PHYSICS
SECTION-A

1. **(d)** Slope of potential energy v/s position curve gives negative of force.
$F_{BC} > F_{AB} > F_{DE} > F_{CD}$

2. **(c)** Water flow rate $= 5\ell/\text{min} = \frac{5}{60}$ kg/s
Power of heater $= \frac{dm}{dt} S\Delta T = \frac{1}{12} \times 4200 \times 60$ W
Let rate of consumption of gas be xg/s.
$$x \times 5.0 \times 10^4 = \frac{1}{12} \times 4200 \times 60$$
$$\Rightarrow x = 4200 \times 10^{-4} = 0.42 \text{ g/s}$$

3. **(c)** Area of the loop $= 1 \text{ m}^2$
$B = \sin(100t)$
$\therefore \phi = BA = \sin(100t)$
$$\therefore \frac{d\phi}{dt} = 100\cos(100t)$$
$$\therefore P = \frac{V^2}{R} = \frac{10^4 \cos^2(100t)}{100}$$
$\therefore$ Thermal energy dissipated in 1 time period
$$= \int_0^T P dt = \int_0^T 100\cos^2(100t) dt$$
$$T = \frac{2\pi}{100} = \frac{\pi}{50} \text{sec}$$
$$T = \frac{2\pi}{100} = \frac{\pi}{50} \text{sec}$$
$$\therefore Q = 100 \int_0^{\pi/50} \cos^2(100t) dt$$
$$= 100 \int_0^{\pi/50} \frac{1 + \cos 200t}{2} dt$$
$$= 100 \left[\frac{\pi}{100}\right] = \pi$$

4. **(c)** From continuity equation
$A_A V_A = A_B V_B \Rightarrow 6V_A = 3V_B \Rightarrow V_B = 2V_A$
Applying Bernoullis equation between A& B,
$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$
$$\Rightarrow \rho g \times 0.05 = \frac{1}{2}\rho\left[V_B^2 - V_A^2\right] = \frac{1}{2}\rho\left(3V_A^2\right)$$
$$\Rightarrow V_A = \sqrt{\frac{2g \times 0.05}{3}} \text{ m/s} = \frac{1}{\sqrt{3}} \text{ m/s} = \frac{100}{\sqrt{3}} \text{ cm/s}$$
$\Rightarrow$ Volume flow rate $= A_A V_A = \frac{6 \times 100}{\sqrt{3}} \text{ cm}^3/\text{sec}$
$= 200\sqrt{3} \text{ cm}^3/\text{sec}$

5. **(d)** For parallel combination of spring,
$K_{eq} = K_1 + K_2 = 30$ N/m
$\Delta K_{eq} = \Delta K_1 + \Delta K_2 = 0.2 + 0.3 = 0.5$ N/m
$$\therefore \% \text{ Error in K} = \frac{0.5}{30} \times 100 = 1.67\%$$

6. **(b)** $\vec{F} = 4t^3\hat{i} - 3t\hat{j}$

$\vec{a} = \frac{\vec{F}}{m} = t^3\hat{i} - \frac{3}{4}\hat{j}$

$a_x = t^3$

$\frac{d\upsilon_x}{dt} = t^3$

$\int_{\upsilon_x=0}^{\upsilon_{x_2}} dv_x = \int_{t=0}^{t=2} t^3 dt$

$\upsilon_{x_2} - 0 = \left[\frac{t^4}{4}\right]_0^2$

$\upsilon_{x_2} = 4$

$a_y = \frac{-3}{4}t$

$\frac{d\upsilon_y}{dt} = -\frac{3}{4}$

$\int_0^{\upsilon_{y_2}} d\upsilon_y = \int_0^2 \frac{-3}{4} tdt$

$\upsilon_{y_2} = \frac{-3}{4}\left[\frac{t^2}{2}\right]_0^2$

$\upsilon_{y_2} = \frac{-3}{2}$

$\vec{\upsilon} = 4i - \frac{3}{2}\hat{j}$

$\upsilon_x = \frac{t^4}{4}$

$\int_0^{x_2} dx = \int_0^2 \frac{t^4}{4} dt$

$x_2 - 0 = \left[\frac{t^5}{20}\right]_0^2$

$x_2 = \frac{8}{5}$

$\upsilon_y = \frac{-3}{8}t^2$

$\int_0^{y_2} dx = \frac{-3}{8}t^2 dt$

$y_2 - 0 = \frac{-3}{8}\left[\frac{t^3}{3}\right]_0^2$

$y_2 = -1$

$\vec{r} = \frac{8}{5}\hat{i} - \hat{j}$

7. **(b)** $\Rightarrow [P] = \left[\frac{A}{Bt^2}\right]$ …………… (i)

$\Rightarrow [h] = [Bt]$ …………….. (ii)

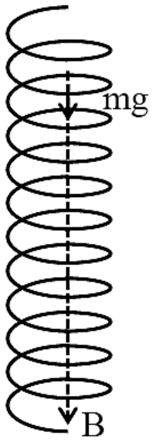
$\Rightarrow [B] = \left[\frac{h}{t}\right] = \left[\frac{L}{T}\right] = [LT^{-1}]$

Putting B is equation (i)

$[ML^{-1}\,T^{-2}] = \left[\frac{A}{LT^{-1} \times T^2}\right]$

$[A] = [ML^0\,T^{-1}]$

8. **(a)** Since the solenoid is placed vertically, the magnetic field inside the solenoid will be either along - y or +y axis.



$\Rightarrow$ Particle will gain velocity along -y axis.

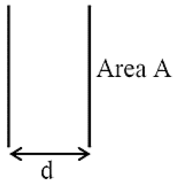
$\Rightarrow \vec{F}_B = q(\vec{v} \times \vec{B})$

$\Rightarrow \vec{F}_B = 0$

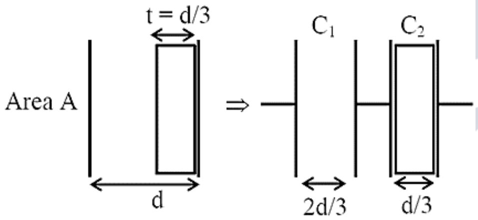
$\Rightarrow \vec{F}_{net} = mg$

$\Rightarrow a_{net} = g$

9. (a)



$C = \frac{A \epsilon_0}{d}$



$C_1 = \frac{3 A\epsilon_0}{2 d}$ $\qquad C_2 = \frac{3 A\epsilon_0 \times K}{d}$

$C_1 = \frac{3}{2}C$ $\qquad C_2 = 3KC$

$$Ceq = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{3}{2}C \times 3KC}{\frac{3}{2}C + 3KC}$$

$$Ceq = \frac{\frac{9}{2}KC^2}{\frac{3}{2}C(2K+1)} = \frac{3KC}{2K+1}$$

10. (a) $\hat{B} = \hat{c} \times \hat{E}$

$\Rightarrow \hat{c} = \hat{i}$ because phase of electric field is function of x .

