1. (a)

2. (c) From energy conservation,

$\begin{bmatrix} \text{after bullet gets embedded till the} \\ \text{system comes momentarily at rest} \end{bmatrix}$

$$(M+m)gh = \frac{1}{2}(M+m)v_1^2$$

[v_1 is velocity after collision]

$$\therefore v_1 = \sqrt{2gh}$$

Applying momentum conservation, (just before and just after collision)

$$mv = (M+m)v_1$$

$$v = \left(\frac{M+m}{m}\right)v_1 = \frac{6}{10\times10^{-3}} \times \sqrt{2\times9.8\times9.8\times10^{-2}}$$

$$\approx 831.55\ m/s$$

3. (c) Since force on a point charge by magnetic field is always perpendicular to $\vec{v}[\vec{F} = q\vec{V}\times\vec{B}]$

$\therefore$ Work by magnetic force on the point charge is zero.

4. (d) The nature of flow is determined by Reynolds Number.

$$R_e = \frac{\rho v D}{\eta}$$

$\begin{bmatrix} \rho \to \text{density of fluid ;} & \eta \to \text{coefficient of} \\ v \to \text{velocity of flow} & \text{viscosity} \\ D \to \text{Diameter of pipe} & \end{bmatrix}$

From NCERT

If $R_e < 1000 \to$ flow is steady

$1000 < R_e < 2000 \to$ flow becomes unsteady

$R_e > 2000 \to$ flow is turbulent

$$R_{einitial} = 10^3 \times \frac{0.18\times10^{-3}}{\pi\times(0.5\times10^{-2})^2\times60} \times \frac{1\times10^{-2}}{10^{-3}}$$

$$= 382.16$$

$$R_{efinal} = 10^3 \times \frac{0.48\times10^{-3}}{\pi\times(0.5\times10^{-2})^2\times60} \times \frac{1\times10^{-2}}{10^{-3}}$$

$$= 1019.09$$

5. (b) Given:

$\vec{v} = 0.5t^2\hat{\imath} + 3t\hat{\jmath} + 9\hat{k}$

$\vec{v}_{att=2} = 2\hat{\imath} + 6\hat{\jmath} + 9\hat{k}$

∴ Angle made by direction of motion of mosquito will be,

$\cos^{-1}\frac{2}{11}$ (from x-axis) $= \tan^{-1}\frac{\sqrt{117}}{2}$

$\cos^{-1}\frac{6}{11}$ (from y-axis) $= \tan^{-1}\frac{\sqrt{85}}{6}$

$\cos^{-1}\frac{9}{11}$ (from z-axis) $= \tan^{-1}\frac{\sqrt{40}}{9}$

6. (a) $\frac{2\,K\lambda}{r} = \frac{\sigma}{\varepsilon_0}$ (x = 3 m)

$\sigma = 0.424 \times 10^{-9}\frac{C}{m^2}$

7. (d)

8. (c) $V_S = \frac{P}{i} = \frac{60}{0.11} = 545.45$

$V_P = 220$

$V_S > V_P$

⇒ Step up transformer

9. (b)

10. (d) $\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$

$\lambda = 102$ nm

11. (b) $\frac{1}{F} = \left[\frac{\mu_L}{\mu_S} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

If $\mu_L = \mu_S \Rightarrow \frac{1}{F} = 0 \Rightarrow F = \infty$

12. (d) $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{Al} = \frac{mg \cdot L}{\pi R^2 \cdot \ell}$

$$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta L}{L} + 2 \cdot \frac{\Delta R}{R} + \frac{\Delta \ell}{\ell}$$

$$\frac{\Delta Y}{Y} \times 100 = 100\left[\frac{1}{1000} + \frac{1}{1000} + 2\left(\frac{0.001}{0.2}\right) + \frac{0.001}{0.5}\right]$$

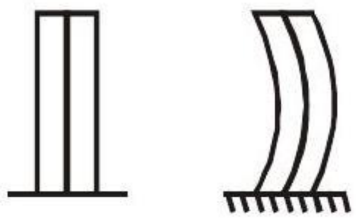
$$= \frac{1}{10} + \frac{1}{10} + 1 + \frac{1}{5} = \frac{14}{10} = 1.4\%$$

13. (d)

$\alpha_A > \alpha_B$

Length of both strips will decrease

$\Delta L_A > \Delta L_B$



14. (d) $500 = (1.5)^2 \times R \times 20$

$E = (3)^2 \times R \times 20$

$E = 2000\ J$

15. Statement I

$v_{max} = \sqrt{\mu Rg} = \sqrt{(0.2) \times 2 \times 9.8}$

$v_{max} = 1.97\ m/s$

$7\ km/h = 1.944\ m/s$

Speed is lower than v_{max}, hence it can take safe turn.

Statement II

$$v_{max} = \sqrt{Rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}$$

$$= \sqrt{2 \times 9.8\left[\frac{1 + 0.2}{1 - 0.2}\right]} = 5.42\ m/s$$

$18.5\ km/h = 5.14\ m/s$

Speed is lower than v_{max}, hence it can take safe turn.

16. (b) $D = 2\sqrt{2Rh}$

$$h = \frac{D^2}{8R} = \frac{45^2}{8 \times 6400}\ km \cong 39.55\ m$$

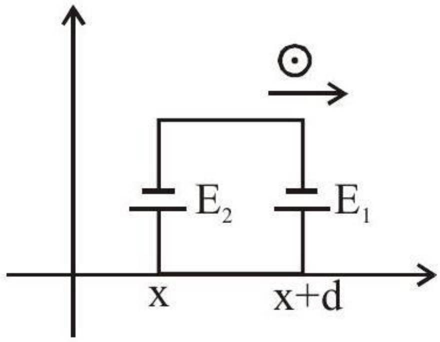
17. (c)

$$E_1 = \frac{B_0(x + d)}{a} v_0 d$$

$$E_2 = \frac{B_0(x)}{a} v_0\ d$$

$E_{net} = E_1 - E_2$

$$E_{net} = \frac{B_0 v_0\ d^2}{a}$$



18. (a) $A = A_0 e^{-\gamma t} = A_0 e^{-\frac{bt}{2m}}$

$\frac{A_0}{2} = A_0 e^{-\frac{bt}{2m}}$

$\frac{bt}{2m} = \ln 2$

$t = \frac{2m}{b}\ln 2 = \frac{2 \times 500 \times 0.693}{20}$

$t = 34.65$ second.

19. (b) $N_1 = N_0 e^{-\lambda t_1}$

$\frac{N_1}{N_0} = e^{-\lambda t_1}$

$0.67 = e^{-\lambda t_1}$

$\ln(0.67) = -\lambda t_1$

$N_2 = N_0 e^{-\lambda t_2}$

$\frac{N_2}{N_0} = e^{-\lambda t_2}$

$0.33 = e^{-\lambda t_2}$

$\ln(0.33) = -\lambda t_2$

$\ln(0.67) - \ln(0.33) = \lambda t_1 - \lambda t_2$

$\lambda(t_1 - t_2) = \ln\left(\frac{0.67}{0.33}\right)$

$\lambda(t_1 - t_2) \cong \ln 2$

$t_1 - t_2 \simeq \frac{\ln 2}{\lambda} = t_{1/2}$

Half life $= t_{1/2} = 20$ minutes.

20. (a) Red light and blue light have different wavelength and different frequency.

