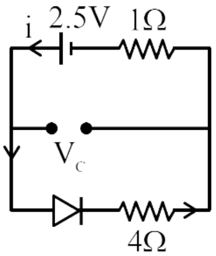
1. **(a)** Using energy conservation

$$(2)\left(\frac{1}{2}mu^2\right)-\frac{Gm^2}{4r}+\frac{KQ^2}{4r}=-\frac{Gm^2}{2r}+\frac{KQ^2}{2r}$$

$$u=\sqrt{\frac{1}{4mr}(KQ^2-Gm^2)}$$

2. **(b)**



in steady state

$i = 2.5/5 = 0.5\ A$

$V_C = 4 \times 0.5$

$V_c = 2\ V$

charge

$Q = CV_c$

$= 5 \times 2$

$= 10\mu C$

3. **(b)** $\frac{6}{R_{AP}}=\frac{4}{R_{PB}}$;

$\ell_{AP}+\ell_{PB}=50$ …………….. (i)

$$\frac{R_{AP}}{R_{PB}}=\frac{\ell_{AP}}{\ell_{PB}}=\frac{3}{2}$$

$$\ell_{AP}=\frac{3}{5}\times 50=30\ cm$$

4. **(b)** $U_{c_f}=75\%U_{c_i}$

$$Q_F^2=\frac{3}{4}Q_I^2$$

$$Q_i\cos\omega t=\frac{\sqrt{3}}{2}Q_i \Rightarrow t=\frac{T}{12}$$

$$t=\frac{\pi}{6}\sqrt{LC}$$

5. **(c)** $V_{rms}=\sqrt{\frac{3RT}{M}}$

$V_{rmsO_2}=V_{rmsH_2}$

$T_{O_2}=273+47=320\ K$

$$\sqrt{\frac{3RT_{O_2}}{M_{O_2}}}=\sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$\frac{T_2}{M_{O_2}}=\frac{T_{H_2}}{M_{H_2}}$$

$$\frac{320}{32}=\frac{T_{H_2}}{2}$$

$T_{H_2}=20\ K$

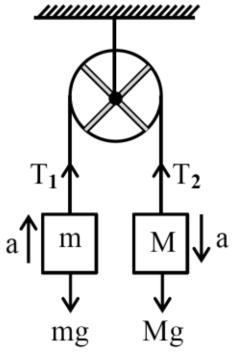
$T_{H_2}=-253°C$

6. **(b)** $L=m.V_{rel}\,r_\perp$

$$=1000\times\left(36\times\frac{5}{18}\right)\times 10$$

$$=10^5\ kg\ m^2/s$$

7. (c)



$Mg - T_2 = Ma$ .... (i)

$T_1 - mg = ma$ .... (ii)

$(T_2 - T_1)r = I\frac{a}{r}$ .... (iii)

(i) + (ii) + (iii)

$(M - m)g = \left(M + m + \frac{I}{r^2}\right)a$

Here $I = Mr^2 + \frac{M\times(2r)^2}{12} \times 2$

$= \left(1 + \frac{2}{3}\right)Mr^2$

$= \frac{5}{3}Mr^2$

$(M - m)g = \left[M + m + \frac{5M}{3}\right]a$

$a = \frac{(M - m)g}{\left[M + m + \frac{5M}{3}\right]}$

8. (a) $\omega = 176 \text{rad/sec}$

$f_k = \frac{\omega}{2\pi} = \frac{176}{2 \times 22} \times 7$

$= \frac{176}{44} \times 7$

$= 4 \times 7 = 28$ Hz

So frequency of oscillator

$= \frac{f_k}{2} = 14$ Hz

9. (c) Angular fringe width $= \frac{\lambda}{d}$

10. (d) For maximum power drawn across load Resistance $R_{Load} = R_{internal}$

$R = r$

11. (d) $L = 52.01 + 153.2 + 0.123$

$= 205.333$

$= 205.3$

12. (b) $v_0 = \frac{2}{9}\frac{r^2 g}{n}(\rho_B - \rho_L)$

$n = \frac{2}{9}\frac{r^2 g}{v_0}(\rho_B - \rho_L)$

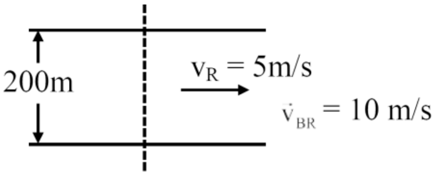
$\frac{\Delta n}{n} = \frac{2\Delta r}{r} + \frac{\Delta v_0}{v_0}$

13. (a) $h = \frac{2T}{\rho g r}$

$h \propto \frac{1}{r}$

If $r_1 > r_2 \Rightarrow h_2 > h_1$

14. (c)



Minimum time :

$t_{min} = \dfrac{200}{10} = 20sec$

For round trip $= 40sec.$

Displacement along river bank $= 40 \times 5 = 200$ m

15. (a) $\frac{(2R+x)\cdot(R)}{3R+x} = x$

$x^2 + 2Rx - 2R^2 = 0$

$x = (\sqrt{3} - 1)R$

16. (c) Angular momentum $L = \frac{nh}{2\pi}$

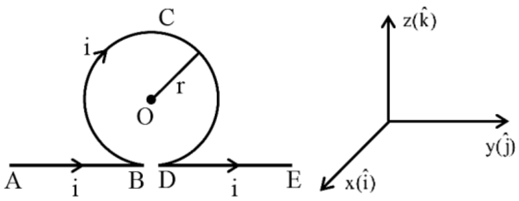
$n = \dfrac{2\pi L}{h}$

Energy $E = -\frac{13.6}{n^2}.z^2$

$E \Rightarrow -\dfrac{E_0}{n^2} = -0.04E_0$

$n^2 = 25, n = 5$

17. (b)



