1. (d) $v_0 = \sqrt{2gh}$

$v = e\sqrt{2gh} = \sqrt{2gh}$

$\Rightarrow e = 0.9$

$S = h + 2e^2 h + 2e^4 h +$

$$t = \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + \cdots \ldots \ldots$$

$v_{av} = \frac{s}{t} = 2.5 \text{ m/s}$

2. (b) $f = 4 + 3 + 3 = 10$

assuming non linear

$\beta = \frac{C_p}{C_v} = 1 + \frac{2}{f} = \frac{12}{10} = 1.2$

3. (d)

4. (d) Conceptual

5. (d) $P = h\rho g$

$$\beta = \frac{p}{\frac{\Delta V}{V}} = \frac{2 \times 10^3 \times 10^3 \times 9.8}{1.36 \times 10^{-2}}$$

$= 1.44 \times 10^9 \text{ N/m}^2$

6. (c) $T \propto R^{3/2}$

$\frac{24}{T} = \left(\frac{12R}{3R}\right)^{3/2} \Rightarrow T = 3\text{hr}$

7. (d) $\omega = 2\pi f$

$= 1.5 \times 10^3$

$A = \frac{6}{2} = 3 \text{ cm} = 0.03 \text{ m}$

8. (b)

9. (b) $v = v_0 + gt + Ft^2$

$\frac{ds}{dt} = v_0 + gt + Ft^2$

$\int ds = \int_0^1 (v_0 + gt + Ft^2)dt$

$$s = \left[v_0 t + \frac{gt^2}{2} + \frac{Ft^3}{3}\right]_0^1$$

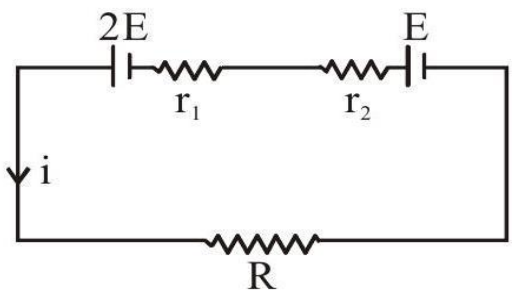
$s = v_0 + \frac{g}{2} + \frac{F}{3}$

10. (d) Band width $= 2f_m$

$\omega_m = 1.57 \times 10^8 = 2\pi f_m$

$BW = 2f_m = \frac{10^8}{2}$ Hz $= 50$MHz

11. (b)



$i = \frac{3E}{R + r_1 + r_2}$

$TPD = 2E - ir_1 = 0$

$2E = ir_1$

$2E = \frac{3E \times r_1}{R + r_1 + r_2}$

$2R + 2r_1 + 2r_2 = 3r_1$

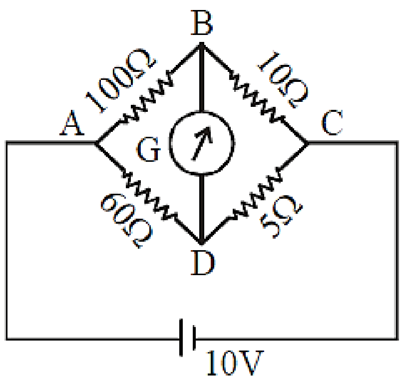
$R = \frac{r_1}{2} - r_2$

12. (b) $B = 2 \times B_{\text{st. wire}} + B_{\text{loop}}$

$B = 2 \times \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{2r}\left(\frac{\pi}{2\pi}\right)$

$B = \frac{\mu_0 i}{4\pi r}(2 + \pi)$

13. (c)



$$\frac{x-10}{100}+\frac{x-y}{15}+\frac{x-0}{10}=0$$

$53x-20y=30$

$$\frac{y-10}{60}+\frac{y-x}{15}+\frac{y-0}{5}=0$$

$17y-4x=10$

on solving (a) & (b)

$x=0.865$

$y=0.792$

$\Delta V=0.073R=15\Omega$

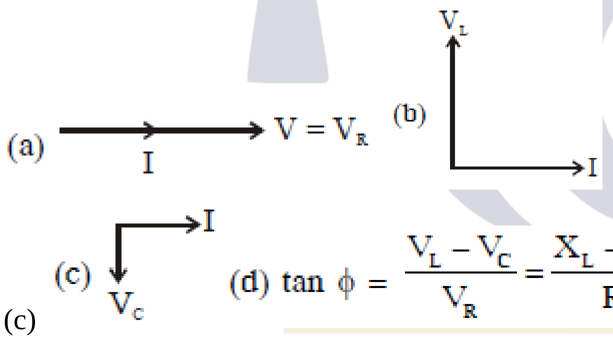
$i=4.87\text{ mA}$

14. (d) $A_1\omega_1=A_2\omega_2$

$$A_1\sqrt{\frac{k_1}{m}}=A_2\sqrt{\frac{k_2}{m}}$$

$$\frac{A_1}{A_2}=\sqrt{\frac{k_2}{k_1}}$$

15. (d)



(d) $\tan\phi=\frac{V_L-V_C}{V_R}=\frac{X_L-X_C}{R}$

(c)

16. (a) C comes to rest

V_{cm} of A& B $=\frac{v}{2}$

$\Rightarrow\frac{1}{2}\text{is }v_{ret}^2=\frac{1}{2}kx^2$

$$x=\sqrt{\frac{\mu\times v^2}{k}}=\sqrt{\frac{m}{2k}}v$$

17. (d)

18. (a) $\frac{1}{2}mv_1^2=hf_1-\phi$

$$\frac{1}{2}mv_2^2 = hf_2 - \phi$$

$$v_1^2 - v_2^2 = \frac{2\,h}{m}(f_1 - f_2)$$

19. (b) $X_L = \omega L$

$$i = \frac{v_0}{\omega L}$$

20. (c) $a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{5}{7} \times \frac{10}{2} = \frac{25}{7}$

$$t = \frac{2v_0}{a} = \frac{2 \times 1 \times 7}{25}$$

$$= 0.56$$

21. (3)

