1. **(c)** Since silicon diode is used so 0.7 Volt is drop across it, only D_1 will conduct so current through cell I $= \frac{5-0.7}{10} = 0.43$ A

2. **(a)** $g_1 = \frac{GM}{(3R)^2} = \frac{GM}{9R^2}$

$$g_2 = \frac{GM}{9R^2} - \frac{G\left(\frac{M}{8}\right)}{\left(3R - \frac{R}{2}\right)^2}$$

$$= \frac{GM}{9R^2} - \frac{GM}{R^2 50} = \frac{41}{9 \times 50}\frac{GM}{R^2}$$

$$\frac{g_1}{g_2} = \frac{41}{50}$$

Force $\Rightarrow \frac{F_1}{F_2} = \frac{mg_1}{mg_2} = \frac{41}{50}$

3. **(b)**

$$\lambda = \frac{v}{f} = \frac{336}{504} = 66.66 \text{ cm}$$

$$\frac{\lambda}{4} = I + e = I + 0.3 \text{ d}$$

$$= I + 1.8$$

$$16.66 = I + 1.8 \text{ cm}$$

$$I = 14.86 \text{ cm}$$

4. **(b)** $C_p = \frac{7}{2}R$

$$C_v = \frac{5}{2}R$$

$$dU = nC_v dT$$

$$dQ = nC_p dT$$

$$dW = nRdT$$

$$dU : dQ : dW$$

$$C_v : C_p : R$$

$$\frac{5}{2}R : \frac{7}{2}R : R$$

$$5 : 7 : 2$$

5. **(d)** Side band $= (f_c - f_m)$ to $(f_c + f_m)$

$= (1000 - 2)$KHz to $(1000 + 2)$KHz

$= 998$KHz to 1002kHz

Band width $= 2f_m$

$= 2 \times 2\text{KHz}$

$= 4\text{KHz}$

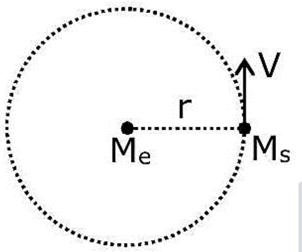
Both statements are true

6. (c)

7. (c)

$$v = \sqrt{\frac{GM_e}{r}}$$

$$T = \frac{2\pi r}{\sqrt{\frac{GM_e}{r}}} = 2\pi r\sqrt{\frac{r}{\mathrm{GM}_e}}$$

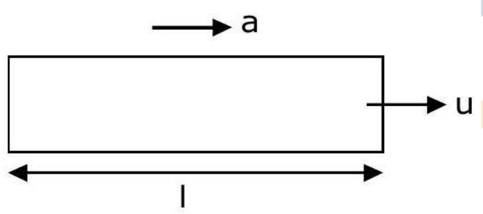


$$T = \sqrt{\frac{4\pi^2 r^3}{GM_e}} = \sqrt{\frac{4\pi^2 r^3}{GM_e}}$$

$$\mathrm{T}_2 - \mathrm{T}_1 = \sqrt{\frac{4\pi^2(8000 \times 10^3)^3}{\mathrm{G} \times 6 \times 10^{24}}} - \sqrt{\frac{4\pi^2(7000 \times 10^3)^3}{G \times 6 \times 10^{24}}}$$

$\cong 1.33 \times 10^3$ s

8. (c)



a = uniform acceleration

u = velocity of first compartment

v = velocity of last compartment

I = length of train

$v^2 = u^2 + 2$ as (3rd equation of motion)

$v^2 = u^2 + 2al$

$$v_{\text{middle}}^2 = u^2 + 2a\frac{1}{2}$$

$$\therefore \mathrm{v}_{\text{middle}} = \mathrm{u}^2 + \mathrm{al}$$

from equation (a) and (b)

$$v_{\text{middle}}^2 = u^2 + \left(\frac{v^2 - u^2}{2}\right)$$

$$= \frac{v^2 + u^2}{2}$$

$$\therefore \mathrm{v}_{\text{middle}} = \sqrt{\frac{\mathrm{v}^2 + \mathrm{u}^2}{2}}$$

9. (a) As $v = \frac{p}{m} \& F = qvB$

$$\therefore \mathrm{F} = \frac{\mathrm{qp}}{\mathrm{m}}\mathrm{B}$$

$$\mathrm{F}_1 = \frac{\mathrm{qpB}}{\mathrm{m}}, \mathrm{v}_1 = \frac{\mathrm{p}}{\mathrm{m}}$$

$$\mathrm{F}_2 = \frac{\mathrm{qpB}}{2\ \mathrm{m}}, \mathrm{v}_2 = \frac{\mathrm{p}}{2\ \mathrm{m}}$$

$$F_3 = \frac{2qpB}{4m}, v_3 = \frac{p}{4m}$$

$$\mathrm{F}_1 : \mathrm{F}_2 : \mathrm{F}_3 \ \& \ \mathrm{V}_1 : \mathrm{V}_2 : \mathrm{V}_3$$

$$1 : \frac{1}{2} : \frac{1}{2}$$

$$\& 1 : \frac{1}{2} : \frac{1}{4}$$

$$2 : 1 : 1$$

$$\& 4 : 2 : 1$$

10. (c) When a rod is free and it is heated then there is no thermal stress produced in it.

The rod will expand due to increase in temperature.

so both a & R are true.

11. (a)

$$\overrightarrow{\mathrm{AO}} + \overrightarrow{\mathrm{OB}} = \overrightarrow{\mathrm{AB}}$$

$$\overrightarrow{\mathrm{AO}} + \overrightarrow{\mathrm{OC}} = \overrightarrow{\mathrm{AC}}$$

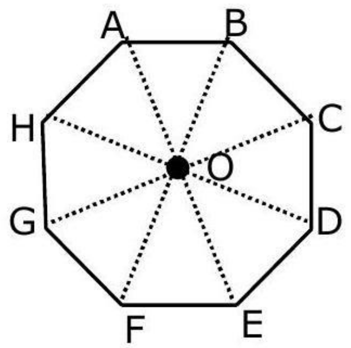
$$\overrightarrow{\mathrm{AO}} + \overrightarrow{\mathrm{OD}} = \overrightarrow{\mathrm{AD}}$$

$\overrightarrow{AO} + \overrightarrow{OE} = \overrightarrow{AE}$

$\overrightarrow{AO} + \overrightarrow{OF} = \overrightarrow{AF}$

$\overrightarrow{AO} + \overrightarrow{OG} = \overrightarrow{AG}$

$\overrightarrow{AO} + \overrightarrow{OH} = \overrightarrow{AH}$



$8\overrightarrow{AO} = (\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH})$

$= 8(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k})$.