$\vec{B}_0 = \vec{B}_{AB} + \vec{B}_{DE} + \vec{B}_{BCD}$

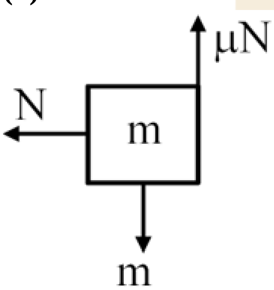
$= \dfrac{\mu_0 i}{4\pi r}\hat{\imath} + \dfrac{\mu_0 i}{4\pi r}\hat{\imath} - \dfrac{\mu_0 i}{2r}\hat{\imath}$

$= \dfrac{\mu_0 i}{2\pi r}\hat{\imath} - \dfrac{\mu_0 i}{2r}\hat{\imath}$

$= \dfrac{\mu_0 i}{2\pi r}(1 - \pi)\hat{\imath}$

$= -\dfrac{\mu_0 i}{2\pi r}(\pi - 1)\hat{\imath}$

18. (d)



$N = m\omega^2 r, mg = \mu N$

$\mu \times m\omega^2 r = mg$

$\omega = \sqrt{\dfrac{g}{\mu r}}$

19. (c) $x(t) = t^2 + t + 1$

$v(t) = 2t + 1$

$a(t) = 2$

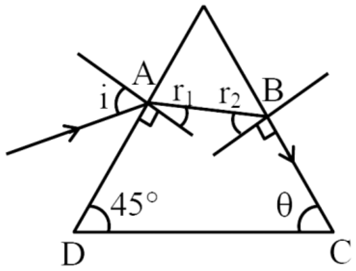
$F = 4$ N.

Displacement $= x(3) - x(2)$

$= 13 - 7 = 6M$

$W = F.S = 4 \times 6 = 24$ J

20. (a)



For grazing emergence

$\sin r_2 = \frac{1}{\mu}$

By Snell's Law at incident surface

$1 \times \frac{1}{\sqrt{2}} = \sqrt{2}\sin r_1$

$r_1 = 30$

$r_1 + r_2 = A$

$A = 75$

$75 + 45 + \theta = 180°$

$\theta = 60°$

21. (1800) A

$j_c = \sigma E$

$E \Rightarrow E_0 \sin(\omega t - kx)$

$j_c = \sigma E_0 \sin(\omega t - kx)$

$\Rightarrow (j_c)_{max} = \sigma E_0$ .... (i)

$J_d \Rightarrow \frac{i_d}{A} = \frac{1}{A} \times \varepsilon_0 \frac{AdE}{dt}$

$\Rightarrow \varepsilon_0 \times E_0 \omega \cos(\omega t - kx)$

$(j_d)_{max} \Rightarrow \varepsilon_0 E_0 \omega$ .... (ii)

(i)/(ii)

$\frac{(j_c)_{max}}{(j_d)_{max}} = \frac{\sigma E_0}{\varepsilon_0 \omega E_0} \Rightarrow \frac{\sigma}{\varepsilon_0 \omega}$

$\Rightarrow \frac{10 \times 4\pi \times 9 \times 10^9}{2\pi \times 100 \times 10^6}$

$\Rightarrow 1800$

22. (5) We know :

Terminal velocity $\propto$ (radius $)^2$

$\frac{(v_T)_1}{(v_T)_2} = \left(\frac{6}{3}\right)^2$

$(v_T)_2 = \frac{(v_T)_1}{4} = 5$ cm/sec

23. (10) $\lambda = \frac{h}{\sqrt{2mqV}}$

$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 18 \times 10^{-46} \times 1.21}}$

$\lambda = 10^{-11}$ m $= 10 \times 10^{-12}$ m

$\alpha = 10$

24. (14) $y_{shift} = \frac{(\mu - 1)t.D}{d}$

$$20910^{-3} = \frac{(\mu - 1) \times 20 \times 10^{-6} \times 1}{0.4 \times 10^{-3}}$$

$(\mu - 1) \Rightarrow 0.4$

$\mu \Rightarrow 1.4$

$\frac{\alpha}{10} = 1.4, \alpha = 14$

25. **(350)** $w = 100\text{ J} = nR\Delta T$ for isobaric process.

$Q = nC_p\Delta T = \left(\frac{f}{2} + 1\right) nR\Delta T$

$= \frac{7}{2}.(100) = 350$ Joule.

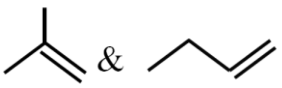
26. **(d)** $\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Series		$\mathbf{n_1}$	$\mathbf{n_2}$
(A)	Paschen (1^{st} line)	3	4
(B)	Balmer (2^{nd} line)	2	4
(C)	Paschen (3^{rd} line)	3	6
(D)	Bracket (4^{th} line)	4	8

So correct ascending order of energy of above lines is :

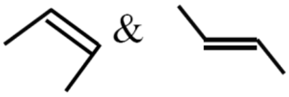
$D < A < C < B$

27. **(b)**



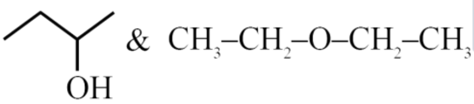
(A)

(III) Chain isomer



(B)