$\Rightarrow \hat{E} = \hat{j}$ (given)

$\Rightarrow \hat{B} = \hat{i} \times \hat{j} = \hat{k}$

$$|B| = \frac{|E|}{c} = \frac{69 \times 0.6 \times 10^3}{1.8 \times 10^{11}} = \frac{69}{3 \times 10^8}$$

$|B| = 2.9 \times 10^{-7}$

$\vec{B}_2 = 2.9 \times 10^{-7} \sin(0.6 \times 10^3 x - 1.8 \times 10^{11} t)$

(phase is same as that of electric field)

11. (a) $d\sin\theta = (\mu - 1)t$

$$d\left[\frac{x}{D}\right] = (\mu - 1)t$$

$$t = \frac{xd}{D(\mu - 1)}$$

$= \dfrac{(0.2)(0.1)}{50(1.5-1)}$

$t = 8 \times 10^{-4}$ cm

12. (a) $\omega_1 = 3 \times 10^{15}$ rad/sec

$\omega_2 = 12 \times 10^{15}$ rad/sec

$\because\ v = \dfrac{\omega}{2\pi}$

$E_{photon} = hv = 6.6 \times 10^{-34} \times 1.91 \times 10^{15}$

$= 1.26 \times 10^{-18}$ J

$E_{max} = \dfrac{1.26 \times 10^{-18}}{1.6 \times 10^{-19}} \simeq 7.9$eV

$K_{max} = E_{max} - \phi_0$

$= 7.9 - 2.8$

$K_{max} = 5.1$eV

13. (a) Energy conservation

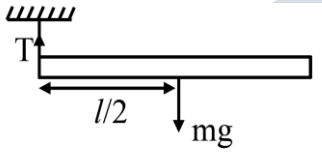
$K_i + U_i = K_f + U_f$

$7.7 \times 10^6 \times 1.6 \times 10^{-19} + 0$

$= 0 + \dfrac{9 \times 10^9 (1.6 \times 10^{-19})(79 \times 1.6 \times 10^{-19})}{r}$

$r = 2.95 \times 10^{-14}$

14. (b)



$mg\dfrac{l}{2} = \dfrac{ml^2}{3}\alpha$

$\alpha = \dfrac{3g}{2l}$ ... (i)

$mg - T = ma_c$

$T = mg - ma_c$

$= mg - m\left(\dfrac{l}{2}\alpha\right)$

$= mg - m\left(\dfrac{l}{2} \cdot \dfrac{3g}{2l}\right)$

$T = \dfrac{mg}{4}$

15. (b) $\ell_{final} = \ell_0(1 + \alpha_A \Delta T) + \ell_0(1 + \alpha_B \Delta T)$

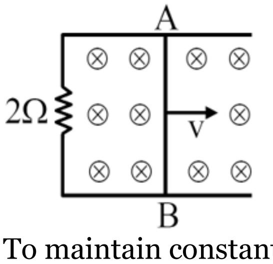
$= \ell_0[2 + (\alpha_A + \alpha_B)\Delta T]$

$= 60[2 + (36 \times 10^{-6}) \times 70]$

$= 60[2 + 0.0025]$

$= 120.15$ cm

16. (d)



To maintain constant speed

$F_{ext} = F_B$

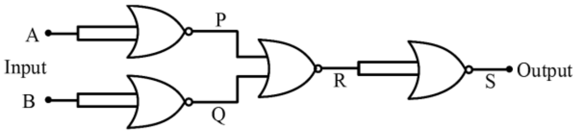
$\Rightarrow\ F_{ext} = ilB$

$= \left(\frac{vBl}{R}\right) lB$

$= \frac{B^2 l^2 v}{R}$

$= \frac{(0.1)^2 \times (1)^2 \times 1.5}{2} = 7.5 \times 10^{-3}\ N$

17. (c)



$P = \overline{A}$

$Q = \overline{B}$

$R = \overline{\overline{A} + \overline{B}} = \overline{\overline{AB}} = AB$

$S = \overline{AB} \Rightarrow$ NAND Gate

18. (d) Given $L_A = 2.5$ m,

$L_B = 1.5$ m

$T = 500$ N

$v_A = \sqrt{\frac{T}{\mu_A}} = \sqrt{\frac{500}{2 \times 10^{-4}}} = 5\sqrt{10} \times \frac{10^2\ m}{s}$

$v_B = \sqrt{\frac{T}{\mu_B}} = \sqrt{\frac{500}{4 \times 10^{-4}}} = 5\sqrt{5} \times 10^2\ m/s$

$t_1 = \frac{L_A}{v_A} = \frac{2.5}{5\sqrt{10}} \times 10^{-2}\ s$

$t_2 = \frac{L_B}{v_B} = \frac{1.5}{5\sqrt{5}} \times 10^{-2}\ s$

$\therefore \frac{t_1}{t_2} = \frac{2.5}{5\sqrt{10}} \times \frac{5\sqrt{5}}{1.5} = \frac{5}{3} \times \frac{1}{\sqrt{2}} = \frac{1.66}{1.41} = 1.18$

19. (d) Energy of a satellite in a circular orbit is given as

$E = \frac{-GM_E m}{2r}$; r = radius of circular orbit Required energy to be supplied $= \Delta E = E_f - E_i$

$\Delta E = \left(\frac{-GM_E\ m}{2(3R_E)}\right) - \left(\frac{-GM_E\ m}{2(1.5R_E)}\right)$