$= 16\hat{\imath} + 24\hat{\jmath} - 32\hat{k}$

12. (a) After n half life no of nuclei undecayed $= \frac{N_0}{2^n}$

given $T_{\frac{1}{2}x} = \frac{T_{\frac{1}{2}y}}{2}$

So 3 half life of y = 6 half life of x

Given, $N_x = N_y \left(\text{after } 3T_{\frac{1}{2}y}\right)$

$\frac{N_1}{2^6} = \frac{N_2}{2^3}$

$\frac{N_1}{N_2} = \frac{2^6}{2^3} = 2^3 = \frac{8}{1}$

13. (a)

K.E. $= [ML^2T^{-2}]$

P(linear momentum) $= [MLT^{-1}]$

h (planck's constant) $= [ML^2\ T^{-1}]$

v (electric potential) $= [ML^2\ T^{-3}I^{-1}]$

14. (c) Least count. $= \frac{\text{pitch}}{\text{no. of div.}} = \frac{1\text{ mm}}{100} = 0.01$ m

ve error $= 8 \times$ L.C. $= +0.08$ mm

measured reading $= 1 \text{ mm} + 72 \times \text{L.C.}$

$= 1 \text{ mm} + 0.72 \text{ mm}$

$= 1.72 \text{ mm}$

True reading $= 1.72 - 0.08$

$= 1.64 \text{ mm}$

Radius $= \frac{1.64}{2} = 0.82 \text{ mm}$

15. (a)

$$T = 2\pi\sqrt{\frac{1}{g}}$$

$$T^2 = \frac{4\pi^2 \, l}{g}$$

$$g = \frac{4\pi^2 \, l}{T^2}$$

$$= \frac{4\pi^2 \times 2}{(2)^2} = 2\pi^2 \text{ ms}^{-2}$$

16. (b)

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqv}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4 \times 2}{1 \times 1}}$$

$$= 2\sqrt{2} = 2.8$$

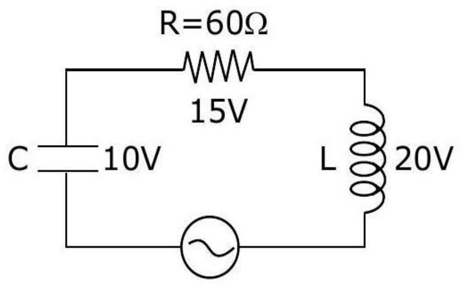
17. (b) V_e = escape velocity

$$v_e = \sqrt{\frac{2GM}{R}}$$

so for same $v_e, \frac{M_1}{R_1} = \frac{M_2}{R_2}$

A is true but R is false

18. (a)



Since key is open, circuit is series

$15 = i_{rms}(60)$

$\therefore i_{rms} = \frac{1}{4}$ A

Now, $20 = \frac{1}{4} X_L = \frac{1}{4}(\omega L)$

$\therefore L = \frac{4}{5} = 0.8H$

$\& 10 = \frac{1}{4} \frac{1}{(100C)}$

$C = \frac{1}{4000}$ F $= 250\mu F$

19. (c) Let $I_1 = 2x$

$I_2 = 1$

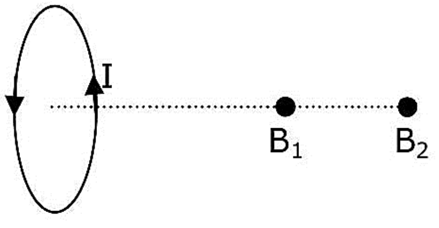
$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(\sqrt{2x}+1)^2 - (\sqrt{2x}-1)^2}{(\sqrt{2x}+1)^2 + (\sqrt{2x}-1)^2}$$

$$= \frac{4\sqrt{2x}}{2+4x} = \frac{2\sqrt{2x}}{1+2x}$$

20. (c)



$$B = \frac{\mu_0 N i R^2}{2(R^2 + x^2)^{3/2}}$$

at $x_1 = 0.05$ m, $B_1 = \frac{\mu_0 NiR^2}{2(R^2+(0.05)^2)^{3/2}}$

at $x_2 = 0.2m, B_2 = \frac{\mu_0 NiR^2}{2(R^2+(0.2)^2)^{3/2}}$

$$\frac{B_1}{B_2} = \frac{(R^2 + 0.04)^{3/2}}{(R^2 + 0.0025)^{3/2}}$$

$$\left(\frac{8}{1}\right)^{2/3} = \frac{R^2 + 0.04}{R^2 + 0.0025}$$

$$4(R^2 + 0.0025) = R^2 + 0.04$$

$$3R^2 = 0.04 - 0.0100$$

$$R^2 = \frac{0.03}{3} = 0.01$$

$$\mathrm{R} = \sqrt{0.01} = 0.1 \text{ m}$$

21. (15)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$m = \frac{v}{u}$$

from (a) and (b) we get

$$m = \frac{f}{f + u}$$

given conditions

$$m_1 = -m_2$$

$$\frac{f}{f - 10} = \frac{-f}{f - 20}$$

$$f - 20 = -f + 10$$

$$2f = 30$$

$$\mathrm{f} = 15 \text{ cm}$$

22. (0.5)

$\phi = \vec{E} \cdot \vec{A}$

$\vec{A}_a = 0.2\hat{\imath}$

$\vec{A}_b = 0.3\hat{\jmath}$

$\phi_a = \left(\frac{3}{5}E_0\hat{\imath} + \frac{4}{5}E_0\hat{\jmath}\right) \cdot 0.2\hat{\imath}$

$\phi_a = \frac{3}{5}E_0 \times 0.2$

$\phi_a = \left(\frac{3}{5}E_0\hat{\imath} + \frac{4}{5}E_0\hat{\jmath}\right) \cdot 0.3\hat{\jmath}$

$\phi_b = \frac{4}{5}E_0 \times 0.3$

$\frac{a}{b} = \frac{\phi_a}{\phi_b} = \frac{\frac{3}{5}E_0 \times 0.2}{\frac{4}{5}E_0 \times 0.3} = \frac{6}{12} = 0.5$

23. (128) Let charge on each drop = q

radius = r

$v = \frac{kq}{r}$

$2 = \frac{kq}{r}$

radius of bigger

$\frac{4}{3}\pi R^3 = 512 \times \frac{4}{3}\pi r^3$

$R = 8r$

$v = \frac{k(512)q}{R} = \frac{512}{8}\frac{kq}{r} = \frac{512}{8} \times 2$

$= 128$ V

24. (1)

