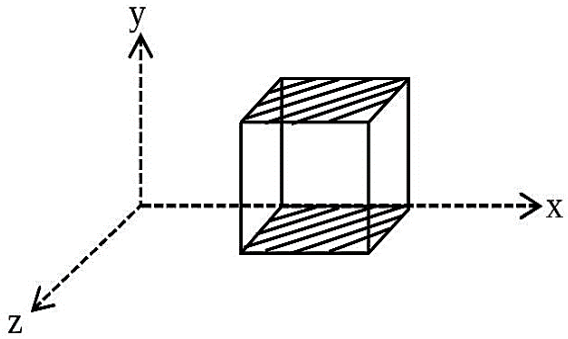
PART-I PHYSICS

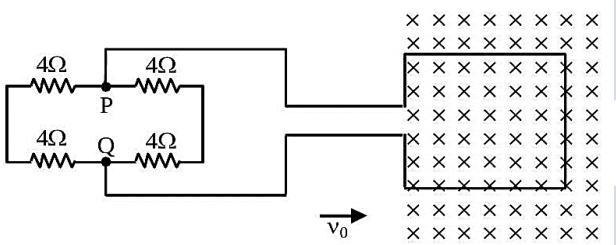
SECTION-A

1. **A cube is placed inside an electric field, $\vec{E} = 150y^2\hat{j}$. The side of the cube is 0.5 m and is placed in the field as shown in the given figure. The charge inside the cube is:**



(a) 3.8×10^{-11}C (b) 8.3×10^{-11}C
(c) 3.8×10^{-12}C (d) 8.3×10^{-12}C

2. **A square loop of side 20 cm and resistance 1Ω is moved towards right with a constant speed v_0. The right arm of the loop is in a uniform magnetic field of 5T. The field is perpendicular to the plane of the loop and is going into it. The loop is connected to a network of resistors each of value 4Ω. What should be the value of v_0 so that a steady current of 2 mA flows in the loop?**



(a) 1 m/s (b) 1 cm/s
(c) 10^2 m/s (d) 10^{-2} cm/s

3. **The temperature of an ideal gas in 3-dimensions is 300 K. The corresponding de-Broglie wavelength of the electron approximately at 300 K, is:**

[m_e = mass of electron = 9×10^{-31} kg

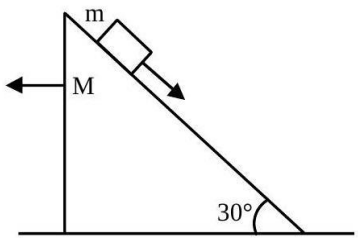
h = Planck constant = 6.6×10^{-34}Js

k_B = Boltzmann constant = 1.38×10^{-23}JK^{-1}]

(a) 6.26 nm (b) 8.46 nm
(c) 2.26 nm (d) 3.25 nm

4. **A body of mass 'm' dropped from a height ' h ' reaches the ground with a speed of $0.8\sqrt{gh}$. The value of workdone by the air-friction is:**
   (a) -0.68mgh (b) mgh
   (c) 1.64mgh (d) 0.64mgh
5. **The ranges and heights for two projectiles projected with the same initial velocity at angles 42° and 48° with the horizontal are R_1, R_2 and H_1, H_2 respectively. Choose the correct option :**
   (a) $R_1 > R_2$ and $H_1 = H_2$
   (b) $R_1 = R_2$ and $H_1 < H_2$
   (c) $R_1 < R_2$ and $H_1 < H_2$
   (d) $R_1 = R_2$ and $H_1 = H_2$

6. A block of mass m slides on the wooden wedge, which in turn slides backward on the horizontal surface. The acceleration of the block with respect to the wedge is : Given $m = 8$ kg, $M = 16$ kg Assume all the surfaces shown in the figure to be frictionless.



(a) $\frac{4}{3}g$ (b) $\frac{6}{5}g$
(c) $\frac{3}{5}g$ (d) $\frac{2}{3}g$

7. Due to cold weather a 1 m water pipe of cross-sectional area 1 cm^2 is filled with ice at $-10°C$. Resistive heating is used to melt the ice. Current of 0.5 A is passed through 4kΩ resistance. Assuming that all the heat produced is used for melting, what is the minimum time required?