21. (2500)

$Q = i^2RT$

$R = \frac{Q}{i^2t} = \frac{10 \times 10^{-3}}{4 \times 10^{-6} \times 1} = 2500\Omega$

22. (3)

$$C = \frac{\varepsilon_0 A}{\frac{d}{2K} + \frac{d}{2}} = \frac{2\varepsilon_0 A}{\frac{d}{K} + d}$$
$$= \frac{2 \times 2\varepsilon_0}{\frac{1}{3.2} + 1} = \frac{4 \times 3.2}{4.2}\varepsilon_0$$
$$= 3.04\varepsilon_0$$

23. (20)

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$\vec{r} = (2\hat{\imath}) - (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) = -3\hat{\jmath} - 4\hat{k}$$

$\&\vec{F} = 4\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$
$$= \hat{i}(-12 + 12) - \hat{j}(0 + 16) + \hat{k}(0 + 12)$$
$$= -16\hat{i} + 12\hat{k}$$
$$\therefore |\vec{\tau}| = \sqrt{16^2 + 12^2} = 20$$

24. (3) Energy given $= U_f - U_i$

$$= 0 - \left(-\frac{3}{5}\frac{GM^2}{R}\right)$$
$$= \frac{3}{5}\frac{GM^2}{R}$$
$$x = 3$$

25. (12)

$\omega_1 = 0.02; \mu_1 = 1.5; \omega_2 = 0.03; \mu_2 = 1.6$

Achromatic combination

$\therefore \theta_{net} = 0$

$\theta_1 - \theta_2 = 0$

$\theta_1 = \theta_2$

$\omega_1\delta_1 = \omega_2\delta_2$

$\&\delta_{net} = \delta_1 - \delta_2 = 2^\circ$

$\delta_1 - \frac{\omega_1\delta_1}{\omega_2} = 2^\circ$

$\delta_1\left(1-\frac{\omega_1}{\omega_2}\right)=2^\circ$

$\delta_1\left(1-\frac{2}{3}\right)=2^\circ$

$\delta_1=6^\circ$

$\delta_1=(\mu_1-1)\mathrm{A}_1$

$6^\circ=(1.5-1)\mathrm{A}_1$

$\mathrm{A}_1=12^\circ$

26. (12)

$$\begin{aligned}\vec{a}&=\frac{\vec{\mathrm{F}}}{\mathrm{m}}=\frac{2\hat{\imath}+3\hat{\jmath}+5\hat{k}}{2}\\&=\hat{\imath}+1.5\hat{\jmath}+2.5\hat{k}\\\vec{\tau}&=\vec{u}t+\frac{1}{2}\vec{a}t^2\\&=0+\frac{1}{2}(\hat{\imath}+1.5\hat{\jmath}+2.5\hat{k})\\&=8\hat{\imath}+12\hat{\jmath}+20\hat{k}\end{aligned}$$

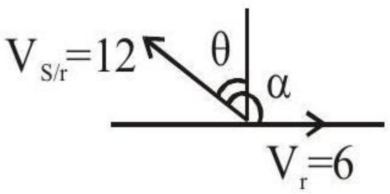
$\mathrm{b}=12$

27. (120)

$12\sin\theta=\mathrm{v_r}$

$\sin\theta=\frac{1}{2}$

$\theta=30^\circ$



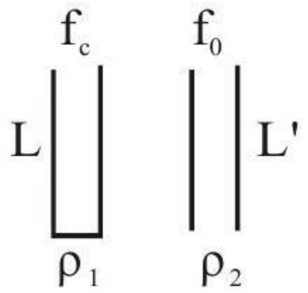
$\therefore\alpha=120^\circ$

28. (4)

$\mathrm{f_c}=\mathrm{f_0}$

$\frac{3\,\mathrm{V_C}}{4\,\mathrm{L}}=\frac{2\,\mathrm{V_0}}{2\,\mathrm{L'}}$

$\frac{3\,\mathrm{V_C}}{4\,\mathrm{L}}=\frac{\mathrm{V_0}}{\mathrm{L'}}$



$L' = \frac{4}{3}\frac{L}{V_C}V_0 = \frac{4L}{3}\sqrt{\frac{B\cdot\rho_1}{\rho_2\cdot B}}$ (B is bulk modulus)

$$= \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

$x = 4$

29. (3)

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}\,g\sin\theta$$

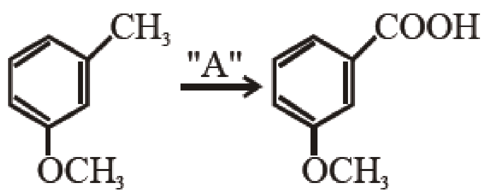
$b = 3$

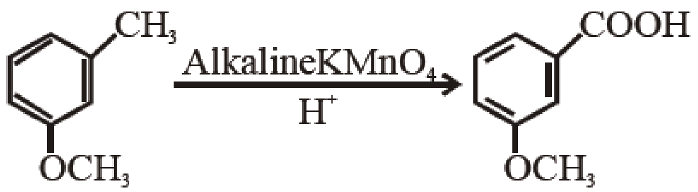
30. (-113)

$$n = 0.60 = 1 = \frac{T_L}{T_H}$$
$$\frac{T_L}{T_H} = 0.4 \Rightarrow T_L = 0.4 \times 400$$
$$= 160\text{ K}$$
$$= -113^\circ\text{C}$$

31. (c) The green- house gases are $CO_2, H_2O_{(vapour)}$ & CH_4.

32. (c)





33. (a) Reduction of $Al_2O_3 \rightarrow Al$ is carried out by electrolytic reduction of its fused salts.

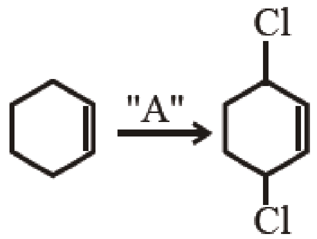
$ZnO, Fe_2O_3 \& Cu_2O$ can be reduce by carbon.

34. (a) $Na \rightarrow [Ne]3\,s^1 IE_1$ is very low but IE_2 is very high due to stable noble gas configuration of Na^+.

$Mg \rightarrow [Ne]3\,s^2 IE_1 \& IE_2 \rightarrow$ Low

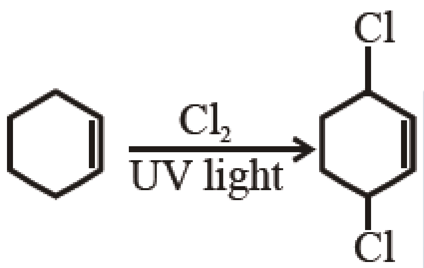
IE_3 is very high.

35. (c)



For substitution at allylic position in the given compound, the reagent used is Cl_2/uv light.

The reaction is free radical halogenation.



36. (c) The secondary structure of protein includes two type:
(a) α-Helix
(b) β-pleated sheet

In α-Helix structure, the poly peptide chain is coil around due to presence of Intramolecular H-Bonding.