$= \frac{GM_E\ m}{6R_E}$

Now, $g = \frac{GM_E}{R_E^2} \Rightarrow \frac{GM_E}{R_E} = gR_E$

$\therefore\ \Delta E = \frac{1}{6} gmR_E$

$= \frac{1}{6} \times 10 \times 100 \times 6 \times 10^6$

$= 1000 \times 10^6$

$\alpha = 1000$

20. (c) Work done by external agent :

$W_{ext} = \Delta U$;

$\Delta U \rightarrow$ Change in potential energy in taking the charge from initial to final configuration

$\Rightarrow W_{ext} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_f} - \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_i}$

Now, $r_f = \sqrt{(2-0)^2 + (2-0)^2 + (1-0)^2} = 3$ m

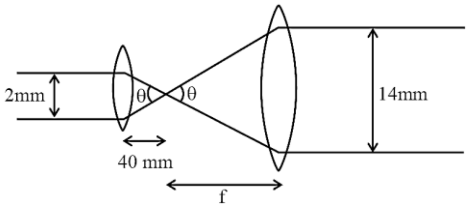
$r_i = \sqrt{(4-0)^2 + (4-0)^2 + (2-0)^2} = 6$ m

$W_{ext} = (9 \times 10^9) \times (10^{-8} \times 2 \times 10^{-6}) \left[\frac{1}{3} - \frac{1}{6}\right]$

$= 3 \times 10^{-5}$

$= 30 \times 10^{-6}$ J

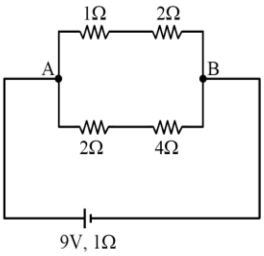
21. (280)



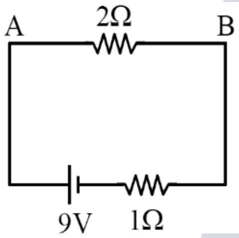
$\frac{40}{2} = \frac{f}{14}$

$\Rightarrow f = 280$ mm

22.(1080)



Balanced Wheatstone bridge

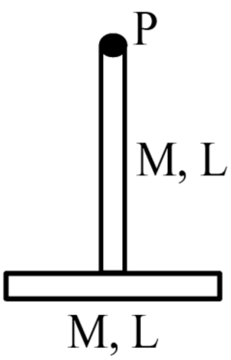


$i = \frac{9}{3} = 3$ A

$\therefore\ H_{AB} = i^2 R_{AB} t$

$= (3)^2 \times 2 \times 60 = 1080$ J

23. (17) $I = \frac{ML^2}{3} + \left(\frac{ML^2}{12} + ML^2\right)$



$= \frac{4ML^2 + ML^2 + 12ML^2}{12}$

$I = \frac{17}{12} ML^2$

$\therefore x = 17$

24. (500) $\Delta U = nC_v \Delta T$

$= n(C_p - R)\Delta T$

$= 10(7 - 2)(40 - 30)$

$\Delta U = 500$

25. (100) $m \simeq \frac{lD}{f_0 f_e}$

$= \frac{32}{2} \times \frac{25}{4}$

$m = 100$

26. (d) $\underset{\text{(Hot solution)}}{PbCl_2} + K_2CrO_4 \rightarrow \underset{\text{(A)}}{PbCrO_4} + 2KCl$

$PbCrO_4 + 4NaOH \text{ (excess)} \rightarrow Na_2[Pb(OH)_4] + Na_2CrO_4$

$PbSO_4 + 4CH_3COONH_4 \rightarrow (NH_4)_2\left[Pb\left(\underset{\text{(X)}}{(CH_3COO)_4}\right.\right] + (NH_4)_2SO_4$

27. (a) ⇒ On moving left to right in a period IE increases and from top to bottom in a group IE decreases.

$F > P > S > B$ (IE order)

⇒ On moving left to right in a period metallic and basic character decreases.

$K > Mg > Al > B$ (Metallic character order)

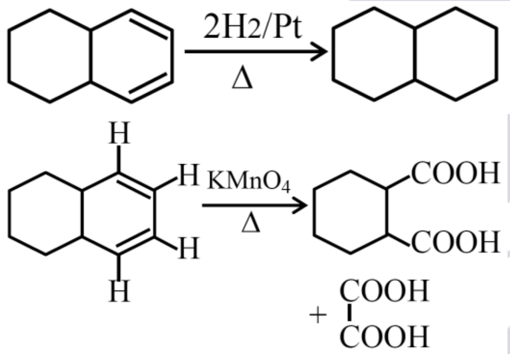
⇒ On moving top to bottom in a group metallic and basic character increases.

$K_2O > Na_2O > MgO > Al_2O_3$

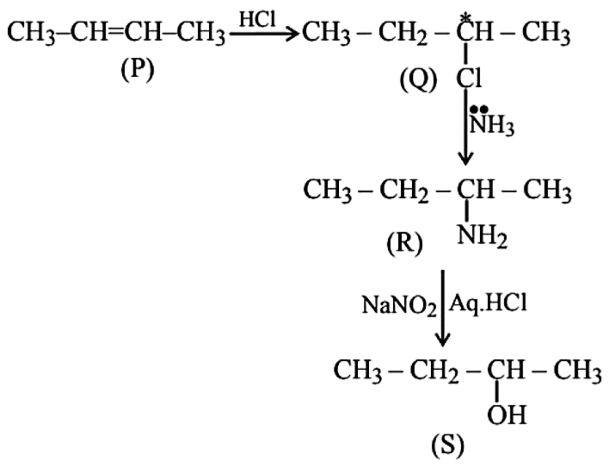
⇒ EA : Group 17 > Group 16 > Group 15

$Cl > F > S > P$

28. (c)



29. (c)



30. (a) ⇒ SiO_2, CO_2, GeO, GeO_2 are acidic in nature. SnO, SnO_2, PbO, PbO_2 are amphoteric in nature.

⇒ BF_3 is lewis acid acoording to lewic octet theory and has sp^2 hybridization with trigonal planar geometry and it can accept lone pair form ammonia to form adduct.

