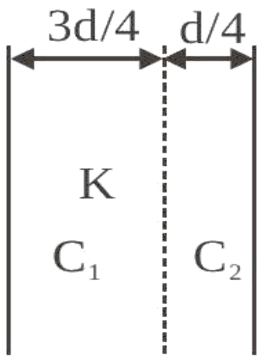
1. (d) $(n-1)a = n(a')$

$$a' = \frac{(n-1)a}{n}$$

$$\therefore \text{ L.C.} = 1\text{MSD} - 1\text{VSD}$$
$$= (a - a')\text{cm}$$
$$= a - \frac{(n-1)a}{n}$$
$$= \frac{na - na + a}{n} = \frac{a}{n}\text{cm}$$
$$= \left(\frac{10a}{n}\right)\text{mm}$$

2. (c)



$$C_0 = \frac{\epsilon_0 A}{d}$$

$C' = C_1$ and C_2 in series.

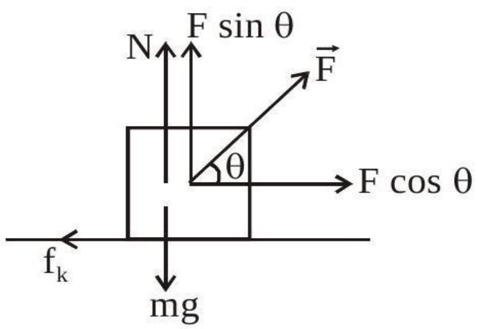
i.e. $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{1}{C'} = \frac{(3\,d/4)}{\epsilon_0 KA} + \frac{d/4}{\epsilon_0\ A}$$

$$\frac{1}{C'} = \frac{d}{4\ \epsilon_0\ A}\left(\frac{3+K}{K}\right)$$

$$C' = \frac{4KC_0}{(3+K)}$$

3. (b)



$N = \text{mg} - \text{f}\sin\theta$

$\mathrm{F}\cos\theta - \mu_k \mathrm{N} = \mathrm{ma}$

$\mathrm{F}\cos\theta - \mu_k(\mathrm{mg} - \mathrm{F}\sin\theta) = \mathrm{ma}$

$$a = \frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$$

4. (a)

$\mathrm{P}_1 = \rho g d + \mathrm{P}_0 = 3 \times 10^5$ Pa

$\therefore \rho g d = 2 \times 10^5$ Pa

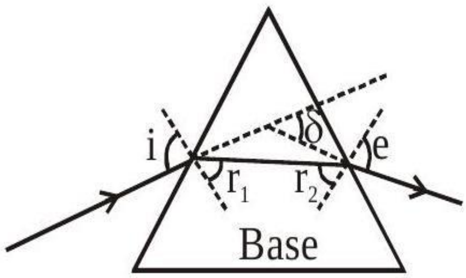
$\mathrm{P}_2 = 2\rho g d + \mathrm{P}_0$

$= 4 \times 10^5 + 10^5 = 5 \times 10^5$ Pa

$\%\text{increase} = \frac{\mathrm{P}_2 - \mathrm{P}_1}{\mathrm{P}_1} \times 100$

$$= \frac{5 \times 10^5 - 3 \times 10^5}{3 \times 10^5} \times 100 = \frac{200}{3}\%$$

5. (a) Deviation is minimum in a prism when: $\mathrm{i} = \mathrm{e}, \mathrm{r}_1 = \mathrm{r}_2$ and ray (b) is parallel to base of prism.



6. (a) $\mathrm{f} = 5 \times 10^8$ Hz

EM wave is travelling towards $+\hat{\jmath}$

$\vec{B} = 8.0 \times 10^{-8}\hat{z}\mathrm{T}$

$\vec{E} = \vec{B} \times \vec{C} = (8 \times 10^{-8}\hat{z}) \times (3 \times 10^8\hat{y})$

$= -24\hat{x}\mathrm{V/m}$

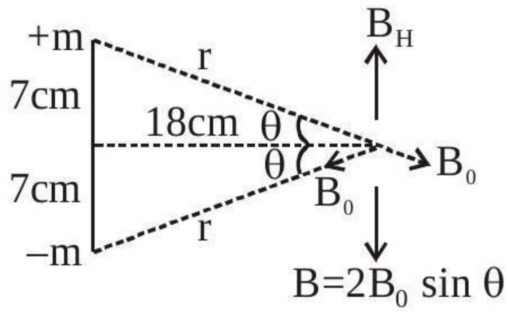
7. (c) By angular momentum conservation:

$\mathrm{mv}_1\mathrm{r}_1 = \mathrm{mv}_2\mathrm{r}_2$

$$\mathrm{v}_1 = \frac{48 \times 10^{14}}{1.6 \times 10^{12}} = 3000 \text{ m/sec}$$

$= 3 \times 10^3$ m/sec.

8. (c)



i.e. $\frac{2\mu_0}{4\pi}\frac{m}{r^2}\times\frac{7}{r}=0.4\times10^{-4}$

$\Rightarrow 2\times10^{-7}\times\frac{m\times7}{(7^2+18^2)^{3/2}}\times10^4$

$=0.4\times10^{-4}$

$m=\frac{4\times10^{-2}\times(373)^{3/2}}{14}$

$M=m\times14\text{ cm}=m\times\frac{14}{100}$

$=\frac{0.04\times(373)^{3/2}}{14}\times\frac{14}{100}$

$=4\times10^{-4}\times7203.82=2.88\text{ J/T}$

9. (c) $PV=(n_1+n_2+n_3)RT$

$P\times V=\left[\frac{16}{32}+\frac{28}{28}+\frac{44}{44}\right]RT$

$PV=\left[\frac{1}{2}+1+1\right]RT$

$P=\frac{5}{2}\frac{RT}{V}$

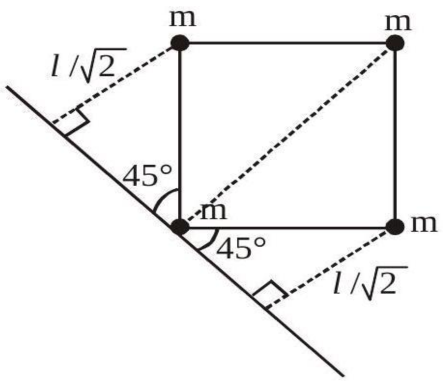
10. (a) Heat and work are treated as path functions in thermodynamics.

$\Delta Q=\Delta U+\Delta W$

Since work done by gas depends on type of process i.e. path and ΔU depends just on initial and final states, so ΔQ i.e. heat, also has to depend on process is path.