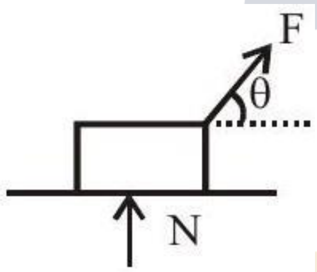
$$c\,\epsilon_0\,E^2 = \frac{100}{4\pi \times 3^2}$$

$$c\,\epsilon_0\left(\sqrt{\frac{x}{5}}E\right)^2 = \frac{60}{4\pi \times 3^2}$$

$$\Rightarrow \frac{x}{5} = \frac{3}{5}$$

$$\Rightarrow x = 3$$

22. (5)



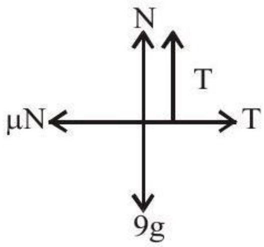
$$F\cos\theta = \mu N$$

$$F\sin\theta + N = mg$$

$$\Rightarrow F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

$$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}} = \frac{\frac{1}{\sqrt{3}} \times 10}{\frac{2}{\sqrt{3}}} = 5$$

23. (30)



$N + T = 90$

$T = \mu N = 0.5(90 - T)$

$1.5\ T = 45$

$T = 30$

24. (25) $4t_T = 100 \times \frac{4}{3}\pi r^3$

$= 100 \times \frac{4\pi}{3} \times \frac{3}{40\pi} \times 10^{-9} = 10^{-8}\ cm^3$

$t_T = 25 \times 10^{-10}$ cm

$= 25 \times 10^{-12}$ m

$t_0 = 0.01 t_T = 25 \times 10^{-14}$ m

$= 25$

25. (3) $F = \frac{-dU}{dr} = -4U_0 r^3 = \frac{mv^2}{r}$

$mv^2 = 4U_0 r^4\ v \propto r^2$

$mvr = \frac{nh}{2\pi}$

$r^3 \propto n$

$r \propto n1/3$

$= 3$

26. (640) $\phi = E_x A \Rightarrow \frac{2}{5} \times 4 \times 10^3 \times 0.4 = 640$

27. (4) C.O.M of quarter disc is at $\frac{4a}{3\pi}, \frac{4a}{3\pi} = 4$

28. (30) $\lambda_m = \frac{\lambda_a}{\mu} \Rightarrow \mu = \frac{3}{2}$

$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$

$\frac{3}{2 \times 10} + \frac{1}{15} = \frac{\frac{3}{2} - 1}{R}$

$R = \frac{30}{13}$

$= 30$

29. (16)

$20 = (C_1 + C_2)V \Rightarrow V = 2$ volt.

$Q_2 = C_2 V = 16\mu C$

$= 16$

30. (6) $J_c = \frac{E}{\rho} = \frac{V}{\rho d}$

$J_d = \frac{1}{A}\frac{dq}{dt}$

$= \frac{C}{A}\frac{dV_c}{dt}$

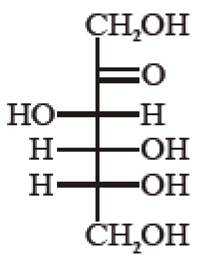
$= \frac{\in dV_c}{d}$

$\Rightarrow \frac{V_0 \sin 2\pi ft}{\rho d} = 10^x \times \frac{80\varepsilon_0}{d} V_0(2\pi f)\cos 2\pi ft$

$\tan\left(2\pi \times \frac{900}{800}\right) = 10^x \times \frac{40}{9 \times 10^9} \times 900$

$= x = 6$

31. (b) Fructose is a ketohexose.



32. (d) $Li - Mg, B - Si, Be - Al$ show diagonal relationship but Li and Na do not show diagonal relationship as both belongs to same group and not placed diagonally.

33. (a) Cation exchanger contains $-SO_3H$ or $-COOH$ groups while anion exchanger contains basic groups like $-NH_2$.

34. (d)

Ore	Formula
(A) Haematite	Fe_2O_3
(B) Bauxite	$Al_2O_3 \cdot xH_2O$
(C) Magnetite	Fe_3O_4
(D) Malachite	$CuCO_3 \cdot Cu(OH)_2$

35. (c) Ambident nucleophile

(A) KCN&AgCN

(C) $AgNO_2$&KNO_2

36. (a) N_2O and NO are neutral oxides of nitrogen NO_2 and N_2O_3 are acidic oxides.

37. (a)

Complex	Type of Isomerism
(A) $[Co(NH_3)_6][Cr(CN)_6]$	Co-ordination isomerism
(B) $[Co(NH_3)_3(NO_2)_3]$	Linkage isomerism
(C) $[Cr(H_2O)_6]Cl_3$	Solvate isomerism
(D) cis- $[CrCl_2(ox)_2]^{3-}$	Optical isomerism

38. (a) Primary amines react with Para Toluene sulfonyl chloride to form a precipitate that is soluble in NaOH. Secondary amines reacts with para toluene sulfonyl chloride to give a precipitate that is insoluble in NaOH. Tertiary amines do not react with para toluen.

39. (a) $Cr(Z = 24)$

[Ar] $4s^1 3d^5$ Cr shows common oxidation states starting from +2 to +6 .

40. (b)

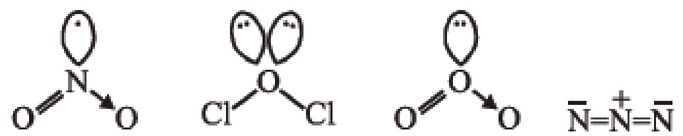
Artificial sweetener: Sucralose
Antiseptic: Bithional
Preservative: Sodium Benzoate
Glyceryl ester of stearic acid: Sodium steasate

41. (c) Alkane are very less reactive, tertiary hydrogen can oxidise to alcohal with $KMnO_4$.

2-methyl-butane

42. (b) Kjeldahl method is not applicable to compounds containing nitrogen in nitrogroup, Azo groups and nitrogen present in the ring (e.g Pyridine) as nitrogen of these compounds does not change to Ammonium sulphate under these conditions.

