1. (c) For Lyman series

$n_1 = 1,\ n_2 = 4$

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{1_1^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda_1} = \frac{15Rz^2}{16}$$

$$\lambda_1 = \frac{16}{15Rz^2}$$

For paschen series

$n_1 = 3,\ n_2 = 4$

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{16-9}{9\times 16}\right)$$

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{7}{9\times 16}\right)$$

$$\lambda_2 = \frac{9\times 16}{7R^2}$$

So,

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{16}{15Rz^2}}{\frac{9\times 16}{7Rz^2}} = \frac{16\times 7}{15\times 9\times 16} = \frac{7}{135}$$

2. (a) Temperature at the junction is θ.

so using the formula

$$\frac{T_2 - T}{R_1} = \frac{T - T_1}{R_2}$$

$$\frac{\theta_2 - \theta}{R_2} = \frac{\theta - \theta_1}{R_1}$$

$$R_1(\theta_2 - \theta) = R_2(\theta - \theta_1)$$

$$R_1\theta_2 - R_1\theta = R_2\theta - R_2\theta_1$$

$R_1\theta + R_2\theta = R_1\theta_2 + R_2\theta_1$

$\theta = \dfrac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$

3. (d) Fringe width $(\beta) = \frac{\lambda D}{d}$

$d = 2 \times 10^{-3}$ m
$\lambda = 500 \times 10^{-9}$ m
$D = 1$ m

Now

$$\beta = \frac{500 \times 10^{-9} \times 1}{2 \times 10^{-3}}$$
$$\beta = \frac{5}{2} \times 10^{-4}$$
$$\beta = 2.5 \times 10^{-4}$$
$$\beta = 0.25 \text{ mm}$$

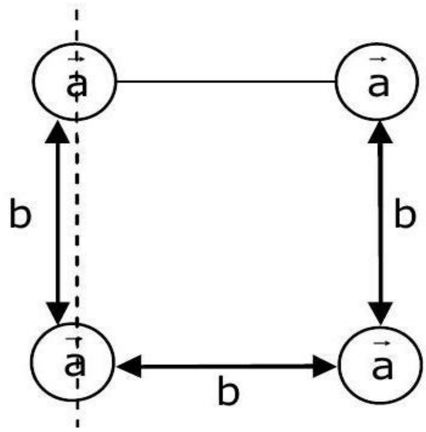
4. (c) Resolution limit $(\Delta\theta) = \frac{1.22\lambda}{d}$

Resolution power $= \frac{1}{\text{Resolution limit}}$

$\lambda \downarrow \Delta\theta \downarrow$

$\Delta\theta \downarrow$ Power $\uparrow$

5. (d)



$$I = \frac{2}{5}ma^2 + \frac{2}{5}ma^2 + \left[\frac{2}{5}ma^2 + mb^2\right] + \frac{2}{5}ma^2 + mb^2$$

$$I = 4 \times \frac{2}{5}ma^2 + 2mb^2$$

$$= \frac{8}{5}ma^2 + 2mb^2$$

6. (d) Bulk modulus $K = \frac{-\Delta P}{\frac{\Delta V}{V}} = \frac{-\Delta pv}{\Delta V}$

We know, $\rho = \frac{M}{V}$

So, $\frac{-\Delta\rho}{\rho} = \frac{\Delta v}{V}$

$$K = \frac{-\Delta P}{\left(-\frac{\Delta\rho}{\rho}\right)} = \frac{\rho\Delta P}{\Delta\rho}$$

$$\Delta\rho = \frac{\rho\Delta P}{K}$$

$$\Delta\rho = \frac{\rho P}{K}$$

7. (a) We know that $E = \frac{hc}{\lambda}$

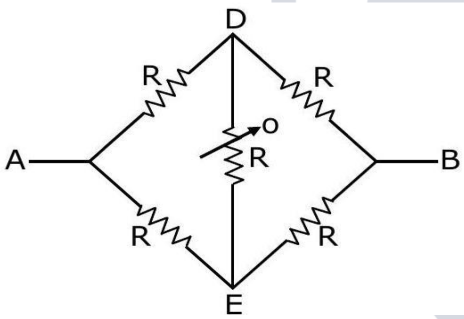
$$\lambda = \frac{hc}{E} \Rightarrow \frac{1240(\text{ in eV })}{E(\text{ in eV })}$$

$$\lambda = \frac{1240}{1.9}$$

$= 652.63\ \text{nm} \approx 654\ \text{nm}$

Wavelength of red light is 620 nm to 750 nm

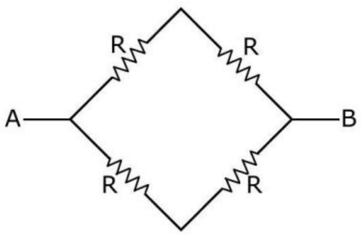
8. (c) It is balanced wheat stone bridge

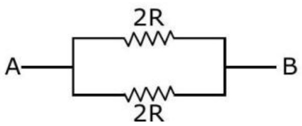


So, we know that

$$R_1R_4 = R_2R_3$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

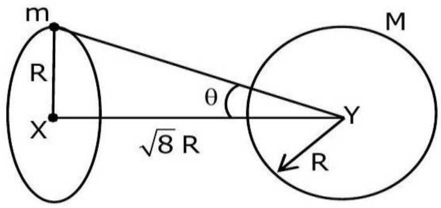




Req $= \frac{2R \times 2R}{2R + 2R}$

$= \frac{4R^2}{4R} \Rightarrow R$

9. (b)