11. (c) Moment of inertia of point mass

$=\text{mass}\times(\text{ Perpendicular distance from axis })^2$



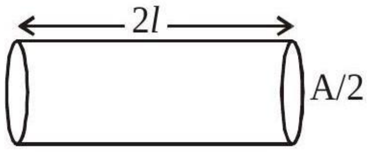
Moment of Inertia

$$= m(0)^2 + m(l\sqrt{2})^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$$

$$= 3\ ml^2$$

12. (a)

As per the question



Resistance $= \frac{\rho(2l)}{(A/2)} = \frac{4\rho l}{A}$

$\Rightarrow$ Current $= \frac{V}{R} = \frac{VA}{4\rho l}$

13. (d) When lift is stationary

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When lift is moving upwards

$\Rightarrow$ Pseudo force acts downwards

$$\Rightarrow g_{eff} = g + \frac{g}{2} = \frac{3g}{2}$$

$\Rightarrow$ New time period

$$T' = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{2L}{3g}}$$

$$T' = \sqrt{\frac{2}{3}}\,T$$

14. (a) For $0 \le x \le 200$

$v = mx + C$

$v = \frac{1}{5}x + 10$

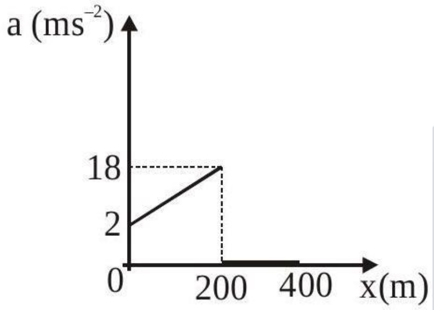
$a = \frac{vdv}{dx} = \left(\frac{x}{5} + 10\right)\left(\frac{1}{5}\right)$

$a = \frac{x}{25} + 2 \Rightarrow$ Straight line till $x = 200$

for $x > 200$

$v = \text{constant}$

$\Rightarrow a = 0$

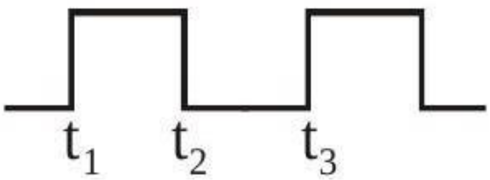
$\Rightarrow$ 

15. (d) Length of Antena $= 25 \text{ m} = \frac{\lambda}{4}$

$\Rightarrow \lambda = 100 \text{ m}$

16. (a) In EMW, Average energy density due to electric (U_e) and magnetic (U_m) fields is same.

17. (c)



For $t_1 - t_2$ Charging graph

$t_2 - t_3$ Discharging graph

18. (d) Stopping potential changes linearly with frequency of incident radiation.

19. (d) $N = m\omega^2 R$

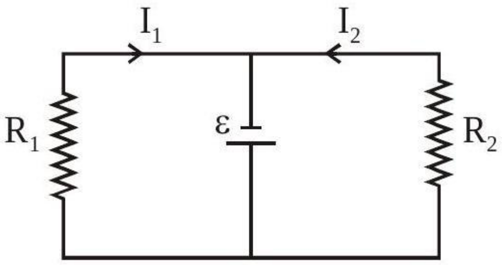
$N = m\left[\frac{4\pi^2}{T^2}\right]R$

Given $m = 0.2 \text{ kg}, T = 40 \text{ S}, R = 0.2 \text{ m}$

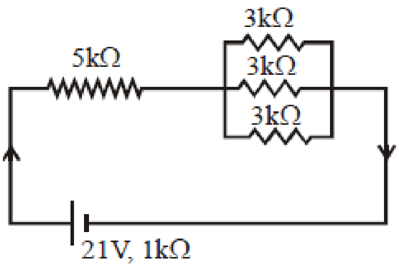
Put values in equation (a)

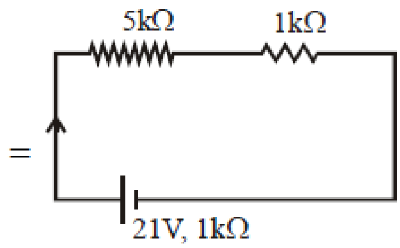
$N = 9.859 \times 10^{-4} \text{ N}$

20. (c)



21. (3)





$$I = \frac{21}{5+1+1} = 3 \text{ mA}$$

22. (600)

$$\beta = \frac{\lambda D}{d}$$

$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{6\times10^{-3}\times10^{-3}}{10}$$

$$\lambda = 6\times10^{-7} \text{ m} = 600\times10^{-9} \text{ m}$$

$$\lambda = 600 \text{ nm}$$

23. (20) $\alpha = \frac{\tau}{I} = \frac{F.R.}{mR^2/2} = \frac{2F}{mR}$

$$\alpha = \frac{2\times200}{20\times(0.2)} = 10 \text{rad/s}^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$(50)^2 = 0^2 + 2(10)\Delta\theta \Rightarrow \Delta\theta = \frac{2500}{20}$$

$$\Delta\theta = 125 \text{rad}$$

No. of revolution $= \frac{125}{2\pi} \approx 20$ revolution

24. (15) For 1st line

$$\frac{1}{\lambda_1} = \text{Rz}^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_1} = \text{Rz}^2\frac{5}{36}$$

For 3rd line

$$\frac{1}{\lambda_3} = \text{Rz}^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$$

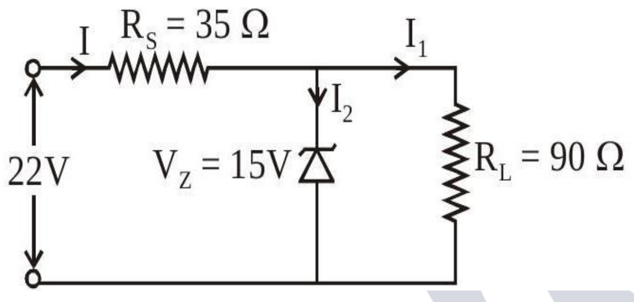
$$\frac{1}{\lambda_3} = \text{Rz}^2\frac{21}{100}$$

(ii) + (i)

$$\frac{\lambda_1}{\lambda_3} = \frac{21}{100} \times \frac{36}{5} = 1.512 = 15.12 \times 10^{-1}$$

$x \approx 15$

25. (5)



Voltage across $R_S = 22 - 15 = 7$ V

Current through $R_S = I = \frac{7}{35} = \frac{1}{5}$ A

Current through $90\Omega = I_2 = \frac{15}{90} = \frac{1}{6}$ A

Current through zener $= \frac{1}{5} - \frac{1}{6} = \frac{1}{30}$ A

Power through zener diode

$P = VI$

$P = 15 \times \frac{1}{30} = 0.5$ watt

$P = 5 \times 10^{-1}$ watt

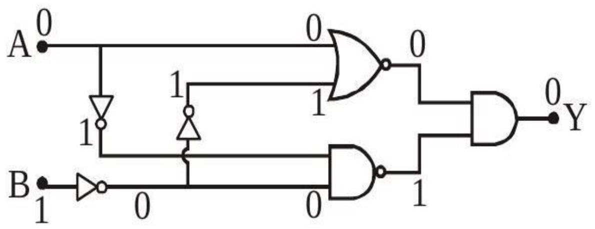
26. (4) At resonance power (P)

$P = \frac{(V_{rms})^2}{R}$

$P = \frac{(250/\sqrt{2})^2}{8} = 3906.25\ W$

$\approx 4\ kW$

27. (0)



28. (5)

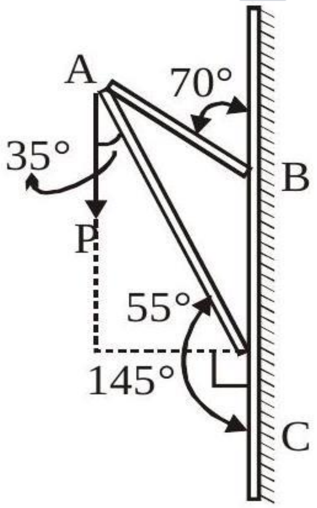
$.\ \frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$

% error in $R = \frac{2}{50} \times 100 + \frac{0.2}{20} \times 100$

% error in $R = 4 + 1$

% error in $R = 5\%$

29. (82)



Component along AC

$= 100\cos 35°N$

$= 100 \times 0.819\ N$

$= 81.9\ N$

$\approx 82\ N$

30. (30)

31. (c) Size of ${}_{97}Bk^{3+}$ ion is less than that of ${}_{93}Np^{3+}$ due to actinoid contraction. As we know that in a period from left to right ionic radius decreases and in actinide series it is due to actinoid contraction.