43. (d)



Bent shape Bent shape Bent shape Linear

44. (c) Informative

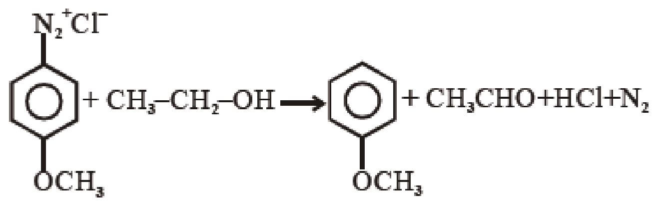
OR

$$C_{12}H_{22}O_{11} + H_2O \xrightarrow{\text{Invertase}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

$$C_6H_{12}O_6 \xrightarrow{\text{Zymase}} 2C_2H_5OH + 2CO_2$$

45. (c)

46. (a)

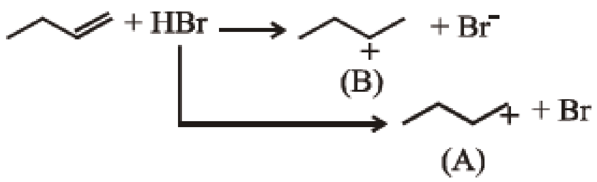


Anisole

47. (b) To coagulate negative sol, cation with higher charge has higher coagulation value.

48. (d) The process in which nutrient enriched water bodies support a dense plant population which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as eutrophication.

49. (a)



This is more stable due to secondary cation formation and formed with faster rate due to low activation energy.

50. (a)

(A) Water $\xrightarrow{0^{\circ}\mathrm{C}}$ ice; $\Delta S = -$ ve

(B) Water $\xrightarrow{-10^{\circ}\mathrm{C}}$ ice; $\Delta S = -$ve

(C) N_2 (g) $+3H_2$(g) $\rightarrow 2NH_3$(g); $\Delta S = -$ve

(D) Adsorption; $\Delta S = -$ve

(E) NaCl (s) $\rightarrow Na^+$(aq) $+Cl^-$(aq) ; $\Delta S = +$ve

51. (57) $\kappa = \frac{1}{R} \cdot G^*$

For same conductivity cell, G^* is constant and hence $\kappa . R. =$ constant.

$\therefore\ 0.14 \times 4.19 = \kappa \times 1.03$

or, κ of HCl solution $= \frac{0.14 \times 4.19}{1.03}$

$= 0.5695 \mathrm{Sm}^{-1}$

$= 56.95 \times 10^{-2} \mathrm{Sm}^{-1} \approx 57 \times 10^{-2} \mathrm{Sm}^{-1}$

52. (6) $Fe^{3+} + 3\,K^+ + 3C_2O_4{}^{2-} \rightarrow K_3[Fe(C_2O_4)_3]$

Secondary valency of Fe in ' A ' is 6.

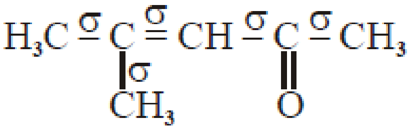
53. (27) Reaction: $2\,A + B_2 \longrightarrow 2AB$

As the reaction is elementary, the rate of reaction is

$r = K \cdot [A]^2 [\,B_2]$

on reducing the volume by a factor of 3 , the concentrations of A and B_2 will become 3 times and hence, the rate becomes $3^2 \times 3 = 27$ times of initial rate.

54. (5) Mesityle oxide

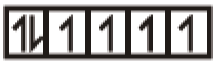


$\therefore \quad C \underset{\sigma}{-} C = 5$

55. (85)

56. (49)

$Fe \rightarrow [Ar]4\,s^2 3\,d^6$ |



Number of unpaired $e^- = 4$

$\mu = \sqrt{4(4+2)}$ B.M.

$\mu = \sqrt{24}$ B.M.

$\mu = 4.89$ B.M.

$\mu = 48.9 \times 10^{-1}$ B.M.

Nearest integer value will be 49

57. (1)

$PV = nRT$

$$1.0 \times \frac{20}{1000} = \frac{N}{6.023 \times 10^{23}} \times 0.083 \times 273$$

$\therefore$ Number of Cl_2 molecules, $N = 5.3 \times 10^{20}$

Hence, Number of Cl-atoms $= 1.06 \times 10^{21}$

$\approx 1 \times 10^{21}$

58. (5) 1 mole $KBr(= 119gm)$ have $\frac{10^{-5}}{100}$ moles $SrBr_2$ and hence, 10^{-7} moles cation vacancy (as $1Sr^{2+}$ will result 1 cation vacancy) $\therefore$ Required number of cation vacancies

$$= \frac{10^{-7} \times 6.023 \times 10^{23}}{119} = 5.06 \times 10^{14} \simeq 5 \times 10^{14}$$

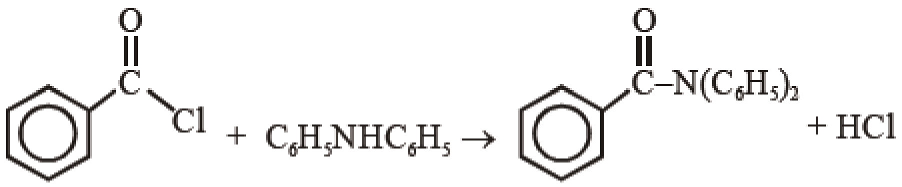
59. (354) $N_2O_4(g) \rightleftharpoons 2NO_2(g); \Delta n_g = 2 - 1 = 1$

Now, $K_p = K_c \cdot (RT)^{\Delta ng^g}$

or, $600.1 = 20.4 \times (0.0831 \times T)$

$\therefore T = 353.99\ K = 354\ K$

60. (77)



$$\begin{array}{lll} 1\ \text{mole} & 1\ \text{mole} & 1\ \text{mole} \\ = 140.5\text{gm} & = 169\text{gm} & = 273\text{gm} \end{array}$$