We know that

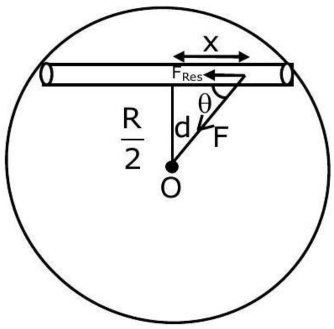
$$F = ME = M\left(\frac{GM\sqrt{8}R}{\left(R^2 + (\sqrt{8}R)^2\right)^{3/2}}\right)$$

$$F = \frac{GMm\sqrt{8}R}{(9R^2)^{3/2}} \Rightarrow \frac{2\sqrt{2}GmM}{(9R^2)^{3/2}}$$

$$= \frac{2\sqrt{2}\mathrm{GmM}}{27\mathrm{R}^2}$$

$$F = \frac{\sqrt{8}GMm}{27R^2}$$

10. (a)



$$\cos\theta = \frac{x}{d}$$

If displaced from equilibrium position,

$$\mathrm{F}_{\text{restoring}} = \left(\frac{\mathrm{GMmd}}{\mathrm{R}^3}\right)\cos\theta$$

$$F_{\text{Res.}} = \frac{GMmd}{R^3}\cdot\frac{x}{d} = \frac{GMmx}{R^3}$$

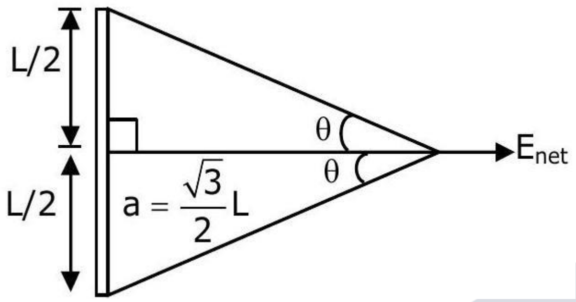
$$\mathrm{a_R} = \frac{\mathrm{GMx}}{\mathrm{R}^3}\ \mathrm{GM_e} = \mathrm{gR}^2$$

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|}$$

$$T = 2\pi\sqrt{\frac{x}{\frac{GMx}{R^3}}}\sqrt{\frac{R^3}{gR^2}}$$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

11. (a)



$$\tan\theta = \frac{L/2}{\frac{\sqrt{3}}{2}L} \Rightarrow \frac{1}{\sqrt{3}}$$

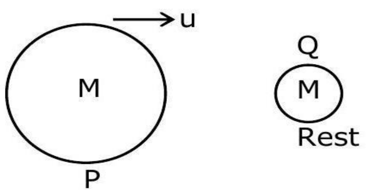
$$\theta = 30^\circ$$

$$E_{\text{net}} = \frac{K\lambda}{\frac{\sqrt{3}}{2}L}(\sin 30^\circ + \sin 30^\circ) \Rightarrow \frac{2KQ}{\sqrt{3}L^2}\left(\frac{1}{2}+\frac{1}{2}\right)$$

$$E_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\sqrt{3L^2}}$$

$$E_{\text{net}} = \frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$$

12. (d)



$m << M$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

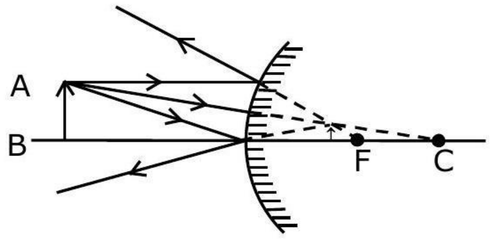
For elastic collision → $e = 1$

$$1 = \frac{v_2 - u}{u - 0}$$

$$u = v_2 - u$$
$$v_2 = 2u$$

In elastic collision kinetic energy & momentum are conserved.

13. (b)



Same orientation so image is virtual. It is combination of real object and virtual image using height it is possible only from convex mirror.

14. (b) $F \propto \frac{1}{R^3}$

$$F = \frac{K}{R^3}$$

$$\frac{mv^2}{R} = \frac{K}{R^3}$$

$$m(\omega R)^2 = \frac{K}{R^2}$$

$$m\omega^2 R^2 = \frac{K}{R^2}$$

$$\omega^2 = \frac{K}{m}\left(\frac{1}{R^4}\right)$$

$$\left(\frac{2\pi}{T}\right)^2 \propto \frac{1}{R^4}$$

$$\frac{4\pi^2}{T^2} \propto \frac{1}{R^4}$$

$$T \propto R^2$$

15. (d) R is the radius of bigger drop.

r is the radius of n water drops.

Water drops are combined to make bigger drop.

So,

Volume of n drops = volume of bigger drop

$$n\left(\frac{4}{3}\pi r^3\right)=\frac{4}{3}\pi R^3$$

$$R=rn^{1/3}\Rightarrow n=\left(\frac{R}{r}\right)^3$$

$\Delta U=T($ Change in surface area $)$

$$\Delta U=T(n4\pi r^2-4\pi R^2)$$

$$\Delta U=4\pi T\left[\left(\frac{R}{r}\right)^3 r^2-R^2\right]\Rightarrow\frac{4\pi T\left(\frac{R^3}{r}-R^2\right)}{J}$$

$$\frac{\Delta U}{V}=\frac{4\pi T\left(\frac{R^3}{r}-R^2\right)}{J\times\frac{4}{3}\pi R^3}=\frac{3T}{J}\left[\frac{1}{r}-\frac{1}{R}\right]$$

16. (b) $\frac{d\vec{A}}{dt}=\frac{\vec{L}}{2\text{ m}}$

17. (b)

$$I_0=\sqrt{I_1^2+I_2^2+2I_1I_2\cos\theta}$$

$$I_0=\sqrt{I_1^2+I_2^2+2I_1I_2\cos 90^\circ}$$

$$I_0=\sqrt{I_1^2+I_2^2+2I_1I_2(0)}\Rightarrow\sqrt{I_1^2+I_2^2}$$

We, know that

$$I_{rms}=\frac{I_0}{\sqrt{2}}$$

So, $I_{rms}=\frac{\sqrt{I_1^2+I_2^2}}{\sqrt{2}}$

18. (d) $C_1+C_2=\frac{15}{4}\left(\frac{C_1C_2}{C_1+C_2}\right)$

$$4(C_1+C_2)^2=15C_1C_2$$

$$4C_1^2+4C_2^2-7C_1C_2=0$$

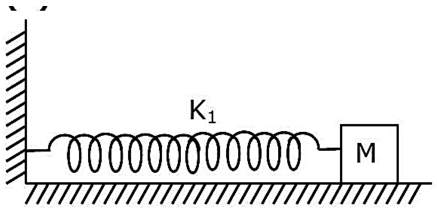
$$4+4\left(\frac{C_2}{C_1}\right)^2-7\frac{C_2}{C_1}=0$$