(Given latent heat of fusion for water/ice $= 3.33 \times 10^5$ J kg^{-1}, specific heat of ice $= 2 \times 10^3$ J kg^{-1} and density of ice $= 10^3$ kg/m^3
(a) 0.353 s (b) 35.3 s
(c) 3.53 s (d) 70.6 s

8. A student determined Young's Modulus of elasticity using the formula $Y = \frac{MgL^3}{4bd^3\delta}$. The value of g is taken to be 9.8 m/s^2, without any significant error, his observation are as following.

Physical Quantity	Least count of the Equipment used for measurement	Observed value
Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (b)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

Then the fractional error in the measurement of Y is:
(a) 0.0083 (b) 0.0155
(c) 0.155 (d) 0.083

9. Two resistors $R_1 = (4 \pm 0.8)\Omega$ and $R_2 = (4 \pm 0.4)\Omega$ are connected in parallel. The equivalent resistance of their parallel combination will be:

(a) $(4 \pm 0.4)\Omega$ (b) $(2 \pm 0.4)\Omega$

(c) $(2 \pm 0.3)\Omega$ (d) $(4 \pm 0.3)\Omega$

10. The half -life period of radioactive element x is same as the mean life time of another radioactive element y. Initially they have the same number of atoms. Then:

(a) x-will decay faster than y.

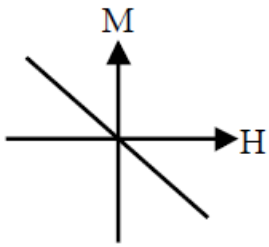
(b) y - will decay faster than x.

(c) x and y have same decay rate initially and later on different decay rate.

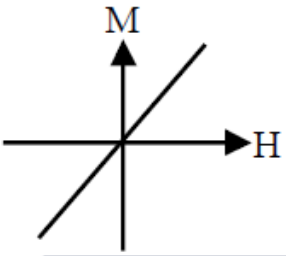
(d) x and y decay at the same rate always.

11. Following plots show Magnetization (M) vs Magnetising field (H) and Magnetic susceptibility (χ) vs temperature (T) graph

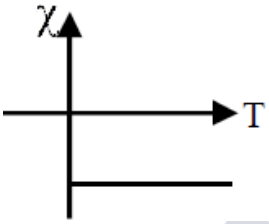
(a)



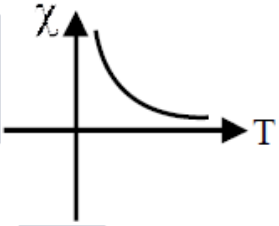
(b)



(c)



(d)



Which of the following combination will be represented by a diamagnetic material?

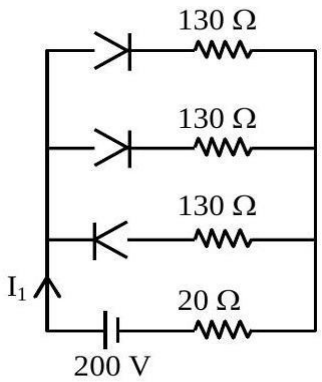
(a) (A), (C) (b) (A), (D)
(c) (B), (D) (d) (B), (C)

12. A glass tumbler having inner depth of 17.5 cm is kept on a table. A student starts pouring water ($\mu = 4/3$) into it while looking at the surface of water from the above. When he feels that the tumbler is half filled, he stops pouring water. Up to what height, the tumbler is actually filled?

(a) 11.7 cm (b) 10 cm

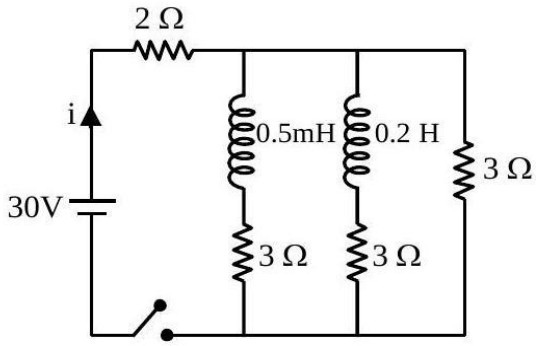
(c) 7.5 cm (d) 8.75 cm

13. In the given figure, each diode has a forward bias resistance of 30Ω and infinite resistance in reverse bias. The current I_1 will be:



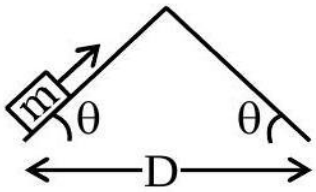
(a) 3.75 A (b) 2.35 A
(c) 2 A (d) 2.73 A