$\therefore 0.140\text{gm}\ \dfrac{169}{140.5} \times 0.140$

L.R. $= 0.168\text{gm} < 0.388\text{gm}$

excess

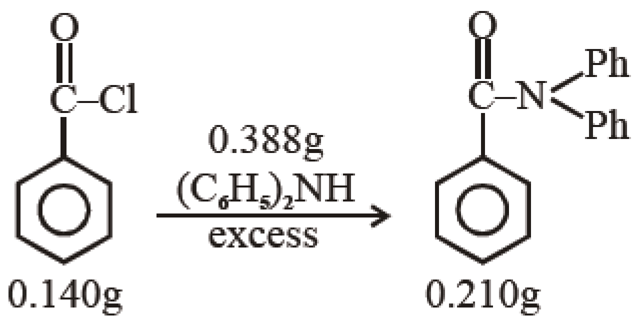
$\therefore$ Theoretical amount of given product formed

$$= \frac{273}{140.5} \times 0.140 = 0.272\text{gm}$$

But its actual amount formed is 0.210gm. Hence, the percentage yield of product.

$$= \frac{0.210}{0.272} \times 100 = 77.20 \approx 77$$

OR



Mole of Ph – CoCl $= \frac{0.140}{140} = 10^{-3}$ mol

Mole of Ph $- \overset{C}{O} - N(Ph)_2$, that should be obtained by mol-mol analysis $= 10^{-3}$ mol.

Theoritical mass of product $= 10^{-3} \times 273 = 273 \times 10^{-3}$ g

Observed mass of product $= 210 \times 10^{-3}$ g

%yield of product $= \frac{210\times10^{-3}}{273\times10^{-3}} \times 100 = 76.9\% = 77$

61. (b)

$f(x) = e^{-x}\sin x$

Now, $F(x)=\int_0^x f(t)dt \;\Rightarrow F'(x)=f(x)$

$$I=\int_0^1 (F'(x)+f(x))e^x dx=\int_0^1 (f(x)+f(x))\cdot e^x dx$$

$$=2\int_0^1 f(x)\cdot e^x dx=2\int_0^1 e^{-x}\sin x\cdot e^x dx$$

$$=2\int_0^1 \sin x dx$$

$$=2(1-\cos 1)$$

$$I=2\left\{1-\left(1-\frac{1}{\underline{2}}+\frac{1}{44}+\frac{1}{\underline{6}}+\frac{1}{\underline{8}}\dots\dots\dots\right)\right\}$$

$$I=1-\frac{2}{44}+\frac{2}{\square 6}-\frac{2}{\underline{9}}+\cdots\dots$$

$$1-\frac{2}{4}<\text{I}<1-\frac{2}{4}+\frac{2}{\underline{6}}$$

$$\frac{11}{12}<\text{I}<\frac{331}{360}$$

$$\Rightarrow \text{I}\in\left[\frac{11}{12},\frac{331}{360}\right]$$

$$\Rightarrow \text{I}\in\left[\frac{330}{360},\frac{331}{360}\right]$$

62. (a)

Let $I=\int_0^{10}\frac{[\sin 2\pi x]}{e^{x-[x]}}dx=\int_0^{10}\frac{[\sin 2\pi x]}{e^{\{x\}}}dx$

Function $\text{f(x)}=\frac{[\sin 2\pi x]}{e^{\{x\}}}$ is periodic with period '1'

Therefore

$$\text{I}=10\int_0^1\frac{[\sin 2\pi x]}{e^{\{x\}}}dx$$

$$=10\int_0^1\frac{[\sin 2\pi x]}{e^x}dx$$

$$=10\left(\int_0^{1/2}\frac{[\sin 2\pi x]}{e^x}dx+\int_{1/2}^1\frac{[\sin 2\pi x]}{e^x}dx\right)$$

$$=10\left(0+\int_{1/2}^1\frac{(-1)}{e^x}dx\right)$$

$$=-10\int_{1/2}^1 e^{-x}dx$$

$$=10\left(e^{-1}-e^{-1/2}\right)$$

Now,

$10 \cdot e^{-1} - 10 \cdot e^{-1/2} = \alpha e^{-1} + \beta e^{-1/2} + \gamma$ (given)

$\Rightarrow \alpha = 10, \beta = -10, \gamma = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$ Ans. (a)

63. (b) $\cos x(3\sin x + \cos x + 3)dy$

$= (1 + y\sin x(3\sin x + \cos x + 3))dx$

$$\frac{dy}{dx} - (\tan x)y = \frac{1}{(3\sin x + \cos x + 3)\cos x}$$

I.F. $= e^{\int -\tan x dx} = e^{\ell n|\cos x|} = |\cos x|$

$= \cos x \forall x \in \left[0, \frac{\pi}{2}\right)$

Solution of D.E.

$$y(\cos x) = \int (\cos x) \cdot \frac{1}{\cos x(3\sin x + \cos x + 3)} dx + C$$

$$y(\cos x) = \int \frac{dx}{3\sin x + \cos x + 3} dx + C$$

$$y(\cos x) = \int \frac{\left(\sec^2 \frac{x}{2}\right)}{2\tan^2 \frac{x}{2} + 6\tan \frac{x}{2} + 4} dx + C$$

Now

Let $I_1 = \int \frac{\left(\sec^2 \frac{x}{2}\right)}{2\left(\tan^2 \frac{x}{2} + 3\tan \frac{x}{2} + 2\right)} dx + C$

Put $\tan \frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2 \frac{x}{2} dx = dt$

$$I_1 = \int \frac{dt}{t^3 + 3t + 2} = \int \frac{dt}{(t+2)(t+1)}$$

$$= \int \left(\frac{1}{t+1} - \frac{1}{t+2}\right) dt$$

$$= \ell n\left|\left(\frac{t+1}{t+2}\right)\right| = \ell n\left|\left(\frac{\tan \frac{x}{2} + 1}{\tan \frac{x}{2} + 2}\right)\right|$$

So solution of D.E.

$$y(\cos x) = \ell n\left|\frac{1 + \tan \frac{x}{2}}{2 + \tan \frac{x}{2}}\right| + C$$

$\Rightarrow y(\cos x) = \ell n\left(\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right) + C$ for $0 \le x < \frac{\pi}{2}$

Now, it is given $y(0) = 0$

$\Rightarrow 0 = \ell n\left(\frac{1}{2}\right) + C \Rightarrow C = \ell n2$

$$\Rightarrow y(\cos x) = \ell n\left(\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right) + \ell n 2$$