31. (c) $C_xH_{y(g)} + \left(x + \frac{y}{4}\right)O_{2(g)} \longrightarrow xCO_{2(g)} + \frac{y}{2}H_2O_{(\ell)}$

$t = 0 \quad 80 \quad 264 \quad 0 \quad -$

$t = t_{final} \quad - \quad 264 - 80\left(x + \frac{y}{4}\right) \quad 80x \quad -$

$264 - 80\left(x + \frac{y}{4}\right) + 80x = 224$

$264 - \dfrac{80y}{4} = 224$

$40 = \dfrac{80y}{4} \Rightarrow y = 2$

$264 - 80\left(x + \dfrac{y}{4}\right) = 64$

$264 - 80\left(x + \dfrac{1}{2}\right) = 64$

$264 - 80x - 40 = 64$

$x = 2$

32. (c) $\underset{\frac{14}{40}}{Ca_{(s)}} + \underset{\text{excess}}{2HCl_{(g)}} \longrightarrow CaCl_{2(s)} + H_{2(g)}$

$= 0.35$ mole 0.35 mole 0.35 mole

Volume of $H_{2(g)}$ evolved $= 0.35 \times 22.4 = 7.84$ L

(c) is wrong because weight of $CaCl_2 = .35 \times 111 = 38.85$ gm

33. (c) $\dfrac{n_{BaSO_4} \times 32}{W_{(\text{unknown comp.})}} \times 100$

$= \dfrac{1.2 \times 32}{233} \times \dfrac{100}{0.75} = 21.97\%$

34. (c) $(\Delta T_b)_{PQ} = K_b m$

$1.176 = 5 \times \dfrac{1}{M_1} \times \dfrac{1000}{50}$

$M_1 = 85.03$

$(\Delta T_b)_{PQ_2} = 5 \times \dfrac{1}{M_2} \times \dfrac{1000}{50} = 0.689$

$M_2 = 145.13$

Let molar mass of P&Q are M_P and M_Q respectively

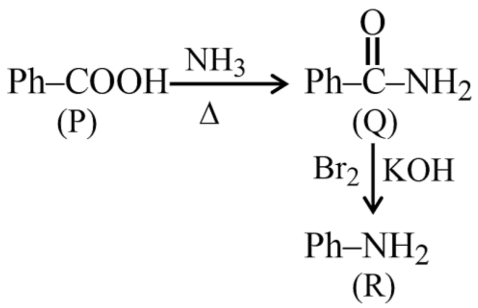
$M_P + M_Q = 85.03$

$M_P + 2M_Q = 145.13$

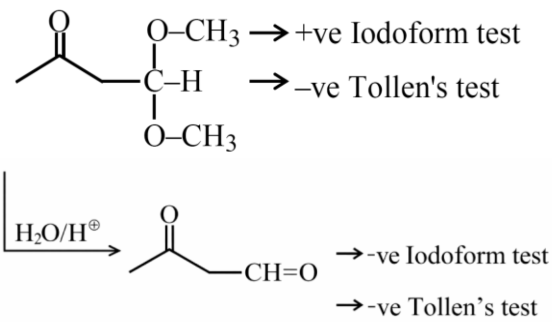
$M_p = 24.93 \approx 25$

$M_Q = 60.1 \approx 60$

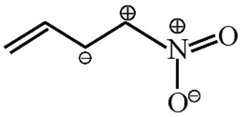
35. (a)



36. (b)



37. (c)



This resonating structure having +ve charge on adjacent atoms so it is least stable.

38. (c) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

39. (c) Statement-I

SF_4 (See-saw)

XeF_4 (square planar),

$[PtCl_4]^{2-}$ (square planar),

$[NiCl_4]^{2-}$ (Tetrahedral),

$[Ni(CN)_4]^{2-}$ (square planar),

SeF_4 (See-saw)

NH_4^+(Tetrahedral)

Statement-II NO_2 (seven electrons on N)

BeH_2 (four electrons on Be)

BF_3 (six electrons on B)

$AlCl_3$ (six electrons on Al)

40. (d)$[Cu(NH_3)_4]^{2+} \Rightarrow d^9, dsp^2$ one unpaired electron

$[Ni(en)_3]^{2+} \Rightarrow d^8, sp^3\ d^2$ two unpaired electrons

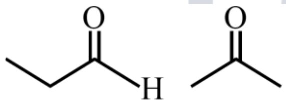
$[Ni(NH_3)_6]^{2+} \Rightarrow d^8, sp^3\ d^2$ two unpaired electrons

$[Mn(H_2O)_6]^{2+} \Rightarrow d^5, sp^3\ d^2$ five unpaired electrons

$[Ni(CO)_4]$ (diamagnetic)

$[NiCl_4]^{-2}$ (paramagnetic)

$[Ni(CN)_4]^{-2}$ (diamagnetic)

41. (c)  are Functional isomer

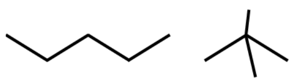
$CH_3 - CH = CH - CH_3$ & $CH_3 - CH_2 - CH = CH_2$

Are Positional isomer

$CH_3 - CH_2 - O - CH_2 - CH_3$&$CH_3 - CH_2 - CH_2 - O - CH_3$

are Metamer

$CH_3 - CH = CH - CH_3$ does not have optical isomer

 Chain isomer

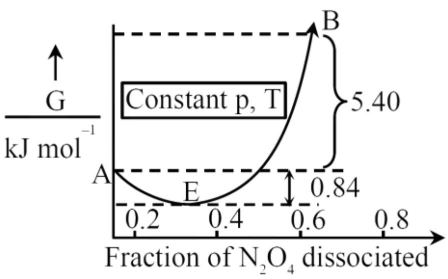
42. (d) • Histidine does contain heterocyclic ring.

• Proline is a five membered cyclic ring amino acid.

• Cysteine has characteristic feature of side chain as $CH_2 - SMe$

43. (d) Area under the Pv/sV curve, is equal to magnitude of work. In option (b) work done is zero while in remaining options net work done is negative due to expansion. NTA has given the answer without considering the negative sign that is considered only magnitude.

44. (a)



(A) $\Delta_r G^\circ = G^\circ{}_B - G^\circ{}_A = +ve$
(B) $\Delta_r G^\circ = +ve, N_2O_4$ will partially dissociates into NO_2.
(C) For reverse reaction It is partially completed as there is equilibrium at E .
(D) For 1 mole $N_2O_4; \Delta_r G^\circ = -0.84$ kJ mol^{-1}
(E) For 2 mole $NO_2; \Delta G^\circ = -5.4 - 0.84 = -6.24$ kJ mol^{-1}.

