1. (a) $\vec{F} = 4\hat{i} - 3\hat{j}$

$\vec{r}_1 = 5\hat{i} + 5\sqrt{3}\hat{j}$ & $\vec{r}_2 = -5\hat{i} + 5\sqrt{3}\hat{j}$

Torque about ' O '

$\vec{\tau}_O = \vec{r}_1 \times \vec{F} = (-15 - 20\sqrt{3})\hat{k} = (15 + 20\sqrt{3})(-\hat{k})$

Torque about 'Q'

$\vec{\tau}_Q = \vec{r}_2 \times \vec{F} = (-15 + 20\sqrt{3})\hat{k} = (15 - 20\sqrt{3})(-\hat{k})$

2. (a) Excess pressure at common surface is given by

$$P_{ex} = 4\,T\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{4\,T}{r}$$

$$\therefore \frac{1}{r} = \frac{1}{a} - \frac{1}{b}$$

$$r = \frac{ab}{b-a}$$

3. (a) Since each vibrational mode has 2 degrees of freedom hence total vibrational degrees of freedom $= 48$

$f = 3 + 3 + 48 = 54$

$$\gamma = 1 + \frac{2}{f} = \frac{28}{27} = 1.03$$

4. (b) We know velocity of electron in n^{th} shell of hydrogen atom is given by

$$v = \frac{2\pi kZe^2}{nh}$$

$$\therefore v \propto \frac{1}{n}$$

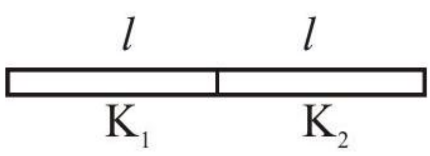
5. (b)

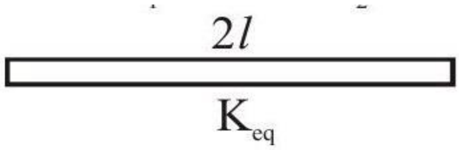
$$\lambda_1 = \frac{h}{\sqrt{2mE}}$$

$$\lambda_2 = \frac{hc}{E}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$$

6. (a)





$$R_{eff} = \frac{l}{K_1\ A} + \frac{l}{K_2\ A} = \frac{2l}{K_{eq}A}$$

$$K_{eq} = \frac{2\ K_1\ K_2}{K_1 + K_2}$$

7. **(b)** Positive zero error = 0.2 mm

Main scale reading = 8.5 cm

Vernier scale reading = $6 \times 0.01 = 0.06$ cm

Final reading = $8.5 + 0.06 - 0.02 = 8.54$ cm

8. **(b)** $I = I_1 \sin \omega t + I_2 \cos \omega t$

$$\therefore I_0 = \sqrt{I_1^2 + I_2^2}$$

$$\therefore I_{rms} = \frac{I_0}{\sqrt{2}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

9. **(b)** $\mu_s N = \frac{mv^2}{R}$

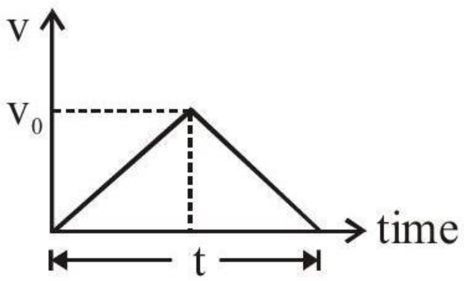
$$N = \frac{mv^2}{\mu_s R} = mg + F_L$$

$$F_L = \frac{mv^2}{\mu_s R} - mg$$

10. **(c)** $v_0 = \alpha t_1$ and $0 = v_0 - \beta t_2 \Rightarrow v_0 = \beta t_2$

$$t_1 + t_2 = t$$

$$v_0\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = t$$



$$\Rightarrow v_0 = \frac{\alpha\beta t}{\alpha + \beta}$$

Distance = area of v − t graph

$$= \frac{1}{2} \times t \times v_0 = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha + \beta} = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$$

11. (a)

$$B = \mu n I = \mu_0 \mu_r n I$$

$$B = 4\pi \times 10^{-7} \times 500 \times 1000 \times 5$$

$B = \pi$ Tesla

12. (d) We know, $\vec{L} = m(\vec{r} \times \vec{v})$

Now with respect to A, we always get direction of $\vec{L}$ along +ve z-axis and also constant magnitude as mvr. But with respect to B, we get constant magnitude but continuously changing direction.

13. (c) KE = PE

$$\frac{1}{2} m\omega^2(A^2 - x^2) = \frac{1}{2} m\omega^2 x^2$$

$$A^2 - x^2 = x^2$$

$$2x^2 = A^2$$

$$x = \pm \frac{A}{\sqrt{2}}$$

14. (d)

$$\eta = \frac{T_2}{T_1} = \frac{Q_2}{Q_1} = \frac{Q_1 - W}{Q_1} \quad (\because W = Q_1 - Q_2)$$