32. (b) Vitamin-A & Vitamin-D

33. (b)

(a) $CaCl_2 \cdot 6H_2O \xrightarrow{\Delta} \underset{\text{(Anhydrous)}}{CaCl_2} + 6H_2O$

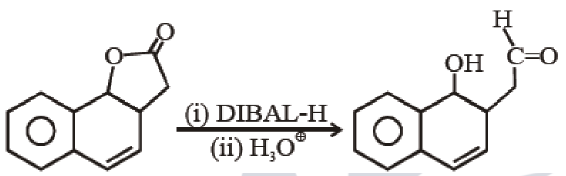
(b) $MgCl_2 \cdot 8H_2O \xrightarrow{\Delta} MgO + 2HCl + 6H_2O$

The dehydration of hydrated chloride of calcium can be achieved. The corresponding hydrated chloride of magnesium on heating suffer hydrolysis.

(c) $BeO \rightarrow$ Amphoteric

$\left.\begin{matrix} MgO \\ CaO \\ SrO \\ BaO \end{matrix}\right] \Rightarrow$ All are basic oxide

34. (b)



DIBAL cannot reduce double bond It can reduce cyclic ester.

35. (c)

(a) Haber's process is used for NH_3 synthesis.

(b) Ostwald's process is used for HNO_3 synthesis.

(c) Contact process is used for H_2SO_4 synthesis.

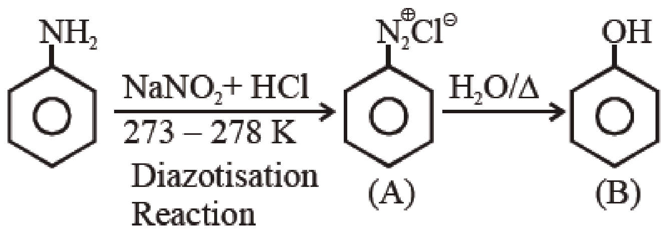
(d) In Hall-Heroult process, electrolytic reduction of impure alumina can be done. (Aluminium extraction)

36. (b)

(A) Non-Aromatic (B) Aromatic

(C) Aromatic (D) Anti-Aromatic

37. (c)



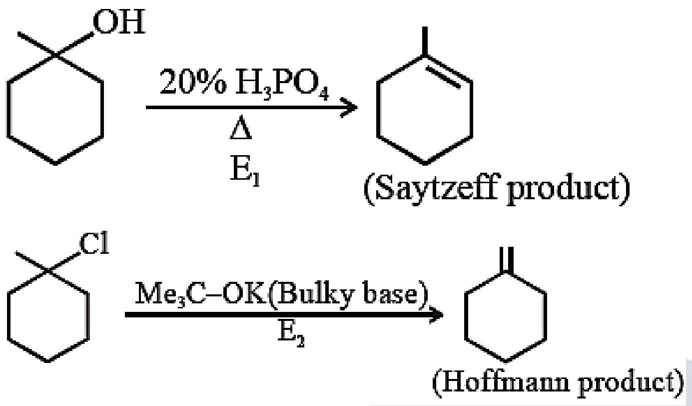
38. **(d)** The E° value for Ce^{4+}/Ce^{3+} is +1.74 V because the most stable oxidation state of lanthanide series elements is +3

It means Ce^{3+} is more stable than Ce^{4+}.

39. **(b)**

40. **(d)** Partially deactivated palladised charcoal $(H_2/pd/CaCO_3)$ is lindlar catalyst.

41. **(c)**



42. **(b)** (a) H_2O_2 can acts as both oxidising and reducing agent in basic medium.

(i) $2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH$

In this reaction, H_2O_2 acts as oxiding agent.

(ii) $2M_{M}^{7}O_4^- + 3H_2O_2 \rightarrow 2M\overset{+4}{n}H_2 + 3O_2 + 2H_2O + 2OH$

In this reaction, H_2O_2 acts as reducing agent.

(b) The basic principle of hydrogen economy is the transportation and storage of energy in the form of liquids or gaseous dihydrogen.

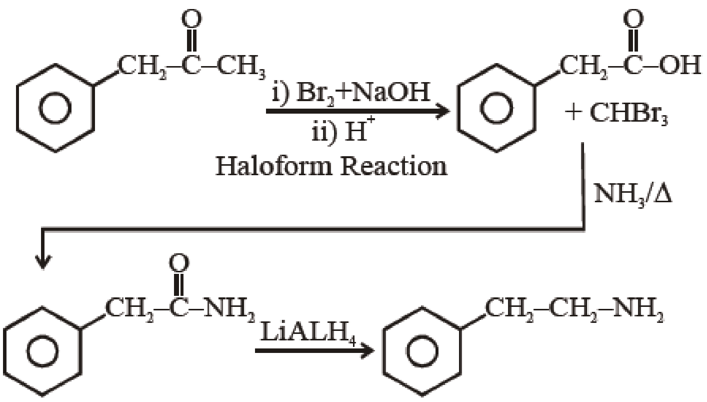
Advantage of hydrogen economy is that energy is transmitted in the form of dihydrogen and not as electric power

43. **(b)** In presence of ozone (O_3), oxidising smog gets increased during the day time because automobiles and factories produce main components of the photochemcial smog (oxidising smog) results from the action of sunlight on unsaturated hydrocarbon and nitrogen oxide.

Ozone is strong oxidising agent and can react with the unburnt hydrocarbons in the polluted air to produce chemicals.

44. **(b)**

45. **(c)**



This reaction does not involve haffmann bromanide degradation.

Rest all options involve haffmann bromamide degradation during the reaction of $Br_2 + NaOH$ with amide.

46. (b) In roasting process, metal sulphide (MS) ore are converted into metal oxide and sulphur is remove in the form of SO_2 gas.

$2MS + 3O_2 \xrightarrow{\Delta} 2MO + 2SO_2 \uparrow$

47. (a)

(a) Hypophosphorus acid: $H_3\underline{PO_2}$

$$(+1)3 + x + (-2)2 = 0$$
$$x = +1$$

(b) Orthophosphoric acid : H_3PO_4

$$(+1)3 + x + (-2)4 = 0$$
$$x = +5$$

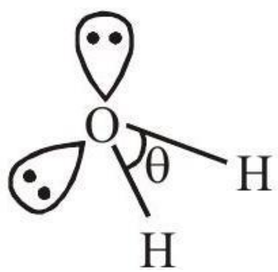
(c) Hypophosphoric acid : $H_4P_2O_6$

$$(+1)4 + 2x + (-2)6 = 0$$
$$x = +4$$

(d) Orthophosphorous acid : H_3PO_3

$$(+1)3 + x + (-2)3 = 0$$
$$x = +3$$

48. (d) H_2O



$\theta = 104.5°$

the hybridisation of oxygen is water molecule is sp^3.