For $x = \frac{\pi}{3}$

$$y\left(\frac{1}{2}\right) = \ell n\left(\frac{1+\frac{1}{\sqrt{3}}}{2+\frac{1}{\sqrt{3}}}\right) + \ell n 2$$

$$y = 2\ell n\left(\frac{2\sqrt{3}+10}{11}\right)$$

64. (d) $\sum_{r=0}^{6} {}^6C_r \cdot {}^6C_{6-r}$

$= {}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + \cdots \ldots + {}^6C_6 \cdot {}^6C_0$

Now,

$$(1+x)^6(1+x)^6$$
$$= ({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + \cdots \ldots + {}^6C_6x^6)$$

$$({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + \cdots \ldots + {}^6C_6x^6)$$

Comparing coefficeint of x^6 both sides

$${}^6C_0 \cdot {}^6C_6 + {}^6C_1 + {}^6C_5 + \cdots \ldots + {}^6C_6 \cdot {}^6C_0 = {}^{12}C_6$$
$$= 924$$

65. (a) We know that

$$r \le [r] < r+1$$
$$\text{and } 2r \le [2r] < 2r+1$$
$$3r \le [3r] < 3r+1$$
$$nr \le [nr] < nr+1$$
$$r + 2r + \cdots . + nr$$
$$\le [r] + [2r] + \cdots + [nr] < (r + 2r + \cdots . + nr) + n$$
$$\frac{\frac{n(n+1)}{2}\cdot r}{n^2} \le \frac{[r]+[2r]+\cdots\ldots+[nr]}{n^2} < \frac{\frac{n(n+1)}{2}r + n}{n^2}$$

Now,

$$\lim_{n\to\infty} \frac{n(n+1)\cdot r}{2\cdot n^2} = \frac{r}{2}$$

and $\lim_{n\to\infty} \frac{\frac{n(n+1)r}{2}+n}{n^2} = \frac{r}{2}$

So, by Sandwich Theorem, we can conclude that

$$\lim_{n\to\infty} \frac{[r]+[2r]+\cdots\ldots+[nr]}{n^2} = \frac{r}{2}$$

66. (b) Given equation

$$\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$$

Now, $\sin^{-1}\left[x^2+\frac{1}{3}\right]$ is defined if $-1\le x^2+\frac{1}{3}<2 \Rightarrow \frac{-4}{3}\le x^2<\frac{5}{3}$

$$\Rightarrow 0\le x^2<\frac{5}{3}$$

and $\cos^{-1}\left[x^2-\frac{2}{3}\right]$ is defined if

$$-1\le x^2-\frac{2}{3}<2 \Rightarrow \frac{-1}{3}\le x^2<\frac{8}{3}$$

$$\Rightarrow 0\le x^2<\frac{8}{3}$$

So, form (1) and (2) we can conclude

$$0\le x^2<\frac{5}{3}$$

Case - I if $0\le x^2<\frac{2}{3}$

$$\sin^{-1}(0)+\cos^{-1}(-1)=x^2$$

$$\Rightarrow x+\pi=x^2$$

$$\Rightarrow x^2=\pi$$

but $\pi\notin\left[0,\frac{2}{3}\right)$

$\Rightarrow$ No value of ' x '

Case - II if $\frac{2}{3}\le x^2<\frac{5}{3}$

$$\sin^{-1}(1)+\cos^{-1}(0)=x^2$$

$$\Rightarrow \frac{\pi}{2}+\frac{\pi}{2}=x^2$$

$$\Rightarrow x^2=\pi$$

but $\pi\notin\left[\frac{2}{3},\frac{5}{3}\right)$

$\Rightarrow$ No value of ' x '

So, number of solutions of the equation is zero.

67. (d)

$\underset{\text{odd place}}{1}\ \underset{\text{even place}}{0}\ \underset{\text{odd place}}{0}\ \underset{\text{even place}}{1}$

or $\underset{\text{even place}}{1}\ \underset{\text{odd place}}{0}\ \underset{\text{even place}}{0}\ \underset{\text{odd place}}{1}$

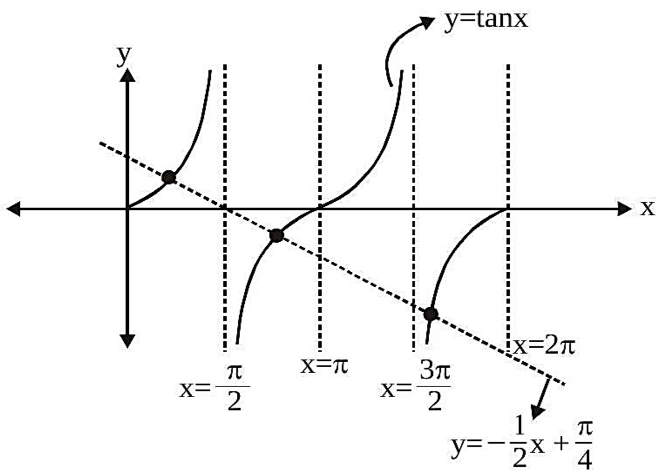
$$\Rightarrow \left(\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}\right)+\left(\frac{2}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\right)$$

$$\Rightarrow \frac{1}{9}$$

68. (a) $x + 2\tan x = \frac{\pi}{2}$

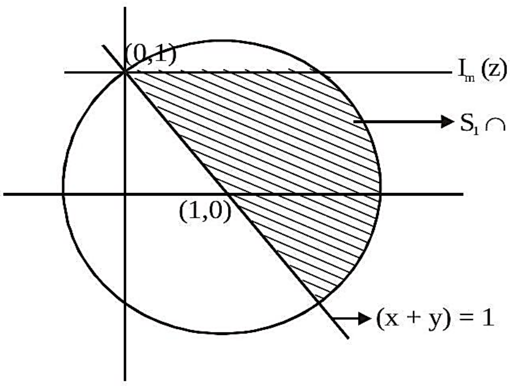
$\Rightarrow 2\tan x = \frac{\pi}{2} - x$

$\Rightarrow \tan x = -\frac{1}{2}x + \frac{\pi}{4}$



Number of soluitons of the given eauation is '3'.

69. (c) For $|z-1| \le \sqrt{2}$, z lies on and inside the circle of radius $\sqrt{2}$ units and centre (1,0).