$$\frac{400}{800} = 1 - \frac{W}{Q_1}$$

$$\frac{W}{Q_1} = 1 - \frac{1}{2} = \frac{1}{2}$$

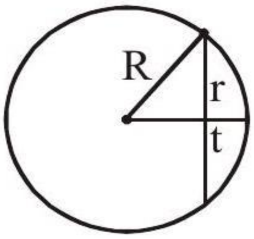
$$Q_1 = 2\,W = 2400\text{ J}$$

15. (d) $R^2 = r^2 + (R - t)^2$

$$R^2 = r^2 + R^2 + t^2 - 2Rt$$

Neglecting t^2, we get

$$R = \frac{r^2}{2t}$$



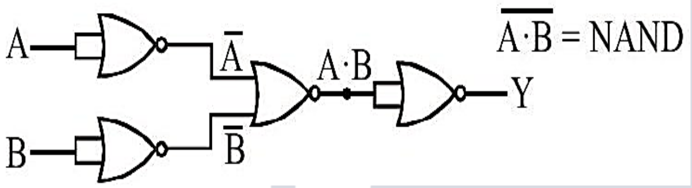
$\therefore \frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\frac{\mu-1}{R}$

$f=\frac{R}{\mu-1}=\frac{r^2}{2t(\mu-1)}=\frac{(3\times10^{-2})^2}{2\times3\times10^{-3}\times\left(\frac{3}{2}-1\right)}$

$=\frac{9\times10^{-4}}{6\times10^{-3}\times1}\times2$

$f=0.3\text{ m}=30\text{ cm}$

16. (b) By De Morgan's theorem, we have



17. (b) Given, $m=0.5$ kg and $u=20$ m/s

Initial kinetic energy $(k_i)=\frac{1}{2}mu^2$

$=\frac{1}{2}\times0.5\times20\times20=100\text{ J}$

After deflection it moves with 5% of k_i

$\therefore k_f=\frac{5}{100}\times k_i \Rightarrow \frac{5}{100}\times100$

$\Rightarrow k_f=5\text{ J}$

Now, let the final speed be ' v ' m/s, then:

$k_f=5=\frac{1}{2}mv^2$

$\Rightarrow v^2=20$

$\Rightarrow v=\sqrt{20}=4.47\text{ m/s}$

18. (b) Energy of H-atom is $E=-13.6Z^2/n^2$ for H-atom $Z=1$ & for ground state, $n=1$

$\Rightarrow E=-13.6\times\frac{1^2}{1^2}=-13.6\text{eV}$

Now for carbon atom (single ionised), $Z=6$

$E = -13.6\frac{Z^2}{n^2} = -13.6$

(given)

$\Rightarrow n^2 = 6^2 \Rightarrow n = 6$

19. (b) Let the final temperature of the mixture be T. Since, there is no loss in energy.

$\Delta U = 0$

$\Rightarrow \frac{F_1}{2} n_1 R\Delta T + \frac{F_2}{2} n_2 R\Delta T = 0$

$\Rightarrow \frac{F_1}{2} n_1 R(T_1 - T) + \frac{F_2}{2} n_2 R(T_2 - T) = 0$

$\Rightarrow T = \frac{F_1 n_1 R\, T_1 + F_2 n_2 R T_2}{F_1 n_1 R + F_2 n_2 R} \Rightarrow \frac{F_1 n_1\, T_1 + F_2 n_2\, T_2}{F_1 n_1 + F_2 n_2}$

20. (b) $i = 10$ A, $A = 5\ \text{mm}^2 = 5 \times 10^{-6}\ \text{m}^2$

and $v_d = 2 \times 10^{-3}$ m/s

We know, $i = neAvd$

$\therefore 10 = n \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 2 \times 10^{-3}$

$\Rightarrow n = 0.625 \times 10^{28} = 625 \times 10^{25}$

21. (1206) $d = \sqrt{2Rh}$

$A = \pi d^2$

$A = \pi 2Rh$

$= 3.14 \times 2 \times 6400 \times \frac{30}{1000}$

$A = 1205.76\ \text{km}^2$

$A \simeq 1206\ \text{km}^2$

22. (728) We know, $\theta = \left(\frac{\omega_1+\omega_2}{2}\right) t$

Let number of revolutions be N

$\therefore 2\pi N = 2\pi \left(\frac{900 + 2460}{60 \times 2}\right) \times 26$

$N = 728$

23. (4)

$R_1 + R_2 = s$

$$\frac{R_1R_2}{R_1+R_2}=p$$

$$R_1R_2=sp$$

$$R_1R_2=npp^2$$

$$R_1+R_2=\frac{nRR_2R_2}{(R_1+R_2)}$$

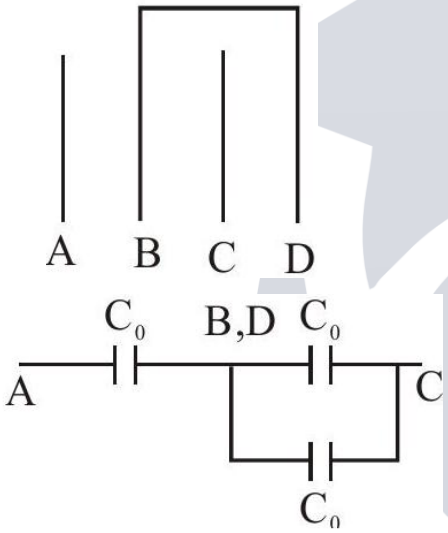
$$\frac{(R_1+R_2)^2}{R_1R_2}=n$$

for minimum value of n

$$R_1=R_2=R$$

$$\therefore n=\frac{(2R)^2}{R^2}=4$$

24. (2)



$$C_{eq}=\frac{2C_0}{3}=\frac{2}{3}\frac{\epsilon_0\,A}{d}$$

$$C_{eq}=\frac{2\epsilon_0}{3\,d}\times\left(2\times\frac{3}{2}\right)=2\ \left(\because A=lb=2\times\frac{3}{2}\right)$$