37. (a)

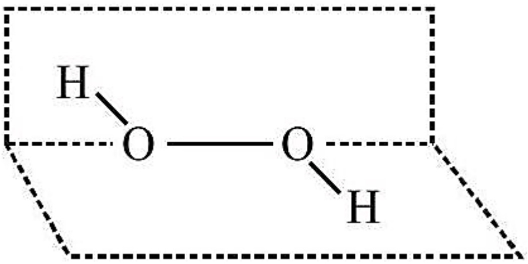
$$\underset{\text{(Unstable)}}{2FeI_3} \longrightarrow \underset{\text{(Stable)}}{2FeI_2} + I_2$$

Due to strong reducing nature of I

$2Fe^{3+} + 2I^- \longrightarrow 2Fe^{2+} + I_2$

remaining halides of Fe^{2+}&Fe^{3+} are stable.

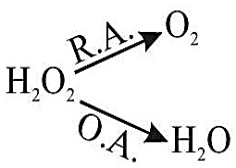
38. (c) Urea –HCHO resin is used in manufacture of wood laminates.
39. (c)
40. (d)
41. (b)



Structure of H_2O_2

(Open book type) → Non planar

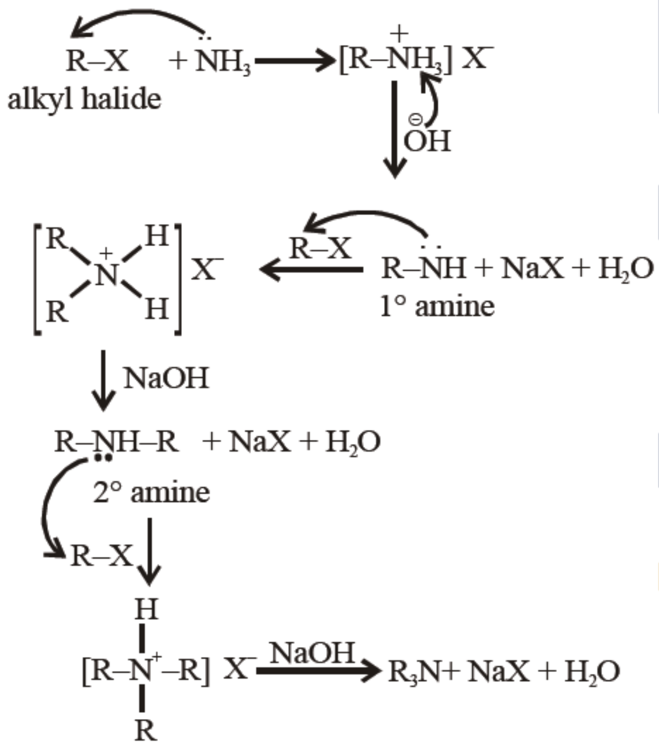
H_2O_2 is used in the treatment of effluents.



act as both O.A & R.A.

H_2O_2 is miscible in water due to hydrogen bonding.

42. (c)



So, the purpose of NaOH in the above reactions in to remove acidic impurities.

43. (c)
44. (c)
45. (b) $X = {}_{33}As$ → Metalloid

$Y = {}_{53}I$ → Nonmetal
$Z = {}_{83}Bi$ → Metal

46. (c) match list

(a) Lassaigne's Test	(iii) N, S, P and Halogen
(b) Cu(II) Oxide	(i) Carbon
(c) $AgNO_3$	(iv) Halogen specifically.
(d) Sodium fusion extract given black precipitate with acetic acid and lead acetate $(CH_3COOH/(CH_3COO)_2Pb)$	(ii) Sulphur

-(a)-(iii) ; (b)-(i) ; (c)-(iv) ; (d)-(ii)

47. (b) Colloidel solution exhibits colligative properties

An ordinary filter can- not stop the flow of colloidal particles.

Flocculating power increases with increase the opposite charge of electrolyte.

Colloidal particles show brownian motion.

48. (d) (a) $_{58}Ce \rightarrow [Xe]4f^2 5\ d^0 6\ s^2$

In complex $Ce^{4+} \rightarrow [Xe]4f^0 5\ d^0 6\ s^0$ there is no unpaired electron so $\mu_m = 0$

(b) $_{64}Gd^{3+} \rightarrow [Xe]4f^7 5\ d^0 6\ s^0$

contain seven unpaired electrons so,

$\mu_m = \sqrt{7(7+2)} = \sqrt{63}$ B.M.

(c) $_{63}Eu^{3+} \rightarrow [\,_{54}Xe]4f^6 5\ d^0 6\ s^0$

contain six unpaired electron

so, $\mu_m = \sqrt{6(6+2)} = \sqrt{48}$ B.M.

Hence, order of spin only magnetic movement

$B > C > A$

49. (c) $H_3PO_3 + 2NaOH \rightarrow Na_2HPO_3 + 2H_2O$

50ml 1M

1M V = ?

$$\Rightarrow \frac{n_{NaOH}}{n_{H_3PO_3}} = \frac{2}{1}$$

$$\Rightarrow \frac{1 \times V}{50 \times 1} = \frac{2}{1} \Rightarrow V_{NaOH} = 100\text{ml}$$

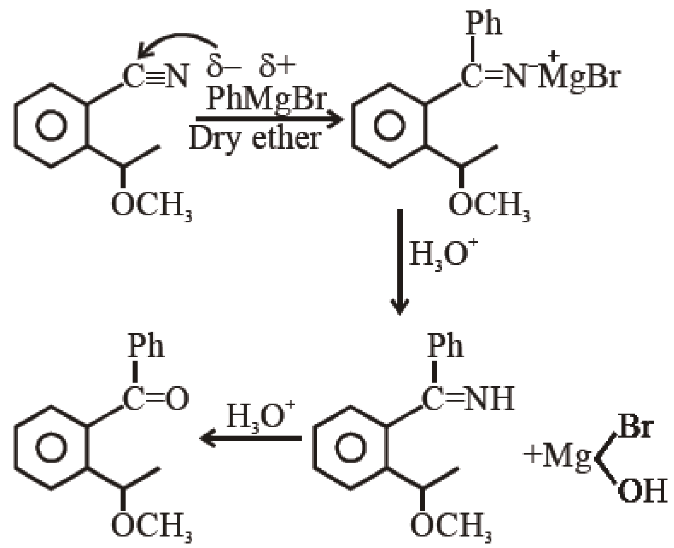
$H_3PO_2 + 2NaOH \rightarrow NaH_2PO_3 + H_2O$

100ml 1M

2M V = ?

$$\Rightarrow \frac{n_{NaOH}}{n_{H_3PO_3}} = \frac{1}{1} \Rightarrow \frac{1 \times V}{2 \times 100} = \frac{1}{1} \Rightarrow V_{NaOH} = 200\text{ml}$$

50. (d)



51. (15) HCP structure: Per atom, there will be one octahedral void (OV) and two tetrahedral voids (TV).

Therefore, total three voids per atom are present in HCP structure.

→ therefore, total no of atoms of Ga will be-

$$= \frac{\text{Mass}}{\text{Molar Mass}} \times N_A = \frac{0.581\text{ g}}{70\text{ g/mol}} \times 6.023 \times 10^{23}$$

→ Now, total Number of voids = 3 × total no. of atoms

$$= 3 \times \frac{0.581}{70} \times 6.023 \times 10^{23} = 14.99 \times 10^{21}$$

$$\simeq 15 \times 10^{21}$$

52. (14.3) Given conc^n of KCl $= \frac{\text{m·mol}}{\text{L}}$

: Conductance (G) = 0.55mS

: Cell constant $\left(\frac{\ell}{A}\right) = 1.3\text{ cm}^{-1}$

To Calculate : Molar conductivity (λ_m) of sol.