$$4\left(\frac{C_2}{C_1}\right)^2-7\frac{C_2}{C_1}+4=0$$

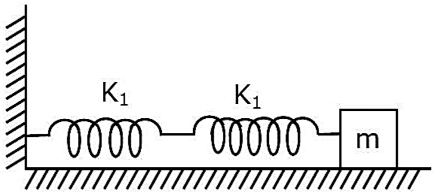
$\frac{C_2}{C_1}$ has not real value

$\frac{C_2}{C_1}=$ imaginary

19. (a)



$$T = 2\pi\sqrt{\frac{m}{K_1}}$$



$$K_{eq} = \frac{K_1 \times K_1}{K_1 + K_1} = \frac{K_1^2}{2\,K_1}$$

$$K_{eq} = \frac{K_1}{2}$$

$$T' = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{\frac{k_1}{2}}}$$

$$= \sqrt{2}\,T$$

20. (a) $\frac{\beta x^2}{KT}$ is dimension less

so

$$KT = \beta x^2 \Rightarrow M^1\,L^2\,T^{-2}$$
$$\beta = \frac{M^1\,L^2\,T^{-2}}{L^2} \Rightarrow M^1\,T^{-2}$$
$$M^1\,L^2\,T^{-2} = \alpha^2 M^1\,T^{-2}$$
$$\alpha^2 = L^2$$
$$\alpha = L$$
$$\alpha = M^0\,L^1\,T^0$$

21. (1215) $y = -0.21\sin(x + 30t)$

$$v = \frac{\omega}{K} = \frac{30}{1} = 30\text{ m/s}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$T = v^2 \times \mu$$

$$T = (30)^2 \times 0.135 \times 10^{-1}$$

$$\mu = 0.135\text{gm/cm}$$

$$T = 900 \times 0.135 \times 10^{-1}$$

$\mu = 0.135 \times \frac{10^{-3}}{10^{-2}} \frac{\text{kg}}{\text{m}}$

$\mathrm{T} = 12.15\ \mathrm{N}$

$\mathrm{T} = 1215 \times 10^{-2}\ \mathrm{N}$

$\mathrm{x} = 1215$

22. (137) Intensity of electromagnetic wave is,

$I = \frac{1}{2} C\varepsilon_0 E_0^2 = \frac{P}{4\pi r^2}$

$\frac{1}{2} 4\pi\varepsilon_0 \times \mathrm{C} \times \mathrm{E}_0^2 = \frac{\mathrm{P}}{\mathrm{r}^2}$

$\frac{1}{2} \times \frac{3 \times 10^5 \times \mathrm{E}_0^2}{9 \times 10^9} = \frac{1000 \times 1.25}{(2)^2} \times \frac{1}{100}$

$\mathrm{E}_0^2 = \frac{60 \times 1000 \times 1.25}{4 \times 100} = \frac{125 \times 3}{2}$

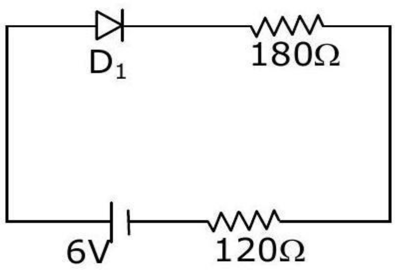
$\mathrm{E}_0^2 = \frac{375}{2} = 187.5$

$\mathrm{E}_0 = 13.69$

$\mathrm{E}_0 \approx 137 \times 10^{-1} \mathrm{v/m}$

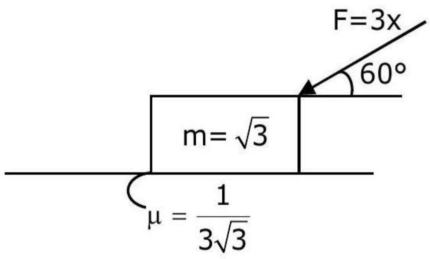
23. (20) D_2 is reverse bias so current does not flow through D_2.

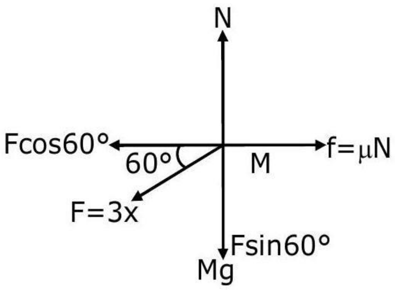
D_1 is forward bias.



$I = \frac{6}{300} \Rightarrow 0.02\ \mathrm{A}$
$= 20\ \mathrm{mA}$

24. (3.33)





$\mathrm{N} = \mathrm{Mg} + \mathrm{Fsin}\,60^\circ$

$N = \sqrt{3}g + \frac{F\sqrt{3}}{2}$

For No slipping

$\mathrm{Fcos}\,60^\circ = \mathrm{Friction}$

$\frac{F}{2} = \mu N = \frac{1}{3\sqrt{3}}\left(\sqrt{3}g + \frac{F\sqrt{3}}{2}\right)$

$\frac{\mathrm{F}}{2} = \frac{\mathrm{g}}{3} + \frac{\mathrm{F}}{6}$

$\frac{\mathrm{F}}{2} - \frac{\mathrm{F}}{6} = \frac{\mathrm{g}}{3}$

$\frac{6\,\mathrm{F} - 2\,\mathrm{F}}{12} = \frac{\mathrm{g}}{3}$

$4\,\mathrm{F} = 4\,\mathrm{g}$

$\mathrm{F} = 10$

$F = 3x$

$x = \frac{F}{3} = \frac{10}{3} = 3.33$

$\mathrm{x} = 3.33$

25. (282.84)

Quality factor $= \frac{\mathrm{X_L}}{\mathrm{R}} = \frac{\omega \mathrm{L}}{\mathrm{R}}$

$$Q=\frac{1}{\sqrt{LC}}\frac{L}{R}$$

$$Q=\left(\frac{1}{\sqrt{C}}\right)\frac{\sqrt{L}}{R}$$

$$Q=\frac{XL}{R}=\frac{\omega L}{R}=\frac{1}{\sqrt{LC}}\frac{L}{R}=\frac{1}{R}\frac{\sqrt{L}}{\sqrt{C}}$$

$$Q'=\frac{\sqrt{2L}}{\left(\frac{R}{2}\right)\sqrt{C}}=2\sqrt{2}Q$$

$Q'=282.84$

26. (300)