14. For the given circuit the current *i* through the battery when the key in closed and the steady state has been reached is

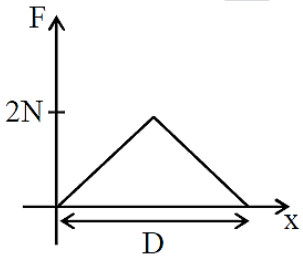


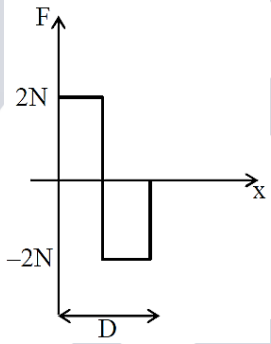
(a) 6 A (b) 25 A
(c) 10 A (d) 0 A

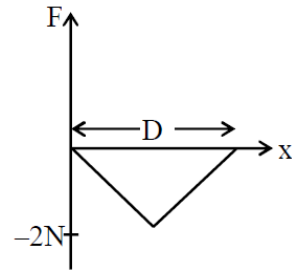
15. An object of mass ' m ' is being moved with a constant velocity under the action of an applied force of 2 N along a frictionless surface with following surface profile.

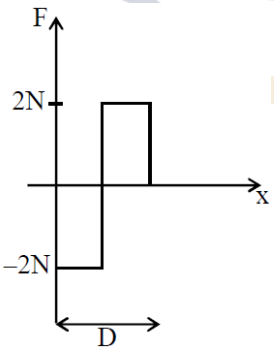


The correct applied force vs distance graph will be:

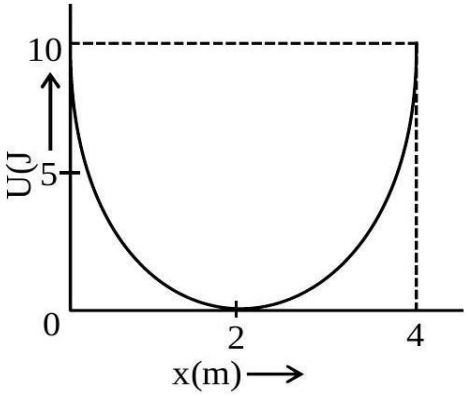
(a)


(b)


(c)


(d)


16. A mass of 5 kg is connected to a spring. The potential energy curve of the simple harmonic motion executed by the system is shown in the figure. A simple pendulum of length 4 m has the same period of oscillation as the spring system. What is the value of acceleration due to gravity on the planet where these experiments are performed?

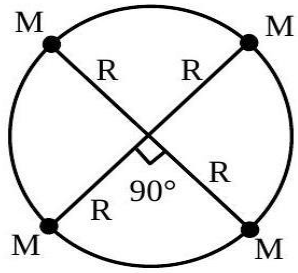


(a) 10 m/s^2 (b) 5 m/s^2
(c) 4 m/s^2 (d) 9.8 m/s^2

17. A capacitor is connected to a 20 V battery through a resistance of 10Ω. It is found that the potential difference across the capacitor rises to 2 V in 1μs. The capacitance of the capacitor is μF. Given: $\ln\left(\frac{10}{9}\right) = 0.105$

(a) 9.52 (b) 0.95
(c) 0.105 (d) 1.85

18. Four particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is:

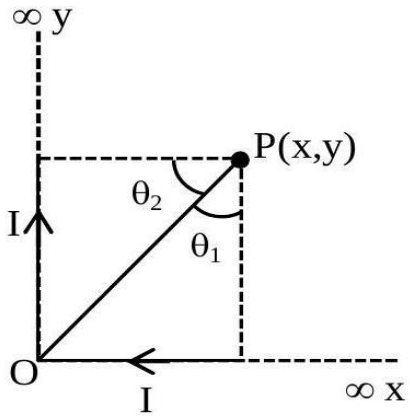


(a) $\frac{1}{2}\sqrt{\frac{GM}{R(2\sqrt{2}+1)}}$ (b) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$
(c) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}-1)}$ (d) $\sqrt{\frac{GM}{R}}$

19. Electric field of plane electromagnetic wave propagating through a non-magnetic medium is given by $E = 20\cos(2\times10^{10}t - 200x)V/m$. The dielectric constant of the medium is equal to: (Take $\mu_r = 1$)