45. (a) $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For Ist line of lyman series in H -atom

$$\frac{1}{\lambda} = R(1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{\lambda} = \frac{3R}{4}$$

for 2^{nd} line of Balmer series of He^+

$$\frac{1}{\lambda'} = R(2)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda'} = \frac{3R}{4}$$

As λ and λ' is equal so frequency of these lines will be also equal.

46. (8) $A \xrightarrow{RX^n(a)}$ product E_1

$B \xrightarrow{RX^n(b)}$ product E_2

Assuming 'A' same for both reaction.

$$\ln k_1 = \ln A - \frac{E_1}{300R}$$

$$\ln k_1 = \ln A - \frac{E_2}{300R}$$

$$\ln\left(\frac{k_2}{k_1}\right) = \frac{E_1 - E_2}{300R} = \frac{20 \times 1000}{300R}$$

$= 8.032$

47. (6) pH = 5

$[H^+] = 10^{-5} = [HX].\alpha$

$$= [HX] \cdot \frac{\Lambda_m}{\Lambda_m^\infty}$$

$$\Lambda_m = \frac{k \times 1000}{[HX]}$$

$$K = G.G^* = 4 \times 10^{-5} \times \frac{15}{1} = 6 \times 10^{-4}\ S.cm^{-1}$$

$$[H^+] = 10^{-5} = [HX] \times \frac{6 \times 10^{-4} \times 1000}{\Lambda_m^\infty \times [HX]}$$

$\Lambda_m^\infty = 60000 S.cm^2\ mol^{-1}$

$\Lambda_m^\infty = 6\ S.m^2\ mol^{-1}$

48. (70) $A_2 + B_2 \xrightarrow{500\ K} 2AB \log K = 2.2$

$\Delta H^\circ = (2 \times 32) - (6 + x) = (58 - x)$kJ

$\Delta S^\circ = (2 \times 222) - (146 + 280) = 18$ Joule

$\Delta G^\circ = -RT\ln K$

$$\Delta G^\circ = -\frac{8.314 \times 500 \times 2.2 \times 2.303}{1000}$$

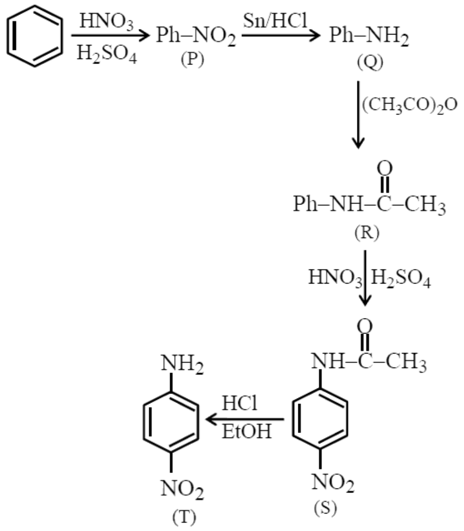
$\Delta G^\circ = -21.06$

$\Delta H^\circ - T\Delta S^\circ = -21.06$

$$58 - x - 500\left(\frac{18}{1000}\right) = -21.06$$

$x = 70.06 KJ/mol$

49. (20)



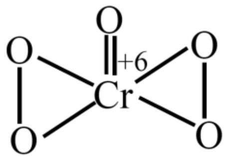
Mol. wt $= 6 \times 12 + (6 \times 1) + (2 \times 14) + (2 \times 16) = 138$

$\% \text{ N} = \frac{28}{138} \times 100 = 20.29\%$

50. (13) $A \rightarrow CrO_2Cl_2$

$B \rightarrow Na_2CrO_4$

$C \rightarrow CrO_5$



$X = 2, Y = 5$ and $Z = 6$

51. (b) $f(x) = \cos^{-1}\left(\frac{2x-5}{11-3x}\right) + \sin^{-1}(2x^2 - 3x + 1)$

$-1 \leq \frac{2x-5}{11-3x} \leq 1$

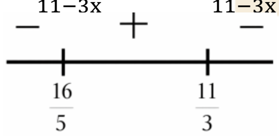
$-1 \leq 2x^2 - 3x + 1 \leq 1$

$2x^2 - 3x + 2 \geq 0, 2x^2 - 3x \leq 0$

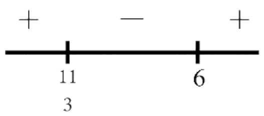
$x \in \left[0, \frac{3}{2}\right]$ .... (i)

$\frac{2x-5}{11-3x} + 1 \geq 0$ $\quad$ $\frac{2x-5}{11-3x} - 1 \leq 0$

$\frac{2x-5+11-3x}{11-3x} \geq 0$ $\quad$ $\frac{5x-16}{11-3x} \leq 0$



$\frac{6-x}{11-3x} \geq 0$



$x \in \left(-\infty, \frac{16}{5}\right] \cup \left(\frac{11}{3}, \infty\right)$

$x \in \left(-\infty, \frac{11}{3}\right) \cup [6, \infty)$

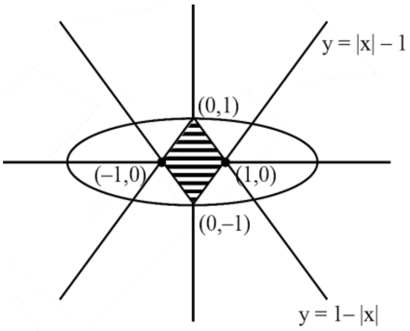
intersection

$x \in \left(-\infty, \frac{16}{5}\right] \cup [6, \infty)$ .... (ii)

Intersection of (i) & (ii) $x \in \left[0, \frac{3}{2}\right]$

$\alpha = 0, \beta = \frac{3}{2} \Rightarrow \alpha + 2\beta = 3$

52. (a)



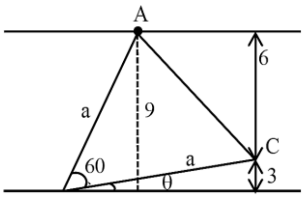
Required area = area of ellipse - shaded area

$= \pi \times 2 \times 1 - 4\left(\frac{1}{2} \times 1 \times 1\right) = 2\pi - 2$

