour of Fe^{2+} is observed green and Fe^{3+} is yellow

42. (b)



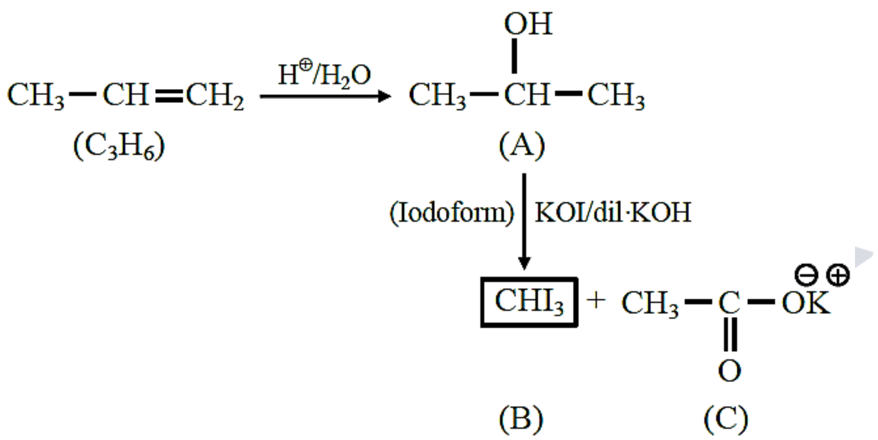
Trans-2-butene

[I]

meso product

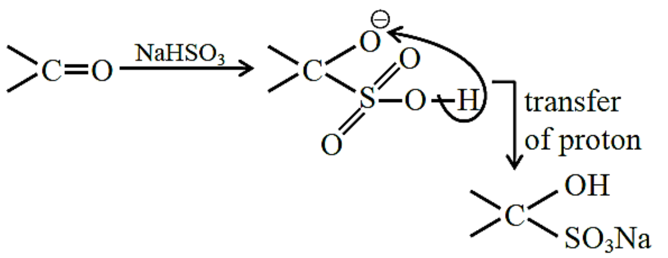
43. (c) $I_2 + H_2O_2 + 2OH^- \longrightarrow 2I^- + 2H_2O + O_2$

44. (d)



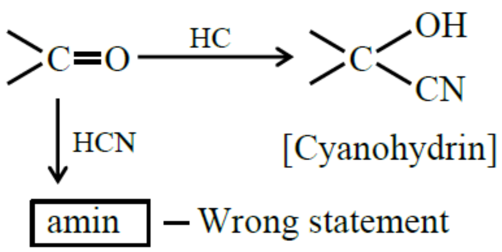
45. (b)

Statement I : Correct



(White crystalline soluble ppt)

Statement II :



— Wrong statement

(Amine not formed)

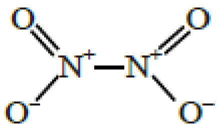
46. (b)

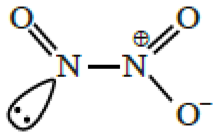
47. (d) $FeCl_3 + K_4[Fe(CN)_6] \rightarrow Fe_4[Fe(CN)_6]_3$

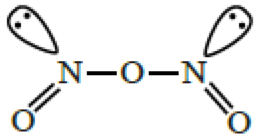
Prussian blue

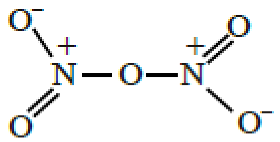
48. (d)

(1) $N \equiv N^{+} - O^{-}$

(2) 

(3)  unsymmetrical

 symmetrical

(4) 

49. (a)

$Li_2O \Rightarrow 2Li^+ \ O^{2-}$ $MgO \Rightarrow Mg^{2+} \ O^{2-}$

$CaO \Rightarrow Ca^{2+} \ O^{2-}$ $K_2O \Rightarrow 2\ K^+ \ O^{2-}$

$Na_2O_2 \Rightarrow 2Na^+ \ O_2^{2-}$

$KO_2 \Rightarrow K^+ \ O_2^-$

$O_2^- \Rightarrow$ Complete octet, diamagnetic

$O^{2-} \Rightarrow \sigma_{1s}^2 \sigma_{1s}^{*2} \sigma_{2s}^2 \sigma_{2s}^{*2} \sigma_{2px}^2 \pi_{2py}^2 \simeq \pi_{2pz}^2 \pi_{2py}^{*2} \simeq \pi_{2pz}^{*2}$ (dia)