25. (64) $V_e=\sqrt{\frac{2Gm}{R}}$

$$10\ V_e=\sqrt{\frac{2Gm}{R'}}$$

$$\therefore 10=\sqrt{\frac{R}{R'}}$$

$\Rightarrow R' = \frac{R}{100} = \frac{6400}{100} = 64$ km

26. (2) $T_a = 2\pi\sqrt{\frac{M}{K}}$

$$T_b = 2\pi\sqrt{\frac{M}{K/2}}$$

$$\frac{T_b}{T_a} = \sqrt{2} = \sqrt{x}$$

$\Rightarrow x = 2$

27. (4) $Mg\sin\theta R = (mk^2 + mR^2)\alpha$

$$\alpha = \frac{Rg\sin\theta}{k^2+R^2} \Rightarrow a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta}\left(1+\frac{k^2}{R^2}\right)}$$

for least time, k should be least & we know k is least for solid sphere.

28. (864)

$U_i = \frac{1}{2} \times 14 \times 12 \times 12$pJ $\left(\because U = \frac{1}{2}CV^2\right)$

$= 1008$pJ

$U_f = \frac{1008}{7}$pJ $= 144$pJ

$(\because C_m = kC_0)$

Mechanical energy $= \Delta U$

$= 1008 - 144$

$= 864$pJ

29. (21) $a_{max} = \mu g = \frac{3}{7} \times 9.8$

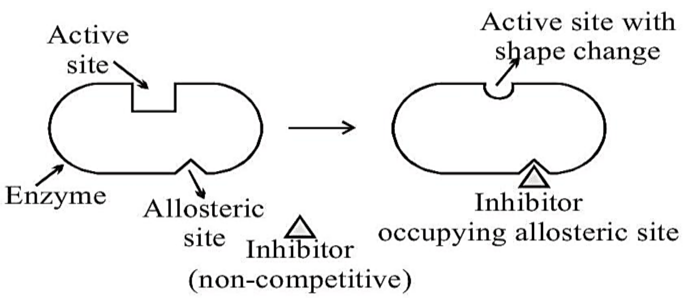
$F = (M+m)a_{max} = 5a_{max}$

$= 21$ Newton

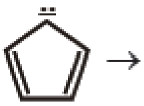
30. (25) $F = \frac{IA}{C}$

$$I = \frac{FC}{A} = \frac{2.5 \times 10^{-6} \times 3 \times 10^{8}}{30} = 25\ \text{W/cm}^2$$

31. **(c)** Some durg do not bind to the Enzyme's active site. These bind to a different site of enzyme which called allosteric site. This binding of inhibitor at allosteric site changes the shape of the active site in such a way that substrate can -not recognise it. Such inhibitor is known as Non-competitive inhibitor.

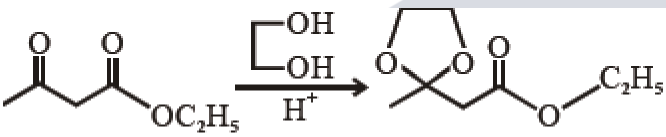


32. **(a)**

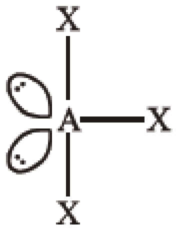


aromatic compound

33. **(b)**



34. **(c)**

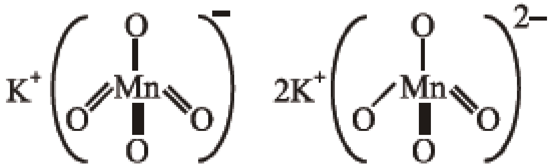


sp^3d hybridised

T-shaped

35. **(a)**

$$\underset{\text{Potassium permanganate}}{2KMnO_4} \xrightarrow{573K} \underset{\text{Potassium manganate}}{K_2MnO_4} + MnO_2 + O_2$$



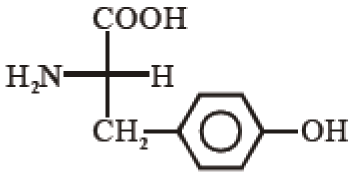
Tetrahedral unit diamagnetic

Tetrahedral unit paramagnetic

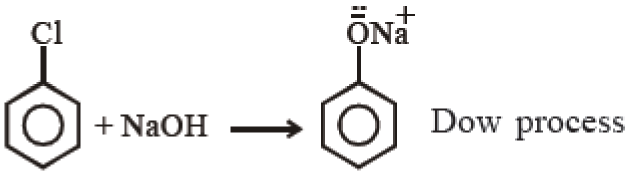
Statement-I is correct.

Statement-II is incorrect.

