

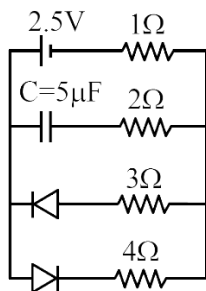
PHYSICS

SECTION-A

1. Consider two identical metallic spheres of radius R each having charge Q and mass m . Their centers have an initial separation of $4R$. Both the spheres are given an initial speed of u towards each other. The minimum value of u , so that they can just touch each other is : (Take $k = \frac{1}{4\pi\epsilon_0}$ and assume $kQ^2 > Gm^2$ where G is the Gravitational constant)

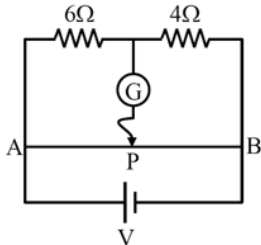
- (a) $\sqrt{\frac{kQ^2}{4mR} \left(1 - \frac{Gm^2}{kQ^2}\right)}$ (b) $\sqrt{\frac{kQ^2}{4mR} \left(1 + \frac{Gm^2}{kQ^2}\right)}$
 (c) $\sqrt{\frac{kQ^2}{2mR} \left(1 - \frac{Gm^2}{kQ^2}\right)}$ (d) $\sqrt{\frac{kQ^2}{2mR} \left(1 - \frac{Gm^2}{2kQ^2}\right)}$

2. The charge stored by the capacitor C in the given circuit in the steady state is ____ μC .



- (a) 12.5 (b) 10
(c) 7.5 (d) 5

3. The total length of potentiometer wire AB is 50 cm in the arrangement as shown in figure. If P is the point where the galvanometer shows zero reading then the length AP is ____ cm .



- (a) 15 (b) 30
(c) 25 (d) 20

4. A capacitor C is first charged fully with potential difference of V_0 and disconnected from the battery. The charged capacitor is connected across an inductor having inductance L . In t s 25% of the initial energy in the capacitor is transferred to the inductor. The value of t is ____ s.

- (a) $\frac{\pi\sqrt{LC}}{3}$ (b) $\frac{\pi\sqrt{LC}}{6}$
(c) $\frac{\pi\sqrt{LC}}{2}$ (d) $\pi\sqrt{\frac{LC}{2}}$

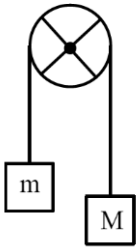
5. The r.m.s speed of oxygen molecules at 47°C is equal to that of the hydrogen molecules kept at ____ $^\circ\text{C}$. (Mass of oxygen molecule/mass of hydrogen molecule = 32/2)

- (a) -235 (b) -100
(c) -253 (d) -20

6. Two cars A and B each of mass 10^3 kg are moving on parallel tracks separated by a distance of 10 m, in same direction with speeds 72 km/h and 36 km/h. The magnitude of angular momentum of car A with respect to car B is ____ J.s.

- (a) 3.6×10^5 (b) 10^5
(c) 3×10^5 (d) 2×10^5

7. The pulley shown in figure is made using a thin rim and two rods of length equal to diameter of the rim. The rim and each rod have a mass of M . Two blocks of mass of M and m are attached to two ends of a light string passing over the pulley, which is hinged to rotate freely in vertical plane about its centre. The magnitudes of the acceleration experienced by the blocks is ____ (assume no slipping of string on pulley.)