$O_2^- \Rightarrow \sigma_{1s}^2 \sigma_{1s}^{*2} \sigma_{2s}^2 \sigma_{2s}^{*2} \sigma_{2px}^2 \pi_{2py}^2 \simeq \pi_{2pz}^2 \pi_{2py}^{*2} \simeq \pi_{2pz}^{*1}$ (para)

50. (d) $Fe^{3+}[Ar]3\ d^5$

51. (2)

Initial mass of gas $= 29 - 14.8 = 14.2$Kg

mass of gas used $= 29 - 23 = 6$Kg

gas left $= 14.2 - 6 = 8.2$Kg

(1) $3.47 \times V = \left(\frac{14.2 \times 10^3}{M}\right) \times R \times T$

(2) $p \times V = \left(\frac{8.2\times10^3}{M}\right) \times R \times T$

Divide:

$\frac{(1)}{(2)} \Rightarrow \frac{3.47}{P} = \frac{14.2}{8.2}$

$P = 2.003$

52. (2) $Zn(OH)_2(s) \rightleftharpoons Zn^{+2}(aq) + 2OH^-(aq)$

$S(0.1 + 2s) \simeq 0.1$

$K_{sp} = S(0.1)^2$

$2 \times 10^{-20} = s \times 10^{-2} \Rightarrow s = 2 \times 10^{-18}$

$= x \times 10^{-18}$

$x = 2$

53. (5)

$\Delta G = \Delta H - T\Delta S \; \Delta G = 57.8 - \frac{298(-176)}{1000}$

$\Delta G = -5.352$ kJ/mole

| Nearest integer value |= 5

54. (2) $\underset{+1}{[}Ag(NH_3)_2]^+ \left[\underbrace{Ag(CN)_2}_{+1}\right]$

55. (2) No. of atoms $= \frac{8}{23} \times 6.02 \times 10^{23} = 2.09 \times 10^{23}$

$\simeq 2 \times 10^{23}$
$= x \times 10^{23}$

$x = 2$

56. (64) Moles of $CuSO_4 \cdot 5H_2O = \frac{80}{249.54}$

Molarity $= \frac{80}{\frac{249.54}{5}} = 64.117 \times 10^{-3}$

Nearest integer, x = 64

57. (2)

Total energy per sec. = 50 J

$$50 = \frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^8}{795 \times 10^{-9}}$$

$n = 1998.49 \times 10^{17}$ [n = no. of photons per second]

$= 1.998 \times 10^{20}$

$\simeq 2 \times 10^{20}$

$= x \times 10^{20}$

$x = 2$

58. (173)

$B_2^+ \Rightarrow \sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\pi_{2py}^1 \simeq \pi_{2pz}^0$

$\Rightarrow 9e^-$

$\mu = \sqrt{1(1+2)} = \sqrt{3}\text{BM}$

$= 1.73\text{BM}$

$= 1.73 \times 10^{-2}\text{BM}$

59. (26)

$k = 1.07 \times 10^6 \text{Sm}^{-1},\ R = 0.243\Omega$

$$G = \frac{1}{R} = \frac{1}{0.243}\Omega^{-1}$$

$k = G \times G^*$

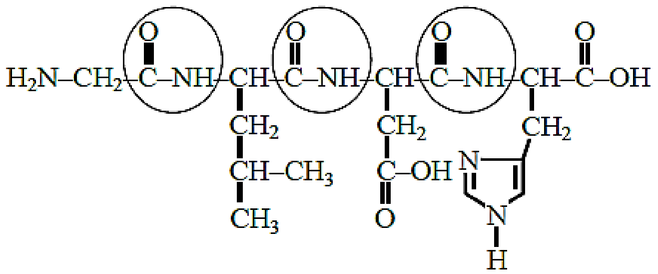
$$G^* = \frac{k}{G} = \frac{1.07 \times 10^6}{\frac{1}{0.243}} \simeq 26 \times 10^4\ \text{m}^{-1}$$

60. (3)

$\boxed{\text{Glycine — leucine — Aspartic acid — Histidine}}$

peptide link　peptide link　peptide link

Total (3) peptide linkages are present



3 peptide linkage

61. (b)

$$\lim_{x\to\frac{\pi}{4}} \frac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)dx}{x^2-\frac{\pi^2}{16}}$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\pi}{4}\cdot\frac{[f(\sec^2 x)\cdot 2\sec x\cdot \sec x\tan x]}{2x}$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\pi}{4} f(\sec^2 x)\cdot \sec^3 x\cdot\frac{\sin x}{x}$$

$$\frac{\pi}{4} f(2)\cdot(\sqrt{2})^3\cdot\frac{1}{\sqrt{2}}\times\frac{4}{\pi}$$

$\Rightarrow$ 2f(2)