$F = -\frac{dU}{dr}$

$F = -\left[-\frac{10\alpha}{r^{11}} + \frac{5\beta}{r^6}\right]$

for equilibrium, $F = 0$

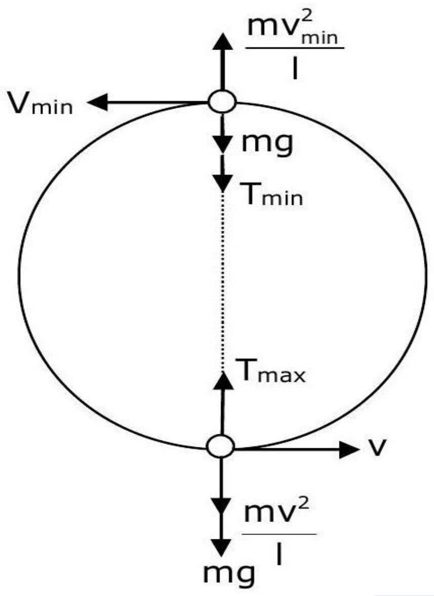
$\frac{10\alpha}{r^{11}} = \frac{5\beta}{r^6}$

$\frac{2\alpha}{\beta} = r^5$

$r = \left(\frac{2\alpha}{\beta}\right)^{1/5}$

$a = 1$

25. (5)



by conservation of energy,

$$V_{min}^2 = V^2 - 4gl$$

$$T_{max} = mg + \frac{mv^2}{l}$$

$$T_{min} = \frac{mv_{min}^2}{l} - mg$$

from equation (a) and (c)

$$T_{min} = \frac{m}{l}(v^2 - 4gl) - mg$$

$$\frac{T_{max}}{T_{min}} = \frac{\frac{v^2}{l} + g}{\frac{v^2}{l} - 5g}$$

$$\frac{5}{1} = \frac{\frac{v^2}{1} + 10}{\frac{v^2}{1} - 50}$$

$$5v^2 - 250 = v^2 + 10$$

$$v^2 = 65$$

from equation (4) and (1)

$$v_{min}^2 = 65 - 40 = 25$$

$$v_{min} = 5$$

26. (50) $P = kv^3$

$pv^{-3} = k$

$x = -3$

$$w = \frac{nR(T_1 - T_2)}{x - 1}$$

$$= \frac{nR(100 - 300)}{-3 - 1}$$

$$= \frac{nR(-200)}{-4}$$

$= 50nR$

27. (a) $\frac{E_1}{E_2} = \frac{I_1}{I_2}$

$$= \frac{3 \times 100 \text{ cm} + (100 - 20)\text{cm}}{7 \times 100 \text{ cm} + 60 \text{ cm}}$$

$$= \frac{380}{760} = \frac{1}{2} = \frac{a}{b}$$

$a = 1$

28. (3600)

$\Delta K_E = \Delta U$

$\Delta U = nCv\Delta T$

$$\frac{1}{2}mv^2 = \frac{3}{2}nR\Delta T$$

$$\frac{mv^2}{3nR} = \Delta T$$

$$\frac{4 \times (30)^2}{3 \times 1 \times R} = \Delta T$$

$$\Delta T = \frac{1200}{R}$$

$$\frac{x}{3R} = \frac{1200}{R}$$

$x = 3600$

29. (144)

$$L\frac{di}{dt} = \varepsilon$$

$= 3t$

$L\int \mathrm{di} = 3\int \mathrm{tdt}$

$Li = \frac{3t^2}{2}$

$i = \frac{3t^2}{2L}$

energy, $E = \frac{1}{2}\mathrm{Li}^2$

$= \frac{1}{2}\mathrm{L}\left(\frac{3\mathrm{t}^2}{2\mathrm{~L}}\right)^2$

$= \frac{1}{2} \times \frac{9\mathrm{t}^4}{4\mathrm{~L}}$

$= \frac{9}{8} \times \frac{(4)^4}{4 \times 2} = 144\mathrm{~J}$

30. (10)

Since resonance

$\omega_r = \frac{1}{\sqrt{\mathrm{LC}}}$

$\therefore 2\pi \mathrm{f} = \frac{1}{\sqrt{\mathrm{LC}}}$

$\therefore 4\pi^2 \frac{\mathrm{C}^2}{\lambda^2} = \frac{1}{\mathrm{LC}}$

$\therefore \frac{4\pi^2 \times 9 \times 10^8 \times 9 \times 10^8}{960 \times 960} = \frac{1}{\mathrm{L} \times 2.56 \times 10^{-6}}$

$L = \frac{375 \times 960}{10^{-6} \times 4 \times \pi^2 \times 9 \times 10^{16}} = \frac{10^3}{10^{10}}$

$= 10^{-7}\mathrm{H}$

$= 10 \times 10^{-8}$

31. (a) Ellingham diagram tells us about the spontanity of a reaction with temperature.
32. (c)
33. (d) $\frac{\mathrm{x}}{\mathrm{m}} = \mathrm{p}^{\mathrm{x}}$

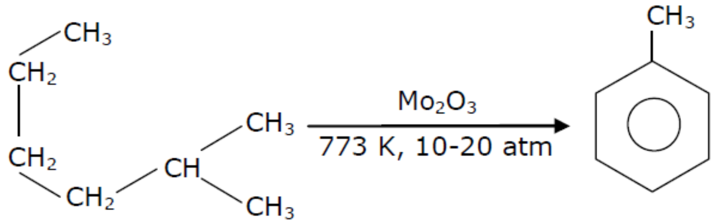
the formula is $\frac{x}{m} = p^{1/n}$

Hence $x = \frac{1}{n}$

The value of ' n ' is any natural number.

34. (d) B.O. of Be_2 is zero, So it does not exist.

35. (d)



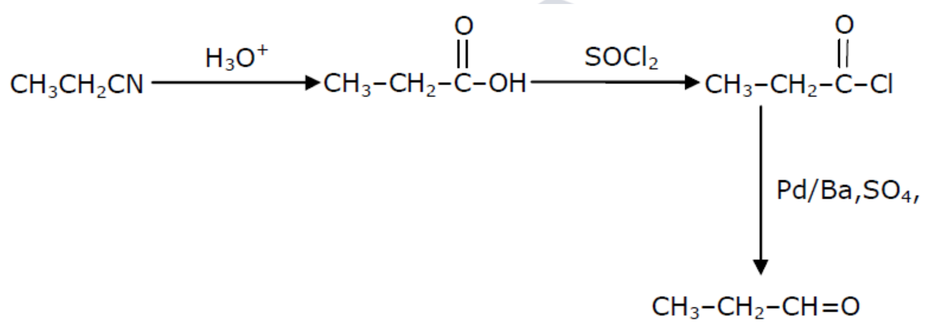
Aromatization reaction or hydroforming reaction.