36. (d) The structure of Tyrosine amino acid is



37. (d)



Temperature = 623 K

Pressure = 300 atm

38. (c) Order of electron gain enthalpy

(Absolute value)

$Cl > F > Br > I$

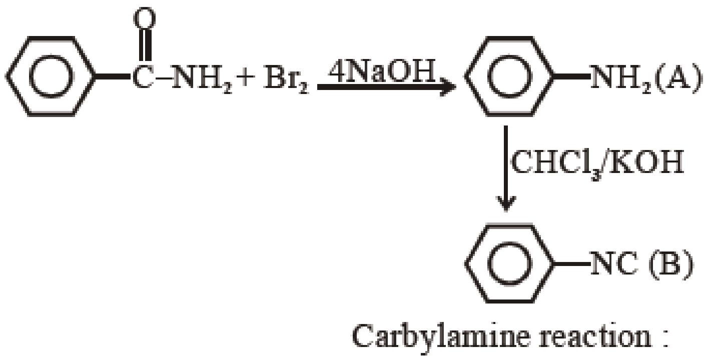
39. (b) Lewis base: Chemical species which has capability to donate electron pair.

In NF_3, SF_4, ClF_3 central atom (i.e. N, S, Cl) having lone pair therefore act as lewis base.

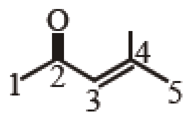
In PCl_5 central atom (P) does not have lone pair therefore does not act as lewis base.

40. (b) Reducing or classical smog is the combination of smoke, fog and SO_2.

41. (b) Hoffmann bromamide degradation reaction:



42. (d)



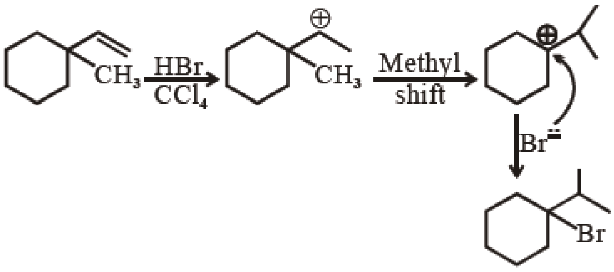
Mesityloxide

IUPAC [4-Methylpent-3-en-2-one]

43. (d) The process of cleavage of the $C-X$ bond by Ammonia molecule is known as ammonolysis.

Ex : $R-CH_2-Cl + \ddot{N}H_3 \longrightarrow R-CH_2-NH_2$

44. (d)



45. (a) Colloid of gas dispersed in solid is called solid sol.

46. (c) The dielectric constant of H_2O is greater than heavy water.

47. (b)

$Li^+ \ Na^+ \ K^+ \ Rb^+ \ Cs^+$

$\longrightarrow$ Hydration energy $\uparrow$

$\longrightarrow$ Ionic mobility $\downarrow$

$\longrightarrow$ Conductivity $\downarrow$

Correct option is $Na^+ > K^+ > Rb^+ > Cs^+$.

OR

As the size of gaseous ion decreases, it get more hydrated in water and hence, the size of aqueous ion increases. When this bulky ion move in solution, it experience greater resistance and hence lower conductivity.

Size of gasesous ion: $Cs^+ > Rb^+ > K^+ > Na^+$

Size of aqueous ion: $Cs^+ < Rb^+ < K^+ < Na^+$

Conductivity: $Cs^+ > Rb^+ > K^+ > Na^+$

48. (a) Electronic configuration of divalent metal ion having atomic number 25 is

$$Mn^{+2}_{(aq)} \Rightarrow \underset{\boxed{\uparrow\downarrow}}{1s^2} \ \underset{\boxed{\uparrow\downarrow}}{2s^2} \ \underset{\boxed{\uparrow\downarrow|\uparrow\downarrow|\uparrow\downarrow}}{2p^6} \ \underset{\boxed{\uparrow\downarrow}}{3s^2} \ \underset{\boxed{\uparrow\downarrow|\uparrow\downarrow|\uparrow\downarrow}}{3p^6} \ \underset{\boxed{\uparrow|\uparrow|\uparrow|\uparrow|\uparrow}}{3d^5}$$

Total number of unpaired electrons = 5

μ (Magnetic moment) $= \sqrt{n(n+2)}BM$

where n = number of unpaired e

$\therefore \mu = \sqrt{5(5+2)} = \sqrt{35}BM = 5.92BM$

49. (c) R_f = retardation factor

Distance travelled by the substance from $R_f = \dfrac{\text{reference line (c.m)}}{\text{Distance travelled by the solvent from}}$ reference line (c.m)

Note: R_f value of different compounds are different.

50. (a)

51. (4)

$P_4 + 3OH^- + 3H_2O \rightarrow PH_3 + 3H_2PO_2^-$

$\underset{\text{1 mole}}{H_2PO_2^-} + \underset{\text{excess}}{4Ag^+} + 2H_2O \rightarrow \underset{\text{4 mole}}{4Ag} + H_3PO_4 + 3H^+$

52. (2)

$HA \rightleftharpoons H^+ + A^-$

Initial conc. 0.01M 0.1M 0

Equ. conc. $(0.01 - x)(0.1 + x)xM$

$\approx 0.01M \approx 0.1M$

Now, $K_a = \frac{[x^+][A^-]}{[HA]} \Rightarrow 2 \times 10^{-6} = \frac{0.1 \times x}{0.01}$

$\therefore x = 2 \times 10^{-7}$

Now, $\alpha = \frac{x}{0.01} = \frac{2 \times 10^{-7}}{0.01} = 2 \times 10^{-5}$

53. (0)

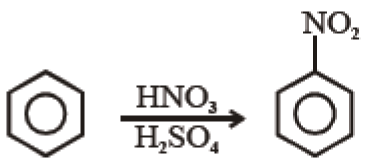
$n = 4$ and $m_\ell = -3$

Hence, ℓ value must be 3 .