62. (c)

$\cos^{-1}(\cos(-5))+\sin^{-1}(\sin(6))-\tan^{-1}(\tan(12))$

$\Rightarrow (2\pi-5)+(6-2\pi)-(12-4\pi)$

$\Rightarrow 4\pi-11.$

63. (c)

$$\Delta=\begin{vmatrix}-1 & 1 & 2\\ 3 & -a & 5\\ 2 & -2 & -a\end{vmatrix}$$

$=-1(a^2+10)-1(-3a-10)+2(-6+2a)$

$=-a^2-10+3a+10-12+4a$

$\Delta=-a^2+7a-12$

$\Delta=-[a^2-7a+12]$

$\Delta=-[(a-3)(a-4)]$

$$\Delta_1=\begin{vmatrix}0 & 1 & 2\\ 1 & -a & 5\\ 7 & -2 & -a\end{vmatrix}$$

$=0-1(-a-35)+2(-2+7a)$

$\Rightarrow a+35-4+14a$

$15a+31$

Now

$\Delta_1=15a+31$

For inconsistent $\Delta=0 \therefore a=3, a=4$

and for a $=3$ and 4 $\Delta_1\neq 0$

$n(S_1) = 2$

For infinite solution: $\Delta = 0$

and $\Delta_1 = \Delta_2 = \Delta_3 = 0$

Not possible

$\therefore n(S_2) = 0$

64. (b) $P_1 : x - 2y - 2z + 1 = 0$

$P_2 : 2x - 3y - 6z + 1 = 0$

$$\left|\frac{x-2y-2z+1}{\sqrt{1+4+4}}\right| = \left|\frac{2x-3y-6z+1}{\sqrt{2^2+3^2+6^2}}\right|$$

$$\frac{x-2y-2z+1}{3} = \pm\frac{2x-3y-6z+1}{7}$$

Since $a_1a_2 + b_1\, b_2 + c_1c_2 = 20 > 0$

$\therefore$ Negative sign will give

acute bisector

$7x - 14y - 14z + 7 = -[6x - 9y - 18z + 3]$

$\Rightarrow 13x - 23y - 32z + 10 = 0$

$\left(-2,0,-\frac{1}{2}\right)$ satisfy it

65. (d)

66. (b) Total ways of choosing square $= {}^{64}C_2$

$$= \frac{64 \times 63}{2 \times 1} = 32 \times 63$$

ways of choosing two squares having common side $= 2(7 \times 8) = 112$

Required probability $= \frac{112}{32\times63} = \frac{16}{32\times9} = \frac{1}{18}$.

67. (d)

$$x^2dy + \left(y - \frac{1}{x}\right)dx = 0 : x > 0, y(1) = 1$$

$$x^2dy + \frac{(xy-1)}{x}dx = 0$$

$$x^2dy = \frac{(xy-1)}{x}dx$$

$$\frac{dy}{dx} = \frac{1-xy}{x^3}$$

$$\frac{dy}{dx}=\frac{1}{x^3}-\frac{y}{x^2}$$

$$\frac{dy}{dx}=\frac{1}{x^2}\cdot y=\frac{1}{x^3}$$

$$\text{If } e^{\int\frac{1}{x^2}dx}=e^{-\frac{1}{x}}$$

$$ye^{-\frac{1}{x}}=\int\frac{1}{x^3}\cdot e^{-\frac{1}{x}}$$

$$ye^{-\frac{1}{x}}=e^{-x}\left(1+\frac{1}{x}\right)+C$$

$$1.e^{-1}=e^{-1}(2)+C$$

$$C=-e^{-1}=-\frac{1}{e}$$

$$ye^{-\frac{1}{x}}=e^{-\frac{1}{x}}\left(1+\frac{1}{x}\right)-\frac{1}{e}$$

$$y\left(\frac{1}{2}\right)=3-\frac{1}{e}\times e^2$$

$$y\left(\frac{1}{2}\right)=3-e$$

68. (a)