(a) 9 (b) 2
(c) $\frac{1}{3}$ (d) 3

20. There are two infinitely long straight current carrying conductors and they are held at right angles to each other so that their common ends meet at the origin as shown in the figure given below. The ratio of current in both conductor is 1: 1. The magnetic field at point *P* is



(a) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}+(x+y)\right]$

(b) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}-(x+y)\right]$

(c) $\frac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}-(x+y)\right]$

(d) $\frac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}+(x+y)\right]$

SECTION-B

21. The temperature of 3.00 mol of an ideal diatomic gas is increased by 40.0°C without changing the pressure of the gas. The molecules in the gas rotate but do not oscillate. If the ratio of change in internal energy of the gas to the amount of workdone by the gas is $\frac{x}{10}$. Then the value of x (round off to the nearest integer) is (Given R = 8.31 J mol^{-1} K^{-1})

22. The width of one of the two slits in a Young's double slit experiment is three times the other slit. If the amplitude of the light coming from a slit is proportional to the slit-width, the ratio of minimum to maximum intensity in the interference pattern is x: 4 where x is

23. Two satellites revolve around a planet in coplanar circular orbits in anticlockwise direction. Their period of revolutions are 1 hour and 8 hours respectively. The radius of the orbit of nearer satellite is 2×10^3 km. The angular speed of the farther satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\frac{\pi}{x}$ rad $^{-1}$ where x is .....

24. When a body slides down from rest along a smooth inclined plane making an angle of 30° with the horizontal, it takes time T. When the same body slides down from the rest along a rough inclined plane making the same angle and through the same distance, it takes time αT, where α is a constant greater than 1. The co-efficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}}\left(\frac{\alpha^2-1}{\alpha^2}\right)$ where x =

25. The average translational kinetic energy of N_2 gas molecules at . °C becomes equal to the K.E. of an electron accelerated from rest through a potential difference of 0.1 volt.

(Given $k_B = 1.38 \times 10^{-23}$ J/K) (Fill the nearest integer).

26. A uniform heating wire of resistance 36Ω is connected across a potential difference of 240 V. The wire is then cut into half and potential difference of 240 V is applied across each half separately. The ratio of power dissipation in first case to the total power dissipation in the second case would be 1: x, where x is

27. A steel rod with $y = 2.0 \times 10^{11} Nm^{-2}$ and $\alpha = 10^{-5}\,^{\circ}C^{-1}$ of length 4 m and area of cross-section 10 cm^2 is heated from $0^{\circ}C$ to $400^{\circ}C$ without being allowed to extend. The tension produced in the rod is $x \times 10^5$ N where the value of x is

28. A 2 kg steel rod of length 0.6 m is clamped on a table vertically at its lower end and is free to rotate in vertical plane. The upper end is pushed so that the rod falls under gravity, Ignoring the friction due to clamping at its lower end, the speed of the free end of rod when it passes through its lowest position is ms^{-1}. (Take $g = 10\ ms^{-2}$)

29. A carrier wave with amplitude of 250 V is amplitude modulated by a sinusoidal base band signal of amplitude 150 V. The ratio of minimum amplitude to maximum amplitude for the amplitude modulated wave is $50 : x$, then value of x is

30. An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 40,000 kg is moving with a speed of $72 kmh^{-1}$ when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, the spring constant is $\times 10^5$ N/m.

PART-II CHEMISTRY

SECTION-A

31. Water sample is called cleanest on the basis of which one of the BOD values given below
(a) 11ppm (b) 15ppm
(c) 3ppm (d) 21ppm

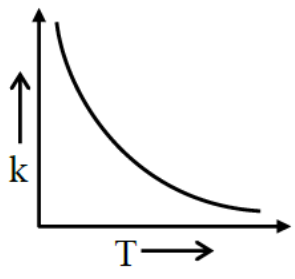
32. Calamine and Malachite, respectively, are the ores of:

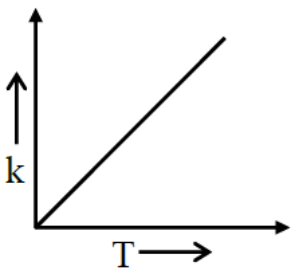
(a) Nickel and Aluminium (b) Zinc and Copper