53. (d) Number of relation which are reply and sym. both $= 1^4 \times 2^6 = 64$

(a, a)	(a, b)	(a, c)	(a, d)
(b, a)	(b, b)	(b, c)	(b, d)
(c, a)	(c, b)	(c, c)	(c, d)
(d, a)	(d, b)	(d, c)	(d, d)

54. (c)



$\sin\theta = \frac{3}{a}$

$\sin(60° + \theta) = \frac{9}{a}$

$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{9}{a}$

$\sqrt{3}\sqrt{1 - \frac{9}{a^2}} + \frac{3}{a} = \frac{18}{a}$

$a = \sqrt{84}$

Area of $\triangle$ ABC $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 84 = 21\sqrt{3}$

55. (d) $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b} = 8\hat{i} + 7\hat{j} - 3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\vec{a} \times \vec{c} = \vec{b} \Rightarrow (2c_3 - 2c_2)\hat{i} + (c_3 + 2c_1)\hat{j} - (c_2 + 2c_1)\hat{k} = 8\hat{i} + 7\hat{j} - 3\hat{k}$

$2c_3 - 2c_2 = 8, c_3 + 2c_1 = 7, c_2 + 2c_1 = 3$

$(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$

$\Rightarrow c_1 + c_2 + c_3 = 4, c_1 = 2, c_2 = -1, c_3 = 3$

$|\vec{a}+\vec{c}|^2 = |\hat{\imath}+\hat{\jmath}+5\hat{k}|^2 = 27$

56. (b) $ar\,.\,ar^2.\,ar^3 = 64$

$a^3r^6 = 64 \;\Rightarrow ar^2 = 4$

$a + ar^2 + ar^4 = \dfrac{813}{7}$

$r^2 = 28$

$ar^2 + ar^4 + ar^6 = ?$

$ar^2(1 + r^2 + r^4) = 4(1 + 28 + 784) = 3252$

57. (b) $|\vec{c}+\vec{d}| = \sqrt{29}$

$\vec{c}+\vec{d} = \lambda((2\hat{\imath}+3\hat{\jmath}+4\hat{k})$

$\lambda = \pm 1$

$\lambda(-14 + 6 + 12) = 4\lambda, \lambda_1 = 4, \lambda_2 = -4$

$k^2x^2 + (k^2 - 5k + 4)xy + (3k - 2)y^2 - 8x + 12y - 4 = 0$

is circle

$k^2 - 5k + 4 = 0 \Rightarrow k = 1,4$

$k^2 = 3k - 2 \Rightarrow k = 1,2$

$k = 1$

58. (a) $\frac{dy}{dx} + \frac{y}{x^2+1} = \frac{\tan^{-1}x}{x^2+1}$

I.f. $= e^{\tan^{-1}x}$

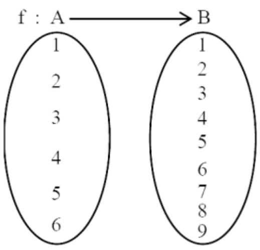
$y \times e^{\tan^{-1}x} = \int e^{\tan^{-1}x} \cdot \dfrac{\tan^{-1}x}{1+x^2}dx$

$y \times e^{\tan^{-1}x} = \tan^{-1}x\left(e^{\tan^{-1}x}\right) - e^{\tan^{-1}x} + c$

$y(o) = 1 \Rightarrow c = 2$

$y(1) = \dfrac{2}{e^{\pi/4}} + \dfrac{\pi}{4} - 1$

59. (d) $f(i) \neq i, f(x)$ is strictly increasing function $f: A \to B$, where $A\{1,2,3 \ldots \ldots .6\}$ $B\ \{1,2,3, \ldots \ldots .9\}$, then number of function $f: A \to B$ is equal to



$f(i) \neq i$ Case - if$(1) = 2 \Rightarrow {}^7C_5 = 21$

Case- ii $f(1) = 3 \Rightarrow {}^6C_5 = 6$

Case- iii $f(1) = 4 \Rightarrow {}^5C_5 = 1$

No of function A to B $= 21 + 6 + 1 = 28$

60. (c) Let $T = \lim_{x\to 1}\left(\frac{f(x+2)}{f(3)}\right)^{\frac{18}{(x-1)^2}}$; 1^∞ form

$\Rightarrow T = e^{\lim_{x\to1}\frac{18}{(x-1)^2}\left(\frac{f(x+2)-f(3)}{f(3)}\right)}$

$\Rightarrow T = e^{\lim_{x\to1}\frac{18}{(x-1)^2}\left(\frac{f(x+2)-f(3)}{18}\right)}$

$\Rightarrow T = e^{\lim_{x\to1}\left(\frac{f(x+2)-f(3)}{(x-1)^2}\right)\frac{0}{0}\text{ form}}$ apply L' pital

$\Rightarrow T = e^{\lim_{x\to1}\frac{f'(x+2)}{2(x-1)};\frac{0}{0}\text{ form}}$ apply L' pital

$\Rightarrow T = e^{\lim_{x\to1}\frac{f''(x+2)}{2}}; = e^{\frac{4}{2}} = e^2$

$\Rightarrow \log_e(T) = 2$

61. (c) Let e_1 be eccentricity of ellipse

$\Rightarrow e_1 = \sqrt{1 - \frac{16}{36}} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

So $ae_1 = 6 \cdot \frac{\sqrt{5}}{3} = 2\sqrt{5}$

Now $H : \frac{x^2}{p^2} - \frac{y^2}{q^2} = 1$

$p.e = ae_1$

$p.5 = 2\sqrt{5}$

$p = \frac{2}{\sqrt{5}} \Rightarrow e^2 = 1 + \frac{q^2}{p^2} \Rightarrow 25 = 1 + \frac{5q^2}{4} \Rightarrow q^2 = \frac{96}{5}$

So length of LR $\frac{2q^2}{p} = \frac{96}{\sqrt{5}}$

62. (b) $= 2\pi\int_0^{\pi/6} \frac{1}{1-\sin\left(x+\frac{\pi}{6}\right)} dx$ let $x + \frac{x}{6} = tdx = dt$

$= 2\pi\int_{\pi/6}^{\pi/3} \frac{dt}{1 - \sin t} = 2\pi\int_{\pi/6}^{\pi/3} \frac{1 + \sin t}{\cos^2 t} dt$