$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1$$

$$2\cos x\left(4\left(\sin^2\frac{\pi}{4}-\sin^2 x\right)-1\right)=1$$

$$2\cos x\left(4\left(\frac{1}{2}-\sin^2 x\right)-1\right)=1$$

$$2\cos x(2-4\sin^2 x-1)=1$$

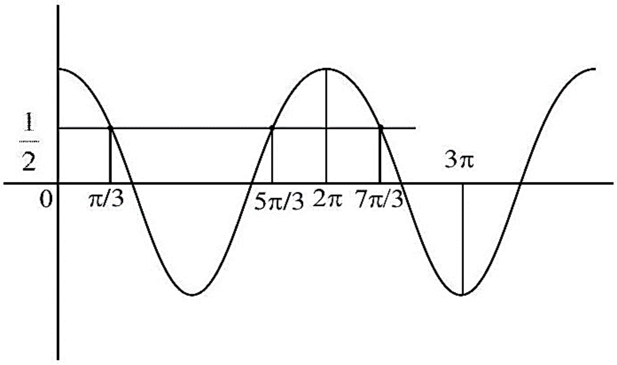
$$2\cos x(1-4\sin^2 x)=1$$

$$2\cos x(4\cos^2 x-3)=1$$

$$4\cos^3 x-3\cos x=\frac{1}{2}$$

$$\cos 3x=\frac{1}{2}$$

$$x\in[0,\pi]\therefore 3x\in[0,3\pi]$$



69. (a)

$f(x) = x^3 - 6x^2 + ax + b$

$f(\mathrm{b}) = 8 - 24 + 2a + b = 0$

$2\mathrm{a} + \mathrm{b} = 16$

$f(\mathrm{d}) = 64 - 96 + 4a + b = 0$

$4a + b = 32$

Solving (1) and (2)

$\mathrm{a} = 8, \mathrm{b} = 0$

$f(x) = x^3 - 6x^2 + 8x$

$f(x) = x^3 - 6x^2 + 8x$

$\mathrm{f}'(\mathrm{x}) = 3\mathrm{x}^2 - 12\mathrm{x} + 8$

$\mathrm{f}''(\mathrm{x}) = 6\mathrm{x} - 12$

$\Rightarrow$ $\mathrm{f}'(\mathrm{x})$ is $\uparrow$ for $\mathrm{x} > 2$, and $\mathrm{f}'(\mathrm{x})$ is $\downarrow$ for $\mathrm{x} < 2$

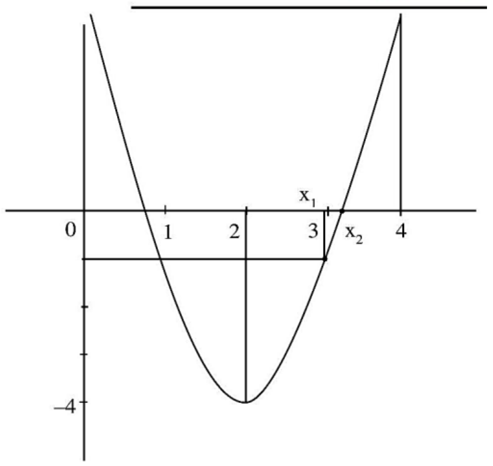
$f'(\mathrm{b}) = 12 - 24 + 8 = -4$

$f'(\mathrm{d}) = 48 - 48 + 8 = 8$

$f'(x) = 3x^2 - 12x + 8$

vertex $(2, -4)$

$f'(\mathrm{b}) = -4, f'(\mathrm{d}) = 8, f'(\mathrm{c}) = 27 - 36 + 8$



$f'(x_1) = -1$, then $x_1 = 3$

$f'(x_2) = 0$

Again

$f'(x) < 0$ for $x \in (2, x_4)$
$f'(x) > 0$ for $x \in (x_4, 4)$

$x_4 \in (3,4)$

$f(x) = x^3 - 6x^2 + 8x$

$f(c) = 27 - 54 + 24 = -3$

$f(d) = 64 - 96 + 32 = 0$

For $x_4(3,4)$

$f(x_4) < -3\sqrt{3}$

and $f'(x_3) > -4$

$2f'(x_3) > -8$

So, $2f'(x_3) = \sqrt{3}f(x_4)$

70. (c)
71. (a)

$$A = \int_0^{\pi/2}((\sin x + \cos x) - |\cos x - \sin x|)dx$$

$$A = \int_0^{\pi/2}((\sin x + \cos x) - (\cos x - \sin x))dx$$

$$+\int_{\pi/4}^{\pi/2}((\sin x + \cos x) - (\sin x - \cos x))dx$$

$$A = 2\int_0^{\pi/2}\sin x dx + 2\int_{\pi/4}^{\pi/2}\cos x dx$$

$$A = -2\left(\frac{1}{\sqrt{2}} - 1\right) + 2\left(1 - \frac{1}{\sqrt{2}}\right)$$

$A=4-2\sqrt{2}=2\sqrt{2}(\sqrt{2}-1)$

72. (c)

$3y-2z-1=0=3x-z+4$
$3y-2z-1=0$
D.R's $\Rightarrow(0,3,-2)$
$3x-z+4=0$
D.R's $\Rightarrow(3,-1,0)$