For S_2

Let $z = x + iy$

Now, $(1-i)(z) = (1-i)(x+iy)$

$\mathrm{Re}((1-i)z) = x + y$

$\Rightarrow x + y \geq 1$

$\Rightarrow S_1 \cap S_2 \cap S_3$ has infinity many elements

70. (c) $\frac{dy}{dx} - \frac{y}{2x} = \frac{x^{9/4}}{x^{5/4}(x^{3/4}+1)}$

$$\mathrm{IF} = e^{-\int \frac{dx}{2d}} = e^{-\frac{1}{2}\ln x} = \frac{1}{x^{1/2}}$$

$$y \cdot x^{-1/2} = \int \frac{x^{9/4} \cdot x^{-1/2}}{x^{5/4}(x^{3/4}+1)} dx$$

$$\int \frac{x^{1/2}}{(x^{3/4}+1)} dx$$

$x = t^4 \Rightarrow dx = 4t^3 dt$

$$\int \frac{t^2 \cdot 4t^3 dt}{(t^3+1)}$$

$$4\int \frac{t^2(t^3+1-1)}{(t^3+1)} dt$$

$$4\int t^2 dt - 4\int \frac{t^2}{t^3+1} dt$$

$$\frac{4t^3}{3} - \frac{4}{3}\ln(t^3+1) + C$$

$$yx^{-1/2} = \frac{4x^{3/4}}{3} - \frac{4}{3}\ln\left(x^{3/4}+1\right) + C$$

$$1 - \frac{4}{3}\log_e 2 = \frac{4}{3} - \frac{4}{3}\log_e 2 + C$$

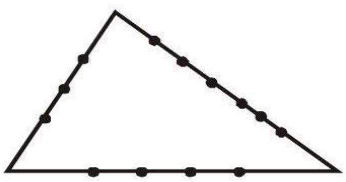
$$\Rightarrow C = -\frac{1}{3}$$

$$y = \frac{4}{3}x^{5/4} - \frac{4}{3}\sqrt{x}\ln\left(x^{3/4}+1\right) - \frac{\sqrt{x}}{3}$$

$$y(16) = \frac{4}{3} \times 32 - \frac{4}{3} \times 4\ln 9 - \frac{4}{3}$$

$$= \frac{124}{3} - \frac{32}{3}\ln 3 = 4\left(\frac{31}{3} - \frac{8}{3}\ln 3\right)$$

71. (333)



Total Number of triangles formed

$= {}^{14}C_3 - {}^{3}C_3 - {}^{5}C_3 - {}^{6}C_3$

$= 333$

72. (a)

$$\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$$

$R_2 \to R_1 + R_3 - 2R_2$

$$\Rightarrow \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 0 & k - 6\sqrt{2} & 0 \\ 5 & k & z \end{vmatrix} = 0$$

$\Rightarrow (k - 6\sqrt{2})(3z - 5x) = 0$

if $3z - 5x = 0 \Rightarrow 3(x + 2\text{ d}) - 5x = 0$

$\Rightarrow x = 3\text{ d}$ (Not possible)

$\Rightarrow k = 6\sqrt{2} \Rightarrow k^2 = 72$

73. (b) $\overrightarrow{OP} \perp \overrightarrow{OQ}$

$\Rightarrow -x + 2y - 3x = 0$

$\Rightarrow y = 2x$

$|\overrightarrow{PQ}|^2 = 20$

$\Rightarrow (x + 1)^2 + (y - 2)^2 + (1 + 3x)^2 = 20$

$\Rightarrow x = 1$

$\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}$ are coplanar.

$$\Rightarrow \begin{vmatrix} x & y & -1 \\ -1 & 2 & 3x \\ 3 & z & -7 \end{vmatrix} = 0$$

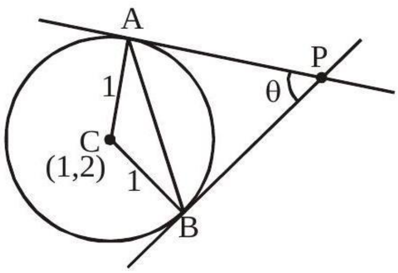
$$\Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \\ 3 & z & -7 \end{vmatrix} = 0$$

$\Rightarrow 1(-14 - 3z) - 2(7 - 9) - 1(-z - 6) = 0$

$\Rightarrow z = -2$

$\therefore x^2 + y^2 + z^2 = 1 + 4 + 4 = 9$

74. (b)



$\tan\theta = \frac{12}{5}$

$\mathrm{PA} = \cot\frac{\theta}{2}$

$\therefore$ area of $\triangle$ PAB $= \frac{1}{2}(\mathrm{PA})^2\sin\theta = \frac{1}{2}\cot^2\frac{\theta}{2}\sin\theta$

$= \frac{1}{2}\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\sin\theta$

$= \frac{1}{2}\left(\frac{1+\frac{5}{13}}{1-\frac{5}{13}}\right)\left(\frac{12}{13}\right) = \frac{1}{2}\frac{18}{18}\times\frac{2}{13} = \frac{27}{26}$

area of $\triangle$ CAB $= \frac{1}{2}\sin\theta = \frac{1}{2}\left(\frac{12}{13}\right) = \frac{6}{13}$

$\therefore \frac{\text{area of }\triangle\text{PAB}}{\text{area of }\triangle\text{CAB}} = \frac{9}{4}$

75. (b)

$$f(x) = \begin{cases} -x\left(2-\sin\left(\frac{1}{x}\right)\right) & x<0 \\ 0 & x=0 \\ x\left(2-\sin\left(\frac{1}{x}\right)\right) & \end{cases}$$

$$f'(x) = \begin{cases} -\left(2-\sin\frac{1}{x}\right) - x\left(-\cos\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right)\right) & x<0 \\ \left(2-\sin\frac{1}{x}\right) + x\left(-\cos\frac{1}{x}\left(-\frac{1}{x^2}\right)\right) & x>0 \end{cases}$$

$$f'(x) = \begin{cases} -2+\sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x} x<0 \\ 2-\sin\frac{1}{x} + \frac{1}{x}\cos\frac{1}{x}\ x>0 \end{cases}$$

$f'(x)$ is an oscillating function which is non-monotonic in $(-\infty, 0)\cup(0,\infty)$.