→ Since $\lambda_m = \frac{1}{1000} \times \frac{k}{m}$

→ Molarity $= 5 \times 10^{-3} \frac{\text{mol}}{\text{L}}$

→ Conductivity $= G \times \left(\frac{\ell}{A}\right) = 0.55\text{mS} \times \frac{1.3}{\frac{1}{100}}\ \text{m}^{-1}$

$= 55 \times 1.3\ \text{mSm}^{-1}$

$$\text{eq}^{n}(a)\lambda_m = \frac{1}{1000} \times \frac{55 \times 1.3}{\left(\frac{5}{1000}\right)} \frac{\text{mSm}^2}{\text{mol}}$$

$$\Rightarrow \lambda_m = 14.3 \frac{\text{mSm}^2}{\text{mol}}$$

53. (108)

Given $t_2 = 54$ min

$T_{1/2} = 18$ min

$t = 0$ 'x' M

$t = 0$ 'x' M

$\Rightarrow$ To calculate : $[A_t] = 16 \times [B_t]$ ..... (a) time = ?

$\Rightarrow$ For I order kinetic : $[A_t] = \frac{A_0}{(2)^n}$

$n \rightarrow$ no of Half lives

$\Rightarrow$ Now from the relation (a)

$[A_t] = 16 \times [B_t]$

$$\Rightarrow \frac{x}{(2)^{n_1}} = \frac{x}{(2)^{n_2}} \times 16 \Rightarrow (2)^{n_2} = (2)^{n_1} \times (2)^4$$

$$\Rightarrow n_2 = n_1 + 4 \Rightarrow \frac{t}{(t_{1/2})_2} = \frac{t}{(t_{1/2})_1} + 4$$

$$\Rightarrow t\left(\frac{1}{18} - \frac{1}{54}\right) = 4 \Rightarrow t = \frac{4 \times 18 \times 54}{36}$$

$\Rightarrow t = 108$ min

54. (19) In Duma's method of estimation of Nitrogen. 0.1840gm of organic compound gave 30 mL of nitrogen which is collected at 287 K&758 mm of Hg.

Given;

Aqueous tension at 287 K = 14 mm of Hg.

Hence actual pressure = (758 – 14)

= 744 mm of Hg

Volume of nitrogen at STP $= \frac{273 \times 744 \times 30}{287 \times 760}$

V = 27.935 mL

$\because$ 22400 mL of N_2 at STP weighs = 28gm.

$\therefore$ 27.94 mL of N_2 at STP weighs =

$$\left(\frac{28}{22400}\times 27.94\right)\text{gm}$$
$$= 0.0349\text{gm}$$

Hence % of Nitrogen = $\left(\frac{0.0349}{0.1840}\times 100\right)$

= 18.97%

Rond off. Answer = 19%

55. (3) For, n = 5

$\ell = (0,1,2,3,4)$

If $\ell = 0$, m = 0

$\ell = 1$, m = $\{-1,0,+1\}$

$\ell = 2$, m = $\{-2,-1,0,+1,+\mathbf{2}\}$

$\ell = 3$, m = $\{-3,-2,-1,0,+1,+\mathbf{2},+3\}$

$\ell = 4$, m = $\{-4,-3,-2,-1,0,+1,+2,+3,+4\}$

5 d, 5f and 5 g subshell contain one-one orbital having $m_\ell = +2$

56. (19) Given $P_A^0 = 21\text{kPa} \Rightarrow P_B^0 = 18\text{kPa} \rightarrow$ An Ideal solution is prepared by mixing 1 mol A and 2 mol B.

$\rightarrow X_A = \frac{1}{3}$ and $X_B = \frac{2}{3}$

$\rightarrow$ Acc to Raoult's low

$P_T = X_A P_A^0 + X_B P_B^0$

$\Rightarrow P_T = \left(\frac{1}{3}\times 21\right) + \left(\frac{2}{3}\times 18\right)$

$\Rightarrow P_T = 7 + 12 = 19\text{KPa}$

57. (1) H_2SO_3 [Dibasic acid]

c = 0.588M

$\Rightarrow$ pH of solution P due to First dissociation only since $K_a, > Ka_2$

$\Rightarrow$ First dissociation of H_2SO_3

$t = 0 \quad H$

$t \quad CO$

$C - x \quad Caq) \rightleftharpoons H^{\oplus}(aq) + HSO_3^-(aq): ka_1 = 1.7 \times 10^{-2}$

$$\Rightarrow \quad Ka_1 = \frac{1.7}{100} = \frac{[H^{\oplus}][HSO_3^-]}{[H_2SO_3]}$$

$$\Rightarrow \frac{1.7}{100} = \frac{x^2}{(0.58 - x)}$$

$$\Rightarrow 1.7 \times 0.588 - 1.7x = 100x^2$$

$$\Rightarrow 100x^2 + 1.7x - 1 = 0$$

$$\Rightarrow [H^{\oplus}] = x = \frac{-1.7 + \sqrt{(1.7)^2 + 4 \times 100 \times 1}}{2 \times 100} = 0.09186$$

Therefore pH of sol. is : $pH = -\log[H^{\oplus}]$

$\Rightarrow pH = -\log(0.09186) = 1.036 \simeq 1$

58. (525)

$3\,Pb(NO_3)_2 + Cr_2(SO_4)_3 \rightarrow 3PbSO_4 + 2Cr(NO_3)_3$ 35ml 20ml

0.15M 0.12M

$= \ 5.25\ m.mol = 2.4\ m \cdot mol 5.25\ m \cdot mol$

$= 5.25 \times 10^{-3}$ mol

Therefore, moles of $PbSO_4$ formed $= 5.25 \times 10^{-3} = 525 \times 10^{-5}$

59. (2218)

$T = 298\ K, R = 8.314 \frac{J}{mol}$

→ Chemical reaction is

$Fe + 2HCl \rightarrow FeCl_2 + H_2(g)$

50 g P = 1 bar

$= \frac{50}{55.85}$ mol

$\frac{50}{55.85}$ mol

→ Work done for 1 mol gas

$= -P_{ext} \times \Delta V$

$= \Delta ngRT$

$= -1 \times 8.314 \times 298$ J

→ Work done for $\frac{50}{55.85}$ mol of gas

$= -1.8314 \times 298 \times \frac{50}{55.85}$ J

$= -2218.059$ J

$\simeq -2218$ J

60. (4) $\lambda_{\text{absorbed}} = 498$ nm (given)

The octahedral spilitting energy

Δ_0 or $E = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^{8}}{498\times10^{-9}}$

$= 0.0399 \times 10^{-17}$ J

$= 3.99 \times 10^{-19}$ J

$= 4.00 \times 10^{-19}$ J (round off)

61. (c)

$C_1 + C_2 \rightarrow C_1$

$$\begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_1 - R_2 \rightarrow R_1$

$$\begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

Open w.r.t. R_1

$-(2\sin 2x - \cos 2x)$

$\cos 2x - 2\sin 2x = f(x)$

$f(x)|_{max} = \sqrt{1+4} = \sqrt{5}$

62. (b) Total cases:

$\underline{6} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2}$

$n(s) = 6 \cdot 6\,!$

Favourable cases:

Number divisible by 3 $\equiv$

Sum of digits must be divisible by 3

Case-I

1,2,3,4,5,6

Number of ways $= 6\,!$

Case-II

0,1,2,4,5,6

Number of ways $= 5 \cdot 5\,!$

Case-III

0,1,2,3,4,5

Number of ways $= 5 \cdot 5\,!$

n(favourable) $= 6! + 2 \cdot 5 \cdot 5!$

$$P = \frac{6! + 2 \cdot 5 \cdot 5!}{6 \cdot 6!} = \frac{4}{9}$$

63. (c) $\operatorname{Lim}_{x \to 0^+} f(x) = f(0) = \operatorname{Lim}_{x \to 0^-}(x)$

$$\operatorname{Lim}_{x \to 0^+} \frac{\cos^{-1}(1-x^2) \cdot \sin^{-1}(1-x)}{x(1-x)(1+x)}$$

$$\operatorname{Lim}_{x \to 0^+} \frac{\cos^{-1}(1-x^2)}{x \cdot 1 \cdot 1} \cdot \frac{\pi}{2}$$

Let $1 - \mathrm{x}^2 = \cos\theta$

$$\frac{\pi}{2} \operatorname{Lim}_{x \to 0^+} \frac{\theta}{\sqrt{1-\cos\theta}}$$

$$\frac{\pi}{2} \operatorname{Lim}_{\theta \to 0^+} \frac{\theta}{\sqrt{2}\sin\frac{\theta}{2}} = \frac{\pi}{\sqrt{2}}$$

Now, $\operatorname{Lim}_{x \to 0^-} \frac{\cos^{-1}(1-(1+x)^2)\sin^{-1}(-x)}{(1+x)-(1+x)^3}$

$$\operatorname{Lim}_{x \to 0^-} \frac{\frac{\pi}{2}(-\sin^{-1}x)}{(1+x)(2+x)(-x)}$$

$$\operatorname{Lim}_{x \to 0^-} \frac{\frac{\pi}{2}}{1 \cdot 2} \cdot \frac{\sin^{-1}x}{x} = \frac{\pi}{4}$$

$\Rightarrow$ RHL $\neq$ LHL

Function can't be continuous

$\Rightarrow$ No value of α exist

64. (b) Plane passing through (42,0,0), (0,42,0), (0,0,42)

From intercept from, equation of plane is

$x + y + z = 42$

$\Rightarrow (x - 11) + (y - 19) + (z - 12) = 0$

let $a = x - 11, b = y - 19, c = z - 12$

$a + b + c = 0$

Now, given expression is $3 + \frac{a}{b^2c^2} + \frac{b}{a^2c^2} + \frac{c}{a^2\,b^2} - \frac{42}{14abc}$

$$3 + \frac{a^3 + b^3 + c^3 - 3abc}{a^2b^2c^2}$$

If $a + b + c = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

$\Rightarrow 3$

65. (c) $I = \int_0^{10} [x] \cdot e^{[x]-x+1}$

$$I = \int_0^1 0dx + \int_1^2 1 \cdot e^{2-x} + \int_2^3 2 \cdot e^{3-x} + \cdots .. + \int_9^{10} 9 \cdot e^{10-x}dx$$

$$\Rightarrow I = \sum_{n=0}^{9} \int_n^{n+1} n \cdot e^{n+1-x}dx$$

$$= -\sum_{n=0}^{9} n(e^{n+1-x})_n^{n+1}$$

$= -\sum_{n=0}^{9} n \cdot (e^0 - e^1)$

$= (e - 1)\sum_{n=0}^{9} n$

$$= (e - 1) \cdot \frac{9 \cdot 10}{2}$$

$= 45(e - 1)$

66. (a) Given $y^2 = 4x$

Mirror image on $y = x \Rightarrow C: x^2 = 4y$

$$2x = 4 \cdot \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$$

$$\left.\frac{dy}{dx}\right|_{P(2,1)} = \frac{2}{2} = 1$$

Equation of tangent at (2,1)

$\Rightarrow y-1=1(x-2)$

$\Rightarrow x-y=1$

67. (b) $\frac{dy}{dx}+(\tan x)y=\sin x; 0 \leq x \leq \frac{\pi}{3}$

I.F. $= e^{\int \tan x dx} = e^{\ln \sec x} = \sec x$

$y\sec x = \int \tan x dx$

$y\sec x = \int \tan x dx$

$y\sec x = \ln|\sec x| + C$

$x=0, y=0 \Rightarrow \therefore c=0$

$y\sec x = \ln|\sec x|$ $y=\cos x \cdot \ln|\sec x|$

$y|_{x=\frac{\pi}{4}} = \left(\frac{1}{\sqrt{2}}\right) \cdot \ln\sqrt{2}$

$y|_{x=\frac{\pi}{4}} = \frac{1}{2\sqrt{2}}\log_e 2$

68. (d) $A=\{2,3,4,5,\ldots,30\}$

$(a,b) \simeq (c,d) \Rightarrow ad=bc$

$(4,3) \simeq (c,d) \Rightarrow 4d=3c$

$\Rightarrow \frac{4}{3}=\frac{c}{d}$

$\frac{c}{d}=\frac{4}{3}$ $\&c,d \in \{2,3,$

$(c,d)=\{(4,3),(8,6),(12,9),(16,12),(20,$

$15),(24,18),(28,21)\}$

No. of ordered pair = 7

69. (c) $x^2+y^2+ax+2ay+c=0$

$$2\sqrt{g^2-c}=2\sqrt{\frac{a^2}{4}-c}=2\sqrt{2}$$

$$\Rightarrow \frac{a^2}{4}-c=2$$

$2\sqrt{f^2-c}=2\sqrt{a^2-c}=2\sqrt{5}$

$\Rightarrow a^2-c=5$

(a) & (b)

$\frac{3a^2}{4} = 3 \Rightarrow a = -2 \ (a < 0)$

$\therefore \ c = -1$

Circle $\Rightarrow x^2 + y^2 - 2x - 4y - 1 = 0$

$\Rightarrow (x-1)^2 + (y-2)^2 = 6$

Given $x + 2y = 0 \Rightarrow m = -\frac{1}{2}$

$m_{\text{tangent}} = 2$

Equation of tangent

$\Rightarrow (y-2) = 2(x-1) \pm \sqrt{6}\sqrt{1+4}$

$\Rightarrow 2x - y \pm \sqrt{30} = 0$

Perpendicular distance from $(0,0) = \left|\frac{\pm\sqrt{30}}{\sqrt{4+1}}\right| = \sqrt{6}$

70. (a) $\exp\left(\frac{(|z|+3)(|z|-1)}{|z|+1}\ln 2\right) \geq \log_{\sqrt{2}}|5\sqrt{7} + 9i|$

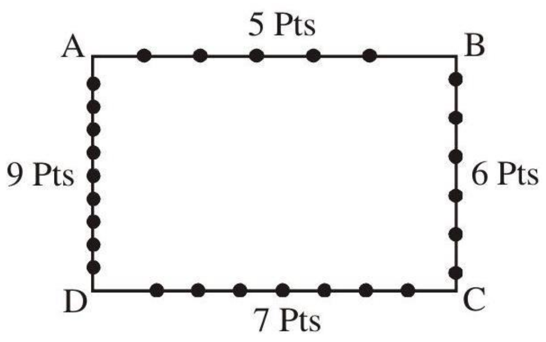
$$\Rightarrow \ 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq \log_{\sqrt{2}}(16)$$
$$\Rightarrow \ 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3$$
$$\Rightarrow \ \frac{(|z|+3)(|z|-1)}{(|z|+1)} \geq 3$$
$$\Rightarrow \ (|z|+3)(|z|-1) \geq 3(|z|+1)$$
$$|z|^2 + 2|z| - 3 \geq 3|z| + 3$$