Let DR's of given line are a, b, c

Now $3b-2c=0 \& 3a-c=0$

$\therefore 6a=3b=2c$

$a:b:c=3:6:9$

Any pt on line

$3K-1, 6K+1, 9K+1$
Now $3(3K-1)+6(6K+1)1+9(9K+1)=0$

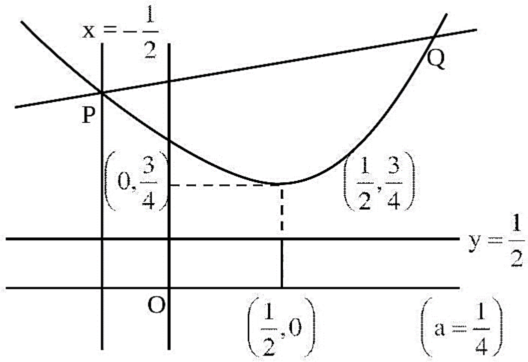
$\Rightarrow K=\frac{1}{3}$

Point on line $\Rightarrow(0,3,4)$

Given point $(2,-1,6)$

$\Rightarrow$ Distance $=\sqrt{4+16+4}=2\sqrt{6}$

73. (b)



$\left(y-\frac{3}{4}\right)=\left(x-\frac{1}{2}\right)^2$

For $x=-\frac{1}{2}$

$y-\frac{3}{4}=1\Rightarrow y=\frac{7}{4}\Rightarrow P\left(-\frac{1}{2},\frac{7}{4}\right)$

Now $y'=2\left(x-\frac{1}{2}\right)$ At $x=-\frac{1}{2}$

$\Rightarrow m_T = -2,\ m_N = \dfrac{1}{2}$

Equation of Normal is

$y - \dfrac{7}{4} = \dfrac{1}{2}\left(x + \dfrac{1}{2}\right)$

$y = \dfrac{x}{2} + 2$

Now put y in equation (a)

$\dfrac{x}{2} + 2 - \dfrac{3}{4} = \left(x - \dfrac{1}{2}\right)^2$

$\Rightarrow x = 2 \& -\dfrac{1}{2}$

$\Rightarrow Q(2,3)$

Now $(PQ)^2 = \frac{125}{16}$

74. (a)

Consider the equation $x^2 + ax + b = 0$

If has two roots (not necessarily real $\alpha \& \beta$)

Either $\alpha = \beta$ or $\alpha \neq \beta$

Case (1) If $\alpha = \beta$, then it is repeated root. Given that $\alpha^2 - 2$ is also a root

So, $\alpha = \alpha^2 - 2 \Rightarrow (\alpha + 1)(\alpha - 2) = 0$

$\Rightarrow \alpha = -1$ or $\alpha = 2$

When $\alpha = -1$ then $(a, b) = (2,1)$

$\alpha = 2$ then $(a, b) = (-4,4)$

Case (2) If $\alpha \neq \beta$ Then

(I) $\alpha = \alpha^2 - 2$ and $\beta = \beta^2 - 2$
Here $(\alpha, \beta) = (2, -1)$ or $(-1,2)$

Hence $(a, b) = (-(\alpha + \beta), \alpha\beta)$

$= (-1, -2)$

(II) $\alpha = \beta^2 - 2$ and $\beta = \alpha^2 - 2$

Then $\alpha - \beta = \beta^2 - \alpha^2 = (\beta - \alpha)(\beta + \alpha)$

Since $\alpha \neq \beta$ we get $\alpha + \beta = \beta^2 + \alpha^2 - 4$

$\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta - 4$

Thus $-1 = 1 - 2\alpha\beta - 4$ which implies

$\alpha\beta = -1$ Therefore $(a, b) = (-(\alpha+\beta), \alpha\beta)$

$= (1, -1)$

(III) $\alpha = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$

$\Rightarrow \alpha = -\beta$

Thus $\alpha = 2, \beta = -2$

$\alpha = -1, \beta = 1$

Therefore $(a, b) = (0, -4) \& (0, -1)$

(IV) $\beta = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$ is same as (III)

Therefore we get 6 pairs of (a,b)

Which are $(2,1), (-4,4), (-1,-2), (1,-1) (0,-4)$

75. (a) Let $T_r = r(n-r)$

$T_r = nr - r^2$

$\Rightarrow S_n = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n} (nr - r^2)$

$$S_n = \frac{n \cdot (n)(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \frac{n(n-1)(n+1)}{6}$$