76. (b) Tangent to parabola

$2y = 2(x+6) - 20$

$\Rightarrow y = x - 4$

Condition of tangency for ellipse.

$16 = 2(1)^2 + b$

$\Rightarrow b = 14$

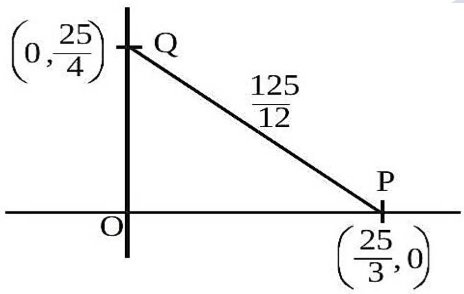
77. (a) $\lim_{\theta\to 0} \frac{\tan(\pi(1-\sin^2\theta))}{\sin(2\pi\sin^2\theta)}$

$= \lim_{\theta\to 0} \frac{-\tan(\pi\sin^2\theta)}{\sin(2\pi\sin^2\theta)}$

$= \lim_{\theta\to 0} -\left(\frac{\tan(\pi\sin^2\theta)}{\pi\sin^2\theta}\right)\left(\frac{2\pi\sin^2\theta}{\sin(2\pi\sin^2\theta)}\right) \times \frac{1}{2}$

$= \frac{-1}{2}$

78. (c) Tangent to circle $3x + 4y = 25$



$OP + OQ + OR = 25$

Incentre $= \left(\frac{\frac{25}{4}\times\frac{25}{3}}{25}, \frac{\frac{25}{4}\times\frac{25}{3}}{25}\right)$

$= \left(\frac{25}{12}, \frac{25}{12}\right)$

$\therefore r^2 = 2\left(\frac{25}{12}\right)^2 = 2 \times \frac{625}{144} = \frac{625}{72}$

79. (a) $(p \wedge q) \longrightarrow (p \rightarrow q)$

$= \sim (p \wedge q) \vee (\sim p \vee q)$

$= (\sim p \vee \sim q) \vee (\sim p \vee q)$

$= \sim p \vee (\sim q \vee q)$

$= \sim p \vee t$

$= t$

Option (b)

$(p \wedge q) \wedge (p \vee q) = (p \wedge q)$ (Not a tautology)

Option (c)

$(p \wedge q) \vee (p \rightarrow q)$

$= (p \wedge q) \vee (\sim p \vee q)$

$= \sim p \vee q$

(Not a tautology)
Option (d)

$(p \wedge q) \wedge (p \rightarrow q)$

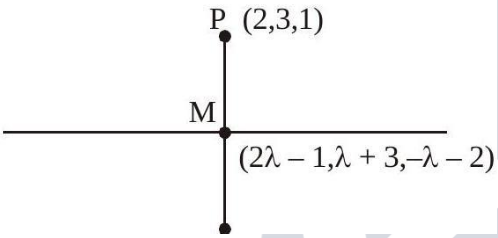
$= (p \wedge q) \wedge (\sim p \vee q)$

$= p \wedge q$

(Not a tautology)

Option (a)

80. (b) Line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$



$\overrightarrow{PM} = (2\lambda - 3, \lambda, -\lambda - 3)$

$\overrightarrow{PM} \perp (2\hat{i} + \hat{j} - \hat{k})$

$4\lambda - 6 + \lambda + \lambda + 3 = 0 \Rightarrow \lambda = \frac{1}{2}$

$\therefore M \equiv \left(0, \frac{7}{2}, \frac{-5}{2}\right)$

$\therefore$ Reflection $(-2, 4, -6)$

Plane : $\begin{vmatrix} x-2 & y-1 & z+1 \\ 3 & -2 & 1 \\ 4 & -3 & 5 \end{vmatrix} = 0$

$\Rightarrow (x-2)(-10+3) - (y-1)(15-4) + (z+1)(-1) = 0$

$\Rightarrow -7x + 14 - 11y + 11 - z - 1 = 0$

$\Rightarrow 7x + 11y + z = 24$

$\therefore \alpha = 7, \beta = 11, \gamma = 1$

$\alpha + \beta + \gamma = 19$

81. (2) $2\log_{10}(4^x-2)=1+\log_{10}\left(4^x+\frac{18}{5}\right)$

$(4^x-2)^2=10\left(4^x+\frac{18}{5}\right)$

$(4^x)^2+4-4(4^x)-32=0$

$(4^x-16)(4^x+2)=0$

$4^x=16$

$x=2$

$\begin{vmatrix}3 & 1 & 4\\1 & 0 & 2\\2 & 1 & 0\end{vmatrix}=3(-2)-1(0-4)+4(a)$

$=-6+4+4=2$

82. (5) $f:[-1,1]\to R$

$f(x)=ax^2+bx+c$

$f(-1)=a-b+c=2$

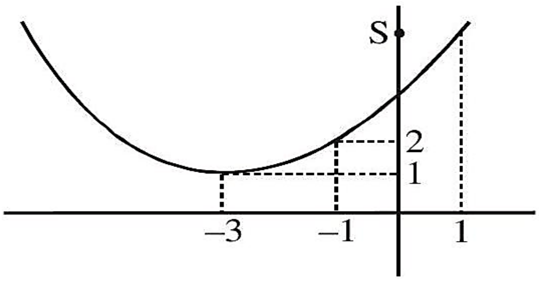
$f'(-1)=-2a+b=1$

$f''(x)=2a$

$\Rightarrow$ Max. value of $f''(x)=2a=\frac{1}{2}$

$\Rightarrow a=\frac{1}{4};\ b=\frac{3}{2};\ c=\frac{13}{4}$

$\therefore\ f(x)=\frac{x^2}{4}+\frac{3}{2}x+\frac{13}{4}$



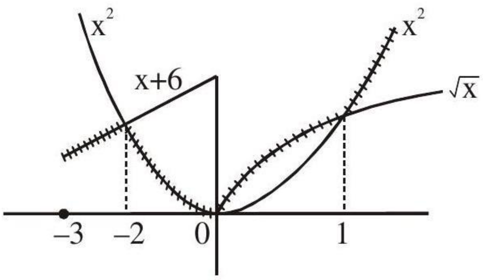
For, $x\in[-1,1]\Rightarrow 2\le f(x)\le 5$