36. (d)

CeO_2 can be used as oxidising agent like seO_2.

Similarly $EuSO_4$ used as a reducing agent.

37. (b)



38. (b)

$$n_{CO_2} = \frac{2.64}{44} = 0.06$$

$n_c = 0.06$

weight of carbon $= 0.06 \times 12 = 0.72$gm

$$n_{H_2O} = \frac{1.08}{18} = 0.06$$

$n_H = 0.06 \times 2 = 0.12$

weight of H $= 0.12$gm

$\therefore$ Weight of oxygen in $C_xH_yO_z$

$= 1.8 - (0.72 + 0.12)$

$= 0.96$gram

% weight of oxygen $= \frac{0.96}{1.8} \times 100$

$= 53.3\%$

39. (c) Terminal bond angle is greater than that of bridge bond angle

Bond angle ∝ S-character

$\propto \frac{1}{p - \text{ character}}$

40. (b)

$Cr^{+} \rightarrow [Ar]3\ d^{5}$

$Mn^{2+} \Rightarrow [Ar]3\ d^{5}$

41. (c) Synthesis of Buna-S needs nascent oxygen.

42. (c) Reducing smog as is acts as reducing agent, the reducing character is due to presence of sulphur dioxide and carbon particles.

43. (a) 3s orbital

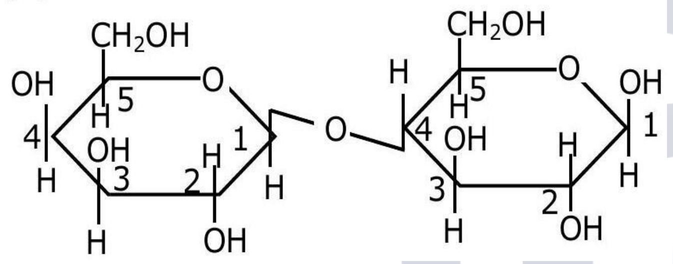
Number of radial nodes $= n - \ell - 1$

For 3 s orbital $n = 3\ \ell = 0$

Number of radial nodes $= 3 - 0 - 1 = 2$

It is correctly represented in graph of option *D*

44. (b)



β-D-Galactose β-D-Glucose

45. (a)

$$CH_3CH_2OH \xrightarrow{CrO_3 - H_2SO_4} CH_3 - COOH$$

46. (a)

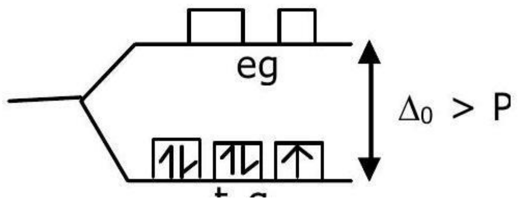
47. (a)

$(Mn(CN)_6)^{4-}$

$Mn^{++} = 3\ d^5$

$\mu = \sqrt{3}$

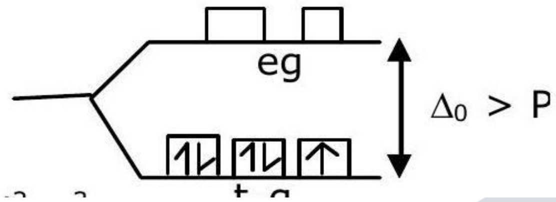
hybridization $= d^2sp^3$



2. $[Fe(CN)_6]^{3-}$

$Fe^{3+} = 3\ d^5$

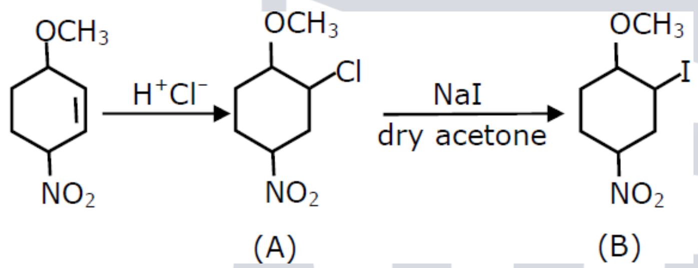
$\mu = \sqrt{3}$



Hybridization $-d^2sp^3$

$t_2\ g$

48. (d)



49. (a) Compounds which are more acidic then H_2CO_3, gives CO_2 gas on reaction with $NaHCO_3$. Compound B i.e. Benzoic acid and compound C i.e. picric acid both are more acidic than H_2CO_3.

50. (d) Let solubility is x

$AgCN \rightleftharpoons Ag^+ + CN^-\ K_{sp} = 2.2 \times 10^{-16}$

$H^+ + CN^- \rightleftharpoons HCN\ K = \frac{1}{K_a} = \frac{1}{6.2 \times 10^{-10}}$

$K_{sp} \times \frac{1}{K_a} = [Ag^+][CN^-] \times \frac{[HCN]}{[H^+][CN^-]}$

$2.2 \times 10^{-16} \times \frac{1}{6.2 \times 10^{-10}} = \frac{[S][S]}{10^{-3}}$

$S^2 = \frac{2.2}{6.2} \times 10^{-9}$

$S^2 = 3.55 \times 10^{-10}$

$S = \sqrt{3.55 \times 10^{-10}}$

$S = 1.88 \times 10^{-5} \Rightarrow 1.9 \times 10^{-5}$

51. (741 kJ/mol)

$$\mathrm{NH_2CN(s)} + \frac{3}{2}\mathrm{O_2(\,g)} \longrightarrow \mathrm{N_2(\,g)} + \mathrm{CO_2(\,g)} + \mathrm{H_2O}(\ell)$$

$$\Delta \mathrm{ng} = (1+1) - \frac{3}{2} = \frac{1}{2}$$

$$\Delta \mathrm{H} = \Delta \mathrm{U} + \Delta \mathrm{ngRT}$$

$$= -742.24 + \frac{1}{2} \times \frac{8.314 \times 298}{1000}$$

$$= -742.24 + 1.24$$

$$= 741 \text{ kJ/mol}$$

52. (173 mL)

$$17\mathrm{H_2O} + 8\mathrm{CrO_4} + 3\ \mathrm{S_2O_3} \longrightarrow 6\mathrm{SO_4} + 8\mathrm{Cr(OH)_4^-} + 2\mathrm{OH^-}$$