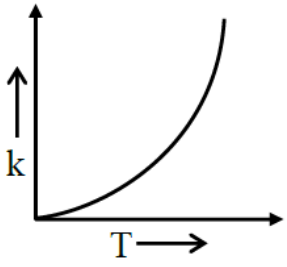
(c) Copper and Iron (d) Aluminium and Zinc

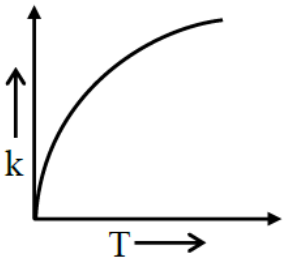
33. Experimentally reducing a functional group cannot be done by which one of the following reagents?
(a) $Pt - C/H_2$ (b) Na/H_2
(c) $Pd - C/H_2$ (d) Zn/H_2O

34. Which one of the following given graphs represents the variation of rate constant (k) with temperature (T) for an endothermic reaction?

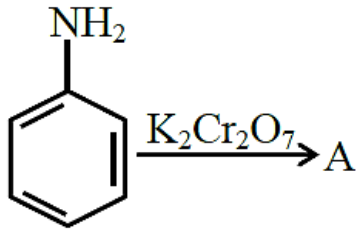
(a) 

(b) 

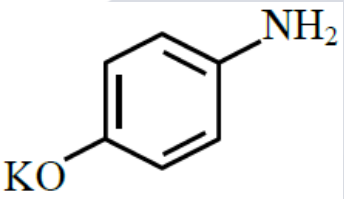
(c) 

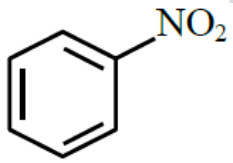
(d) 

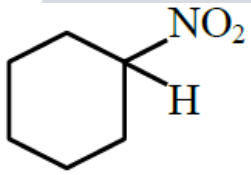
35. Identify A in the following reaction.



(a) 

(b) 

(c) 

(d) 

36. In the following sequence of reactions a compound

A, (molecular formula $C_6H_{12}O_2$) with a straight chain structure gives a C_4 carboxylic acid. A is :

$A \xrightarrow[H_3O^+]{LiAlH_4} B \xrightarrow{Oxidation} C_4$ - carboxylic acid

(a) $CH_3 - CH_2 - COO - CH_2 - CH_2 - CH_3$

(b) $CH_3 - CH_2 - CH - CH_2 - O - CH = CH_2$

(c) $CH_3 - CH_2 - CH_2 - COO - CH_2 - CH_3$

(d) $CH_3 - CH_2 - CH_2 - O - CH = CH - CH_2 - OH$

37. Match List - I with List - II.

List -I (Colloid Preparation Method)		List -II (Chemical Reaction)	
(a)	Hydrolysis	(i)	$2AuCl_3 + 3HCHO + 3H_2O \rightarrow 2Au(sol) + 3HCOOH + 6HCl$
(b)	Reduction	(ii)	$As_2O_3 + 3H_2S \rightarrow As_2S_3(sol) + 3H_2O$
(c)	Oxidation	(iii)	$SO_2 + 2H_2S \rightarrow 3S(sol) + 2H_2O$
(d)	Double Decomposition	(iv)	$FeCl_3 + 3H_2O \rightarrow Fe(OH)_3(sol) + 3HCl$

Choose the most appropriate answer from the options given below.

(a) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

(b) (a)-(iv), (b)-(i), (c)-(iii), (d)-(ii)

(c) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(d) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

38. The Crystal Field Stabilization Energy (CFSE) and magnetic moment (spin-only) of an octahedral aqua complex of a metal ion (M^{2+}) are $-0.8\Delta_0$ and 3.87 BM, respectively. Identify (M^{z+}) :

(a) V^{3+} (b) Cr^{3+}

(c) Mn^{4+} (d) Co^{2+}

39. Monomer units of Dacron polymer are:

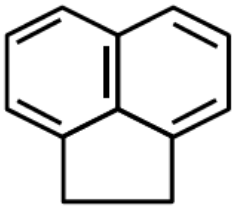
(a) ethylene glycol and phthalic acid

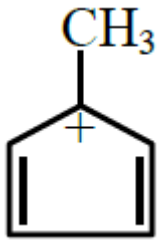
(b) ethylene glycol and terephthalic acid