Charge flown $(Q)=20\mathrm{C}$

Potential difference (V) $=15$ V

Work done (w) $=$ Q.V

$=20\times15=300$ J

$w=300$ J

27. (25) By energy Conservation

$$\frac{3}{2}n_1RT_1+\frac{3}{2}n_2RT_2=\frac{3}{2}(n_1+n_2)RT$$

Using PV $=nRT$

$P_1V_1+P_2V_2=P(V_1+V_2)$

$$P=\frac{P_1V_1+P_2V_2}{V_1+V_2}=\frac{2\times4.5+3\times5.5}{4.5+5.5}$$

$$P=\frac{9+16.5}{10}=\frac{25.5}{10}$$

$\approx 25\times10^{-1}$ atm

28. (500)

$F=20\hat{\imath}+10\hat{\jmath}$

$F_x=20$ N

$F_y=10N$

$$a_x=\frac{F_x}{M}=\frac{20}{2}=10\ \mathrm{m/s^2}$$

$$a_y=\frac{F_y}{M}=\frac{10}{2}=5\ \mathrm{m/s^2}$$

displacement on x axis is

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$S = 0 \times 10 + \frac{1}{2} \times 10 \times (10)^2$$

$$S = 500 \text{ m}$$

29. (33)

$$A_m = \frac{A_{\max} - A_{\min}}{2}$$

$$A_C = \frac{A_{\max} + A_{\min}}{2} \quad \begin{bmatrix} A_{\max} = 16 \text{ V} \\ A_{\min} = 8 \text{ V} \end{bmatrix}$$

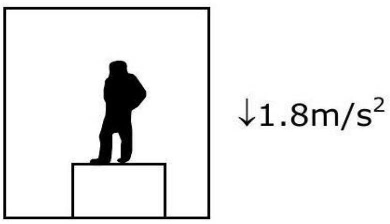
Modulation index (mi) $= \frac{A_m}{A_c} = \frac{\frac{A_{\max} - A_{\min}}{2}}{\frac{A_{\max} + A_{\min}}{2}} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$

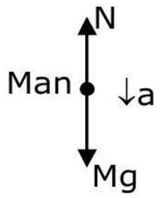
$$\text{mi} = \frac{16 - 8}{16 + 8} = \frac{8}{24} = \frac{1}{3} = 0.33$$

$$\text{mi} = 33 \times 10^{-2}$$

$$x = 33$$

30. (492)





$$Mg - N = Ma$$

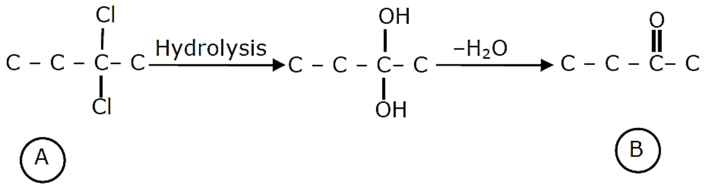
$$N = M(g - a)$$

$$N = 60(10 - 1.8)$$

$$N = 60 \times 8.2 = 492 \text{ N}$$

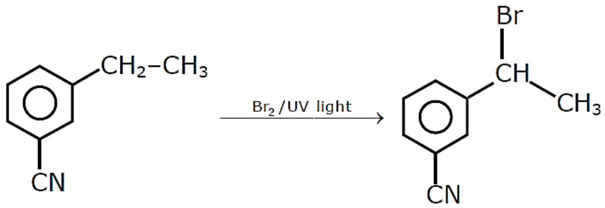
$$N = 492$$

31. (b)



Compound 'B' does not gives Tollen's test due to presence of kenotic group but react with hydroxyl amine

32. (b)
33. (d)



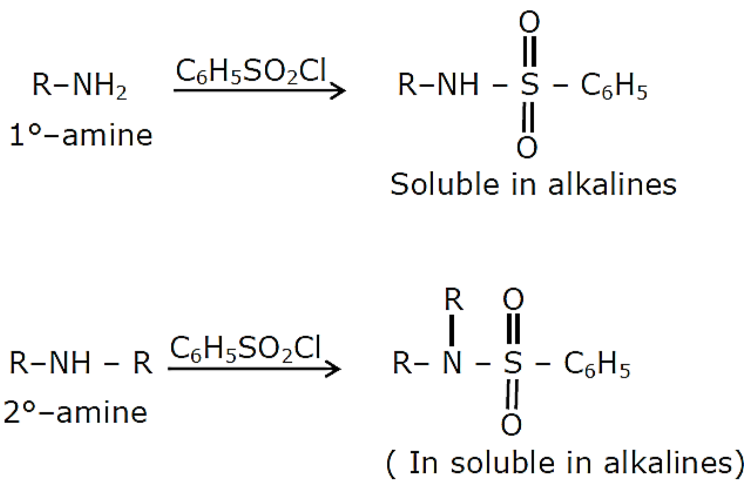
It is bezylic substitution reaction

34. (a) A.N. $= \ell$
R. $N = \text{n} - \ell - 1$

Orbital	Angular Node	Radial Node
5 d	2	2
4f	3	0
3p	1	1
4 d	0	1

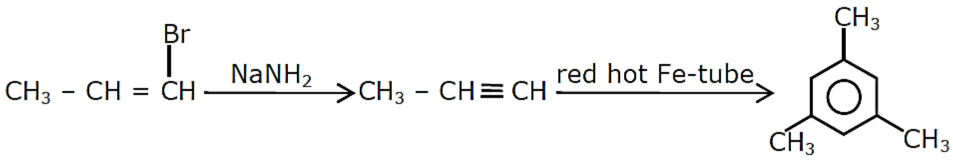
35. (d) o-Nitrophenol is steam volatile due to intramolecular hydrogen H-bonding. but m-Nitrophenol has more melting point due to its symmetry.