Applying mole-mole analysis

$$\frac{0.154 \times v}{8} = \frac{40 \times 0.25}{3}$$

$$V = 173 \text{ mL}$$

53. (526 K)

$$\log_{10}\ \mathrm{K} = \log_{10}\ \mathrm{A} - \frac{\mathrm{E_a}}{2.303\mathrm{RT}}$$

$$\text{Slope} = \frac{E_a}{2.303R} = -10000$$

$$\log_{10} \frac{\mathrm{K_2}}{\mathrm{K_1}} = \frac{\mathrm{E_a}}{2.303\mathrm{R}} \times \left[\frac{1}{\mathrm{T_1}} - \frac{1}{\mathrm{T_2}}\right]$$

$$\log_{10} \frac{10^{-4}}{10^{-5}} = 10000 \times \left[\frac{1}{500} - \frac{1}{\mathrm{T}}\right]$$

$$1 = 10000 \times \left[\frac{1}{500} - \frac{1}{\mathrm{T}}\right]$$

$$\frac{1}{10000} = \frac{1}{500} - \frac{1}{\mathrm{T}}$$

$$\frac{1}{T} = \frac{1}{500} - \frac{1}{10000}$$

$$\frac{1}{T} = \frac{20-1}{10000} = \frac{19}{10000}$$

$$T = \frac{10,000}{19} \Rightarrow 526 \text{ K}$$

54. (3)

1^{st} **end point reaction**

$NaOH + HCl \longrightarrow NaCl + H_2O$

$nf = 1$

$NaCO_3 + HCl \longrightarrow NaHCO_3$

$nf = 1$

Eq of HCl used $= n_{NaOH} \times 1 + n_{Na_2CO_3} \times 1$

$17.5 \times \frac{1}{10} \times 10^{-3} = n_{NaOH} + n_{Na_2CO_3}$

2^{nd} end point

$NaHCO_3 + HCl \longrightarrow H_2CO_3$

$1.5 \times \frac{1}{10} \times 10^{-3} = n_{NaHCO_3} \times 1 = n_{NaHCO_3}$

$0.15\text{mmol} = n_{Na_2CO_3}$

$0.15 = n_{Na_2CO_3}$

$w_{Na_2CO_3} = \frac{0.15 \times 106 \times 10^{-3}}{0.5} \times 100 \times 10$

$= 3 \times 106 \times 10^{-2}$

$= 3 \times 1.06 = 3.18\%$

55. (5576 kJ)

$Na(s) \longrightarrow Na^+(g)$ $\quad \Delta H = 495.8$
$\frac{1}{2}Br_2(\ell) + e^- \longrightarrow Br^-(g)$ $\quad \Delta H = 325$
$Na^+(g) + Br^-(g) \longrightarrow NaBr(s)$ $\quad \Delta H = -728.4$
$Na(s) + \frac{1}{2}Br_2(\ell) \longrightarrow NaBr(s).\ \Delta H = ?$
$\Delta H = 495.8 - 325 - 728.4$
$-557.6 \text{ kJ} = -5576 \times 10^{-1} \text{ kJ}$

56. (7)
57. ($69.85^\circ C \simeq 70^\circ C$)

$\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$\frac{35}{300} = \frac{40}{T_2}$

$$T_2 = \frac{40 \times 300}{35}$$

$= 342.86\ K$

$= 69.85°C \simeq 70°C$

58. (d)

59. (375 K)

$A_2B_3 \longrightarrow 2A^{+3} + 3B^{-2}$

No. of ions $= 2 + 3 = 5$

$i = 1 + (n-1) \propto$

$= 1 + (5-1) \times 0.6$

$= 1 + 4 \times 0.6 = 1 + 2.4 = 3.4$

$\Delta T_b = K_b \times m \times i$

$= 0.52 \times 1 \times 3.4 = 1.768°C$

$\Delta T_b = (T_b)_{solution} - [(T_b)_{H_2O}]_{solution}$

$1.768 = (T_b)_{solution} - 100$

$(T_b)_{solution} = 101.768°C$

$= 375\ K$

60. (4)

$$R_f = \frac{\text{Dis tan ce travelled by compound}}{\text{Dis tan ce travelled by solvent}}$$

On chromatogram distance travelled by compound is → 2 cm

Distance travelled by solvent = 5 cm

So $R_f = \frac{2}{5} = 4 \times 10^{-1} = 0.4$

61. (d) $ax^2 + bx + c = 0$

$a, b, c \in \{1,2,3,4,5,6\}$

$n(s) = 6 \times 6 \times 6 = 216$

$D = 0 \Rightarrow b^2 = 4ac$

$ac = \frac{b^2}{4}$ If $b = 2, ac = 1 \Rightarrow a = 1, c = 1$

If $b = 4, ac = 4 \Rightarrow a = 1, c = 4$

$a = 4, c = 1$

$a = 2, c = 2$

If $b = 6, ac = 9 \Rightarrow a = 3, c = 3$

$\therefore$ probability $= \frac{5}{216}$

62. (c) $r^2 + m^2 + n^2 = 1$

$\therefore 2n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore I^2 + m^2 = \frac{1}{2} \& I + m = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{2} - 2lm = \frac{1}{2}$

$\Rightarrow m = 0 \text{ or } m = 0$

$\therefore I = 0, m = \frac{1}{\sqrt{2}}$ or $I = \frac{1}{\sqrt{2}}$

$< 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ or $< \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$

$\therefore \cos\alpha = 0 + 0 + \frac{1}{2} = \frac{1}{2}$

$\therefore \sin^4\alpha + \cos^4\alpha = 1 - \frac{1}{2}\sin^2(2\alpha) = 1 - \frac{1}{2}, \frac{3}{4} = \frac{5}{8}$

63. (c)

$$\int \frac{2\sin^2\theta\cos\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{2\sin^2\theta} d\theta$$

Let $\sin\theta = t, \cos\theta d\theta = dt$

$= \int (t^6 + t^4 + t^2)\sqrt{2t^4 + 3t^2 + 6}dt = \int (t^5 + t^3 + t)\sqrt{2t^6 + 3t^4 + 6t^2}dt$