So electron geometry of water molecule is tetrahedral and the bond angle should be $109°28''$ but as we know that lone pair-lone pair repulsion of electrons is higher than the bond pair-bond pair repulsion because lone pair is occupied more space areound central atom than that of bond pair.

49. (d) In chromotography technique, the purification of a compound is independent of the physical state of the pure compound.

50. (d) In group 15

$\left.\begin{matrix} N \\ P \end{matrix}\right] \rightarrow$ Non metal

$\left.\begin{matrix} As \\ Sb \end{matrix}\right] \rightarrow$ Metalloid

Bi] → Metal

Hydrides of group 15 elements are

NH_3

PH_3

AsH_3

SbH_3

BiH_3

In NH_3, hydrogen atom gets partial positive charge due to less electronegativity.

But in BiH_3, hydrogen atom gets partial negative charge because hydrogen is more electronegative than bismuth.

i.e. BiH_3 is a strong reducing agent than others because we know that $H^-$is a strong reducing agent.

51. (20)

$\Delta G° = -RT \ln K_{eq}$

Given $\Delta G° = -9.478 KJ/mole$

$T = 495$ K $R = 8.314$ J mol^{-1}

So $-9.478 \times 10^3 = -495 \times 8.314 \times \ell n K_{eq}$

$\ln K_{eq} = 2.303$

$= \ln 10$

So $K_{eq} = 10$

Now $A(g) \rightleftharpoons B(g)$

$t = 0$ 22 0

$t = t$ $22 - x$ x

$$K_{eq} = \frac{[B]}{[C]} = \frac{x}{22 - x} = 10$$

or $x = 20$

So millmoles of B $= 20$

52. (3) $44gmCO_2$ have $12gm$ carbon

So, $420gmCO_2 \Rightarrow \frac{12}{44} \times 420$

$\Rightarrow \frac{1260}{11}$gm carbon

$\Rightarrow 114.545$ gram carbon

So, % of carbon $= \frac{114.545}{750} \times 100$

$\simeq 15.3\%$

$18gmH_2O \Rightarrow 2gmH_2$

$210gm \Rightarrow \frac{2}{18} \times 210$

$= 23.33gmH_2$

So, $\%H_2 \Rightarrow \frac{23.33}{750} \times 100 = 3.11\%$

$\approx 3\%$

53. (16) Writting the half reaction

oxidation half reaction

$MnO_4^- \rightarrow Mn^{2+}$

balancing oxygen

$MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

balancing Hydrogen

$8H^+ + MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

balancing charge

$5e^- + 8H^+ + MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

Reduction half

$C_2O_4^{2-} \rightarrow CO_2$

Balancing carbon

$C_2O_4^{2-} \rightarrow 2CO_2$

Balancing charge

$C_2O_4^{2-} \rightarrow 2CO_2 + 2e^-$

Net equation

$16H^+ + 2MnO_4^- + 5C_2O_4^{2-} \rightarrow 10CO_2 + 2Mn^{2+} + 8H_2O$

So $c = 16$

54. (106) $AB_2 \rightarrow A^{2+} + 2\ B^-$

$t = 0\ a\ 0\ 0$

$t = t\ a - a\alpha\ a\ \alpha\ 2a\alpha$

$n_T = a - a\alpha + a\alpha + 2a\alpha$

$= a(1 + 2\alpha)$

so $i = 1 + 2\alpha$

Now $\Delta T_b = i \times m \times K_b$

$\Delta T_b = (1 + 2\alpha) \times m \times K_b$

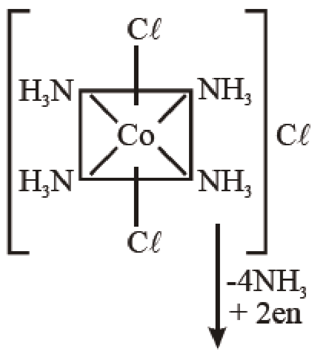
$\alpha = 0.1\ \ m = 10\ \ K_b = 0.5$

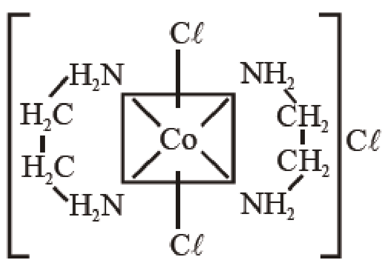
$\Delta T_b = 1.2 \times 10 \times 0.5 = 6$

So boiling point $= 106$

55. (2) trans $-CoCl_3 \cdot 4NH_3$

Or trans-$[Co(NH_3)_4Cl_2]C\ell$





As we know that ethylene diamine is a bidentate ligand and ammonia is a mono dentate ligand.

It means overall two ethylene diamine is required to replace the all neutral ligands (four ammonia) from the coordination sphere of this complex.

56. (9)

6.5 molal KOH = 1000gm solvent has 6.5 moles KOH

so wt of solute $= 6.5 \times 56$

$= 364\text{gm}$

wt of solution $= 1000 + 364 = 1364$

Volume of solution $= \frac{1364}{1.89}\ \text{m}\ell$

Molarity $= \frac{\text{mole of solute}}{V_{\text{solution}}\ \text{in Litre}}$

$$= \frac{6.5 \times 1.89 \times 1000}{1364}$$

$= 9.00$

57. (9) Energy incident $= \frac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{248 \times 10^{-9} \times 1.6 \times 10^{-19}}\text{eV}$$

$$= \frac{6.63 \times 3 \times 100}{248 \times 1.6}$$

$= 0.05\text{eV} \times 100 = 5\text{eV}$

Now using

$E = \phi + K.E.$

$5 = 3 + K.E.$

$K.E. = 2\text{eV} = 3.2 \times 10^{-19}\ J$

for debroglie wavelength $\lambda = \frac{h}{mv}$

$K.E = \frac{1}{2}mv^{2}$

so $v = \sqrt{\frac{2KE}{m}}$

hence $\lambda = \frac{h}{\sqrt{2KE \times m}}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 3.2 \times 10^{-19} \times 9.1 \times 10^{-31}}}$$

$= \frac{6.63}{7.6} \times \frac{10^{-34}}{10^{-25}} = \frac{66.3 \times 10^{-10}\ \text{m}}{7.6}$

$= 8.72 \times 10^{-10}$ m

$\approx 9 \times 10^{-10}$ m

$= 9\text{Å}$

58. (50)

For A_2X

$A_2X \rightarrow 2\ A^+ + X^{2-}$
$\quad\quad\quad 2\ S_1 \quad S_1$

$K_{sp} = 4\ S_1^3 = 4 \times 10^{-12}$

$S_1 = 10^{-4}$

for MX

$MX \rightarrow M^+ + X^-$

$S_2 \quad S_2$

$K_{sp} = S_2^2 = 4 \times 10^{-12}$

$S_2 = 2 \times 10^{-6}$

so $\frac{S_{A_2X}}{S_{MX}} = \frac{10^{-4}}{2 \times 10^{-6}} = 50$

59. (33) For BCC $\sqrt{3}a = 4r$

so $r = \frac{\sqrt{3}}{4} \times 27$

for FCC $a = 2\sqrt{2}r$

$= 2 \times \sqrt{2} \times \frac{\sqrt{3}}{4} \times 27$

$= \frac{\sqrt{3}}{\sqrt{2}} \times 27$

$= 33$

60. (10)