Now $\sum_{r=4}^{\infty} \left(\frac{2\,S_n}{n!} - \frac{1}{(n-2)!}\right)$

$$= \sum_{r=4}^{\infty} \left(2 \cdot \frac{n(n-1)(n+1)}{6 \cdot n(n-1)(n-2)!} - \frac{1}{(n-2)!}\right)$$

$$= \sum_{r=4}^{\infty} \left(\frac{1}{3}\left(\frac{n-2+3}{(n-2)!}\right) - \frac{1}{(n-2)!}\right)$$

$$= \sum_{r=4}^{\infty} \frac{1}{3} \cdot \frac{1}{(n-3)!} = \frac{1}{3}(e-1)$$

76. (c) Total Number of Triangles $= {}^{15}C_3$

$i + j + k = 15$ (Given)

5 Cases

i	j	k
1	2	12
1	3	11
1	4	10
1	5	9
1	6	8

4 Cases

i	j	k
2	3	10
2	4	9
2	5	8
2	6	7

3 Cases

i	j	k
3	4	8
3	5	7

1 Cases

i	j	k
4	5	6

Number of Possible triangles using the vertices P_i, P_j, P_k such that $i+j+k \neq 15$ is equal to ${}^{15}C_3 - 12 = 443$

77. (d)

$\mathrm{f(x)} = \log_{\sqrt{5}}$

$$\left(3+\cos\left(\frac{3\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{3\pi}{4}-x\right)\right)$$

$$f(x)=\log_{\sqrt{5}}\left[3+2\cos\left(\frac{\pi}{4}\right)\cos(x)-2\sin\left(\frac{3\pi}{4}\right)\sin(x)\right]$$

$$f(x)=\log_{\sqrt{5}}[3+\sqrt{2}(\cos x-\sin x)]$$

Since $-\sqrt{2} \leq \cos x - \sin x \leq \sqrt{2}$

$$\Rightarrow \log_{\sqrt{5}}[3+\sqrt{2}(-\sqrt{2}) \leq \mathrm{f(x)} \leq \log_{\sqrt{5}}[3+\sqrt{2}(\sqrt{2})]]$$

$$\Rightarrow \log_{\sqrt{5}}(\mathrm{a}) \leq \mathrm{f(x)} \leq \log_{\sqrt{5}}(5)$$

So Range of $f(x)$ is $[0,2]$

78. (b) $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \sum_{n=1}^{20} \frac{1}{a_n(a_n+d)}$

$$=\frac{1}{\mathrm{d}}\sum_{\mathrm{n}=1}^{20}\left(\frac{1}{\mathrm{a_n}}-\frac{1}{\mathrm{a_n}+\mathrm{d}}\right)$$

$$\Rightarrow \frac{1}{\mathrm{d}}\left(\frac{1}{\mathrm{a_1}}-\frac{1}{\mathrm{a_{21}}}\right)=\frac{4}{9}\text{(Given)}$$

$$\Rightarrow \frac{1}{\mathrm{d}}\left(\frac{\mathrm{a_{21}}-\mathrm{a_1}}{\mathrm{a_1 a_{21}}}\right)=\frac{4}{9}$$

$$\Rightarrow \frac{1}{\mathrm{d}}\left(\frac{\mathrm{a_1}+20\,\mathrm{d}-\mathrm{a_1}}{\mathrm{a_1 a_2}}\right)=\frac{4}{9}\Rightarrow \mathrm{a_1 a_2}=45$$

Now sum of first 21 terms $=\frac{21}{2}(2a_1+20d)=189$

$\Rightarrow \mathrm{a_1} + 10\,\mathrm{d} = 9$ ... (2)

For equation (1) & (2) we get

$\mathrm{a_1} = 3$& $\mathrm{d} = \frac{3}{5}$

OR

$a_1 = 15\&\ d = -\frac{3}{5}$

So, $a_6 \cdot a_{16} = (a_1 + 5\,d)(a_1 + 15\,d)$

$\Rightarrow a_6 a_{16} = 72$

79. (d) $f(x) = x + \int_0^{\pi/2} \sin x \cos y f(y) dy$

$f(x) = x + \sin x \underbrace{\int_0^{\pi/2} \cos y f(y) dy}_{K}$

$\Rightarrow f(x) = x + K \sin x$

$\Rightarrow f(y) = y + K \sin y$

Now $K = \int_0^{\pi/2} \cos y (y + K \sin y) dy$

$K = \int_0^{\pi/2} y \cos dy + \int_0^{\pi/2} \underset{\text{Apply IBP}}{\cos y} \sin y dy$