36. (a)



According to the question the amine should be 2°-amine, in which one of the alkyl group should be ethyl, because it can be formed by ammonolysis of ethyl chloride

37. (c)



(mesitylene)

38. (c)
39. (d)
40. (b) A suitable method for separating a mixture of aniline and chloro form would be steam distillation. Steam distillation is the process used to separate aromatic compound from a mixture because of their temperature sensitivity. Therefore, steam distillation is an ideal method for their separation
41. (c) Vitamin K is used by the body to help blood clot.
42. (b) The presence of ozone in troposphere protect earth from ultra violet rays
43. (a)

$$\underset{(X)}{SO_2} + K_2Cr_2O_7 + H_2SO_4 \longrightarrow \underset{(Y)}{Cr_2(SO_4)_3} + K_2SO_4 + H_2O$$

44. (a)

$$(1)\ NH_3 + \underset{(A)}{H_2O} + CO_2 \rightarrow (NH_4)_2CO_3$$

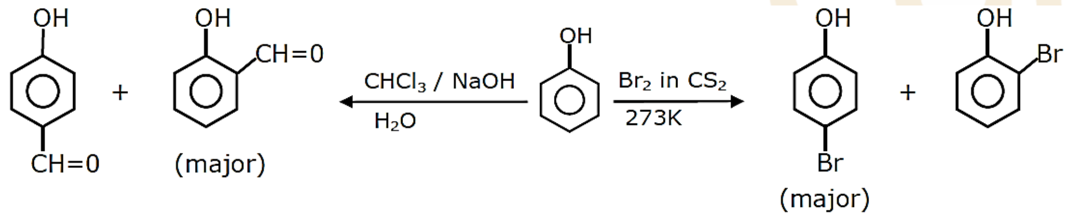
$$(2)\ (NH_4)_2CO_3 + H_2O + \underset{(B)}{CO_2} \rightarrow NH_4HCO_3$$

$$(3)\ NH_4HCO_3 + NaCl \rightarrow NH_4Cl + \underset{(C)}{NaHCO_3}$$

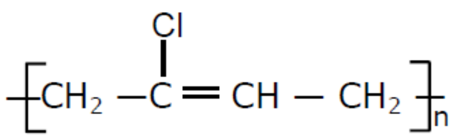
45. (c)
46. (a) Order of I.E. in second period

$$\underset{2p^1}{Li < B} \quad \underset{2\,s^2 2p^2 2p^4 2p^3}{< Be < C < O < N < F < Ne}$$

47. (b)
48. (a)



49. (b)



neoprene

50. (b) lead storage batteries PbO_2 is used. In this O.S. of Pb is +4 so it is always reduced and behaves as oxidizing agent

51. (0.18) The balanced equation is

$SO_2 + 2NaOH \longrightarrow Na_2SO_3 + H_2O$

moles of NaOH = molarity × volume (in litre)

$= 0.1 \times 0.1$
$= 0.01\text{moles}$

Here NaOH is limiting Reagent

2 mole NaOH $\longrightarrow$ 1 mole Na_2SO_3

0.01 mole NaOH $\longrightarrow \frac{1}{2} \times 0.01\ \text{mole}^{2}_{\text{Na}}SO_3$

Moles of $Na_2SO_3 \longrightarrow 0.005$ mole

$Na_2SO_3 \longrightarrow 2Na^+ + SO_3^{2-}$

$i = 3$

Moles of $H_2O = \frac{36}{18} = 2$ moles

Accoding to RLVP -

$$\frac{P_A^0 - P_A}{P_A^\circ} = iX_B$$

$$\frac{P_A^\circ - P_A}{P_A^\circ} = \frac{inn_A}{in_A + n_B}(ini_A \simeq 0)$$

$n_B << n_A$

$\{n_A + n_B \simeq n_A\}$

$$\frac{P_A^\circ - P_A}{P_A^\circ} = i \times \frac{n_B}{n_A}$$

$$\frac{2H - P_A}{2H} = 3 \times \frac{0.005}{2}$$

$\Rightarrow 2H - P_A = 0.18$

Lowering in pressure = 0.18 mm of Hg

lowering in pressure $= 18 \times 10^{-12}$ mm of Hg

$x = 18$

52. (25) $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O$

1 mole of MnO_4^- require 5 faraday charge

5 moles of MnO_4^- will require 25 faraday charge.

53. (2) Moles of $O_2 = \frac{3.12}{32} = 0.0975$

volume of $O_2 = \frac{nRT}{p} = \frac{0.0975 \times 0.082 \times 300}{1}$

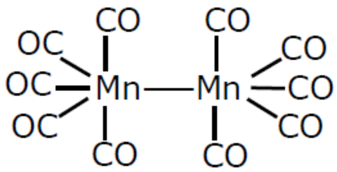
$= 2.3985 \text{ L} \simeq 2.4 \text{ L}$

volume of O_2 absorbed per gm of pt $= \frac{2.4}{1.2} = 2$

54. (7) $\underline{50000.020} \times 10^{-3}$

Number of significant figure $= 7$

55. (0)



56. (200) $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

To make the process spontaneous

$\Delta G^\circ < 0$

$\Delta H^\circ - T\Delta S^\circ < 0$

$T > \frac{\Delta H^\circ}{\Delta S^\circ}$

$T > \frac{80000}{2\text{ T}}$

$2T^2 > 80000$

$T^2 > 40000$

$T > 200$

The minimum temperature to make it spontaneous is 200 K.