$K_{300} = 10^{-4}$ $K_{200} = ?$

$E_a = 11.488\text{KJ/mole}$ $R = 8.314\ \text{J/mole} - \text{K}$

so $\ln\left(\frac{K_{300}}{K_{200}}\right) = \frac{E_a}{R}\left(\frac{1}{200} - \frac{1}{300}\right)$

$\ln\left(\frac{K_{300}}{K_{200}}\right) = \frac{11.488 \times 1000 \times 100}{8.314 \times 200 \times 300}$

$= 2.303$

$= \ell n10$

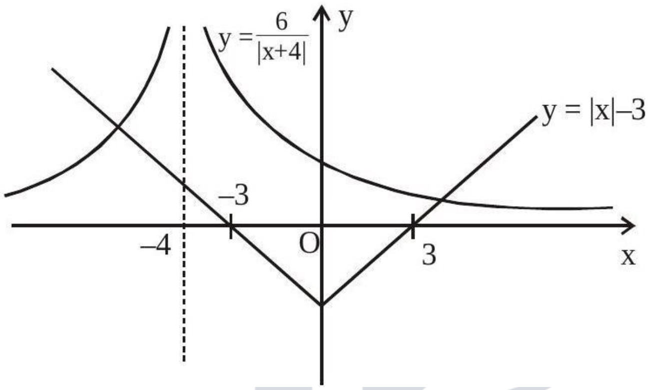
so $\frac{K_{300}}{K_{200}} = 10$

$K_{200} = \frac{1}{10} \times K_{300} = 10^{-4}$
$= 10 \times 10^{-5} sec^{-1}$

61. (b) $x \neq -4$

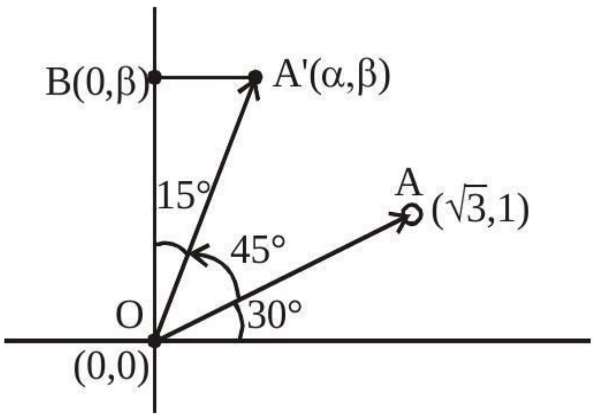
$(|x| - 3)(|x + 4|) = 6$

$\Rightarrow |x| - 3 = \frac{6}{|x + 4|}$



No. of solutions = 2

62. (a)

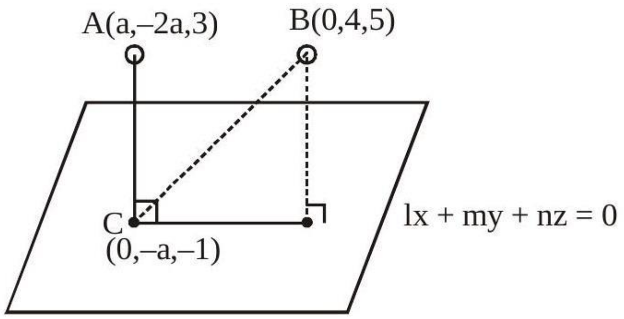


Area of $\Delta(OA'B) = \frac{1}{2} OA' \cos 15° \times OA' \sin 15°$

$= \frac{1}{2}(OA')^2 \frac{\sin 30°}{2}$

$= (3 + 1) \times \frac{1}{8} = \frac{1}{2}$

63. (d)



C lies on plane $\Rightarrow -ma - n = 0 \Rightarrow \frac{m}{n} = -\frac{1}{a}$ ..... (a)

$\overrightarrow{CA} \parallel l\hat{i} + m\hat{j} + n\hat{k}$

$$\frac{a-0}{l} = \frac{-a}{m} = \frac{4}{n} \Rightarrow \frac{m}{n} = -\frac{a}{4}$$

From (a) & (b)

$-\frac{1}{a} = \frac{-a}{4} \Rightarrow a^2 = 4 \Rightarrow a = 2$ (since $a > 0$)

From (b) $\frac{m}{n} = \frac{-1}{2}$

Let $m = -t \Rightarrow n = 2t$

$$\frac{2}{l} = \frac{-2}{-t} \Rightarrow l = t$$

So plane : $t(x - y + 2z) = 0$

$$BD = \frac{6}{\sqrt{6}} = \sqrt{6}\ C \cong (0, -2, -1)$$

$$CD = \sqrt{BC^2 - BD^2}$$

$$= \sqrt{(0^2 + 6^2 + 6^2) - (\sqrt{6})^2}$$

$$= \sqrt{66}$$

64. (d) For a, b, c

mean $= \frac{a+b+c}{3} (= \bar{x})$

$b = a + c$

$$\Rightarrow \bar{x} = \frac{2b}{3}$$

S.D. $(a+2, b+2, c+2) =$ S.D. $(a, b, c) = d$

$$\Rightarrow d^2 = \frac{a^2 + b^2 + c^2}{3} - (\bar{x})^2$$

$\Rightarrow d^2 = \frac{a^2+b^2+c^2}{3} - \frac{4\,b^2}{9}$

$\Rightarrow 9\,d^2 = 3(a^2+b^2+c^2) - 4\,b^2$

$\Rightarrow b^2 = 3(a^2+c^2) - 9\,d^2$

65. (b) $x \in \left(0, \frac{\pi}{2}\right)$

$\log_{10} \sin x + \log_{10} \cos x = -1$

$\Rightarrow \log_{10} \sin x \cdot \cos x = -1$

$\Rightarrow \sin x \cdot \cos x = \frac{1}{10}$

$\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \sin x + \cos x = 10^{\left(\log_{10}\sqrt{n} - \frac{1}{2}\right)} = \sqrt{\frac{n}{10}}$

by squaring

$1 + 2\sin x \cdot \cos x = \frac{n}{10}$

$\Rightarrow 1 + \frac{1}{5} = \frac{n}{10} \Rightarrow n = 12$

66. (c)

67. (d) For standard parabola

For more than 3 normals (on axis)

$x > \frac{L}{2}$ (where L is length of L.R.)

For $y^2 = 2x$

L.R. $= 2$

for $(a, 0)$

$a > \frac{L \cdot R.}{2} \Rightarrow a > 1$

68. (b) $P(3, -1, 2)$

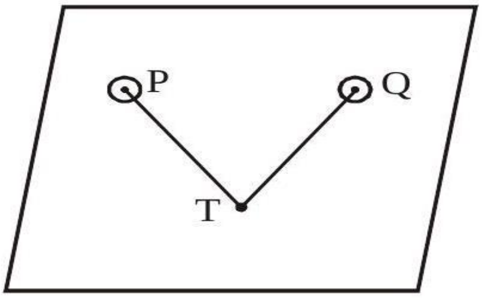
$Q(1, 2, -4)$

$\overrightarrow{PR} \parallel 4\hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{QS} \parallel -2\hat{i} + \hat{j} - 2\hat{k}$

dr 's of normal to the plane containing P, T&Q will be proportional to :

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}$$



$\therefore \ \frac{\ell}{0} = \frac{m}{4} = \frac{n}{2}$

For point, T: $\overrightarrow{PT} = \frac{x-3}{4} = \frac{y+1}{-1} = \frac{z-2}{2} = \lambda$