Now, number of radial nodes $= n - \ell - 1$

$= 4 - 3 - 1 = 0$

54. (80)



1 mole 1 mole

78gm 123gm

3.9gm $\frac{123}{78} \times 3.9 = 6.15gm$

But actual amount of nitrobenzene formed is 4.92gm and hence. Percentage yield $= \frac{4.92}{6.15} \times 100 = 80\%$

55. (64) 100 molal aqueous solution means there is 100 mole solute in 1 kg = 1000gm water.

Now,

mole-fraction of solute $= \frac{n_{solute}}{n_{solute} + n_{solvent}}$

$$= \frac{100}{100 + \frac{1000}{18}} = \frac{1800}{2800} = 0.6428$$

$$= 64.28 \times 10^{-2}$$

56. (2)

For 1st order reaction,

$$K = \frac{2.303}{t} \cdot \log \frac{[A_0]}{[A_t]} = \frac{2.303}{570\text{sec}} \cdot \log\left(\frac{100}{32}\right)$$
$$= 1.999 \times 10^{-3}\text{sec}^{-1} \approx 2 \times 10^{-3}\text{sec}^{-1}$$

57. (230) $3CaO + Al \rightarrow Al_2O_3 + 3Ca$

Now, $\Delta_r H^\circ = \Sigma\Delta_f H^\circ_{\text{Products}} - \Sigma\Delta_f H^\circ_{\text{Reactants}} = [1 \times (-1675) + 3 \times 0] - [3 \times (-635) + 2 \times 0] = +230 \text{ kJ mol}^{-1}$

58. (24)

$$n_{eq}Fe^{2+} = n_{eq}Cr_2O_7{}^{2-}$$
$$\text{or, } \left(\frac{15 \times M_{Fe^{2+}}}{1000}\right) \times 1 = \left(\frac{20 \times 0.03}{1000}\right) \times 6$$
$$\therefore\ M_{Fe^{2+}} = 0.24\text{M} = 24 \times 10^{-2}\text{M}$$

59. (1389)

$P = K_H \cdot x$

or, $20 \times 10^3 = (8 \times 10^4 \times 10^3) \times \frac{n_{O_2}}{n_{O_2} + n_{\text{water}}}$

or, $\frac{1}{4000} = \frac{n_{O_2}}{n_{O_2} + n_{\text{water}}} = \frac{n_{O_2}}{n_{\text{water}}}$

Means 1 mole water (= 18gm = 18ml) dissolves

$\frac{1}{4000}$ moles O_2. Hence, molar solubility

$$= \frac{\left(\frac{1}{4000}\right)}{18} \times 1000 = \frac{1}{72}\text{moldm}^{-3}$$

$$= 1388.89 \times 10^{-5} \text{ moldm}^{-3} \approx 1389 \text{ moldm}^{-3}$$

60. (150)

Total moles of gases, $n = n_{CH_4} + n_{CO_2}$

$$= \frac{6.4}{16} + \frac{8.8}{44} = 0.6$$

Now,

$$P = \frac{nRT}{V} = \frac{0.6 \times 8.314 \times 300}{10 \times 10^{-3}}$$

$= 1.49652 \times 10^5$ Pa $\quad = 149.652$kPa

≈ 150kPa

61. (c)

62. (a) $\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$

$\Rightarrow \vec{r} \times (\vec{a} - \vec{b}) = 0$

$\Rightarrow \vec{r} = \lambda(\vec{a} - \vec{b})$

$\Rightarrow \vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$

Also $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$

$\Rightarrow \lambda(-5 - 8 + 10) = -3$

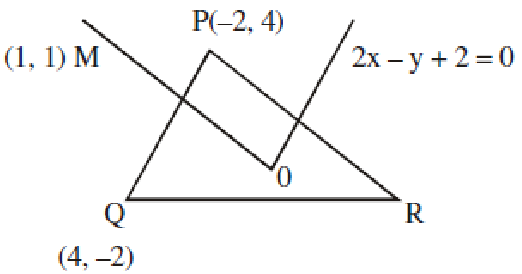
$\lambda = 1$

Now $\vec{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$

$= \vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$

$= -10 + 12 + 10 = 12$

63. (b)



Equation of perpendicular bisector of PR is

$y = x$

Solving with $2x - y + 2 = 0$ will give

$(-2,2)$

64. (d)

$kx + y + z = 1$

$x + ky + z = k$

$x + y + zk = k^2$

$$\Delta = \begin{vmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{vmatrix} = K(K^2 - 1) - 1(K - 1) + 1(1 - K)$$

$= K^3 - K - K + 1 + 1 - K$

$= K^3 - 3K + 2$

$= (K-1)^2(K+2)$

For $K = 1$

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

But for $K = -2$, at least one out of $\Delta_1, \Delta_2, \Delta_3$ are not zero

Hence for no soln, $K = -2$

65. (a)

$\text{Cot}^{-1}(\alpha) = \cot^{-1}(b) + \cot^{-1}(8) + \cot^{-1}(18) + \cdots \ldots$