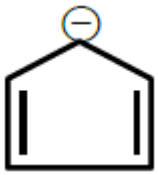
(c) glycerol and terephthalic acid

(d) glycerol and phthalic acid

40. Which one of the following compounds is aromatic in nature?

(a)  (b) 

(c) 

(d) 

41. In the given chemical reaction, colors of the Fe^{2+} and Fe^{3+} ions, are respectively:

$$5Fe^{2+} + MnO_4^- + 8H^+ \rightarrow Mn^{2+} + 4H_2O + 5Fe^{3+}$$

(a) Yellow, Orange (b) Yellow, Green
(c) Green, Orange (d) Green, Yellow

42. The stereoisomers that are formed by electrophilic addition of bromine to trans-but-2-ene is/are :

(a) 2 enantiomers and 2 mesomers

(b) 2 identical mesomers

(c) 2 enantiomers

(d) 1 racemic and 2 enantiomers

43. Hydrogen peroxide reacts with iodine in basic medium to give:

(a) IO_4^- (b) IO^-
(c) I (d) IO_3^-

44. In the following sequence of reactions,

$$C_3H_6 \xrightarrow{H^+/H_2O} A \xrightarrow[\text{dil KOH}]{KIO} B + C$$

The compounds B and C respectively are :

(a) $CI_3COOK, HCOOH$ (b) CI_3COOK, CH_3I
(c) CH_3, HCOOK (d) CHI_3, CH_3COOK

45. Given below are two statements:

Statement I : The nucleophilic addition of sodium hydrogen sulphite to an aldehyde or a ketone involves proton transfer to form a stable ion.

Statement II : The nucleophilic addition of hydrogen cyanide to an aldehyde or a ketone yields amine as final product.

In the light of the above statements, choose the most appropriate answer from the options given below :

(a) Both Statement I and Statement II are true.

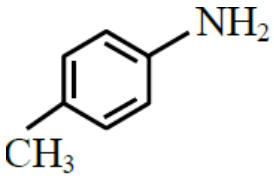
(b) Statement I is true but Statement II is false.

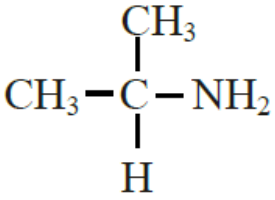
(c) Statement I is false but Statement II is true.

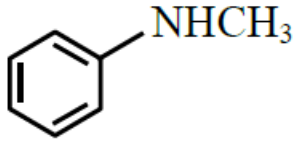
(d) Both Statement I and Statement II are false.

46. Which one of the following gives the most stable Diazonium salt?

(a) $CH_3–CH_2–CH_2–NH_2$

(b) 

(c) 

(d) 

47. The potassium ferrocyanide solution gives a Prussian blue colour, when added to :
(a) $CoCl_3$ (b) $FeCl_2$
(c) $CoCl_2$ (d) $FeCl_3$

48. The oxide without nitrogen-nitrogen bond is:
(a) N_2O (b) N_2O_4
(c) N_2O_3 (d) N_2O_5

49. Number of paramagnetic oxides among the following given oxides is

$Li_2O, CaO, Na_2O_2, KO_2, MgO$ and K_2O
(a) 1 (b) 2
(c) 3 (d) 0

50. Identify the element for which electronic configuration in +3 oxidation state is $[Ar]3d^5$:
(a) Ru (b) Mn
(c) Co (d) Fe

SECTION-B

51. An empty LPG cylinder weighs 14.8 kg. When full, it weighs 29.0 kg and shows a pressure of 3.47 atm. In the course of use at ambient temperature, the mass of the cylinder is reduced to 23.0 kg. The final pressure inside of the cylinder is atm. (Nearest integer)

(Assume LPG of be an ideal gas)

52. The molar solubility of $Zn(OH)_2$ in $0.1 MNaOH$ solution is $x \times 10^{-18}$M. The value of x is (Nearest integer) (Given: The solubility product of $Zn(OH)_2$ is 2×10^{-20})

53. For the reaction $2NO_2(g) \rightleftharpoons N_2O_4(g)$, when $\Delta S = -176.0 JK^{-1}$ and $\Delta H = -57.8$ kJ mol^{-1}, the magnitude of ΔG at 298 K for the reaction is $kJmol^{-1}$. (Nearest integer)