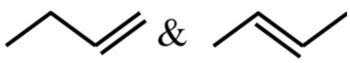
(I) Streoisomers



(C)

& $CH_3 - CH_2 - O - CH_2 - CH_3$

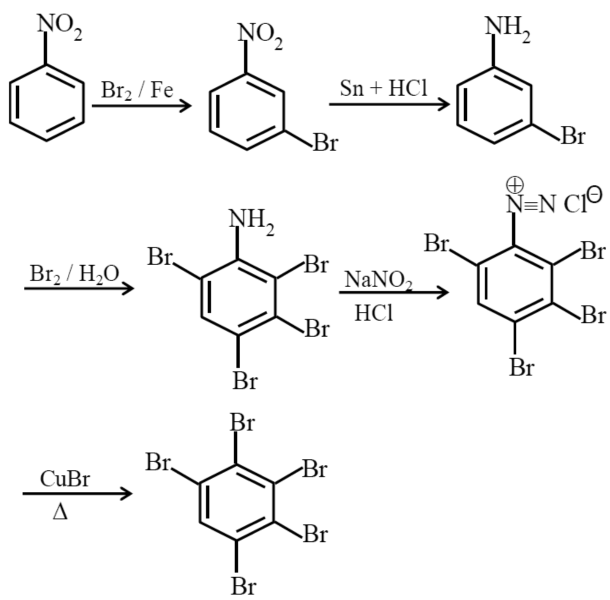
(IV) Functional isomers



(D)

(IV) Positional isomers

28. **(c)**



Number of Br atom in major product (P) = 5

29. **(a)** $MnO_2 + 4HCl \rightarrow MnCl_2 + Cl_2 + 2H_2O$

$\frac{8.7}{87}$ Excess

$= 0.1$ mole 0.1 mole

Wt. of Cl_2 obtained $= 0.1 \times 71 = 7.1$ g

30. **(a)** % of P $= \frac{n_{Mg_2P_2O_7} \times 2 \times 31}{W_{(\text{unknown compound})}} \times 100$

$= \frac{\left(\frac{1.79}{222} \times 2 \times 31\right)}{1} \times 100$

$= 49.99\% \approx 50\%$

31. **(a)** $C(s) + 2H_2(g) \rightarrow CH_4(g)$

$-x = (\Delta H_{sub}$ of carbon$) + 2 \times ($B.E. of H $-$ H$)$

$-4 \times ($ B.E. of C $-$ H$)$

$-x = y + 2z - 4($ B.E. of C $-$ H$)$

B.E. of C $-$ H $= \frac{y+2z+x}{4}$.

32. **(d)**

33. **(b)** $A_g \rightarrow B_g + C_g$

1 - -

1-P P P

$P_{total} = 1 + P$

$1.5 = 1 + P$

$P = 0.5$

$K = \frac{1}{100} \ln \frac{1}{0.5}$

$= \frac{0.693}{100}$

$= 6.9 \times 10^{-3}\ \text{min}^{-1}$

34. **(b)** This is S_N1 reaction.

Rate of S_N1 reaction ∝ stability of carbocation

35. **(b)** Correct order of size is $Mg > Al > Mg^{2+} > Al^{3+}$

Atomic size depends mainly upon $Z_{effective}$ and shell number.

Generally on moving down the group electron affinity decreases and on moving across the period electron affinity increase.

In the periodic table Cl has maximum electron affinity. Halogen has higher electron affinity than Chalcogen.

$Cl > Br > S > O$

36. **(c)** Activation energy for enzyme catalysed hydrolysis of sucrose is lower than that of acid catalysed hydrolysis. During denaturation secondary and tertiary structures of a protein are destroyed but primary structure remains intact.

37. **(d)** Stability order of anion

$$ClO_4^- > CH_3-\overset{\overset{\displaystyle O}{||}}{C}-O^- > Ph\text{–}O^- > OH^-$$

and nucleophlicity order reverse of stability of anion

$$OH^- > Ph\text{–}O^- > CH_3-\overset{\overset{\displaystyle O}{||}}{C}-O^- > ClO_4^-$$

38. **(a)**$[Mn(H_2O)_6]^{2+}$ $\Rightarrow$ CFSE value is zero because of d^5 configuraiton with WFL in coordination number 6 $[Cr(H_2O)_6]^{2+}$.

$[Cr(H_2O)_6]^{2+}$ $\Rightarrow$ CFSE value is $-0.6\Delta_0$ because of d^4 configuraiton with WFL in coordination number 6.

For : $K_3[Fe(CN)_6]$, $\mu = \sqrt{1(1+2)} = \sqrt{3}$ B.M.

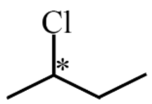
For : $Na_4[Fe(CN)_6]$, $\mu = \sqrt{0}$ B.M.

39. **(b)** $C-H(A)$ 107pm

$C \equiv N(D)$ 116pm

C-O(B) 143 pm

$C = O(C)$ 121pm

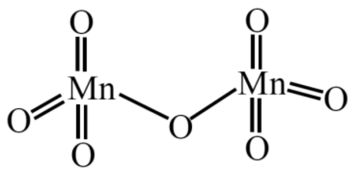
40. **(a)**  2-Chlorobutane is optically active and chiral molecule

Molecular formula $\Rightarrow C_4H_9Cl$

Molar mass $= 48 + 9 + 35.5 = 92.5$

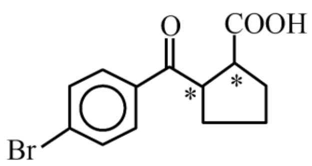
$\%OC = \dfrac{48}{92.5} \times 100 = 51.89\%$

41. **(b)**Mn_2O_7: Mn in +7 oxidation state.