57. (50) $\Delta H = E_{a,f} - E_{a,b}$

$-20 = 30 - E_{a,b}$

$E_{a,b} = 50 \text{ kJ/mole}$

58. (1) $P(v - b) = RT$

$PV - Pb = RT$

$$\frac{PV}{RT}-\frac{Pb}{RT}=1$$

$$Z=1+\frac{PV}{RT}$$

$$\frac{dz}{dp}=0+\frac{b}{RT}$$

$$\Rightarrow\frac{b}{RT}=\frac{xb}{RT}$$

$x=1$

59. (74) $AB_{2(g)} \rightleftharpoons A_{(g)}+2B_{(g)}$

initial $1-x\ x\ 2x$

at eq. $\frac{1}{1+2x}\ \ \frac{1}{1.9}$

By ratio of pressure & mole

$$\frac{1}{1+2x}=\frac{0.985}{1.9}$$

$1.9=0.985+1.9x$

$0.915=1.9x$

$$\frac{0.915}{1.9}=x;\ K_p=\frac{4x^2\cdot x}{(1-x)}\left[\frac{P_{total}}{n_{total}}\right]^2$$

$$\Rightarrow\frac{4x^3}{1-x}\left(\frac{RT}{V}\right)^2$$

On substituting the values

$K_p=74\times10^{-2}$

60. (26)

$Cr_2O_7^{2-}+2OH^- \rightleftharpoons 2CrO_4^{2-}+H_2O$

$2CrO_7^{2-}$

$x+(-2\times4)=-2$

$x=6$

61. (a)

CASE-I: 1, 1, 1, 1, 1, 2, 3

WAYS $=\frac{7!}{5!}=42$

CASE-II: 1,1,1,1,2,2,2

$\text{WAYS} = \frac{7!}{4!\cdot 3!} = 35$

$\text{TOTAL WAYS} = 42 + 35 = 77$

62. (d) $T_{r+1} = {}^{10}C_r\left(tx^{1/5}\right)^{10-r}\left[\frac{(1-x)^{1/10}}{t}\right]^r$

$= {}^{10}C_r t^{(10-2r)} \times x^{\frac{10-r}{5}} \times (1-x)^{\frac{r}{10}}$

$\Rightarrow 10 - 2r = 0 \Rightarrow r = 5$

$T_6 = {}^{10}C_5 x\sqrt{1-x}$

$\frac{dT_6}{dx} = {}^{10}C_5\left[\sqrt{1-x} - \frac{x}{2\sqrt{1-x}}\right] = 0$

$= 1 - x = x/2 \Rightarrow 3x = 2$

$T_6|_{\max} = \frac{10!}{5!\,5!} \times \frac{2}{3\sqrt{3}}$

63. (a) $\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}dx$

$$\begin{aligned}
&= \int_0^1 e^{\{x\}}dx + \int_1^2 e^{\{x\}}dx + \int_2^3 e^{\{x\}}dx + \cdots \ldots \int_{99}^{100} e^{\{x\}}dx \ (\because \{x\} = x - [x]) \\
&= e^x\big|_0^1 + e^{(x-1)}\big|_1^2 + e^{(x-2)}\big|_2^3 + \cdots \ldots + e^{(x-99)}\big|_{99}^{100} \\
&= (e-1) + (e-1) + (e-1) + \cdots \ldots (e-1) \\
&= 100(e-1)
\end{aligned}$$

64. (a) $\frac{dx}{dt} \propto x$

$\frac{dx}{dt} = \lambda x$

$\int_{1000}^{x}\frac{dx}{x} = \int_0^t \lambda dt$

$\ln x - \ln 1000 = \lambda t$

$\ln\left(\frac{x}{1000}\right) = \lambda t$

Put $t = 2, x = 1200$

$\ln\left(\frac{12}{10}\right) = 2\lambda \Rightarrow \lambda = \frac{1}{2}\ln\frac{6}{5}$

Now $\ln\left(\frac{x}{1000}\right) = \frac{t}{2}\ln\left(\frac{6}{5}\right)$

$x = 1000e^{\frac{t}{2}\ln\left(\frac{6}{5}\right)}$

$x = 2000$ at $t = \frac{k}{\ln\left(\frac{6}{5}\right)}$

$\Rightarrow 2000 = 1000e^{\frac{k}{2\ln(6/5)}\ln(6/5)}$

$\Rightarrow 2 = e^{k/2}$

$\Rightarrow \ln 2 = \frac{k}{2}$

$\Rightarrow \frac{k}{\ln 2} = 2$

$\Rightarrow \left(\frac{k}{\ln 2}\right)^2 = 4$

65. (c) $\vec{a} \times \left(\vec{a} \times \left((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2\vec{b}\right)\right)$

$\vec{a} \times \left(-|\vec{a}|^2(\vec{a} \times \vec{b})\right) = -|\vec{a}|^2\left((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2\vec{b}\right)$

$= -(\vec{a} \cdot \vec{b})\vec{a}|\vec{a}|^2 + |\vec{a}|^4\vec{b}$

$= |\vec{a}|^4\vec{b}$ $(\because \vec{a} \cdot \vec{b} = 0)$

66. (a) $ar + ar^5 = \frac{25}{2}$

$ar^2 \times ar^4 = 25$

$a^2r^6 = 25$

$ar^3 = 5$

$a = \frac{5}{r^3}$

$\frac{5r}{r^3} + \frac{5r^5}{r^3} = \frac{25}{2}$

$\frac{1}{r^2} + r^2 = \frac{5}{2}$

Put $r^2 = t$

$\frac{t^2 + 1}{t} = \frac{5}{2}$

$2t^2 - 5t + 2 = 0$

$2t^2 - 4t - t + 2 = 0$

$(2t - 1)(t - 2) = 0$

$t = \frac{1}{2}, 2 \Rightarrow r^2 = \frac{1}{2}, 2$

$r = \sqrt{2}$

$= ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 5[1 + 2 + 4] = 35$

67. (a) $P_1 = x + 5y + 7z = 3$

$P_2 = x - 3y - z = 5$

$P_3 = x + 5y + 7z = 5/2$

$\Rightarrow P_1 \parallel P_3$

68. (c) $s = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \cdots.$

$$\frac{s}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \cdots \ldots \infty$$

$$\frac{2s}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \cdots \ldots \infty$$

$$\frac{2s}{3} = \frac{4}{3} + \frac{5}{3}\left\{\frac{1/3}{1 - \frac{1}{3}}\right\} = \frac{5}{6} + \frac{4}{3} = \frac{13}{6}$$

$$s = \frac{13}{4}$$

69. (a) $c_1 \to c_1 - c_2, c_2 \to c_2 - c_3$

$$= \begin{vmatrix} (a+2)a & a+1 & 1 \\ (a+3)(a+1) & a+2 & 1 \\ (a+4)(a+2) & a+3 & 1 \end{vmatrix}$$