$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{2}{4n^2}\right)$

$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

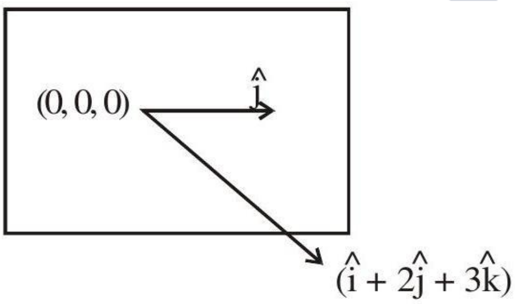
$= \tan^{-1} 201 - \tan^{-1} 1$

$= \tan^{-1}\left(\frac{200}{202}\right)$

$\therefore \cot^{-1}(\alpha) = \cot^{-1}\left(\frac{202}{200}\right)$

$\alpha = 1.01$

66. (d)



$\vec{n} = \hat{j} \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$= -3\hat{\imath} + 0\hat{\jmath} + \hat{k}$

So, $(-3)(x-1) + 0(y-2) + (a)(z-3) = 0$

$\Rightarrow -3x + z = 0$

Option 4

Alternate:

Required plane is

$$\begin{vmatrix} x & y & z \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 3x - z = 0$

67. (c) $A^2 = \sin^2 \alpha I$

So, $\left|A^2 - \frac{I}{2}\right| = \left(\sin^2 \alpha - \frac{1}{2}\right)^2 = 0$

$\Rightarrow \sin \alpha = \pm \frac{1}{\sqrt{2}}$

68. (a) $\because p \rightarrow q \equiv \sim p \vee q$

So, $* \equiv v$

Thus, $p^*(\sim q) \equiv pv(\sim q)$

$\equiv q \rightarrow p$

69. (b) $n(E) = 5 + 4 + 4 + 3 + 1 = 17$

So, $P(E) = \frac{17}{36}$

70. (a) ${}^7C_3 x^4\, x^{(3\log_2^x)} = 4480$

$\Rightarrow x^{(4+3\log_2^x)} = 2^7$

$\Rightarrow (4 + 3t)t = 7; t = \log_2^x$

$\Rightarrow t = 1, \frac{-7}{3} \Rightarrow x = 2$

71. (c)

72. (a) $\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\frac{8}{31}$

Taking tangent both sides :-

$\frac{(x+1)+(x-1)}{1-(x^2-1)} = \frac{8}{31}$

$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$

$\Rightarrow 4x^2 + 31x - 8 = 0$

$\Rightarrow x = -8, \frac{1}{4}$

But, if $x = \frac{1}{4}$

$\tan^{-1}(x+1) \in \left(0, \frac{\pi}{2}\right)$

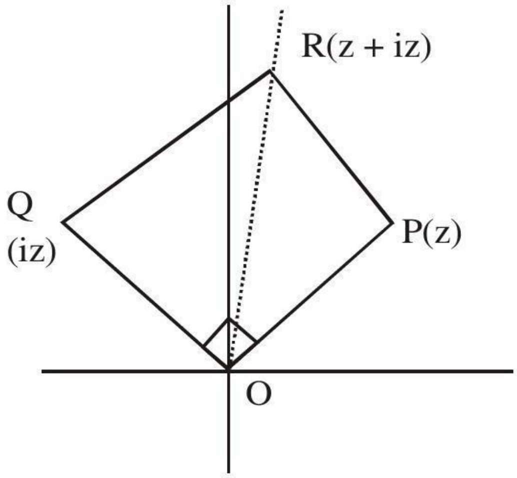
$\&\cot^{-1}\left(\frac{1}{x-1}\right) \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow$ LHS $> \frac{\pi}{2}$ & RHS $< \frac{\pi}{2}$

(Not possible)

Hence, $x = -8$

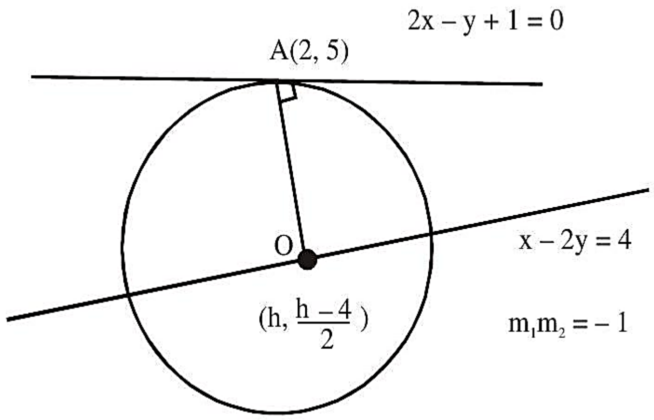
73. (b)



$A = \frac{1}{2}|z||iz|$

$= \frac{|z|^2}{2}$

74. (a)



$\left(\frac{h - \frac{(h-4)}{2}}{2-h}\right)(b) = -1$

$h = 8$

center (8,2)