$\Rightarrow |z|^2 + |z| - 6 \geq 0$

$\Rightarrow (|z|-3)(|z|+2) \geq 0 \Rightarrow |z| - 3 \geq 0$

$\Rightarrow |z| \geq 3 \Rightarrow |z|_{\min} = 3$

71. (d)



α = Number of triangles

$\alpha = 5 \cdot 6 \cdot 7 + 5 \cdot 7 \cdot 9 + 5 \cdot 6 \cdot 9 + 6 \cdot 7 \cdot 9$

$= 210 + 315 + 270 + 378$

$= 1173$

β = Number of Quadrilateral

$\beta = 5 \cdot 6 \cdot 7 \cdot 9 = 1890$

$\beta - \alpha = 1890 - 1173 = 717$

72. (a) $y^2 = 3x^2$

and $x^2 + y^2 = 4b$

Solve both we get

so

$$x^2 = b$$
$$\frac{x^2}{16} + \frac{3x^2}{b^2} = 1$$

$$\frac{b}{16} + \frac{3}{b} = 1$$
$$b^2 - 16\,b + 48 = 0$$
$$(\,b - 12)(b - 4) = 0$$
$$b = 12,\ b > 4$$

73. (c) $\sin^{-1}\frac{3x}{5} + \sin^{-1}\frac{4x}{5} = \sin^{-1} x$

$$\sin^{-1}\left(\frac{3x}{5}\sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5}\sqrt{1 - \frac{9x^2}{25}}\right) = \sin^{-1} x$$

$$\frac{3x}{5}\sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5}\sqrt{1 - \frac{9x^2}{25}} = x$$

$x = 0, 3\sqrt{25 - 16x^2} + 4\sqrt{25 - 9x^2} = 25$

$4\sqrt{25 - 9x^2} = 25 - 3\sqrt{25 - 16x^2}$ squaring we get

$16(25 - 9x^2) = 625 + 9(25 - 16x^2) - 150\sqrt{25 - 16x^2}$

$400 = 625 + 225 - 150\sqrt{25 - 16x^2}$

$\sqrt{25 - 16x^2} = 3 \Rightarrow 25 - 16x^2 = 9$

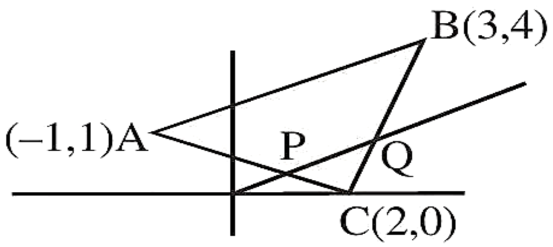
$\Rightarrow x^2 = 1$

Put $x = 0, 1, -1$ in the original equation

We see that all values satisfy the original equation.

Number of solution = 3

74. (b)



$P \equiv (x_1, mx_1)$

$Q \equiv (x_2, mx_2)$

$$A_1 = \frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ 2 & 0 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{13}{2}$$

$$A_2 = \frac{1}{2}\begin{vmatrix} x_1 & mx_1 & 1 \\ x_2 & mx_2 & 1 \\ 2 & 0 & 1 \end{vmatrix}$$

$$A_2 = \frac{1}{2}|2(mx_1 - mx_2)| = m|x_1 - x_2|$$

$$A_1 = 3\,A_2 \Rightarrow \frac{13}{2} = 3\,m|x_1 - x_2|$$

$$\Rightarrow |x_1 - x_2| = \frac{16}{6\,m}$$

$AC: x + 3y = 2$

$BC: y = 4x - 8$

$$P: x + 3y = 2 \,\&\, y = mx \Rightarrow x_1 = \frac{2}{1 + 3m}$$

$$Q: y = 4x - 8 \,\&\, y = mx \Rightarrow x_2 = \frac{8}{4 - m}$$

$$|x_1 - x_2| = \left|\frac{2}{1 + 3\,m} - \frac{8}{4 - m}\right|$$

$$= \left|\frac{-26m}{(1 + 3m)(4 - m)}\right| = \frac{26m}{(3m + 1)|m - 4|}$$
$$= \frac{26m}{(3m + 1)(4 - m)}$$

$$|x_1 - x_2| = \frac{13}{6\,m}$$

$$\frac{26m}{(3m + 1)(4 - m)} = \frac{13}{6m}$$

$$\Rightarrow\ 12\,m^2 = -(3\,m + 1)(m - 4)$$

$\Rightarrow 12\text{ m}^2 = -(3\text{ m}^2 - 11\text{ m} - 4)$

$\Rightarrow 15\text{ m}^2 - 11\text{ m} - 4 = 0$

$\Rightarrow 15\text{ m}^2 - 15\text{ m} + 4\text{ m} - 4 = 0$

$\Rightarrow (15\text{ m} + 4)(\text{m} - 1) = 0$

$\Rightarrow \text{m} = 1$

75. (a) $(x) = 3\ln(x-1) - 3\ln(x+1) - \frac{2}{x-1}$

$$f'(x) = \frac{3}{x-1} - \frac{3}{x+1} + \frac{2}{(x-1)^2}$$

$$f'(x) = \frac{4(2x-1)}{(x-1)^2(x+1)}$$

$f'(x) \geq 0$

$\Rightarrow x \in (-\infty, -1) \cup \left[\frac{1}{2}, 1\right) \cup (1, \infty)$

76. (a) $\ln f(x+1) = \ln(xf(x))$

$\ln f(x+1) = \ln x + \ln f(x)$

$\Rightarrow \text{g}(\text{x}+1) = \ln \text{x} + \text{g}(\text{x})$

$\Rightarrow \text{g}(\text{x}+1) - \text{g}(\text{x}) = \ln \text{x}$

$$\Rightarrow \text{g}''(\text{x}+1) - \text{g}''(\text{x}) = -\frac{1}{\text{x}^2}$$

Put $\text{x} = 1,2,3,4$

$$g''(\text{b}) - g''(\text{a}) = -\frac{1}{1^2}$$

$$g''(\text{c}) - g''(\text{b}) = -\frac{1}{2^2}$$

$$g''(\text{d}) - g''(\text{c}) = -\frac{1}{3^2}$$

$$g''(5) - g''(\text{d}) = -\frac{1}{4^2}$$

Add all the equation we get

$$g''(5) - g''(\text{a}) = -\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2}$$

$$|\text{g}''(5) - \text{g}''(\text{a})| = \frac{205}{144}$$

77. (c) . $\int_0^1 (\text{x}^2 + \text{bx} + \text{c})\text{dx} = 1$

$\frac{1}{3}+\frac{b}{2}+c=1 \Rightarrow \frac{b}{2}+c=\frac{2}{3}$

$3b+6c=4$

$P(b)=5$

$4+2b+c=5$

$2\,b+c=1$

From (a) & (b)