Let $2t^6 + 3t^4 + 6t^2 = z$

$12(t^5 + t^3 + t)dt = dz$

$= \frac{1}{12}\int \sqrt{z}dz = \frac{1}{18}z^{3/2} + c$

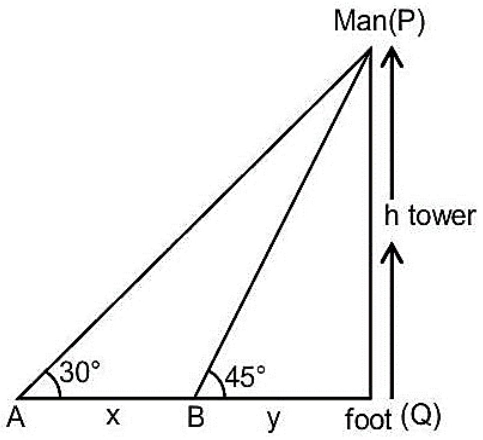
$= \frac{1}{18}[(2\sin^6\theta + 3\sin^4\theta + 6\sin^2\theta)^{3/2} + C$

$= \frac{1}{18}[(1 - \cos^2\theta)(2(1 - \cos^2\theta)^2 + 3 - 3\cos^2\theta + 6)]^{3/2} + C$

$= \frac{1}{18}[(1 - \cos^2\theta)(2\cos^4\theta - 7\cos^2\theta + 11)]^{3/2} + C$

$= \frac{1}{18}[-2\cos^6\theta + 9\cos^4\theta - 18\cos^2\theta + 11]^{3/2} + C$

64. (d)



$\frac{h}{x+y} = \tan 30°$

$x + y = \sqrt{3}h$

Also

$\frac{\mathrm{h}}{\mathrm{y}} = \tan 45°$

$\mathrm{h} = \mathrm{y}$

put in (a)

$x + y = \sqrt{3}y$

$x = (\sqrt{3} - 1)y$

$\frac{\mathrm{x}}{20} =$ 'v'speed

∴ time taken to reach

Foot from B

$\Rightarrow \frac{\mathrm{y}}{\mathrm{V}}$

$\Rightarrow \frac{\mathrm{x}}{(\sqrt{3}-1)\cdot \mathrm{x}} \times 20$

$\Rightarrow 10(\sqrt{3} + 1)$

65. (d) $x = 1 + \cos^2\theta + \cdots \ldots\ldots$

$x = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$

$y = 1 + \sin^2\phi + \cdots \ldots\ldots$

$y = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi}$

$$z=\frac{1}{1-\cos^2\theta\cdot\sin^2\phi}=\frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)}=\frac{xy}{xy-(x-1)(y-1)}$$

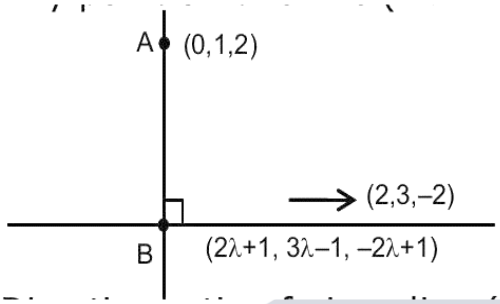
$\mathrm{xz}+\mathrm{yz}-\mathrm{z}=\mathrm{xy}$

$xy+z=(x+y)z$

66. (a)

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}=\lambda$$

Any point on this line $(2\lambda+1, 3\lambda-1, -2\lambda+1)$



Direction ratio of given line $(2,3,-2)$

Direction ratio of line to be found $(2\lambda+1, 3\lambda-2, -2\lambda-1)$

$\therefore \vec{\mathrm{d}}_1\cdot\vec{\mathrm{d}}_2=0$

$\lambda=2/17$

Direction ratio of line $(21,-28,-21)\equiv(3,-4,-3)\equiv(-3,4,3)$

67. (b) $A\rightarrow(B\rightarrow A)$

$\Rightarrow A\rightarrow(\sim B\vee A)$

$\Rightarrow\sim A\vee(\sim B\vee A)$

$\Rightarrow\sim B\vee(\sim A\vee A)$

$\Rightarrow\sim B\vee\mathrm{t}$

$=\mathrm{t}$ (tantology)

From options:

(b) $A\rightarrow(A\vee B)$

$\Rightarrow\sim A\vee(A\vee B)$

$\Rightarrow(\sim A\vee A)\vee B$

$\Rightarrow\mathrm{t}\vee\mathrm{B}$

$\Rightarrow\mathrm{t}$

68. (a) $D < 0$

$(2(3k-1))^2 - 4(8k^2-7) < 0$
$4(9k^2-6k+1) - 4(8k^2-7) < 0$
$k^2 - 6k + 8 < 0$
$(k-4)(k-2) < 0$
$2 < k < 4$
then $k = 3$

69. (d) Equation of tangent: $y = mx + \frac{3}{2m}$

$m_T = \frac{1}{2}$($\because$ perpendicular to line $2x + y = 1$)

$\therefore$ tangent is : $y = \frac{x}{2} + 3 \Rightarrow x - 2y + 6 = 0$

70. (c) $f(n+1) = f(n) + 1$

$f(\mathrm{b}) = 2f(\mathrm{a})$

$f(\mathrm{c}) = 3f(\mathrm{a})$

$f(\mathrm{d}) = 4f(\mathrm{a})$

….

$f(n) = nf(\mathrm{a})$

$f(x)$ is one-one

71. (c)

$(2-i)z = (2+i)\bar{z}$

$\Rightarrow (2-\mathrm{i})(\mathrm{x}+\mathrm{iy}) = (2+\mathrm{i})(\mathrm{x}-\mathrm{iy})$

$\Rightarrow 2\mathrm{x} - \mathrm{ix} + 2\mathrm{iy} + \mathrm{y} = 2\mathrm{x} + \mathrm{ix} - 2 - \mathrm{iy} + \mathrm{y}$

$\Rightarrow 2\mathrm{ix} - 4\mathrm{iy} = 0$

$\mathrm{L}_1 : \mathrm{x} - 2\mathrm{y} = 0$

$\Rightarrow (2+\mathrm{i})\mathrm{z} + (\mathrm{i}-2)\bar{\mathrm{z}} - 4\mathrm{i} = 0.$

$\Rightarrow (2+\mathrm{i})(\mathrm{x}+\mathrm{iy}) + (\mathrm{i}-2)(\mathrm{x}-\mathrm{iy}) - 4\mathrm{i} = 0.$

$\Rightarrow 2\mathrm{x} + \mathrm{ix} + 2\mathrm{iy} - \mathrm{y} + \mathrm{ix} - 2\mathrm{x} + \mathrm{y} + 2\mathrm{iy} - 4\mathrm{i} = 0$