$R_2 \to R_2 - R_1 \,\&R_3 \to R_3 - R_1$

$$= \begin{vmatrix} a^2+2a & a+1 & 1 \\ 2a+3 & 1 & 0 \\ 4a+8 & 2 & 0 \end{vmatrix}$$

$= 6 - 8 = -2$

70. (b)

$$\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c}$$

$$\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\sin^{-1}x + \cos^{-1}x}{a+b} = \frac{\pi}{2(a+b)}$$

Now, $\frac{\tan^{-1}y}{c} = \frac{\pi}{2(a+b)}$

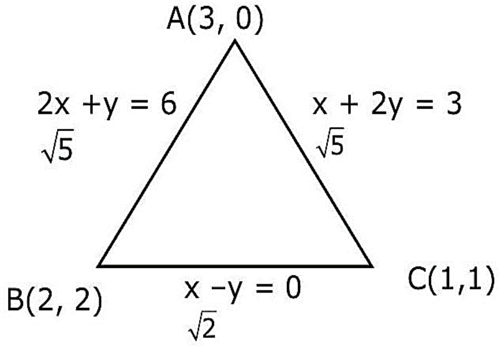
$$2\tan^{-1}y = \frac{\pi c}{a+b}$$

$\Rightarrow \cos\left(\frac{\pi c}{a+b}\right) = \cos(2\tan^{-1} y) = \frac{1-y^2}{1+y^2}$

71. (c) Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} a^2+b^2 & ab+bc \\ ab+bc & c^2+b^2 \end{bmatrix}$$
$= a^2 + 2b^2 + c^2 = 1$
$a = 1, b = 0, c = 0$
$a = 0, b = 0, c = 1$
$a = -1, b = 0, c = 0$
$c = -1, b = 0, a = 0$

72. (c)



73. (b) $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Let $f(x) = 2x^3 - 15x^2 + 36x - 19$

$f'(x) = 6x^2 - 30x + 36 = 0$

$x^2 - 5x + 6 = 0$

$x = 2,3$

$f''(x) = 12x - 30$

$f''(x) < 0$ for $x = 2$

At x = 2

$y = 8 - 40 + 72 - 38$

$y = 72 - 70 = 2$

$\Rightarrow (2,2)$

74. (d) $|f(x) - f(y)| \leq |(x-y)^2|, \forall (x,y) \in R$

$$\left|\frac{f(x)-f(y)}{x-y}\right| \le |x-y|$$

$$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right| \le 0$$

$|f'(y)| \le 0 \Rightarrow f'(y) = 0$

$f(y) = C$

$\Rightarrow C = 1$

$\Rightarrow f(x) = 1$

75. (d) Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$

$$I = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$$

$2I = \int_{-\pi/2}^{\pi/2} \cos^2 x dx$

$I = \int_0^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$

76. (c) $\lim_{h\to 0} 2 \times 2 \left\{\frac{\sin\left(\frac{\pi}{6}+h-\frac{\pi}{6}\right)}{2\sqrt{3h}\left(\cos\left(h+\frac{\pi}{6}\right)\right)}\right\}$

$$= \frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}} = \frac{4}{3}$$

77. (b) $p(x=9) = p(x=7)$

$${}^{n}C_9 \left(\frac{1}{2}\right)^{n-9} \times \left(\frac{1}{2}\right)^9 = {}^{n}C_7 \left(\frac{1}{2}\right)^{n-7} \times \left(\frac{1}{2}\right)^7$$

$${}^{n}C_9 \times \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2 \times {}^{n}C_7$$

$$x + y = n = 2) = {}^{16}C_2 \times \left(\frac{1}{2}\right)^{14} \times \left(\frac{1}{2}\right)^2$$

$$= {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16} = \frac{15}{2^{13}}$$

78. (d) Let $P(1,5,35), Q(7,5,5), R(1,\lambda,7), S(2\lambda,1,2)$

$[\overrightarrow{PQ} \quad \overrightarrow{PR} \quad \overrightarrow{PS}] = 0$

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & -5 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$\{-33\lambda + 165 - 112\} + 5(\lambda-5)(2\lambda-1) = 0$

$53 - 33\lambda + 5\{2\lambda^2 - 11\lambda + 5\} = 0$

$16\lambda^2 - 88\lambda + 78 = 0$

$5\lambda^2 - 44\lambda + 39 = 0 <^{\lambda_1}_{\lambda_2}$

$\Rightarrow \lambda_1 + \lambda_2 = 44/5$

79. (d) $P(a,b), Q(c,d), PO = QO$

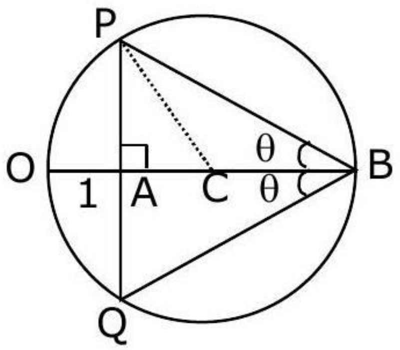
$\Rightarrow \mathrm{a}^2 + \mathrm{b}^2 = \mathrm{c}^2 + \mathrm{d}^2$

$R(x,y)\ s = (1,-1) \Rightarrow RO = SO$

($\because$ equivalence class)

$x^2 + y^2 = 2$

80. (c)



$$\mathrm{OC} = \frac{13}{2} = 6.5$$
$$\mathrm{AC} = \mathrm{CO} - \mathrm{AO}$$
$$= 6.5 - 1$$

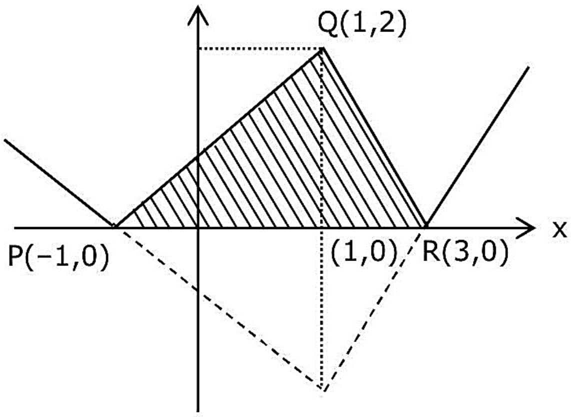
In $\triangle$ PAC

$$PA = \sqrt{6.5^2 - 5.5^2}$$
$$PA = \sqrt{12}$$
$$\Rightarrow PQ = 2PA = 2\sqrt{12}$$