$b=\frac{2}{9}$ & $c=\frac{5}{9}$

$9(b+c)=7$

78. (b) (3,5,7) satisfy the line L_1

$\frac{3-a}{\ell}=\frac{5-2}{3}=\frac{7-b}{4}$

$\frac{3-a}{\ell}=1$ & $\frac{7-b}{4}=1$

$a+\ell=3$ ... (a) & $b=3$

$\vec{v}_1=<4,3,8>-<3,5,7>$

$\vec{v}_1=<1,-2,1>$

$\vec{v}_2=<\ell,3,4>$

$\vec{v}_1\cdot\vec{v}_2=0 \Rightarrow \ell-6+4=0 \Rightarrow \ell=2$

$a+\ell=3 \Rightarrow a=1$

$L_1:\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}\; L_2:\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$

$A=\langle 1,2,3\rangle$

$B=\langle 2,4,5>$

$\overrightarrow{AB}=<1,2,2>$

$\vec{p}=2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{q}=3\hat{i}+4\hat{j}+5\hat{k}$

$\vec{p}\times\vec{q}=-\hat{i}+2\hat{j}-\hat{k}$

Shortest distance $=\left|\frac{\overrightarrow{AB}\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|=\frac{1}{\sqrt{6}}$

79. (b) . $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$, $x\in(0,\infty)$

put $y = vx$

$$x\frac{dv}{dx} + v = \frac{v^2 - 1}{2v}$$
$$\frac{2v}{v^2 + 1}dv = -\frac{dx}{x}$$

Integrate,

$$\ln(v^2 + 1) = -\ln x + C$$
$$\ln\left(\frac{y^2}{x^2} + 1\right) = -\ln x + C$$

put $x = 1, y = 1, C = \ln 2$

$$\ln\left(\frac{y^2}{x^2} + 1\right) = -\ln x + \ln 2$$

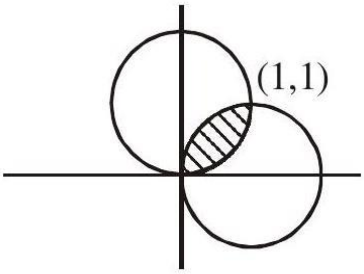
$\Rightarrow x^2 + y^2 - 2x = 0$ (Curve C_1)

Similarly,

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

Put $y = vx$

$x^2 + y^2 - 2y = 0$



required area $= 2\int_0^1 \left(\sqrt{2x - x^2} - x\right)dx = \frac{\pi}{2} - 1$

80. (b)

$\vec{r} \times \vec{a} = \vec{b} \times \vec{r} \Rightarrow \vec{r} \times (\vec{a} + \vec{b}) = 0$

$\vec{r} = \vec{\lambda}(\vec{a} + \vec{b}) \Rightarrow \vec{r} = \vec{\lambda}(\hat{\imath} + 2\hat{\jmath} - 3\hat{k} + 2\hat{\imath} - 3\hat{\jmath} + 5\hat{k})$

$\vec{r} = \vec{\lambda}(3\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\vec{r} \cdot (\alpha\hat{\imath} + 2\hat{\jmath} + \hat{k}) = 3$

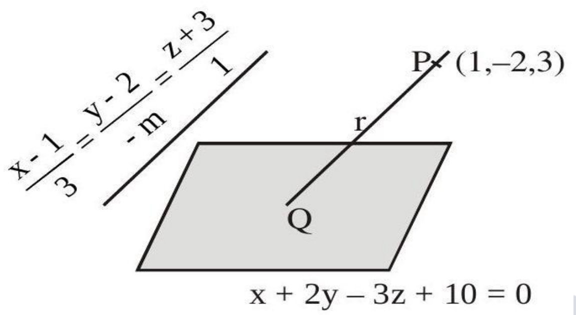
Put $\vec{r}$ from (a) $\alpha\lambda = 1$

$\vec{r} \cdot (2\hat{\imath} + 5\hat{\jmath} - \alpha\hat{k}) = -1$

Put $\vec{r}$ from (a) $2\lambda\alpha - \lambda = 1$
Solve (b) & (c)

$\alpha = 1, \lambda = 1$
$\Rightarrow \vec{r} = 3\hat{i} - \hat{j} + 2\hat{k}$
$|\vec{r}|^2 = 14$ & $\alpha = 1$
$\alpha + |\vec{r}|^2 = 15$

81. (2)



DC of line $\equiv \left(\frac{3}{\sqrt{m^2+10}}, \frac{-m}{\sqrt{m^2+10}}, \frac{1}{\sqrt{m^2+10}}\right)$

$$Q \equiv \left(1 + \frac{3r}{\sqrt{m^2+10}}, -2 + \frac{-mr}{\sqrt{m^2+10}}, 3 + \frac{r}{\sqrt{m^2+10}}\right)$$

Q lies on $x + 2y - 3z + 10 = 0$

$$1 + \frac{3r}{\sqrt{m^2+10}} - 4 - \frac{2mr}{\sqrt{m^2+10}} - 9 - \frac{3r}{\sqrt{m^2+10}} + 10 = 0$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(3 - 2\,m - 3) = 2$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(-2\,m) = 2$$
$$r^2m^2 = m^2 + 10$$

$$\frac{7}{2}\,m^2 = m^2 + 10 \Rightarrow \frac{5}{2}\,m^2 = 10 \Rightarrow m^2 = 4$$

$|m| = 2$

82. (5)

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1n_2}{(n_1+n_2)}(\bar{x}_1 - \bar{x}_2)^2$$

$n_1 = 10, n_2 = n, \sigma_1^2 = 2, \sigma_2^2 = 1$

$\bar{x}_1 = 2, \bar{x}_2 = 3, \sigma^2 = \frac{17}{9}$

$$\frac{17}{9} = \frac{10 \times 2 + n}{n + 10} + \frac{10n}{(n+10)^2}(3-2)^2$$

$\Rightarrow \dfrac{17}{9} = \dfrac{(n+20)(n+10)+10n}{(n+10)^2}$

$\Rightarrow 17n^2 + 1700 + 340n = 90n + 9(n^2 + 30n + 200)$

$\Rightarrow 8n^2 - 20n - 100 = 0$

$2n^2 - 5n - 25 = 0$

$\Rightarrow (2n+5)(n-5) = 0 \Rightarrow n = \dfrac{-5}{2}, 5$

(Rejected)

Hence $n = 5$

83. (1)

$A = XB$

$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \dfrac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$\begin{bmatrix} \sqrt{3}a_1 \\ \sqrt{3}a_2 \end{bmatrix} = \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$

$b_1 - b_2 = \sqrt{3}a_1$

$b_1 + kb_2 = \sqrt{3}a_2$

Given, $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$

$(1)^2 + (2)^2$

$(b_1 + b_2)^2 + (b_1 + kb_2)^2 = 3(a_1^2 + a_2^2)$

$a_1^2 + a_2^2 = \dfrac{2}{3}b_1^2 + \dfrac{(1+k^2)}{3}b_2^2 + \dfrac{2}{3}b_1 b_2 (k-1)$

Given, $a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{2}{3}b_2^2$

On comparing we get

$\dfrac{k^2+1}{3} = \dfrac{2}{3} \Rightarrow k^2 + 1 = 2$

$\Rightarrow k = \pm 1$

$\& \dfrac{2}{3}(k-1) = 0 \Rightarrow k = 1$

From both we get $k = 1$

84. (6)

$$\int \frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}+\int \frac{dx}{x^4+3x^2+1}$$

$$\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)}+\frac{1}{2}\int \frac{(x^2+1)-(x^2-1)dx}{x^4+3x^2+1}$$