$= 2\pi\left[\int_{\pi/6}^{\pi/3} \sec^2 t dt + \int_{\pi/6}^{\pi/3} \sec t \tan t dt\right]$

$= 2\pi\left[(\tan t)_{\pi/6}^{\pi/3} + (\sec t)_{\pi/6}^{\pi/3}\right]$

$= 2\pi\left[\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) + \left(2 - \frac{2}{\sqrt{3}}\right)\right]$

$= 2\pi[\sqrt{3} + 2 - \sqrt{3}] = 4\pi$

63. (b) Mean $(\bar{x}) = 8$ (Given)

$\Rightarrow \frac{2 + 4 + 10 + x + 12 + 14 + y}{7} = 8$

$\Rightarrow x + y = 14 \ldots (1) \ldots.$

Variance $(\sigma^2) = 16$ (Given)

$\Rightarrow 16 = \frac{2^2 + 4^2 + 10^2 + x^2 + 12^2 + 14^2 + y^2}{7} - 8^2$

$\Rightarrow x^2 + y^2 = 100 \ \ldots \ldots (2) \ldots.$

$\because (x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow xy = 48$ (sum is 14 product is 48)

Since problem states $x > y$

$\therefore x = 8$ and $y = 6$

Now set $X = \{1,2,3,4,6,5\}$

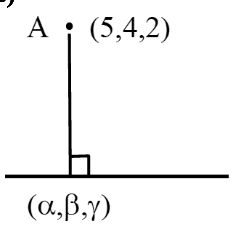
Now we choose two numbers one after author without replacement total outcomes $= 6 \times 5 = 30$

We want the prob. That the smaller number among the two is less than 4

$P(\text{smaller} < 4) = 1 - P(\text{smaller} \geq 4)$

$= 1 - \frac{6}{30} = \frac{4}{5}$

64. (c)



$\bar{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$

$\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}=\lambda$

Any general point P on the line is

$(2\lambda-1, 3\lambda+3, -\lambda+1)$

Let the given point is $A(5,4,2)$

$\overline{AP}(2\lambda-6)\hat{i}+(3\lambda-1)\hat{j}+(-\lambda-1)\hat{k}$

$\because \overline{AP} \perp^r$ Line (L)

$\therefore \overline{AP}\cdot(2\hat{i}+3\hat{j}-\hat{k})=0$

$2(2\lambda-6)+3(3\lambda-1)-1(-\lambda-1)=0$

$\Rightarrow \lambda=1$

$\therefore \alpha=1$

$\beta=6$

$\gamma=0$

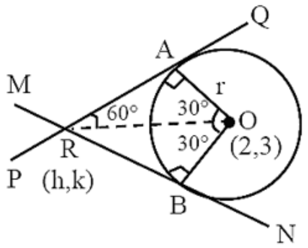
Let the vector $\overline{u}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$

$\overline{u}=\hat{i}+6\hat{j}+0\hat{k}$

$\&\overline{w}=6\hat{i}+2\hat{j}+3\hat{k}$

So projection $=\frac{|\overline{u}\cdot\overline{w}|}{|\overline{w}|}=\frac{18}{7}$

65. **(d)** Given circle



$x^2+y^2-4x-6y-3=0$

$C(2,3)\&r=4$

$\cos30^\circ=\frac{r}{OR}=\frac{4}{OR}$

$\Rightarrow OR=\frac{8}{\sqrt{3}}$

Now

$OR^2=(h-2)^2+(k-3)^2$

$\Rightarrow 3(x^2+y^2)-12x-18y-25=0$

66. **(c)** $(ax^2+bc+c)\sum_{r=0}^{26}{}^{26}C_r(-2x)^r$

Coeff. of x^2: a. ${}^{26}C_0(-2)^0$ + b. ${}^{26}C_1(-2)$ + c. ${}^{26}C_2(-2)^2=0$

$\Rightarrow a-52b+1300c=0$ .... (i)

Coeff. of x^3: a. ${}^{26}C_1(-2)$ + b. ${}^{26}C_2(-2)^2$ + c. ${}^{26}C_3(-2)^3=0$

$\Rightarrow -52a+1300b-20800c=0$ .... (ii)

Coeff. of $x=-56$

$\Rightarrow$ b. ${}^{26}C_0(-2)^0$ + c. ${}^{26}C_1(-2)^1=-56$

$b-52c=-56$ .... (iii)

After solving (i), (ii) & (iii)

$a=1300, b=100, c=3$

$\Rightarrow a+b+c=1403$

67. **(d)** $x^2+x+1=0$

$\Rightarrow x=\omega$ or ω^2

$\therefore \alpha=\omega\&\beta=\omega^2$

$=(\omega+\omega^2)^4+(\omega^2+\omega^4)^4+(\omega^3+\omega^6)^4+\cdots+(\omega^{25}+\omega^{50})^4$

$=[(\omega+\omega^2)^4+(\omega^2+\omega^4)^4+(\omega^4+\omega^8)^4+\cdots.+(\omega^{25}+\omega^{50})^4]$

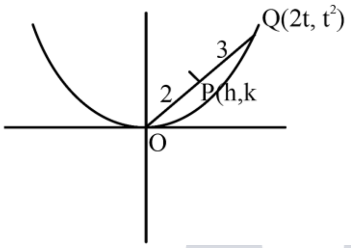
$+[(\omega^3+\omega^6)^4+(\omega^6+\omega^{12})^4+(\omega^9+\omega^{18})^4+\cdots+(\omega^{24}+\omega^{48})^4]$

$= \underbrace{[1+1+1\ldots\ldots+1]}_{17\text{ times}} + \underbrace{[(1+1)^4+(1+1)^4+\cdots..(1+1)^4]}_{8\text{ times}}$
$= 17 + 128$
$= 145$

68. (a) $= \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$
$= \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ}$
$= 4\left[\frac{\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ}{2\sin 10^\circ \cos 10^\circ}\right]$
$= 4\left[\frac{\sin(30^\circ - 10^\circ)}{\sin 20^\circ}\right]$
$= 4$