Now, area of $\triangle$ PQB $= \frac{1}{2} \times \mathrm{PQ} \times \mathrm{AB}$

$$= \frac{1}{2} \times 2\sqrt{12} \times 12$$
$$= 12\sqrt{12}$$
$$= 24\sqrt{3}$$

81. (8)



82. (11) $3\sin x + 4\cos x = k + 1$

$-5 \le \mathrm{k} + 1 \le 5$

$-6 \le \mathrm{k} \le 4$

$-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4 \Rightarrow$ 11 integral values

83. (45) Let $S = \sum_{r=0}^{30} (30 - r)\ {}^{30}\mathrm{C}_\mathrm{r}$

$$= 30 \sum_{r=0}^{30} {}^{30}\mathrm{C}_\mathrm{r} - \sum_{\mathrm{r}=0}^{30} \mathrm{r}\ {}^{30}\mathrm{C}_\mathrm{r}$$

$$= 20 \times 2^{30} - \sum_{r=1}^{30} r \cdot \frac{30}{4} \cdot\ {}^{29}\mathrm{C}_{\mathrm{r}-1}$$

$$= 30 \times 2^{30} - 30.2^{29}$$

$$= (30 \times 2 - 30) \cdot 2^{29} = 30 \cdot 2^{29} \Rightarrow 15.2^{30}$$

$$= \mathrm{n} = 15, \mathrm{m} = 30$$

$$n + m = 45$$

84. (1)

$$e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$$

Put $\mathrm{e}^{\sin y} = \mathrm{t}$

$$e^{\sin y} \times \cos y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dt}{dx} + t\cos x = \cos x$$

I.F. $= \mathrm{e}^{\int \cos x dx} = e^{\sin x}$

Solution of diff equation:
t. $e^{\sin x} = \int e^{\sin x} \cdot \cos x dx$

$$\mathrm{e}^{\sin y} \cdot \mathrm{e}^{\sin x} = \mathrm{e}^{\sin x} + c$$

at $x=0, y=0$

$1=1+c \Rightarrow c=0$

$e^{\sin x+\sin y}=e^{\sin x}$

$\sin x+\sin y=\sin x$

$y=0$

$\Rightarrow y\left(\frac{\pi}{6}\right)=0,\ y\left(\frac{\pi}{3}\right)=0,\ y\left(\frac{\pi}{4}\right)=0$

$\Rightarrow 1+0+0+0=1$

85. (1) $\frac{1}{2}\log_2(x-1)=\log_2(x-3)$

$x-1=(x-3)^2$

$x^2-6x+9=x-1$

$x^2-7x+10=0$

$x=2,5$

$X=2$ Not possible as $\log_2(x-3)$ is not defined

$\Rightarrow$ No. of solution $=1$

86. (1) $\sqrt{3}t^2-(\sqrt{3}-1)t-1=0$(where $t=\cos x$)

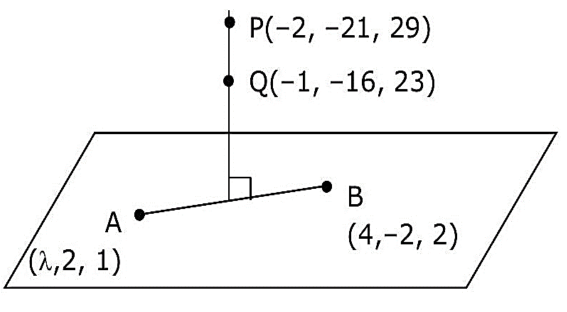
Now, $t=\frac{(\sqrt{3}-1)\pm\sqrt{4+2\sqrt{3}}}{2\sqrt{3}}$

$t=\cos x=1$ or $-\frac{1}{\sqrt{3}} \rightarrow$ rejected as $x\in\left[0,\frac{\pi}{2}\right]$

$\Rightarrow \cos x=1$

$\Rightarrow$ No. of solution $=1$

87. (8)



$\overrightarrow{AB}\perp\overrightarrow{PQ}$

$[(4-\lambda)\hat{\imath}-4\hat{\jmath}+\hat{k}]\cdot[+\hat{\imath}+5\hat{\jmath}-6\hat{k}]=0$

$4-\lambda-20-6=0$

$\lambda=-22$

Now , $\frac{\lambda}{11}=-2$

$\Rightarrow \left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4$

$\Rightarrow 4+8-4=8$

88. (2) $y^2=a\left(x+\frac{\sqrt{a}}{2}\right)$

$2yy'=a$

$y^2=2yy'\left(x+\frac{\sqrt{2yy'}}{2}\right)$

$y=2y'\left(x+\frac{\sqrt{yy'}}{\sqrt{2}}\right)$

$y-2xy'=\sqrt{2}y'\sqrt{yy'}$

$\left(y-2x\frac{dy}{dx}\right)^2=2y\left(\frac{dy}{dx}\right)^3$

$D=3\&O=1$

$D-O=3-1=2$

89. (3) Let roots of $x^3-2x^2+2x-1=0$ are α,β,γ

$(x-1)(x^2-x+1)\ =0$

$x=\underset{\alpha}{\underset{\downarrow}{1}},\ \underset{\beta}{\underset{\downarrow}{-\omega}},\ \underset{\gamma}{\underset{\downarrow}{-\omega^2}}$

Now $\alpha^{162}+\beta^{162}+\gamma^{162}$

$=1+\omega^{162}+(\omega^2)^{162}$

$=1+(\omega^3)^{54}+(\omega^3)^{108}$

$=3$

90. (2) $I=\int_0^{\pi}|\sin 2x|dx$

$$I=2\int_0^{\pi/2}|\sin 2x|dx=2\int_0^{\pi/2}\sin 2xdx$$

$$I=2\left[\frac{-\cos(2x)}{2}\right]_0^{\pi/2}=2$$