$\overrightarrow{QT} = \frac{x-1}{-2} = \frac{y-1}{1} = \frac{z+4}{-2} = \mu$

T: $(4\lambda + 3, -\lambda - 1, 2\lambda + 2)$

$\cong (2\mu + 1, \mu + 2, -2\mu - 4)$

$4\lambda + 3 = -2\mu + 1 \Rightarrow 2\lambda + \mu = -1$

$\lambda + \mu = -3 \Rightarrow \lambda = 2$

& $\mu = -5\ \lambda + \mu = -3 \Rightarrow \lambda = 2$

So point T: $(11, -3, 6)$

$\overrightarrow{OA} = (11\hat{\imath} - 3\hat{\jmath} + 6\hat{k}) \pm \left(\frac{2\hat{\jmath} + \hat{k}}{\sqrt{5}}\right)\sqrt{5}$

$\overrightarrow{OA} = (11\hat{\imath} - 3\hat{\jmath} + 6\hat{k}) \pm (2\hat{\jmath} + \hat{k})$

$\overrightarrow{OA} = 11\hat{\imath} - \hat{\jmath} + 7\hat{k}$

or

$9\hat{\imath} - 5\hat{\jmath} + 5\hat{k}$

$|\overrightarrow{OA}| = \sqrt{121 + 1 + 49} = \sqrt{171}$

or

$\sqrt{81 + 25 + 25} = \sqrt{131}$

69. (b) $f(g(x)) = \begin{cases} g(x) + 2, & g(x) < 0 \\ (g(x))^2, & g(x) \geq 0 \end{cases}$

$$= \begin{cases} x^3+2, & x<0 \\ x^6, & x \in [0,1) \\ (3x-2)^2, & x \in [1,\infty) \end{cases}$$

$$(f \circ g(x))' = \begin{cases} 3x^2, & x<0 \\ 6x^5, & x \in (0,1) \\ 2(3x-2) \times 3, & x \in (1,\infty) \end{cases}$$

At 'O'

L.H.L. ≠ R.H.L. (Discontinuous)

At '1'

L.H.D. = 6 = R.H.D.

$\Rightarrow$ $f \circ g(\mathrm{x})$ is differentiable for x $\in R - \{0\}$

70. (d)

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

$(p \wedge q) \rightarrow (p \rightarrow q)$ is tautology

71. (b) $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \le 2$

$$\frac{|z|+11}{(|z|-1)^2} \ge \frac{1}{2}$$

$2|z| + 22 \ge (|z|-1)^2$

$2|z| + 22 \ge |z|^2 + 1 - 2|z|$

$|z|^2 - 4|z| - 21 \le 0$

$\Rightarrow$ $|z| \le 7$

$\therefore$ Largest value of |z| is 7

72. (a)

$\left(3^{1/4} + 5^{1/8}\right)^{60}$

${}^{60}C_r\left(3^{1/4}\right)^{60-r} \cdot \left(5^{1/8}\right)^{r}$

${}^{60}C_r(3)^{\frac{60-r}{4}} \cdot 5^{\frac{r}{8}}$

For rational terms.

$\frac{r}{8} = k; 0 \le r \le 60$

$0 \le 8k \le 60$

$0 \le k \le \frac{60}{8}$

$0 \le k \le 7.5$

$k = 0,1,2,3,4,5,6,7$

$\frac{60-8k}{4}$ is always divisible by 4 for all value of k.

Total rational terms = 8

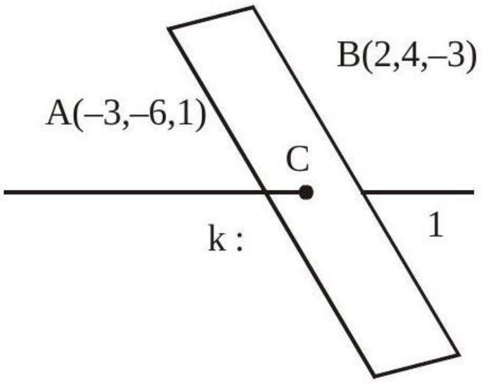
Total terms = 61

irrational terms = 53

$n - 1 = 53 - 1 = 52$

52 is divisible by 26

73. (c)



Point C is

$$\left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$$

$$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$$

Plane $lx + my + nz = 0$

$l(-1) + m(b) + n(c) = 0$

$-l + 2m + 3n = 0$

It also satisfy point $(1, -4, -2)$

$l - 4m - 2n = 0$

Solving (a) and (b)

$2m + 3n = 4m + 2n$

$n = 2m$

$l-4\mathrm{~m}-4\mathrm{~m}=0$

$l=8\mathrm{~m}$

$\frac{l}{8}=\frac{\mathrm{m}}{1}=\frac{\mathrm{n}}{2}$

$l:\mathrm{m}:\mathrm{n}=8:1:2$

Plane is $8\mathrm{x}+\mathrm{y}+2\mathrm{z}=0$

It will satisfy point C

$$8\left(\frac{2\mathrm{k}-3}{\mathrm{k}+1}\right)+\left(\frac{4\mathrm{k}-6}{\mathrm{k}+1}\right)+2\left(\frac{-3\mathrm{k}+1}{\mathrm{k}+1}\right)=0$$

$16\mathrm{k}-24+4\mathrm{k}-6-6\mathrm{k}+2=0$

$14\mathrm{k}=28\ \therefore\ \mathrm{k}=2$

74. (b)

$f(x)=(4a-3)(x+\log_e 5)+(a-7)\sin x$

$f(x)=(4a-3)(\mathrm{a})+(a-7)\cos x=0$

$$\Rightarrow\ \cos\mathrm{x}=\frac{3-4\mathrm{a}}{\mathrm{a}-7}$$

$$-1\leq\frac{3-4\mathrm{a}}{\mathrm{a}-7}<1$$

$$\frac{3-4a}{a-7}+1\geq 0\quad \frac{3-4a}{a-7}<1$$

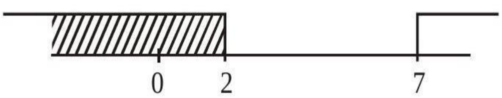
$$\frac{3-4\mathrm{a}+\mathrm{a}-7}{\mathrm{a}-7}\geq 0\quad \frac{3-4\mathrm{a}}{\mathrm{a}-7}-1<0$$

$$\frac{-3a-4}{a-7}\geq 0\quad \frac{3-4a-a+7}{a-7}<0$$

$$\frac{3a+4}{a-7}\leq 0\quad \frac{-5a+10}{a-7}<0$$

$$\frac{5a-10}{a-7}>0$$

$$\frac{5(\mathrm{a}-2)}{\mathrm{a}-7}>0$$



$\alpha\in\left[-\frac{4}{3},2\right)$

Check end point $\left[-\frac{4}{3}, 2\right)$

75. (c) E_1 : Event denotes spade is missing

$$P(E_1) = \frac{1}{4}; P(\overline{E}_1) = \frac{3}{4}$$

A : Event drawn two cards are spade

$$P(A) = \frac{\frac{1}{4} \times \left(\frac{{}^{12}C_2}{{}^{51}C_2}\right) + \frac{3}{4} \times \left(\frac{{}^{13}C_2}{{}^{51}C_2}\right) + \frac{3}{4} \times \left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}{\frac{1}{4} \times \left(\frac{{}^{12}C_2}{{}^{51}C_2}\right) + \frac{3}{4} \times \left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}$$

$$= \frac{39}{50}$$

76. (c) $(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$

$(1 - x + x^3)^n = a_0 + a_1 x + a_2 x^2 \ldots\ldots.. + a_{3n} x^{3n}$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} =$ Sum of $a_0 + a_2 + a_4 \ldots \ldots$