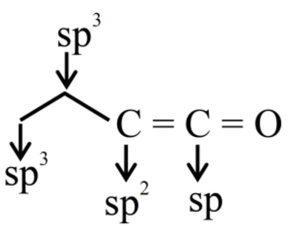


42. **(b)** E^0_{cell} remain constant with time.

43. **(c)**



Two chiral centre and due to presence of -COOH compound dissolves in $NaHCO_3$.



44. **(a) Statement-I :**

Bond energy order is $Cl_2 > Br_2 > F_2 > I_2$

Bond energy increases with increase in bond order.

Statement-II

Correct order of covalent character

According to the Fajan's rule, higher the charge on cation, greater is the covalent character.

$PbCl_2 < PbCl_4$,

$UF_6 > UF_4$,

$SnCl_4 > SnCl_2$

45. **(b)** $4NaCl + K_2Cr_2O_7 + 6H_2SO_4 \longrightarrow 2KHSO_4 + \underset{\text{(Chromyl chloride)}}{2CrO_2Cl_2} + 4NaHSO_4 + 3H_2O$

In Chromyl chloride Cr is in +6 oxidation state.

46. **(100)** $H_2X \rightleftharpoons H^+ + HX^-$

$0.1 - x \quad x + y \quad x - y$

$2.5 \times 10^{-8} = \frac{(x+y)(x-y)}{0.1 - x}$

$HX \rightleftharpoons H^+ + X^{2-}$

$x - y \quad x + y \quad y$

$1 \times 10^{-13} = \frac{(x+y)(y)}{x-y}$

Approximate : $K_{a_1} \gg K_{a_2} \Rightarrow$ So $x \gg y$.

$x + y \approx x, x - y \approx x$

$10^{-13} = \frac{x \cdot y}{x}$

$y = 10^{-13}$

$[X^{2-}] = 10^{-13}$

$[X^{2-}] = 100 \times 10^{-15}$

47. **(15)** $\pi = iCRT$

$12 = 2 \times C \times 0.08 \times 300$

$12 = 2 \times C \times 24$

$C = \frac{1}{4}$ mole/L

then strength of NaCl solution

$= \frac{1}{4} \times 58.5$ g/L

$= 14.625$ g/L

$= 15$ g/L

48. **(3)** $\Delta T_b = i \times K_b \times m$

$0.5 = i \times m \times 5$

$i \times m = \frac{0.5}{5} = 0.1$

$i \times a = \frac{15}{1000}$

(where a = moles of solute)

Now,

$\frac{P_o - P_S}{P^\circ} = iX_{\text{soltot}} = i \times \frac{a}{a + \frac{150}{300}}$

$= i \times \frac{a}{1/2} = \frac{15/1000}{1/2} = \frac{30}{1000} = 3 \times 10^{-2} = 3$

49. **(7)** $V^{3+} = (Ar)_{18} 3\,d^2$

$Ti^{2+} = (Ar)_{18} 3\,d^2$

$Ni^{2+} = (Ar)_{18} 3\,d^8$

$Fe^{2+} = (Ar)_{18} 3\,d^6$

$Co^{2+} = (Ar)_{18} 3\,d^7$

Only for Fe^{2+} and Co^{2+} μ is more than 3.0 B.M.

$\boxed{\uparrow}\boxed{\uparrow}$

$Fe^{2+}\ 3d^6$ $\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}$ $n = 4, \mu > 3$

$\boxed{\uparrow}\boxed{\uparrow}$

$Co^{2+}\ 3d^7$ $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}$ $n = 3,\ \mu > 3$

$\therefore$ Number of unpaired electrons $= 4 + 3 = 7$

50. (200) $E^0_{\frac{X^-}{\frac{MX(s)}{M}}} = E^0_{\frac{M^+}{M}} + \frac{0.0591}{n}\log K_{sp}$

$= 0.79 + \frac{0.059}{1}\log 10^{-10}$

$= 0.79 - 0.59$

$= 0.20\text{ V} = 200\text{mV}$

51. (a) $x(x+2) + (x+2)(x+4) + \cdots. + (x+2n-2)(x+2n) = \frac{8n}{3}$

$\Rightarrow \sum_{r=1}^{n}(x+2r-2)(x+2r) = \frac{8n}{3}$

$nx^2 + 2x\sum_{r=1}^{n}(2r-1) + 4\sum_{r=1}^{n}r(r-1) = \frac{8n}{3}$

$nx^2 + 2x\cdot n^2 + \frac{4n(n^2-1)}{3} - \frac{8n}{3} = 0$

$x^2 + 2nx + \frac{4(n^2-1)}{3} - \frac{8}{3} = 0 <^{\alpha}_{\beta}$

$\because |\alpha - \beta| = 2 \Rightarrow \frac{\sqrt{D}}{|a|} = 2 \Rightarrow D = 4$