$K = (y \sin y)_0^{\pi/2} - \int_0^{\pi/2} \sin dy + K \int_0^1 t dt$

$\Rightarrow K = \frac{\pi}{2} - 1 + K\left(\frac{1}{2}\right)$

$\Rightarrow K = \pi - 2$

So $f(x) = x + (\pi - 2) \sin x$

80. (b) The point of intersection of the curves $\frac{x^2}{9} + \frac{y^2}{1} = 1$ and $x^2 + y^2 = 3$ in the first quadrant is $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

Now slope of tangent to the ellipse $\frac{x^2}{9} + \frac{y^2}{1} = 1$ at $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$ is $m_1 = -\frac{1}{3\sqrt{3}}$

And slope of tangent to the circle at $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$ is $m_2 = -\sqrt{3}$

So, if angle between both curves is θ then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\frac{1}{3\sqrt{3}} + \sqrt{3}}{1 + \left(-\frac{1}{3\sqrt{3}}(-\sqrt{3})\right)}\right| = \frac{2}{\sqrt{3}}$$

81. (781)

x	-2	-1	3	4	6
$P(X = x)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

$\bar{X} = 2.3$

$-a + 6\,b = \frac{9}{10}$

$\sum P_i = \frac{1}{5} + a + \frac{1}{3} + \frac{1}{5} + b = 1$

$a + b = \frac{4}{15}$

From equation (a) and (b)

$a = \frac{1}{10}, \quad b = \frac{1}{6}$

$\sigma^2 = \Sigma p_i x_i^2 - (\bar{X})^2$

$\frac{1}{5}(d) + a(a) + \frac{1}{3}(9) + \frac{1}{5}(16) + b(36) - (2.3)^2$

$= \frac{4}{5} + a + 3 + \frac{16}{5} + 36\,b - (2.3)^2$

$= 4 + a + 3 + 36\,b - (2.3)^2$

$= 7 + a + 36\,b - (2.3)^2$

$= 7 + \frac{1}{10} + 6 - (2.3)^2$

$= 13 + \frac{1}{10} - \left(\frac{23}{10}\right)^2$

$= \frac{131}{10} - \left(\frac{23}{10}\right)^2$

$= \frac{1310 - (23)^2}{100}$

$= \frac{1310 - 529}{100}$

$\sigma^2 = \frac{781}{100}$

$100\sigma^2 = 781$

82. (7) $\lim_{x\to 1} \frac{x^n f(a) - f(x)}{x-1} = 44$

$\lim_{x\to 1} \frac{9x^n - (x^6 + 2x^4 + x^3 + 2x + 3)}{x - 1} = 44$

$\lim_{x\to 1} \frac{9nx^{n-1} - (6x^5 + 8x^3 + 3x^2 + 2)}{1} = 44$

$\Rightarrow 9n - (19) = 44$

$\Rightarrow 9n = 63$

$\Rightarrow n = 7$

83. (5) $|z - 2 - 2i| \leq 1$

$|x + iy - 2 - 2i| \leq 1$

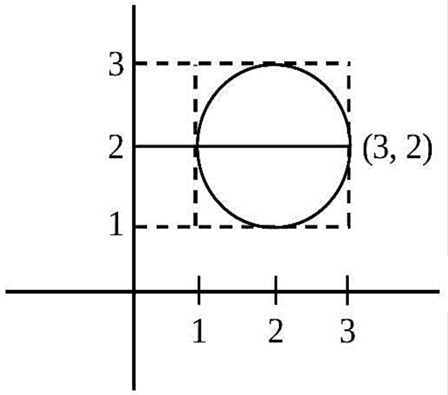
$|(x - 2) + i(y - 2)| \leq 1$

$(x - 2)^2 + (y - 2)^2 \leq 1$

$|3iz + 6|_{max}$ at $a + ib$

$|3i|\left|z + \frac{6}{3i}\right|$

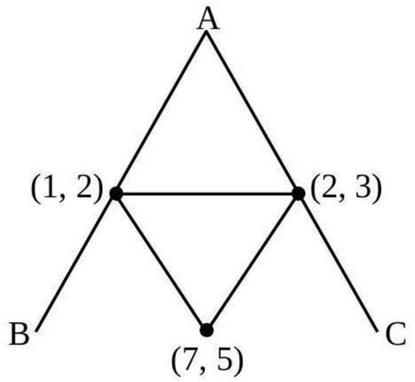
$3|z - 2i|_{max}$



From Figure maximum distance at $3 + 2i$

$a + ib = 3 + 2i = a + b = 3 + 2 = 5$

84. (6) intersection point of give lines are $(1,2), (7,5), (2,3)$



$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & 5 & 1 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(5 - 3) - 2(7 - 2) + 1(21 - 10)]$$