$\sum_{j=0}^{\left[\frac{3n-1}{2}\right.} a_{2j} + 1 =$ Sum of $a_1 + a_3 + a_5 \ldots \ldots$

put $x = 1$

$1 = a_0 + a_1 + a_2 + a_3 \ldots \ldots \ldots + a_{3n}$

Put $x = -1$

$1 = a_0 - a_1 + a_2 - a_3$

$+(-1)^{3n} a_{3n}$

Solving (A) and (B)

$a_0 + a_2 + a_4 \ldots.. = 1$

$a_1 + a_3 + a_5 \ldots \ldots. = 0$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$

77. (d) $\frac{dy}{dx} + 2y\tan x = \sin x$

$\text{I.F.} = e^{\int 2\tan x dx} = e^{2\ln \sec x}$

$\text{I.F.} = \sec^2 x$

$y \cdot (\sec^2 x) = \int \sin x \cdot \sec^2 x dx$

$y.(\sec^2 x) = \int \sec x \tan x dx$

$y \cdot (\sec^2 x) = \sec x + C$

$x = \frac{\pi}{3}; y = 0$

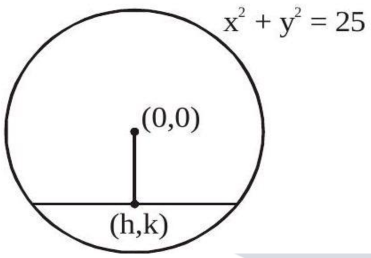
$\Rightarrow C = -2$

$\Rightarrow y = \frac{\sec x - 2}{\sec^2 x} = \cos x - 2\cos^2 x$

$y = t - 2t^2 \Rightarrow \frac{dy}{dt} = 1 - 4t = 0 \Rightarrow t = \frac{1}{4}$

$\therefore \ \max = \frac{1}{4} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$

78. (d)



Equation of chord

$y - k = -\frac{h}{k}(x - h) \ ky - k^2 = -hx + h^2$

$hx + ky = h^2 + k^2$

$y = -\frac{hx}{k} + \frac{h^2 + k^2}{k}$

tangent to $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$c^2 = a^2 m^2 - b^2$

$\left(\frac{h^2 + k^2}{k}\right)^2 = 9\left(-\frac{h}{k}\right)^2 - 16$

$(x^2 + y^2)^2 = 9x^2 - 16y^2$

79. (b)

$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

$(81)^{\sin^2 x} + \frac{(81)^1}{(18)^{\sin^2 x}} = 30$

$(81)^{\sin^2 x} = t$

$t + \frac{81}{t} = 30$

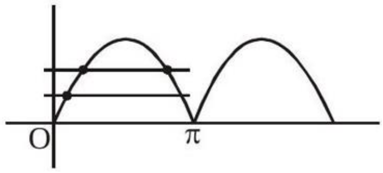
$t^2 - 30t + 81 = 0$

$(t-3)(t-27) = 0$

$(81)^{\sin^2 x} = 3^1$ or $(81)^{\sin^2 x} = 3^3$

$3^{4\sin^2 x} = 3^1$ or $3^{4\sin^2 x} = 3^3$

$\sin^2 x = \frac{1}{4}$ or $\sin^2 x = \frac{3}{4}$



Total SOL; = 4

80. (c) $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$

Divide by 3^{2r}

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{\left(\frac{2}{3}\right)^{2r} .2+3}\right)$$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{3\left(\left(\frac{2}{3}\right)^{2r+1}+1\right)}\right)$$

Let $\left(\frac{2}{3}\right)^r = t$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\frac{t}{3}}{1+\frac{2}{3}t^2}\right)$$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{t-\frac{2t}{3}}{1+t\cdot\frac{2t}{3}}\right)$$

$$\sum_{r=1}^{k} \left(\tan^{-1}(t) - \tan^{-1}\left(\frac{2t}{3}\right)\right)$$

$$\sum_{r=1}^{k} \left(\tan^{-1}\left(\frac{2}{3}\right)^r - \tan^{-1}\left(\frac{2}{3}\right)^{r+1}\right)$$

$$S_k = \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}\left(\frac{2}{3}\right)^{k+1}$$

$$S_\infty = \lim_{k\to\infty}\left(\tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}\left(\frac{2}{3}\right)^{k+1}\right)$$

$= \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}(0)$

$\therefore\ S_\infty = \tan^{-1}\left(\frac{2}{3}\right) = \cot^{-1}\left(\frac{3}{2}\right)$

81. (3)

GP: 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192

AP: 11, 16, 21, 26, 31, 36

Common terms: 16, 256, 4096 only

82. (1) $E = 2\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$

$$E = \frac{2}{\ln 2}\int_0^1 \ln\left(1+\tan\frac{\pi x}{4}\right)dx$$

replacing $x \to 1 - x$

$$E = \frac{2}{\ln 2}\int_0^1 \ln\left(1+\tan\frac{\pi}{4}(1-x)\right)dx$$

$$E = \frac{2}{\ln 2}\int_0^1 \ln\left(1+\tan\left(\frac{\pi}{4}-\frac{\pi}{4}x\right)\right)dx$$

$$E = \frac{2}{\ln 2}\int_0^1 \ln\left(1+\frac{1+\tan\frac{\pi}{4}x}{1+\tan\frac{\pi}{4}x}\right)dx$$

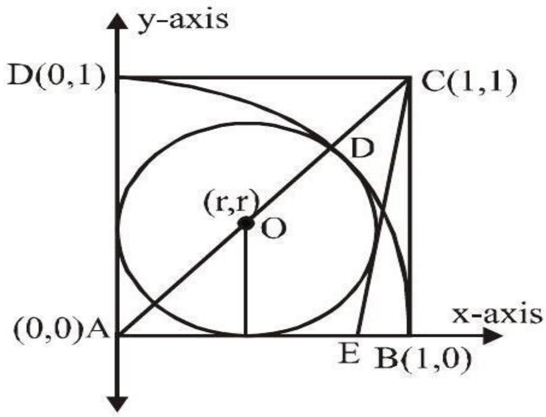
$$E = \frac{2}{\ln 2}\int_0^1 \ln\left(\frac{2}{1+\tan\frac{\pi x}{4}}\right)dx$$

$$E = \frac{2}{\ln 2}\int_0^1 \left(\ln 2 - \ln\left(1+\tan\frac{\pi x}{4}\right)\right)dx$$

equation (i) + (ii)

$E = 1$

83. (1)



Here AO + OD = 1 or $(\sqrt{2}+1)\mathrm{r} = 1$

$\Rightarrow\ \mathrm{r} = \sqrt{2-1}$

equation of circle $(x-r)^2 + (y-r)^2 = r^2$

Equation of CE

$\mathrm{y} - 1 = \mathrm{m}(\mathrm{x} - 1)$

$\mathrm{mx} - \mathrm{y} + 1 - \mathrm{M} = 0$

It is tangent to circle

$\therefore\ \left|\frac{\mathrm{mr} - \mathrm{r} + 1 - \mathrm{m}}{\sqrt{\mathrm{m}^2+1}}\right| = \mathrm{r}$