$\Rightarrow 2\mathrm{ix} + 4\mathrm{iy} - 4\mathrm{i} = 0$

$\mathrm{L}_2 : \mathrm{x} + 2\mathrm{y} - 2 = 0$

Solve L_1 and L_2 $4y = 2, y = \frac{1}{2}$

$\therefore \mathrm{x} = 1$

Centre $\left(1,\frac{1}{2}\right)$

$L_3: iz+\bar{z}+1+i=0$

$\Rightarrow i(x+iy)+x-iy+1+i=0$

$\Rightarrow ix-y+x-iy+1+i=0$

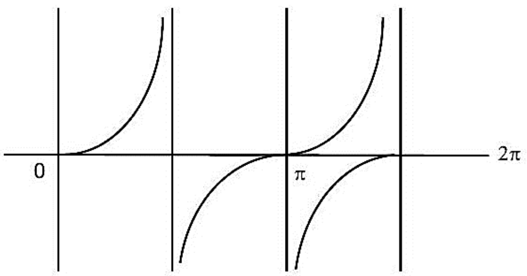
$\Rightarrow (x-y+1)+i(x-y+1)=0$

Radius = distance from $\left(1,\frac{1}{2}\right)$ to $x-y+1=0$

$$r=\frac{1-\frac{1}{2}+1}{\sqrt{2}}$$

$$r=\frac{3}{2\sqrt{2}}$$

72. (b)



$\tan 2\theta(1+\cos 2\theta)>0$

$$2\theta\in\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)\cup\left(2\pi,\frac{5\pi}{2}\right)\cup\left(3\pi,\frac{7\pi}{2}\right)$$

$$\Rightarrow \theta\in\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$$

73. (a) Image of $P(3,5)$ on the line $x-y+1=0$ is

$$\frac{x-3}{1}=\frac{y-5}{-1}=\frac{-2(3-5+1)}{2}=1$$

$x=4, y=4$

$\therefore$ Image is $(4,4)$

Which lies on

$(x-2)^2+(y-4)^2=4$

74. (b) $f(\mathrm{a})=f(\mathrm{b})$

$\Rightarrow 1-a+b-4=8-4a+2b-4$

$3a-b=7$

$f'(x) = 3x^2 - 2ax + b$

$\Rightarrow f'\left(\frac{4}{3}\right) = 0 \Rightarrow 3 \times \frac{16}{9} - \frac{8}{3}a + b = 0$

$\Rightarrow -8a + 3b = -16$

$a = 5, b = 8$

75. (c) $\frac{x^2}{a} + \frac{y^2}{b} = 1$

diff : $\frac{2x}{a} + \frac{2y}{b}\frac{dy}{dx} = 0 \Rightarrow \frac{y}{b}\frac{dy}{dx} = \frac{-x}{a}$

$\frac{dy}{dx} = \frac{-bx}{ay}$

$\frac{x^2}{c} + \frac{y^2}{d} = 1$

Diff: $\frac{dy}{dx} = \frac{-dx}{cy}$

$m_1 m_2 = -1 \Rightarrow \frac{-bx}{ay} \times \frac{-dx}{cy} = -1$

$\Rightarrow bdx^2 = -acy^2$

(a) $-$(c) $\Rightarrow \left(\frac{1}{a} - \frac{1}{c}\right)x^2 + \left(\frac{1}{b} - \frac{1}{d}\right)y^2 = 0$

$\Rightarrow \frac{c-a}{ac}x^2 + \frac{d-b}{bd} \times \left(\frac{-bd}{ac}\right)x^2 = 0$ (using 5)

$\Rightarrow (c - a) - (d - b) = 0$

$\Rightarrow c - a = d - b$

$\Rightarrow c - d = a - b$

76. (c) It is 1^∞ form

$$L = e^{\lim_{n\to\infty}\left(\frac{1+\frac{1}{2}+\frac{1}{3}+\cdots..+\frac{1}{n}}{n}\right)}$$

$$S = 1 + \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}\right) + \left(\frac{1}{8} + \cdots \ldots \ldots + \frac{1}{15}\right)$$

$$S < 1 + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) \ldots \ldots \ldots + \underbrace{\left(\frac{1}{2^P} + \cdots \ldots . + \frac{1}{2^P}\right)}_{2^P \text{ times}}$$

$$S < 1 + 1 + 1 + 1 + \cdots \ldots \ldots + 1$$

$$S < P + 1$$

$$\therefore L = e^{\lim_{n\to\infty}\frac{(P+1)}{2^P}}$$

$$\Rightarrow L = e^\circ = 1$$

77. (d) $x \cdot y.z = 24$

$x \cdot y \cdot z=2^3 \cdot 3^1$

Now using beggars method.

3 things to be distributed among 3 persons

Each may receive none, one or more

$\therefore\ {}^5C_2$ ways

Similarly, for ' 1 ' $\therefore\ {}^3C_2$ ways

Total ways $= {}^5C_2 \cdot {}^3C_2=30$ ways

78. (d)

$$\frac{dy}{dx}=\frac{(x-2)^2+y+4}{(x-2)}=(x-2)+\frac{y+4}{(x-2)}$$

Let $x-2=t \Rightarrow dx=dt$

and $y+4=u \Rightarrow dy=du$

$$\frac{dy}{dx}=\frac{du}{dt}$$
$$\frac{du}{dt}=t+\frac{u}{t} \Rightarrow \frac{du}{dt}-\frac{u}{t}=t$$
$$\text{I.F} = e^{\int \frac{-1}{t} dt}=e^{-Int}=\frac{1}{t}$$
$$u \cdot \frac{1}{t}=\int t \cdot \frac{1}{t} dt \Rightarrow \frac{u}{t}=t+c$$
$$\frac{y+4}{x-2}=(x-2)+c$$

Passing through $(0,0)$

$c=0$

$\Rightarrow(y+4)=(x-2)^2$

79. (c) $I=\int_{-1}^0 x^2 \cdot e^{-1} dx+\int_0^1 x^2 dx$

$$\therefore \mathrm{I}=\left.\frac{\mathrm{x}^3}{3 \mathrm{e}}\right|_{-1} ^0+\left.\frac{\mathrm{x}^3}{3}\right|_0 ^1$$

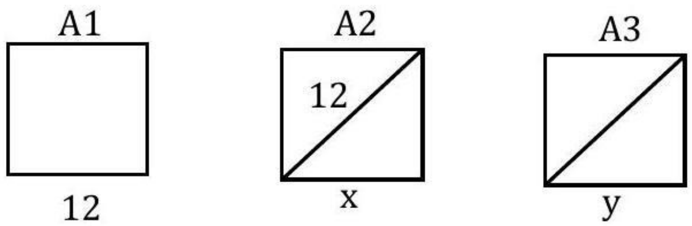
$$\Rightarrow \mathrm{I}=\frac{1}{3 \mathrm{e}}+\frac{1}{3}$$