54. The sum of oxidation states of two silver ions in $[Ag(NH_3)_2][Ag(CN)_2]$ complex is

55. The number of atoms in 8 g of sodium is $x \times 10^{23}$.

The value of x is (Nearest integer)

[Given: $N_A = 6.02 \times 10^{23}$ mol^{-1}

Atomic mass of Na $= 23.0$u]

56. If 80 g of copper sulphate $CuSO_4 \cdot 5H_2O$ is dissolved in deionised water to make 5 L of solution. The concentration of the copper sulphate solution is $x \times 10^{-3}$ mol L^{-1}. The value of x is

[Atomic masses Cu: 63.54u, S: 32u, O: 16u, H: 1u]

57. A 50-watt bulb emits monochromatic red light of wavelength of 795 nm. The number of photons emitted per second by the bulb is $x \times 10^{20}$. The value of x is

[Given: $h = 6.63 \times 10^{-34}$Js and $c = 3.0 \times 10^8$ ms^{-1}]

58. The spin-only magnetic moment value of B_2^+ species is $\times 10^{-2}$BM. (Nearest integer)

[Given: $\sqrt{3} = 1.73$]

59. If the conductivity of mercury at 0°C is 1.07×10^6 Sm^{-1} and the resistance of a cell containing mercury is 0.243Ω, then the cell constant of the cell is $x \times 10^4$ m^{-1}. The value of x is .(Nearest integer)

60. A peptide synthesized by the reactions of one molecule each of Glycine, Leucine, Aspartic acid and Histidine will have peptide linkages.

PART-III MATHEMATICS

SECTION-A

61. Let $f: R \to R$ be a continuous function. Then

$\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(x)dx}{x^2 - \frac{\pi^2}{16}}$ is equal to:

(a) $f(2)$ (b) $2f(2)$
(c) $2f(\sqrt{2})$ (d) $4f(2)$

62. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$ is equal to:

(The inverse trigonometric functions take the principal values)
(a) $3\pi - 11$ (b) $4\pi - 9$
(c) $4\pi - 11$ (d) $3\pi + 1$

63. Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S be the set of all $a \in R$ for which the system is inconsistent and S_2 be the set of all $a \in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

(a) $n(S_1) = 2, n(S_2) = 2$ (b) $n(S_1) = 1, n(S_2) = 0$

(c) $n(S_1) = 2, n(S_2) = 0$ (d) $n(S_1) = 0, n(S_2) = 2$

64. Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then which of the following points lies on P?

(a) $\left(3,1,-\frac{1}{2}\right)$ (b) $\left(-2,0,-\frac{1}{2}\right)$

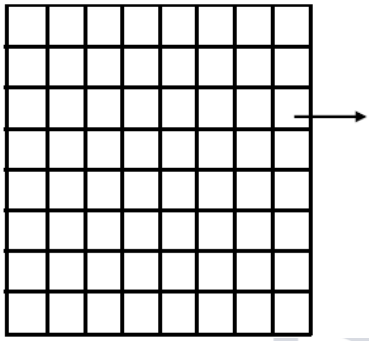
(c) $(0,2,-4)$ (d) $(4,0,-2)$

65. Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?

(a) $\sim (q \rightarrow p)$ (b) $\sim p \rightarrow \sim q$

(c) $\sim (p \rightarrow \sim q)$ (d) $\sim (p \rightarrow q)$

66. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is:



(a) $\frac{2}{7}$ (b) $\frac{1}{18}$

(c) $\frac{1}{7}$ (d) $\frac{1}{9}$

67. If $y = y(x)$ is the solution curve of the differential equation $x^2 dy + \left(y - \frac{1}{x}\right) dx = 0 ; x > 0$ and $y(a) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to:

(a) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (b) $3 + \frac{1}{\sqrt{e}}$

(c) $3 + e$ (d) $3 - e$

68. If n is the number of solutions of the equation

$2\cos x \left(4\sin\left(\frac{\pi}{4} + x\right) \sin\left(\frac{\pi}{4} - x\right) - 1\right) = 1, x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is:

(a) $(3, 13\pi/9)$ (b) $(2, 2\pi/3)$

(c) $(2, 8\pi/9)$ (d) $(3, 5\pi/3)$

69. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(b) = f(d) = 0$. Consider two statements.

(S1) there exists $x_1, x_2 \in (2, 4), x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S2) there exists $x_3, x_4 \in (2, 4), x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3} f(x_4)$.