$\left|\frac{(\mathrm{m}-1)\mathrm{r} + 1 - \mathrm{m}}{\sqrt{\mathrm{m}^2+1}}\right| = \mathrm{r}$

$\frac{(\mathrm{m}-1)^2(\mathrm{r}-1)^2}{\mathrm{m}^2+1} = \mathrm{r}^2$

Put $\mathrm{r} = \sqrt{2} - 1$

On solving $m = 2-\sqrt{3}, 2+\sqrt{3}$
Taking greater slope of CE as

$2+\sqrt{3}$

$\mathrm{y} - 1 = (2+\sqrt{3})(\mathrm{x} - 1)$

Put $\mathrm{y} = 0$

$-1 = (2+\sqrt{3})(\mathrm{x}-1)$

$\frac{-1}{2+\sqrt{3}} \times \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = \mathrm{x} - 1$

$\mathrm{x} - 1 = \sqrt{3} - 1$

$\mathrm{EB} = 1 - \mathrm{x} = 1 - (\sqrt{3} - 1)$

$EB = 2 - \sqrt{3}$

84. (4)

$$\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$$

$$\Rightarrow \lim_{x\to 0} \frac{a\left(1 + x + \frac{x^2}{2!}\ldots\right) - b\left(1 - \frac{x^2}{2!} + \cdots\right) + c\left(1 - x + \frac{x^2}{2!}\right)}{\left(\frac{x\sin x}{x}\right)x} = 2$$

$a - b + c = 0$

$a - c = 0$

$\& \frac{a+b+c}{2} = 2$

$\Rightarrow a + b + c = 4$

85. (766)

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

diagonal elements of

AA^T, $a^2 + b^2 + c^2$, $d^2 + e^2 + f^2$, $g^2 + b^2 + c^2$

Sum $= a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 9$ a, b, c, d, e, f, g, h, i $\in \{0,1,2,3\}$

	Case	No. of Matrices
(a)	All -1 s	$\frac{9!}{9!} = 1$
(b)	One $\to$ 3 remaining-0	$\frac{9!}{1! \times 8!} = 9$
(c)	One-2 five-1s three-0s	$\frac{9!}{1! \times 5! \times 3!} = 8 \times 63$
(d)	two -2 's one-1 six-0's	$\frac{9!}{2! \times 6!} = 63 \times 4$

Total no. of ways $= 1 + 9 + 8 \times 63 + 63 \times 4 = 766$

86. (36) Let $M = (P^{-1}AP - I)^2$

$= (P^{-1}AP)^2 - 2P^{-1}AP + I$

$= P^{-1}\, A^2P - 2P^{-1}AP + I$

$PM = A^2P - 2AP + P$

$= (A^2 - 2\, A \cdot I + I^2)P$

$\Rightarrow \text{Det}(PM) = \text{Det}((A - I)^2 \times P)$

$\Rightarrow\ \text{DetP} \cdot \text{Det}\, M = \text{Det}(A - I)^2 \times \text{Det}(P)$

$\Rightarrow\ \text{Det}\, M = (\text{Det}(A - I))^2$

Now $A - I = \begin{bmatrix} 1 & 7 & w^2 \\ -1 & -w-1 & 1 \\ 0 & -w & -w \end{bmatrix}$

$\text{Det}(A - I) = (w^2 + w + w) + 7(-w) + w^3 = -6w$

$\text{Det}((A - I))^2 = 36w^2$

$\Rightarrow\ \alpha = 36$

87. (406)

$y(x) = \int_0^x (2t^2 - 15t + 10)dt$

$y'(x)]_{x=a} = [2x^2 - 15x + 10]_a = 2a^2 - 15a + 10$

Slope of normal $= -\frac{1}{3}$

$\Rightarrow\ 2a^2 - 15a + 10 = 3 \Rightarrow a = 7$

$\&\ \ a = \frac{1}{2}$ (rejected)

$b = y(7) = \int_0^7 (2t^2 - 15t + 10)dt$

$= \left[\frac{2t^3}{3} - \frac{15t^2}{2} + 10t\right]_0^7$

$\Rightarrow 6\, b = 4 \times 7^3 - 45 \times 49 + 60 \times 7$

$|a + 6\, b| = 406$

88. (2)

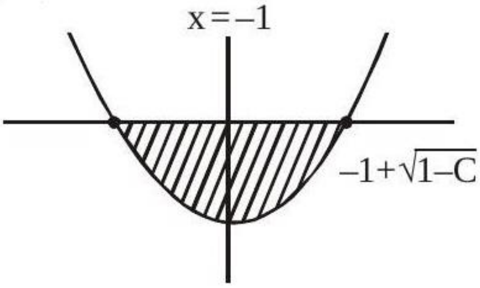
$\frac{dy}{dx} = 2(x + 1)$

$\Rightarrow \int dy = \int 2(x+1)dx$

$\Rightarrow y(x) = x^2 + 2x + C$

Area $= \frac{4\sqrt{8}}{3}$

$-1 + \sqrt{1-C}$



$\Rightarrow \ 2\int_{-1}^{-1+\sqrt{1-C}} (-(x+1)^2 - C + 1)dx = \frac{4\sqrt{8}}{3}$

$\Rightarrow \ 2\left[-\frac{(x+1)^3}{3} - Cx + x\right]_{-1}^{-1+\sqrt{1-C}} = \frac{4\sqrt{8}}{3}$

$\Rightarrow \ -(\sqrt{1-C})^3 + 3c - 3C\sqrt{1-C}$

$-3 + 3\sqrt{1-C} - 3C + 3 = 2\sqrt{8}$

$\Rightarrow C = -1$

$\Rightarrow f(x) = x^2 + 2x - 1, f(a) = 2$

89. (16) $f(x) + f(x+1) = 2$

$\Rightarrow f(x)$ is periodic with period $= 2$

$I_1 = \int_0^8 f(x)dx = 4\int_0^2 f(x)dx$

$= 4\int_0^1 (f(x) + f(1+x))dx = 8$

Similarly, $I_2 = 2 \times 2 = 4$

$I_1 + 2I_2 = 16$

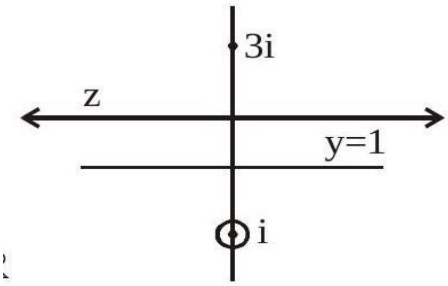
90. (4)

$\omega = z\bar{z} - 2z + 2$

$\left|\frac{z+i}{z-3i}\right| = 1$

$\Rightarrow |z+i| = |z-3i|$

$\Rightarrow z = x + i, \ x \in \mathbb{R}$



$\omega = (x+i)(x-i) - 2(x+i) + 2$

$= x^2 + 1 - 2x - 2i + 2$

$\mathrm{Re}(\omega) = x^2 - 2x + 3$

For $\min(\mathrm{Re}(\omega)), x = 1$

$\Rightarrow \omega = 2 - 2i = 2(1-i) = 2\sqrt{2}e^{-i\frac{\pi}{4}}$

$\omega^n = (2\sqrt{2})^n e^{-i\frac{n\pi}{4}}$

For real & minimum value of n, $n = 4$