$= \frac{1}{2}[2 - 10 + 11]$

$\Delta DEF = \frac{1}{2}(c) = \frac{3}{2}$

$\Delta ABC = 4\Delta DEF = 4\left(\frac{3}{2}\right) = 6$

85. (26)

$kf(k) + 2 = \lambda(x - 2)(x - 3)(x - 4)(x - 5) \ldots (a)$

put $x = 0$

we get $\lambda = \frac{1}{60}$

Now put λ in equation (a)

$\Rightarrow kf(k) + 2 = \frac{1}{60}(x - 2)(x - 3)(x - 4)(x - 5)$

Put $x = 10$

$\Rightarrow 10f(10) + 2 = \frac{1}{60}(8)(7)(6)(5)$

$\Rightarrow 52 - 10f(10) = 52 - 26 = 26$

86. (77)

87. (924)

$(x + y)^n \Rightarrow 2^n = 4096\ 2^{10} = 1024 \times 2$

$\Rightarrow 2^n = 2^{12}\ 2^{11} = 2048$

$n = 12\ 2^{12} = \underline{4096}$

${}^{12}C_6 = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= 11 \times 3 \times 4 \times 7$

$= 924$

88. (1494)

$\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\vec{c} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\vec{v} = x\vec{a} + y\vec{b}\ \vec{v}(3\hat{\imath} + 2\hat{\jmath} - k) = 0$

$\vec{v} \cdot \hat{a} = 19$

$\vec{v} = \lambda\vec{c} \times (\vec{a} \times \vec{b})$

$\vec{v} = \lambda[(\vec{c} \cdot \vec{b})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b}]$

$= \lambda\left[(3 + 4 + 1)(2\hat{\imath} - \hat{\jmath} + 2\hat{k}) - \left(\frac{6 - 2 - 2}{2}\right)(\hat{\imath} + 2\hat{\jmath} + \hat{k})\right.$

$= \lambda[16\hat{\imath} - 8\hat{\jmath} + 16\hat{k} - 2\hat{\imath} - 4\hat{\jmath} + 2\hat{k}]$

$\vec{v} = \lambda[14\hat{\imath} - 12\hat{\jmath} + 18\hat{k}]$

$\lambda[14\hat{\imath} - 12\hat{\jmath} + 18\hat{k}] \cdot \frac{(2\hat{\imath} - \hat{\jmath} + 2\hat{k})}{\sqrt{4 + 1 + 4}} = 19$

$\lambda\frac{[28 + 12 + 36]}{3} = 19$

$\lambda\left(\frac{76}{3}\right) = 19$

$4\lambda = 3 \Rightarrow \lambda = \frac{3}{4}$

$|2v^2| = \left|2 \times \frac{3}{4}(14\hat{\imath} - 12\hat{\jmath} + 18\hat{k})\right|^2$

$\frac{9}{4} \times 4(7\hat{\imath} - 6\hat{\jmath} + 9\hat{k})^2$

$= 9(49 + 36 + 81)$

$= 9(166)$

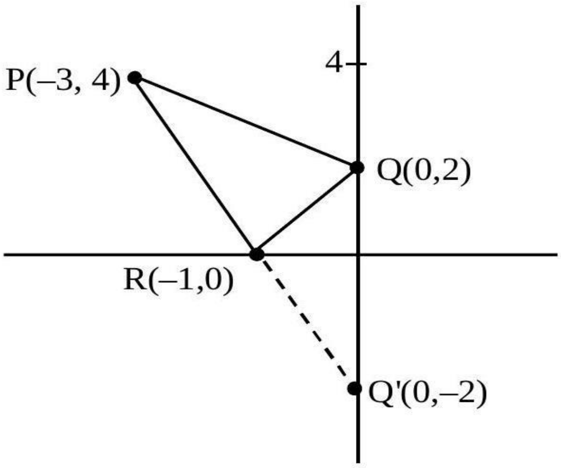
$= 1494$

89. **(2)** $f(x) = [x]|x^2 - 1| + \sin\frac{\pi}{[x+3]} - [x + 1]$

$$f(x) = \begin{cases} 3 - 2x^2, & -2 < x < -1 \\ x^2, & -1 \le x < 0 \\ \frac{\sqrt{3}}{2} + 1 & 0 \le x < 1 \\ x^2 + 1 + \frac{1}{\sqrt{2}}, & 1 \le x < 2 \end{cases}$$

discontinuous at $x = 0,1$

90. **(1250)**



$50(PR^2 + RQ^2)$

$50(20 + 5)$

$50(25)$

$= 1250$