Radius $= \sqrt{(8-2)^2 + (2-5)^2} = 3\sqrt{5})$

75. (d) Total matches between boys of both team

$= {}^7C_1 \times {}^4C_1 = 28$

Total matches between girls of both

team $= {}^nC_1\,{}^6C_1 = 6n$

Now, $28 + 6n = 52$

$\Rightarrow n = 4$

76. (a) $y = 4 + \frac{1}{\left(5+\frac{1}{y}\right)}$

$$y - 4 = \frac{y}{(5y+1)}$$

$$5y^2 - 20y - 4 = 0$$

$$y = \frac{20 + \sqrt{480}}{10}$$

$y = \frac{20-\sqrt{480}}{10} \rightarrow$ rejected

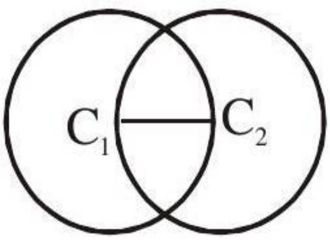
$$y = 2 + \sqrt{\frac{480}{100}}$$

77. (b)

$r_1 = 3, c_1(5,5)$

$r_2 = 3, c_2(8,5)$

$C_1C_2 = 3, r_1 = 3, r_2 = 3$



78. (d)

79. (a) $\frac{dy}{dx} = (1+y)(x-1)$

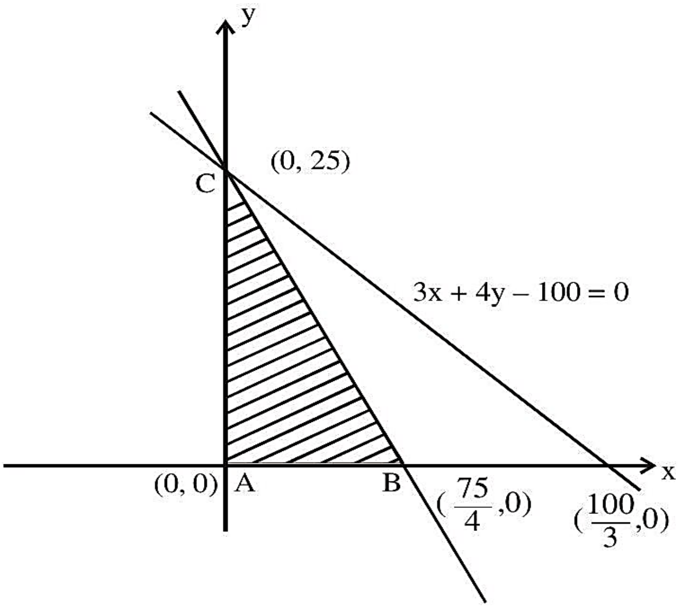
$$\frac{dy}{(y+1)} = (x-1)dx$$

Integrate $\ln(y+1) = \frac{x^2}{2} - x + c$

$(0,0) \Rightarrow c = 0 \Rightarrow y = e^{\left(\frac{x^2}{2}-x\right)} - 1$

80. (d) $\lim_{x\to 0^+} \frac{\cos^{-1}x}{(1-x^2)} \times \frac{\sin^{-1}x}{x} = \frac{\pi}{2}$

81. (904)



$z = 6xy + y^2 = y(6x + y)$

$3x + 4y \le 100$

$4x + 3y \le 75$

$x \ge 0$

$y \ge 0$

$x \le \frac{75-3y}{4}$

$Z = y(6x + y)$

$z \le y\left(6 \cdot \left(\frac{75-3y}{4}\right) + y\right)$

$Z \le \frac{1}{2}(225y - 7y^2) \le \frac{(225)^2}{2\times 4\times 7}$

$= \frac{50625}{56}$
≈ 904.0178
≈ 904.02

It will be attained at $y = \frac{225}{14}$

82. (6)

$$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}=f(0)$$

$$\Rightarrow \lim_{x\to 0}\frac{2\sin\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}=\frac{1}{K}$$

$$\Rightarrow \lim_{x\to 0}2\left(\frac{\sin x+x}{2x}\right)\left(\frac{x-\sin x}{2x^3}\right)=\frac{1}{K}$$

$$\Rightarrow 2\times\frac{(1+1)}{2}\times\frac{1}{2}\times\frac{1}{6}=\frac{1}{\mathrm{K}}$$

$$\Rightarrow \mathrm{K}=6$$

83. (481)

$$\mathrm{f(x)}=\sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)\text{ at x}=1;\ 2^{2x}=4$$

$$\text{for }\sin\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right);$$

$$\text{Let }\tan^{-1}\mathrm{x}=\theta;\ \theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

$$\therefore \sin(\cos^{-1}\cos 2\theta)=\sin 2\theta$$

$$\left\{\begin{array}{c}\text{If } x>1\Rightarrow\frac{\pi}{2}>\theta>\frac{\pi}{4}\\ \therefore\ \pi>2\theta>\frac{\pi}{2}\end{array}\right\}$$

$$=2\sin\theta\cos\theta=\frac{2\tan\theta}{1+\tan^2\theta}$$

$$=\frac{2x}{1+x^2}$$

$$\text{Hence, } f(x)=\frac{2\cdot 2^x}{1+2^{2x}}$$

$$\therefore f'(x)=\frac{(1+2^{2x})(2.2^x\ln 2)-2^{2x}\cdot 2\cdot\ln 2\cdot 2\cdot 2^x}{(1+2^{2x})}$$

$$\therefore\ \mathrm{f}^1(1)=\frac{20\ln 2-32\ln 2}{25}=-\frac{12}{25}\ln 2$$

$$\text{So, a}=25,\ \text{b}=12\Rightarrow|\mathrm{a}^2-\mathrm{b}^2|=25^2-12^2$$

$$=625-144$$
$$=481$$

84. (6)

$$\text{Let }\mathrm{P(E_1)}=\mathrm{P_1};\ \mathrm{P(E_2)}=\mathrm{P_2};\ \mathrm{P(E_3)}=\mathrm{P_3}$$