Put $\tan^{-1}\left(x+\frac{1}{x}\right)=t$

$$\int \frac{dt}{t}+\frac{1}{2}\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+5}-\frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1}$$

Put $x-\frac{1}{x}=y, x+\frac{1}{x}=z$

$$\log_e t+\frac{1}{2}\int \frac{dy}{y^2+5}-\frac{1}{2}\int \frac{dz}{z^2+1}$$

$$=\log_e \tan^{-1}\left(x+\frac{1}{x}\right)+\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)$$

$$-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

$\alpha=1, \beta=\frac{1}{2\sqrt{5}}, \gamma=\frac{1}{\sqrt{5}}, \delta=\frac{-1}{2}$ or

$$\alpha=1, \beta=\frac{-1}{2\sqrt{5}}, \gamma=\frac{-1}{\sqrt{5}}, \delta=\frac{-1}{2}$$

$$10(\alpha+\beta\gamma+\delta)=10\left(1+\frac{1}{10}-\frac{1}{2}\right)=6$$

85. (1)

$$g[f(x)]=\begin{cases} f(x)+1 & f(x)<0 \\ (f(x)-1)^2+b & f(x)\ge 0 \end{cases}$$

$$g[f(x)]=\begin{cases} x+a+1 & x+a<0 \,\&\, x<0 \\ |x-1|+1 & |x-1|<0 \,\&\, x\ge 0 \\ (x+a-1)^2+b & x+a\ge 0 \,\&\, x<0 \\ (|x-1|-1)^2+b & |x-1|\ge 0 \,\&\, x\ge 0 \end{cases}$$

$$g[f(x)]=\begin{cases} x+a+1 & x\in(-\infty,-a) \,\&\, x\in(-\infty,0) \\ |x-1|+1 & x\in\phi \\ (x+a-1)^2+b & x\in[-a,\infty) \,\&\, x\in(-\infty,0) \\ (|x-1|-1)^2+b & x\in R \,\&\, x\in[0,\infty) \end{cases}$$

$$g[f(x)]=\begin{cases} x+a+1 & x\in(-\infty,-a) \\ (x+a-1)^2+b & x\in[-a,0) \\ (|x-1|-1)^2+b & x\in[0,\infty) \end{cases}$$

g(f(x)) is continuous

at $x = -a$ & at $x = 0$

$1 = b + 1$ & $(a-1)^2 + b = b$

$b = 0$ a $a = 1$

$\Rightarrow a + b = 1$

86. (14)

. $a^2 = \frac{b}{16} \Rightarrow \frac{1}{b} = \frac{1}{16a^2}$

$\frac{2}{b} = \frac{1}{a} + 6$

$\frac{1}{8a^2} = \frac{1}{a} + 6$

$\frac{1}{a^2} - \frac{8}{a} - 48 = 0$

$\frac{1}{a} = 12, -4 \Rightarrow a = \frac{1}{12}, -\frac{1}{4}$

$a = \frac{1}{12}, a > 0$

$b = 16a^2 = \frac{1}{9}$

$\Rightarrow 72(a+b) = 6 + 8 = 14$

87. (15)

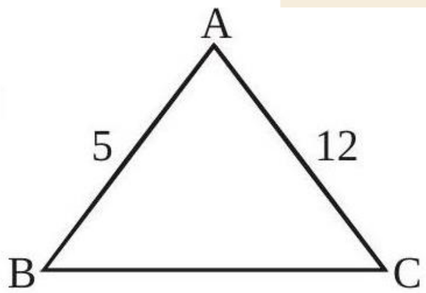
$\Delta = \frac{1}{2} \cdot 5 \cdot 12 \cdot \sin A = 30$

$\sin A = 1$

$A = 90° \Rightarrow BC = 13$

$BC = 2R = 13$

$r = \frac{\Delta}{S} = \frac{30}{15} = 2$



$2R + r = 15$

88. (6)

$A = \sum_{k=0}^{n} {}^nC_k \left[\left(-\frac{1}{2}\right)^k + \left(\frac{-3}{4}\right)^k + \left(\frac{-7}{8}\right)^k + \left(\frac{-15}{16}\right)^k + \left(\frac{-37}{32}\right)^k\right]$

$A=\left(1-\frac{1}{2}\right)^{n}+\left(1-\frac{3}{4}\right)^{n}+\left(1-\frac{7}{8}\right)^{n}+\left(1-\frac{15}{16}\right)^{n}+\left(1-\frac{31}{32}\right)^{n}$

$A=\frac{1}{2^{n}}+\frac{1}{4^{n}}+\frac{1}{8^{n}}+\frac{1}{16^{n}}+\frac{1}{32^{n}}$

$A=\frac{1}{2^{n}}\left(\frac{1-\left(\frac{1}{2^{n}}\right)^{5}}{1-\frac{1}{2^{n}}}\right)\Rightarrow A=\frac{\left(1-\frac{1}{2^{5n}}\right)}{(2^{n}-1)}$

$(2^{n}-1)A=1-\frac{1}{2^{5n}}$, Given $63A=1-\frac{1}{2^{30}}$

Clearly $5n=30$

$n=6$

89. (28) $\vec{c}=\lambda(\vec{a}\times\vec{b})$

$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 1 & -1\\ 1 & 2 & 1\end{vmatrix}$

$(\vec{a}\times\vec{b})=3\hat{\imath}-2\hat{\jmath}+\hat{k}$

$\vec{c}\cdot(\hat{\imath}+\hat{\jmath}+3\hat{k})=\lambda(3\hat{\imath}-2\hat{\jmath}+\hat{k})\cdot(\hat{\imath}+\hat{\jmath}+3\hat{k})$
$\Rightarrow\lambda(\mathrm{d})=8\Rightarrow\lambda=2$

$\vec{c}=2(\vec{a}\times\vec{b})$

$\vec{c}.(\vec{a}\times\vec{b})=2|\vec{a}\times\vec{b}|^{2}=28$

90. (16) $S_n(x)=(2+3+6+11+18+27+\cdots\ldots+\text{n-terms})\log_a x$

Let $S_1=2+3+6+11+18+27+\cdots.+T_n$ $S_1=2+3+6+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+T_n$

$T_n=2+1+3+5+\cdots\ldots.+\text{n terms}$

$T_n=2+(n-1)^2$

$S_1=\Sigma T_n=2n+\frac{(n-1)n(2n-1)}{6}$

$\Rightarrow S_n(x)=\left(2n+\frac{n(n-1)(2n-1)}{6}\right)\log_a x$ $S_{24}(x)=1093$ (Given)

$\log_a x\left(48+\frac{23\cdot 24.47}{6}\right)=1093$

$\log_a x=\frac{1}{4}$

$S_{12}(2x)=265$

$S_{12}(2x)=265$

$\log_a(2x)\left(24+\frac{11.12.23}{6}\right)=265$

$\log_a 2x=\frac{1}{2}$

$(2)-(1)$

$\log_a 2x-\log_a x=\frac{1}{4}$

$\log_a 2=\frac{1}{4}\Rightarrow a=16$