$\Rightarrow 4n^2 - 4\left(4\frac{(n^2-1)}{3} - \frac{8}{3}\right) = 4$

$\Rightarrow n^2 - \frac{4n^2}{3} = -3$

$\Rightarrow n^2 = 9$

$\Rightarrow n = 3$

52. (a) $g(x) = f((\tan x - 1)^2 + a - 1)$

$g'(x) = f'((\tan x - 1)^2 + a - 1)\cdot 2(\tan x - 1)\sec^2 x$

$\because f'(a-1) = 0$ and $f''(x) > 0$

$\therefore f'((\tan x - 1)^2 + a - 1) > 0$

$g'(x) > 0$ if $(\tan x - 1) > 0$

g is increasing in $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$g'(x) < 0$ if $\tan x - 1 < 0$

g is decreasing in $x \in \left(0, \frac{\pi}{4}\right)$

53. (d) $f'(x) = 3x^2 + 2xf'(1) + 2f''(2)$

$f''(x) = 6x + 2f'(1)$

$f''(2) = 12 + 2f'(1)$

$\therefore f'(x) = 3x^2 + 2xf'(1) + 2(12 + 2f'(1))$

$f'(x) = 3x^2 + 2(x+2)f'(1) + 24$

Putting, $x = 1$

$f'(1) = 3 + 6f'(1) + 24$

$-5f'(1) = 27 \Rightarrow f'(1) = \frac{-27}{5}$

$\therefore f''(2) = 12 + 2\left(\frac{-27}{5}\right) = 12 - \frac{54}{5} = \frac{6}{5}$

$\therefore f'(x) = 3x^2 - \frac{54}{5}x + \frac{12}{5}$

$\therefore f'(5) = 75 - 54 + \frac{12}{5} = \frac{117}{5}$

54. (d) $\alpha x + 4y - \sqrt{7} = 0$ touches $3x^2 + 4y^2 = 1$

$\therefore c^2 = a^2 m^2 + b^2$

$\frac{7}{16} = \frac{1}{3} \times \frac{\alpha^2}{16} + \frac{1}{4} \Rightarrow \alpha = 3, -3$

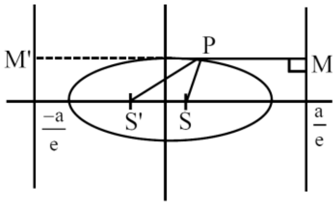
Tangent is $3x + 4y - \sqrt{7} = 0$

Let the point of contact is $P(x_1 y_1)$

$\therefore$ Tangent is $3xx_1 + 4yy_1 = 1$

$\therefore \frac{3x_1}{3} = \frac{4y_1}{4} = \frac{1}{\sqrt{7}} \qquad \therefore P\left(\frac{1}{\sqrt{7}}, \frac{1}{\sqrt{7}}\right)$

$e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$



$PS = e(PM)$

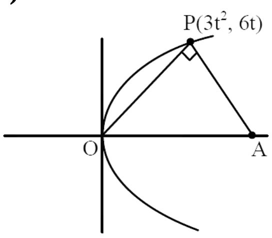
$= e\left(\frac{a}{e} - \frac{1}{\sqrt{7}}\right)$

$= \frac{1}{2}\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{7}}\right) = \frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{7}}$

$PS' = e(PM') = \frac{1}{2}\left(\frac{a}{e} + \frac{1}{\sqrt{7}}\right) = \frac{1}{2}\left(\frac{1}{\sqrt{7}} + \frac{2}{\sqrt{3}}\right)$

$= \frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$

55. (c)



$m_{AP} = \frac{-t}{2}$

Equation of AP is

$y - 6t = \frac{-t}{2}(x - 3t^2)$

Put $y = 0 \Rightarrow x = 12 + 3t^2$

$\Rightarrow A(12 + 3t^2, 0)$

Let centroid of $\triangle$ OPA be $G(h, k)$

$\Rightarrow 3h = 0 + 3t^2 + 12 + 3t^2$

$3k = 0 + 6t + 0$

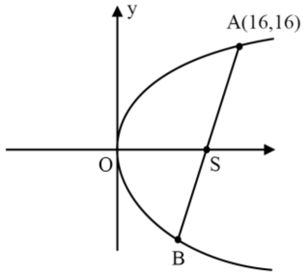
$\Rightarrow t = \frac{k}{2}, h = 2t^2 + 4$

$\Rightarrow h = 2\frac{k^2}{4} + 4$

$\Rightarrow$ Locus of (h, k) is

$y^2 = 2x - 8$

56. (b) $y^2 = 16x$

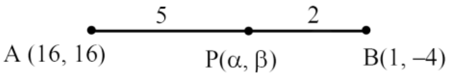


$\because$ parameter of point A is $t = 2$

$\Rightarrow$ Parameter of point B is $t = -\frac{1}{2}$

$\Rightarrow$ Coordinates of B is $(1, -4)$

Case 1:

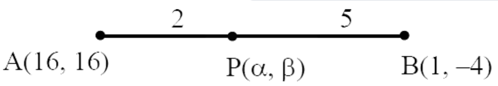


$$\alpha = \frac{5 + 32}{7} = \frac{37}{7}$$

$$\beta = \frac{-20 + 32}{7} = \frac{12}{7}$$

$\Rightarrow \alpha + \beta = 7$

Case 2 :



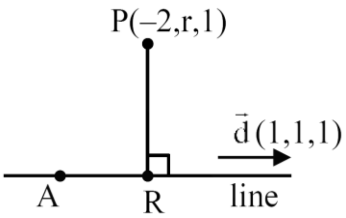
$\alpha = \frac{2+80}{7}$, $\quad\quad \beta = \frac{-8+80}{7}$

$\alpha + \beta = 22$

So minimum value of $\alpha + \beta = 7$

57. (d) Equation line is : $\frac{x+3}{1} = \frac{y-5}{1} = \frac{z-2}{1} = \lambda$