69. (a) Let $|x-1| = t$
$t^2 - 5t + 6 = 0$
$t = 2 \& t = 3$
$|x-1| = 2 \& |x-1| = 3$
$x - 1 \pm 2 \& x - 1 = \pm 3$
$x = 1 \pm 2 \& x = 1 \pm 3$
$\therefore$ root $= 3, -1, 4, -2$
$\therefore$ Sum of root $= 3 + (-1) + 4 + (-2) = 4$

70. (d) $h = \frac{4t}{5}$



$k = \frac{2t^2}{5} = \frac{2}{5}\cdot\left(\frac{5\text{ h}}{4}\right)^2$
$8k = 5\text{ h}^2$
$\Rightarrow 5x^2 = 8y$
$T = S_1$
$5(xx_1) - 4(y + y_1) = 5x_1{}^2 - 8y_1$
$5(x) - 4(y+2) = 5 - 8.2$
$5x - 4y + 3 = 0$

71. (a) Given quadratic equation has equal roots, thus
$D = 0 \Rightarrow (f'(x))^2 = f''(x)\cdot f(x)$
$\frac{f'(x)}{f(x)} = \frac{f''(x)}{f'(x)}$
Integrate
$\ell n(f(x)) = \ell n(f'(x)) + \ell nC \Rightarrow f(x) = c.f'(x)$
Put $= x = 0$
$1 = c.2 \Rightarrow c = \frac{1}{2}$
Now $2f(x) = f'(x)$
$\Rightarrow \frac{f'(x)}{f(x)} = 2$
Integrate

$\ell n(f(x)) = 2x + d$
$\Rightarrow d = 0$
$\Rightarrow \ln(f(x)) = 2x \Rightarrow f(x) = e^{2x}$
Now let $g(x) = f(\ln x - x) = e^{2(nx-x)}$
$\therefore g'(x) = 2e^{2(\ell nx-x)}\left(\frac{1}{x} - 1\right) \geq 3$
$\Rightarrow \frac{1-x}{x} \geq 0$
$\Rightarrow x \in (0,1]$
$\Rightarrow \alpha = 0, \beta = 1$
$\alpha + \beta = 1.$

72. (2) $a_{n+1} - \frac{1}{2}a_n = \frac{n^2-2n-1}{n^2(n+1)^2} = \frac{2n^2-(n+1)^3}{n^2(n+1)^2}$

$\Rightarrow a_{n+1} - \frac{1}{2}a_n = \frac{2}{(n+1)^2} - \frac{1}{n^2}$

$n = 1a_2 - \frac{1}{2}a_1 = \frac{2}{2^2} - \frac{1}{1^2}$

$2\left[a_3 - \frac{1}{2}a_2 = \frac{2}{3^2} - \frac{1}{2^2}\right]$

$2^2\left[a_4 - \frac{1}{2}a_3 = \frac{2}{4^2} - \frac{1}{3^2}\right]$

$\vdots$

$2^{n-2}\left[a_n - \frac{1}{2}a_{n-1} = \frac{2}{n^2} - \frac{1}{(n-1)^2}\right]$

$2^{n-1}\left[a_{n+1} - \frac{1}{2}a_n = \frac{2}{(n+1)^2} - \frac{1}{n^2}\right]$

Adding

$a_{n+1} = \frac{2}{(n+1)^2} - \frac{1}{2^n} \Rightarrow a_n = \frac{2}{n^2} - \frac{1}{2^{n-1}}$

$\Rightarrow \left|\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right| \Rightarrow \left|\sum_{n=1}^{\infty} - \frac{1}{2^{n-1}}\right| = 2$

73. (1333) $S = \{1,2,3, \ldots .50\}$
$p = (6^m + 9^n)$ is divisible by 5
No. of ways
$6^m = (5\lambda + 1)^m = 5k + 1$
$9^n = (10 - 1)^n = 10\mu - 1$ if n is odd
$\Rightarrow$ n must be odd
$10\mu + 1$ if n is even
$\Rightarrow$ No. of ways $= 50 \times 25 = 1250$
$q \Rightarrow (m + n)$ is square of a prime

	$m + n = 4$	$m + n = 9$	$m + n = 25$	$m + n = 49$
No.of	↓	↓	↓	↓
ways	3	8	24	48

$q = 3 + 8 + 24 + 48 = 83$
$= p + q = 1250 + 83 = 1333$

74. (225) $\text{Tr}(A) = 4 \Rightarrow \alpha + 2 = 4 \Rightarrow \alpha = 2$
$\text{Tr}(B) = 3 \Rightarrow \beta + 1 = 3 \Rightarrow \beta = 2$
$A^2 - 4A + 2I = O$

$A^3 = 4A^2 - 2A = 16A - 8I - 2A$

14A-8I

$= \begin{bmatrix} 28 & 28 \\ 14 & 28 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ 0 & -8 \end{bmatrix}$

$= A^3 = \begin{bmatrix} 20 & 28 \\ 14 & 20 \end{bmatrix}$

$B^2 - 3B + I = O$

$B^2 = 3B - I$

$\Rightarrow B^3 = 3B^2 - B = 3(3B - I) - B = 8B - 3I$

$B^3 = \begin{pmatrix} 8 & 8 \\ 8 & 16 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix}$

$A^3 - B^3 = \begin{pmatrix} 20 & 28 \\ 14 & 20 \end{pmatrix} - \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix} = \begin{pmatrix} 15 & 20 \\ 6 & 7 \end{pmatrix}$

$\Rightarrow |A^3 - B^3| = 105 - 120 = -15$

$\Rightarrow |\mathrm{adj}(A^3 - B^3)| = |A^3 - B^3| = -15$

$\Rightarrow |\mathrm{adj}(A^3 - B^3)|^2 = 225$

75. (17) $6\int_0^{\pi} |2\sin 2x\cos x + \sin 2x|dx$

$6\int_0^{\pi} |4\sin x\cos^2 x + 2\sin x\cos x|dx$

$I = 12\int_0^{\pi} \sin x|2\cos^2 x + \cos x|$

Put $\cos x = t, -\sin x dx = dt$

$I = -12\int_1^{-1} |2t^2 + t|dt$

$I = 12\left(\int_{-1}^{-1/2} (2t^2 + t)dt + \int_{-1/2}^{0} -(2t^2 + t)dt + \int_0^1 (2t^2 + t)dt\right)$

$I = 17$