$P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = \alpha = P_1(1-P_2)(1-P_3)$

$P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) = \beta = (1-P_1)P_2(1-P_3)$

$P(\bar{E}_1 \cap \bar{E}_2 \cap E_3) = \gamma = (1-P_1)(1-P_2)P_3$

$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = P = (1-P_1)(1-P_2)(1-P_3)$
Given that, $(\alpha - 2\beta)P = \alpha\beta$

$\Rightarrow \left(P_1(1-P_2)(1-P_3) - 2(1-P_1)P_2(1-P_3)\right)P = P_1P_2\,(1-P_1)(1-P_2)(1-P_3)^2$

$\Rightarrow (P_1(1-P_2) - 2(1-P_1)P_2) = P_1P_2$

$\Rightarrow (P_1 - P_1P_2 - 2P_2 + 2P_1P_2) = P_1P_2$

$\Rightarrow P_1 = 2P_2$

and similarly, $(\beta - 3\gamma)P = 2\,B\gamma$

$P_2 = 3P_3$

So, $P_1 = 6P_3 \Rightarrow \frac{P_1}{P_3} = 6$

85. (2) $\vec{a} \cdot \vec{b} = 1 \Rightarrow -\alpha\beta - \alpha\beta - 3 = 1$

$\Rightarrow -2\alpha\beta = 4 \Rightarrow \alpha\beta = -2$

$\vec{b} \cdot \vec{c} = -3 \Rightarrow -\beta + 2\alpha + 1 = -3$

$\beta - 2\alpha = 4$

Solving (1) & (2), $(\alpha, \beta) = (-1,2)$

$$\frac{1}{3}[\vec{a}\vec{b}\vec{c}] = \frac{1}{3}\begin{vmatrix} \alpha & \beta & 3 \\ -\beta & -\alpha & -1 \\ 1 & -2 & -1 \end{vmatrix}$$
$$= \frac{1}{3}\begin{vmatrix} -1 & 2 & 3 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix}$$
$$= \frac{1}{3}\begin{vmatrix} 0 & 0 & 2 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix} = \frac{1}{3}[2(4-1)] = 2$$

86. (16)

$2\,A\operatorname{adj}(2\,A) = |2\,A|I$

$\Rightarrow A\operatorname{adj}(2\,A) = -4I \quad \ldots..(i)$

Now, $E = |A^4| + |A^{10} - (\operatorname{adj}(2\,A))^{10}|$

$= (-2)^4 + \frac{|A^{20} - A^{10}(\operatorname{adj} 2\,A)^{10}|}{|A|^{10}}$

$= 16 + \frac{|A^{20} - (A\operatorname{adj}(2\,A))^{10}|}{|A|^{10}}$

$= 16 + \frac{|A^{20} - 2^{10} I|}{2^{10}}$ (from (1))

Now, characteristic roots of A are 2 and -1 . So, characteristic roots of A^{20} are 2^{10} and 1 . Hence, $(A^{20} - 2^{10} I)(A^{20} - I) = 0$

$\Rightarrow |A^{20} - 2^{10} I| = 0 ($ as $A^{20} \neq I)$

$\Rightarrow E = 16$

87. (1)

$I = \int_0^{\sqrt{\pi/2}} ([x^2] + [-\cos x]) dx$

$= \int_0^1 0 dx + \int_1^{\sqrt{\pi/2}} dx + \int_0^{\sqrt{\pi/2}} (-1) dx$

$= \sqrt{\frac{\pi}{2}} - 1 - \sqrt{\frac{\pi}{2}} = -1$

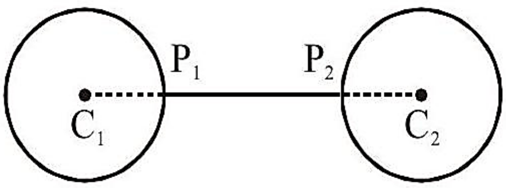
$\Rightarrow |I| = 1$

88. (1)

Given $C_1(5,5), r_1 = 3$ and $C_2(12,5), r_2 = 3$

Now, $C_1C_2 > r_1 + r_2$

Thus, $(P_1P_2)_{min} = 7 - 6 = 1$



89. (4)

Required plane is

$p_1 + \lambda p_2 = (2 + 3\lambda)x - (7 + 5\lambda)y$

$+(4 + 4\lambda)z - 3 + 11\lambda = 0;$

which is satisfied by $(-2,1,3)$.

Hence, $\lambda = \frac{1}{6}$

Thus, plane is $15x - 47y + 28z - 7 = 0$

So, $2a + b + c - 7 = 4$

90. (4)

$(2023 - 2)^{3762} = 2023k_1 + 2^{3762}$

$= 17k_2 + 2^{3762}$(as $2023 = 17 \times 17 \times 9$)

$= 17k_2 + 4 \times 16^{940}$

$= 17k_2 + 4 \times (17 - 1)^{940}$

$= 17k_2 + 4(17k_3 + 1)$

$= 17k + 4 \Rightarrow$ remainder $= 4$