Then

(a) both (S1) and (S2) are true

(b) (S1) is false and (S2) is true

(c) both (S1) and (S2) are false

(d) (S1) is true and (S2) is false

70. Let $J_{n,m} = \int_0^{\frac{1}{2}} \frac{x^n}{x^m - 1} dx$, $\forall n > m$ and $n, m \in N$.

Consider a matrix $A = [a_{ij}]_{3\times 3}$ where

$a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}$. Then $|adjA^{-1}|$ is :

(a) $(15)^2 \times 2^{42}$ (b) $(15)^2 \times 2^{34}$

(c) $(105)^2 \times 2^{38}$ (d) $(105)^2 \times 2^{36}$

71. The area, enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ and the lines $x = 0, x = \frac{\pi}{2}$, is:

(a) $2\sqrt{2}(\sqrt{2} - 1)$ (b) $2(\sqrt{2} + 1)$

(c) $4(\sqrt{2} - 1)$ (d) $2\sqrt{2}(\sqrt{2} + 1)$

72. The distance of line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point $(2, -1, 6)$ is:

(a) $\sqrt{26}$ (b) $2\sqrt{5}$

(c) $2\sqrt{6}$ (d) $4\sqrt{2}$

73. Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to:

(a) $\frac{75}{8}$ (b) $\frac{125}{16}$

(c) $\frac{25}{2}$ (d) $\frac{15}{2}$

74. The numbers of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0, a^2 - 2$ is also a root of this equation, is:

(a) 6 (b) 2

(c) 4 (d) 8

75. Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \cdots + (n-1) \cdot 1, n \ge 4$.

The sum $\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal to:

(a) $\frac{e-1}{3}$ (b) $\frac{e-2}{6}$

(c) $\frac{e}{3}$ (d) $\frac{e}{6}$

76. Let $P_1, P_2, \ldots, P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \ne 15$, is:

(a) 12 (b) 419

(c) 443 (d) 455

77. The range of the function,

$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$ is:

(a) $(0, \sqrt{5})$ (b) $[-2, 2]$

(c) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$ (d) $[0, 2]$

78. Let $a_1, a_2, \ldots \ldots, a_{21}$ be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$.

If the sum of this AP is 189, then $a_6 a_{16}$ is equal to:

(a) 57 (b) 72

(c) 48 (d) 36

79. The function f(x), that satisfies the condition $f(x) = x + \int_0^{\pi/2} \sin x \cdot \cos y f(y) dy$, is :

(a) $x + \frac{2}{3}(\pi - 2)\sin x$ (b) $x + (\pi + 2)\sin x$

(c) $x + \frac{\pi}{2}\sin x$ (d) $x + (\pi - 2)\sin x$

80. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{1} = 1$ and the circle $x^2 + y^2 = 3$ at their point of intersection in the first quadrant. Then $\tan\theta$ is equal to:

(a) $\frac{5}{2\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{4}{\sqrt{3}}$ (d) 2

SECTION-B

81. Let X be a random variable with distribution.

x	-2	-1	3	4	6
P(X = x)	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then $100\sigma^2$ is equal to:

82. Let $f(x) = x^6 + 2x^4 + x^3 + 2x + 3, x \in R$. Then the natural number n for which $\lim_{x\to 1} \frac{x^n f(a) - f(x)}{x-1} = 44$ is

83. If for the complex numbers z satisfying $|z - 2 - 2i| \leq 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then $a + b$ is equal to

84. Let the points of intersections of the lines $x - y + 1 = 0$, $x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is

85. Let $f(x)$ be a polynomial of degree 3 such that $f(k) = -\frac{2}{k}$ for $k = 2, 3, 4, 5$. Then the value of $52 - 10f(10)$ is equal to:

86. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is

87. If the sum of the coefficients in the expansion of $(x + y)^n$ is 4096, then the greatest coefficient in the expansion is

88. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - \hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to

89. Let [t] denote the greatest integer ≤ t. The number of points where the function

$$f(x) = [x]|x^2 - 1| + \sin\left(\frac{\pi}{[x] + 3}\right) - [x + 1], x \in (-2, 2)$$

is not continuous is

90. A man starts walking from the point $P(-3,4)$, touches the x-axis at R, and then turns to reach at the point $Q(0,2)$. The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50\left((PR)^2+(RQ)^2\right)$ is equal to