$\therefore$ Least value of α is 5

83. (41)

$f:[-3,1]\to R$

$f(x)=\begin{cases}\min\{(x+6),x^2\} & ,-3\le x\le 0\\ \max\{\sqrt{x},x^2\} & ,0\le x\le 1\end{cases}$



area bounded by $y = f(x)$ and x-axis

$= \int_{-3}^{-2}(x+6)dx + \int_{-2}^{0} x^2 dx + \int_{0}^{1}\sqrt{x}dx$

$A = \frac{41}{6}$

$6\text{ A} = 41$

84. (144) Since ortho-centre and circumcentre both lies on y-axis

$\Rightarrow$ Centroid also lies on y-axis

$\Rightarrow \Sigma\cos\alpha = 0$

$\cos\alpha + \cos\beta + \cos\gamma = 0$

$\Rightarrow \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$

$\therefore \frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}$

$= \frac{4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3(\cos\alpha + \cos\beta + \cos\gamma)}{\cos\alpha\cos\beta\cos\gamma}$

$= 12$

85. (68.00) Let number be $a_1, a_2, a_3, \ldots \ldots . a_{2n}, b_1, b_2, b_3 \ldots . b_n$

$\sigma^2 = \frac{\sum a^2 + \sum b^2}{3n} - (5)^2$

$\Rightarrow \sum a^2 + \sum b^2 = 87n$

Now, distribution becomes

$a_1 + 1, a_2 + 1, a_3 + 1, \ldots \ldots a_{2n} + 1,\ b_1 - 1,\ b_2 - 1 \ldots . b_n - 1$

Variance

$= \frac{\sum(a+1)^2 + \sum(b-1)^2}{3n} - \left(\frac{12n + 2n + 3n - n}{3n}\right)^2$

$= \frac{(\sum a^2 + 2n + 2\sum a) + (\sum b^2 + n - 2\sum b)}{3n}$

$= \frac{(\sum a^2 + 2n + 2\sum a) + (\sum b^2 + n - 2\sum b)}{3n} - \left(\frac{16}{3}\right)^2$

$$= \frac{87n + 3n + 2(12n) - 2(3n)}{3n} - \left(\frac{16}{3}\right)^2$$

$$\Rightarrow \mathrm{k} = \frac{108}{3} - \left(\frac{16}{5}\right)^2$$

$$\Rightarrow 9\mathrm{k} = 3(108) - (16)^2 = 324 - 256 = 68$$

86. (4)

$$T_{r+1} = {}^{n}C_r(x)^{n-r}\left(\frac{a}{x^2}\right)^{\mathrm{r}}$$

$$= {}^{n}C_r a^r x^{n-3r}$$

$${}^{n}\mathrm{C}_2\mathrm{a}^2 : {}^{\mathrm{n}}\mathrm{C}_3\mathrm{a}^3 : {}^{\mathrm{n}}\mathrm{C}_4\mathrm{a}^4 = 12:8:3$$

After solving

$$\mathrm{n} = 6, \mathrm{a} = \frac{1}{2}$$

For term independent of ' x ' $\Rightarrow \mathrm{n} = 3\mathrm{r}$

$\mathrm{r} = 2$

$\therefore$ Coefficient is ${}^{6}\mathrm{C}_2\left(\frac{1}{2}\right)^2 = \frac{15}{4}$

Nearest integer is 4.

87. (2020)

$$\mathrm{A} = \begin{bmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{bmatrix}, \mathrm{B} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$\mathrm{AB} = \mathrm{B}$

$\Rightarrow (\mathrm{A} - \mathrm{I})\mathrm{B} = \mathrm{O}$

$\Rightarrow |\mathrm{A} - \mathrm{I}| = 0$, since $\mathrm{B} \neq \mathrm{O}$

$$\begin{vmatrix} \mathrm{a} - 1) & \mathrm{b} \\ \mathrm{c} & (\mathrm{d} - 1) \end{vmatrix} = 0$$

$\mathrm{ad} - \mathrm{bc} = 2020$

88. (486) Let $\vec{\mathrm{x}} = \lambda\vec{\mathrm{a}} + \mu\vec{\mathrm{b}}$ (λ and μ are scalars)

$$\vec{x} = \hat{\imath}(2\lambda + \mu) + \hat{\jmath}(2\mu - \lambda) + \hat{k}(\lambda - \mu)$$

Since $\vec{x} \cdot (3\hat{\imath} + 2\hat{\jmath} - \hat{k}) = 0$

$3\lambda + 8\mu = 0$

Also Projection of $\vec{x}$ on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$

$$\frac{\vec{x}\cdot\vec{a}}{|\vec{a}|}=\frac{17\sqrt{6}}{2}$$

$6\lambda-\mu=51$

From (a) and (b)

$\lambda=8, \mu=-3$

$\vec{x}=13\hat{i}-14\hat{j}+11\hat{k}$

$|\vec{x}|^2=486$

89. (1)
90. (0) Let point P is (α, β, γ)

$$\left(\frac{\alpha+\beta+\gamma}{\sqrt{3}}\right)^2+\left(\frac{\ell\alpha-n\gamma}{\sqrt{\ell^2+n^2}}\right)^2+\left(\frac{\alpha-2\beta+\gamma}{\sqrt{6}}\right)^2=9$$

Locus is

$$\frac{(x+y+z)^2}{3}+\frac{(\ell x-nz)^2}{\ell^2+n^2}+\frac{(x-2y+z)^2}{6}=9$$

$$x^2\left(\frac{1}{2}+\frac{\ell^2}{\ell^2+n^2}\right)+y^2+z^2\left(\frac{1}{2}+\frac{n^2}{\ell^2+n^2}\right)+2zx\left(\frac{1}{2}-\frac{\ell n}{\ell^2+n^2}\right)-9=0$$

Since its given that $x^2+y^2+z^2=9$

After solving $\ell=n$