80. (a) Probability of not getting intercepted $=\frac{2}{3}$

Probability of missile hitting target $=\frac{3}{4}$

$\therefore$ Probability that all 3 hit the target $=\left(\frac{2}{3} \times \frac{3}{4}\right)^3=\frac{1}{8}$

81. (9)



$x = \frac{12}{\sqrt{2}}$ $y = \frac{12}{(\sqrt{2})^2}$

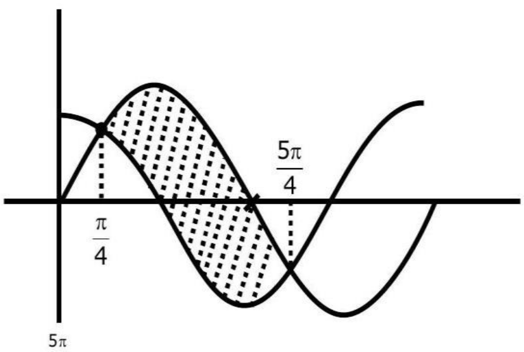
$\therefore$ Side lengths are in G.P.

$$T_n = \frac{12}{(\sqrt{2})^{n-1}}$$

$\therefore$ Area $= \frac{144}{2^{n-1}} < 1 \Rightarrow 2^{n-1} > 144$

Smallest $n = 9$

82. (64)



$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)dx = [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$

$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$

$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$

$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$

$\Rightarrow A^4 = (2\sqrt{2})^4 = 64$

83. (2)

$\sqrt{3}kx + ky = 4\sqrt{3}$

$\sqrt{3}\text{kx} - \text{ky} = 4\sqrt{3}\text{k}^2$

Adding equation (a) & (b)

$2\sqrt{3}\text{kx} = 4\sqrt{3}(\text{k}^2 + 1)$

$x = 2\left(k + \frac{1}{k}\right)$

Substracting equation (a) & (b)

$y = 2\sqrt{3}\left(\frac{1}{k} - k\right)$

$\therefore \frac{x^2}{4} - \frac{y^2}{12} = 4$

$\frac{x^2}{16} - \frac{y^2}{48} = 1$ Hyperbola

$\therefore e^2 = 1 + \frac{48}{16}$

$e = 2$

84. (13)

$$A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$$

$$\Rightarrow I + A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$\Rightarrow I - A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix} \{\therefore |I - A| = \sec^2\theta/2\}$$

$$\Rightarrow (I - A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$\Rightarrow (I + A)(I - A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$= \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 - \tan^2\frac{\theta}{2} & -2\tan\frac{\theta}{2} \\ 2\tan\frac{\theta}{2} & 1 - \tan^2\frac{\theta}{2} \end{bmatrix}$$

$$a = \frac{1 - \tan^2\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$$b = \frac{2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$$\therefore a^2 + b^2 = 1$$

85. (144)

$f(x) = x^6 + ax^5 + bx^4 + x^3$

$\therefore f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

Roots $1 \& -1$

$\therefore 6 + 5a + 4b + 3 = 0 \& -6 + 5a - 4b + 3 = 0$ solving

$a = -\frac{3}{5} \quad b = -\frac{3}{2}$

$\therefore f(x) = x^6 - \frac{3}{5}x^5 - \frac{3}{2}x^4 + x^3$

$\therefore 5.f(b) = 5\left[64 - \frac{96}{5} - 24 + 8\right] = 144$

86. (2)

$f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$$f(x) = \begin{cases} x^2 - 7; & x > 1 \\ -x^2 - 2x - 3; & -\frac{1}{2} < x < 1 \\ -x^2 - 6x - 5; & -2 < x < \frac{-1}{2} \\ x^2 + 2x + 3; & x < -2 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 2x; & x > 1 \\ -2x - 3; & -\frac{1}{2} < x < 1 \\ -2x - 6; & -2 < x < \frac{-1}{2} \\ 2x + 2; & x < -2 \end{cases}$$

Check at $1, -2$ and $\frac{-1}{2}$

Non. Differentiable at $x = 1$ and $\frac{-1}{2}$

87. (21)

$D = 0$

$$\Rightarrow \begin{vmatrix} k & 1 & 2 \\ 3 & -1 & -2 \\ -2 & -2 & -4 \end{vmatrix} = 0$$

$\Rightarrow k(4 - 4) - 1(-12 - 4) + 2(-6 - 2)$

$\Rightarrow 16-16=0$

Also. $D_1=D_2=D_3=0$

$\Rightarrow \mathrm{D}_2=\begin{vmatrix} \mathrm{k} & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix}=0$

$\Rightarrow \mathrm{k}(-8+6)-1(-12-4)+2(9+4)=0$

$\Rightarrow -2\mathrm{k}+16+26=0$

$\Rightarrow 2\mathrm{k}=42$

$\Rightarrow \mathrm{k}=21$

88. (12)

$\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$

$\vec{\mathrm{r}}\times\vec{\mathrm{a}}-\vec{\mathrm{c}}\times\vec{\mathrm{a}}=0$

$(\vec{\mathrm{r}}-\vec{\mathrm{c}})\times\vec{\mathrm{a}}=0$

$\therefore \vec{\mathrm{r}}-\vec{\mathrm{c}}=\lambda\vec{\mathrm{a}}$

$\vec{\mathrm{r}}=\lambda\vec{\mathrm{a}}+\vec{\mathrm{c}}$

$\vec{\mathrm{r}}\cdot\vec{\mathrm{b}}=\lambda\vec{\mathrm{a}}\cdot\vec{\mathrm{b}}+\vec{\mathrm{c}}\cdot\vec{\mathrm{b}}=0$

$\Rightarrow \lambda(1-2)+2=0$

$\Rightarrow \lambda=2$

$\therefore \vec{\mathrm{r}}=2\vec{\mathrm{a}}+\vec{\mathrm{c}}$

$\vec{\mathrm{r}}\cdot\vec{\mathrm{a}}=2|\vec{\mathrm{a}}|^2+\vec{\mathrm{a}}\cdot\vec{\mathrm{c}}$

$=2(1+4+1)+(1-2+1)$

$=12$

89. (7)

$A=\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix} \therefore |A|=(x^3+y^3+z^3-3xyz)$

$A^2=I_3$

$|A^2|=1$

$\therefore (x^3+y^3+z^3-3xyz)^2=1$

$\Rightarrow x^3+y^3+z^3-3xyz=1$

$\Rightarrow x^3+y^3+z^3=6+1=7$

only as $(x + y + z > 0)$

90. (32)