$\therefore$ General point R on line is $R(\lambda - 3, \lambda + 5, \lambda + 2)$



$\overrightarrow{PR} \equiv (\lambda - 1, \lambda + 5 - r, \lambda + 1)$

Now $\overrightarrow{PR} \cdot \vec{d} = 0$

$\Rightarrow (\lambda - 1)1 + (\lambda + 5 - r)1 + (\lambda + 1)1 = 0$

$\Rightarrow 3\lambda - r + 5 = 0$

$$\Rightarrow \lambda = \frac{r - 5}{3}$$

$$\therefore R \equiv \left(\frac{r - 5}{3} - 3, \frac{r - 5}{3} + 5, \frac{r - 5}{3} + 2\right)$$

$$R \equiv \left(\frac{r - 14}{3}, \frac{r + 10}{3}, \frac{r + 1}{3}\right)$$

Now

$$PR = \sqrt{\frac{14}{3}} \Rightarrow (PR)^2 = \frac{14}{3}$$

$$\Rightarrow \left(\frac{r - 14}{3} + 2\right)^2 + \left(\frac{r + 10}{3} - r\right)^2 + \left(\frac{r + 1}{3} - 1\right)^2 = \frac{14}{3}$$

$$\Rightarrow \frac{(r - 8)^2}{9} + \frac{(10 - 2r)^2}{9} + \frac{(r - 2)^2}{9} = \frac{14}{3}$$

$\Rightarrow (r^2 - 16r + 64) + (100 + 4r^2 - 40r) + (r^2 - 4r + 4) = 42$
$\Rightarrow 6r^2 - 60r + 126 = 0$
$\Rightarrow r^2 - 10r + 21 = 0$
$\Rightarrow r = 3,7$
sum of possible value of r is $= 10$

58. (b) $L_1 : \frac{x-2}{-3} = \frac{y-6}{2} = \frac{z-7}{4}$
Point C on $L_1 : (-3\lambda_1 + 2, 2\lambda_1 + 6, 4\lambda_1 + 7)$
$L_2 : \frac{x-4}{2} = \frac{y-3}{1} = \frac{z-5}{3}$
Point D on $L_2 : (2\lambda_2 + 4, \lambda_2 + 3, 3\lambda_2 + 5)$
Dr's of line L_3 :
$L_3 : \frac{2\lambda_2 + 3\lambda_1 + 2}{-3} = \frac{\lambda_2 - 2\lambda_1 - 3}{5} = \frac{3\lambda_2 - 4\lambda_1 - 2}{16}$
$\lambda_1 = -3, \lambda_2 = 2$
C (11, 0, -5)
D (8, 5, 11)
$|\overrightarrow{CD}|^2 = 3^2 + 5^2 + 16^2 = 290$

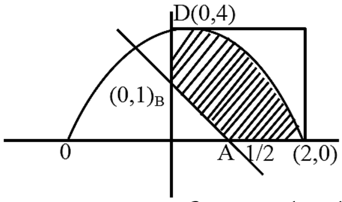
59. (d) $\because \alpha < 1 < \beta$
$f(1) < 0$
$\Rightarrow 1 + 2a + (3a + 10) < 0$
$\Rightarrow 5a + 11 < 0$
$a < \frac{-11}{5}$
$\therefore a \in \left(-\infty, \frac{-11}{5}\right)$

60. (c)

x	0	1	2	3
p(x)	$\frac{2a+1}{30}$	$\frac{8a-1}{30}$	$\frac{4a+1}{30}$	b

$\sigma^2 = \sum x_i^2 p(x_i) - \mu^2$
$\sigma^2 + \mu^2 = \sum x_i^2 p(x_i)$
$= 0 + 1\left(\frac{8a-1}{30}\right) + 4\left(\frac{4a+1}{30}\right) + 9b$
$\Rightarrow \frac{24a + 270b + 3}{30} = 2$
$24a + 270b = 57$
$8a + 90b = 19 \ldots.(1)$
Also
$\sum p\ (i) = 1$
$\frac{2a+1}{30} + \frac{8a-1}{30} + \frac{4a+1}{30} + b = 1$
$14a + 30\ b = 29 \quad \ldots\ldots\ldots\ldots\ldots (2)$
Solving (1) & (2)
$a = 2,\ b = \frac{1}{30}, \frac{a}{b} = 60$

61. (a)



Required area $= \frac{2}{3} \times 8 - \frac{1}{2} \times \frac{1}{2} \times 1$

$= \frac{16}{3} - \frac{1}{4} = \frac{61}{12} = \frac{\alpha}{\beta}$

$\Rightarrow \alpha + \beta = 73$

62. (a) $\frac{a_2}{2a_1} = \frac{a_3}{2a_2} = \frac{a_4}{2a_3} = \cdots . = \frac{a_{10}}{2a_9} = \frac{1}{\sqrt{2}}$

$\therefore a_1, a_2, a_3, ., a_{10}$ are in G.P. with common ratio $\sqrt{2}$.

$\sum_{i=1}^{10} a_i = \frac{a_1\left((\sqrt{2})^{10} - 1\right)}{\sqrt{2} - 1} = 62$

$\Rightarrow a_1 = 2(\sqrt{2} - 1)$

63. (d) $|x^2 - 10| \leq 6$

$-6 \leq x^2 - 10 \leq 6$

$4 \leq x^2 \leq 16$

$A = [-4, -2] \cup [2,4]$

$|x - 2| > 1$

$B = (-\infty, 1) \cup (3, \infty)$

$A \cup B = (-\infty, 1) \cup [2, \infty)$

$A \cap B = [-4, -2] \cup (3,4]$

$A - B = [2,3]$

$B - A = (-\infty, -4) \cup (-2,1) \cup (4, \infty)$

64. (c) Here $A^n = \begin{bmatrix} 2n+1 & -4n \\ n & -2n+1 \end{bmatrix}$

$\Rightarrow A^{15} = \begin{bmatrix} 31 & -60 \\ 15 & -29 \end{bmatrix}$

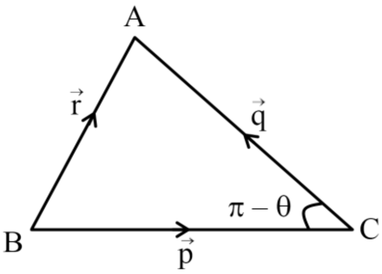
$\Rightarrow A^{15} + B = \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix}$

Now $(A^{15} + B)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow 2x - 11y = 0$

65. (d)



$\vec{p} + \vec{q} = \vec{r}$

$\cos(\pi - \theta) = \frac{|\vec{p}|^2 + |\vec{q}|^2 - |\vec{r}|^2}{2|\vec{p}||\vec{q}|}$

$\frac{-1}{\sqrt{3}} = \frac{12 + 4 - |\vec{r}|^2}{2 \cdot 2\sqrt{3} \cdot 2}$

$|\vec{r}|^2 = 24$

$\therefore |\vec{p} \times (\vec{q} - 3\vec{r})|^2 + 3|\vec{r}|^2$

$= |\vec{p} \times (\vec{q} - 3\vec{p} - 3\vec{q})|^2 + 72$
$= |\vec{p} \times (-3\vec{p} - 2\vec{q})|^2 + 72$
$= |-2\vec{p} \times \vec{q}|^2 + 72$
$= 4|\vec{p}|^2|\vec{q}|^2 \times \sin^2\theta + 72$
$= 4 \cdot 12.4 \cdot \dfrac{2}{3} + 72$
$= 200$

66. (a) $\frac{dy}{dx} - 2y\cos x = 2\cos x + 3\sin x \cdot \cos x$

I.F. $= e^{-2\sin x}$

$e^{-2\sin x} \cdot y = \int e^{-2\sin x}(3\sin x\cos x + 2\cos x)dx$

$y.e^{-2\sin x} = e^{-2\sin x}\left(-\dfrac{3}{2}\sin x - \dfrac{7}{4}\right) + C$

$\Rightarrow y = -\dfrac{3}{2}\sin x - \dfrac{7}{4} + C.e^{2\sin x}$

$\because y(0) = -\dfrac{7}{4} \Rightarrow C = 0$

$y\left(\dfrac{\pi}{6}\right) = \dfrac{-3}{2} \cdot \dfrac{1}{2} - \dfrac{7}{4} = \dfrac{-5}{2}$

67. (b) $A = \{2,3,5,7,9\}$

$y \geq \dfrac{2x}{3}$

$$\left.\begin{array}{ll} x = 2, & y = 2,3,5,7,9 \\ x = 3, & y = 2,3,5,7,9 \\ x = 5, & y = 5,7,9 \\ x = 7, & y = 5,7,9 \\ x = 9 & y = 7,9 \end{array}\right] \rightarrow \ell = 18$$

to make it symmetric elements to be added are $\{(5,2),(7,2),(9,2),(5,3),(7,3),(9,3),(9,5)\}$

$m = 7$

$\therefore\ \ell + m = 25$

68. (a) for no solution $\Delta = 0$

$$\begin{vmatrix} 3 & 1 & 4 \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 3(a+2) + 1(-1-2) + 4(4-\alpha) = 0$

$\Rightarrow 19 - \alpha = 0 \Rightarrow \alpha = 19$

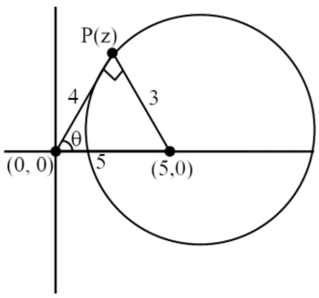
& for $\alpha = 19$

$$\Delta_x = \begin{vmatrix} 3 & 1 & 4 \\ -3 & 19 & -1 \\ 4 & 2 & 1 \end{vmatrix} = 3(21) + 1(-1) + 4(-82)$$

$\neq 0$

$\therefore$ no solution for $\alpha = 19$

69. (d)



$|z - 5| \leq 3$

For $\arg(z)$ to be maximum, z lies at P .

$z \equiv (4\cos\theta, 4\sin\theta)$

$\equiv\left(4\cdot\left(\frac{4}{5}\right),4\left(\frac{3}{5}\right)\right)=\left(\frac{16}{5},\frac{12}{5}\right)=\frac{16}{5}+\frac{12i}{5}$

Now, $34\left|\frac{5z-12}{5iz+16}\right|^2=34\left|\frac{(16+12i)-12}{(16i-12)+16}\right|^2$

$=34\left|\frac{4+12i}{16i+4}\right|^2$

$=34\left(\frac{16+144}{256+16}\right)=34\left(\frac{160}{272}\right)=20$

70. (a) Exponent of 7 in 101!

$=\left[\frac{101}{7}\right]+\left[\frac{101}{7^2}\right]+\left[\frac{101}{7^3}\right]+\cdots\ldots$

$=14+2=16$

71. (2) $\sum_{k=1}^{n}\left(\frac{k^2}{3^x}-1\right)<\sum_{k=1}^{n}\left[\frac{k^2}{3^x}\right]\le\sum_{k=1}^{n}\frac{k^2}{3^x}$

$\frac{n(n+1)(2n+1)}{6.3^x}<\sum_{k=1}^{n}\left[\frac{k^2}{3^x}\right]\le\frac{n(n+1)(2n+1)}{6.3^x}$

$\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3\cdot3^x}<\lim_{n\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}\left[\frac{k^2}{3^x}\right]\le\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6\cdot3^x\cdot n^3}$

$\frac{1}{3^{x+1}}<\lim_{n\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}\left[\frac{k^2}{3^x}\right]\le\frac{1}{3^{x+1}}$

$\Rightarrow f(x)=\frac{1}{3^{x+1}}$

$\Rightarrow 12\sum_{j=1}^{\infty}f(j)=12\sum_{j=1}^{\infty}\frac{1}{3^{j+1}}=12\left[\frac{1}{9}+\frac{1}{27}+--\infty\right]$

$=12\cdot\left(\frac{\frac{1}{9}}{1-\frac{1}{3}}\right)=2$

72. (9) Let $I=\int_0^1\cot^{-1}(1-2x+4x^2)dx$

$I=\int_0^1(\cot^{-1}(2x-1)-\cot^{-1}(2x))dx$ …………….. (1)

Applying king

$I=\int_0^1(-\cot^{-1}(2x-1)+\cot^{-1}(2x-2))dx$ …………….. (2)

From (1) & (2)

$2I=\int_0^1(\cot^{-1}(2x-2)-\cot^{-1}(2x))dx$

$=\int_0^1\cot^{-1}(2x-2)dx-\int_0^1\cot^{-1}(2x)dx$

Applying King

$=\int_0^1\cot^{-1}(-2x)dx-\int_0^1\cot^{-1}(2x)dx$

$=\int_0^1(\pi-\cot^{-1}(2x))dx-\int_0^1\cot^{-1}(2x)dx$

$=\int_0^1(\pi-2\cot^{-1}(2x))dx$

$=\pi-2\int_0^1(\cot^{-1}2x)\cdot1dx$

By parts

$=\pi-2\left[(x\cot^{-1}2x)_0^1+\int_0^1\frac{2x}{1+4x^2}dx\right]$

Let $1+4x^2=t$

$8xdx=dt$

$=\pi-2\left[\cot^{-1}2+\frac{1}{4}\int_1^5\frac{dt}{t}\right]$

$=\pi-2\cot^{-1}2-\frac{1}{2}\ell n5$

$2I = 2\tan^{-1}2 - \frac{1}{2}\ln 5$

$\Rightarrow 4I = 4\tan^{-1}2 - \ell n5$

$\therefore 2a + b = 8 + 1 = 9$

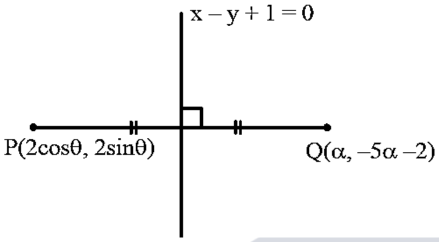
73. (32) $\prod_{r=0}^{12}\left(\frac{1}{^{15}C_r} + \frac{1}{^{15}C_{r+1}}\right) = \prod_{r=0}^{12}\frac{\frac{16}{r+1}\cdot {^{15}C_r}}{^{15}C_r\cdot {^{15}C_{r+1}}}$

$= \prod_{r=0}^{12}\frac{16}{(r+1)\cdot\frac{15}{r+1}\cdot {^{14}C_r}} = \prod_{r=0}^{12}\frac{\left(\frac{16}{15}\right)}{^{14}C_r}$

$= \frac{\left(\frac{16}{15}\right)^{13}}{^{14}C_0\cdot {^{14}C_1}\ {^{14}C_{12}}} \Rightarrow \alpha = \frac{16}{15}$

$\Rightarrow 30\alpha = 32$

74. (2)



Mid point of PQ lies on $x - y + 1 = 0$

$\frac{2\cos\theta + \alpha}{2} - \frac{2\sin\theta - 5\alpha - 2}{2} + 1 = 0$

$2\cos\theta + \alpha - 2\sin\theta + 5\alpha + 2 + 2 = 0$

$\cos\theta - \sin\theta + 3\alpha + 2 = 0$ ………… (i)

$\because$ Slope of PQ is -1

$\frac{2\sin\theta + 5\alpha + 2}{2\cos\theta - \alpha} = -1$

$2\sin\theta + 5\alpha + 2 = -2\cos\theta + \alpha$

$\sin\theta + \cos\theta + 2\alpha + 1 = 0$ ………… (ii)

eliminate α from (i) and (ii)

$\Rightarrow \cos\theta + 5\sin\theta = 1, \theta \in [0,2\pi]$

$\Rightarrow 5 \times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\sin^2\frac{\theta}{2}$

$\therefore \sin\frac{\theta}{2} = 0 \Rightarrow \cos\theta = 1$

or

$\sin\frac{\theta}{2} = 5 \Rightarrow \cos\theta = -\frac{12}{13}$

Sum of all possible values of abscissa of point P is

$= 2\times 1 + 2\left(\frac{-12}{13}\right) = \frac{2}{13}$

$\therefore$ 13 times sum of all possible values of abscissa of point P is 2 .