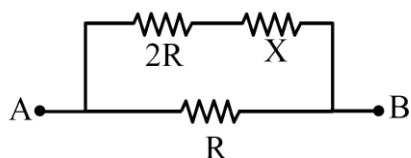
- (a) $\frac{(M-m)g}{\left[\left(\frac{13}{6}\right)M+m\right]}$ (b) $\frac{(M-m)g}{M+m}$
 (c) $\frac{(M-m)g}{\left[\left(\frac{8}{3}\right)M+m\right]}$ (d) $\frac{(M-m)g}{2M+m}$
8. The kinetic energy of a simple harmonic oscillator is oscillating with angular frequency of 176rad/s . The frequency of this simple harmonic oscillator is ____ Hz. [take $\pi = \frac{22}{7}$]
- (a) 14 (b) 88
 (c) 28 (d) 176
9. Given below are two statements:
Statement I: In a Young's double slit experiment, the angular separation of fringes will increase as the screen is moved away from the plane of the slits
Statement II : In a Young's double slit experiment, the angular separation of fringes will increase when monochromatic source is replaced by another monochromatic source of higher wavelength
In the light of the above statements, choose the correct answer from the options given below:
 (a) Both Statement I and Statement II are true
 (b) Both Statement I and Statement II are false
 (c) Statement I is false but Statement II is true
 (d) Statement I is true but Statement II is false
10. A battery with EMF E and internal resistance r is connected across a resistance R . The power consumption in R will be maximum when :
 (a) $R = 2r$ (b) $R = \frac{r}{2}$
 (c) $R = \sqrt{2}r$ (d) $R = r$
11. Keeping the significant figures in view, the sum of the physical quantities 52.01 m , 153.2 m and 0.123 m is :
 (a) 205 m (b) 205.333 m
 (c) 205.33 m (d) 205.3 m
12. A spherical body of radius r and density σ falls freely through a viscous liquid having density ρ and viscosity η and attains a terminal velocity v_0 . Estimated maximum error in the quantity η is : (Ignore errors associated with σ , ρ and g , gravitational acceleration)
 (a) $2 \frac{\Delta r}{r} - \frac{\Delta v_0}{v_0}$ (b) $\frac{2\Delta r}{r} + \frac{\Delta v_0}{v_0}$
 (c) $2 \left[\frac{\Delta r}{r} + \frac{\Delta v_0}{v_0} \right]$ (d) $2 \left[\frac{\Delta r}{r} - \frac{\Delta v_0}{v_0} \right]$
13. Surface tension of two liquids (having same densities), T_1 and T_2 , are measured using capillary rise method utilizing two tubes with inner radii of r_1 and r_2 where $r_1 > r_2$. The measured liquid heights in these tubes are h_1 and h_2 respectively. [Ignore the weight of the liquid about the lowest point of meniscus]. The heights h_1 and h_2 and surface tensions T_1 and T_2 satisfy the relation :
 (a) $h_1 < h_2$ and $T_1 = T_2$
 (b) $h_1 = h_2$ and $T_1 = T_2$
 (c) $h_1 > h_2$ and $T_1 = T_2$

(d) $h_1 > h_2$ and $T_1 < T_2$

14. A river of width 200 m is flowing from west to east with a speed of 18 km/h. A boat, moving with speed of 36 km/h in still water, is made to travel one-round trip (bank to bank of the river). Minimum time taken by the boat for this journey and also the displacement along the river bank are ___ and ___ respectively.

(a) 20 s and 100 m (b) 40 s and 0 m
(c) 40 s and 200 m (d) 40 s and 100 m

15. Two known resistance of $R\Omega$ and $2R\Omega$ and one unknown resistance $X\Omega$ are connected in a circuit as shown in the figure. If the equivalent resistance between points A and B in the circuit is $X\Omega$, then the value of X is ___ Ω .



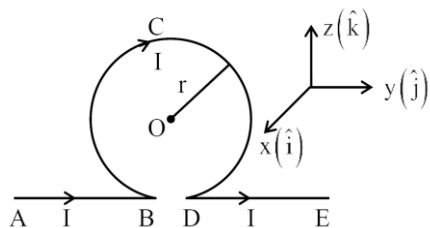
(a) $(\sqrt{3} - 1)R$ (b) R
(c) $2(\sqrt{3} - 1)R$ (d) $(\sqrt{3} + 1)R$

16. The energy of an electron in an orbit of the Bohr's atom is $-0.04E_0$ eV where E_0 is the ground state energy.

If L is the angular momentum of the electron in this orbit and h is the Planck's constant, then $\frac{2\pi L}{h}$ is ___ :

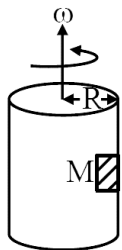
(a) 2 (b) 4
(c) 5 (d) 6

17. An infinitely long straight wire carrying current I is bent in a planer shape as shown in the diagram. The radius of the circular part is r . The magnetic field at the centre O of the circular loop is :



(a) $\frac{\mu_0 I}{2\pi r}(\pi + 1)\hat{i}$ (b) $-\frac{\mu_0 I}{2\pi r}(\pi - 1)\hat{i}$
(c) $\frac{\mu_0 I}{2\pi r}(\pi - 1)\hat{i}$ (d) $-\frac{\mu_0 I}{2\pi r}(\pi + 1)\hat{i}$

18. A large drum having radius R is spinning around its axis with angular velocity ω , as shown in figure. The minimum value of ω so that a body of mass M remains stuck to the inner wall of the drum, taking the coefficient of friction between the drum surface and mass M is μ , is :

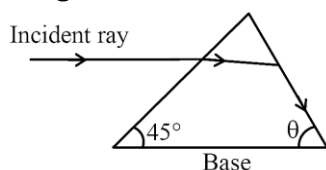


(a) $\sqrt{\frac{\mu g}{R}}$ (b) $\sqrt{\frac{2g}{\mu R}}$
(c) $\sqrt{\frac{g}{2\mu R}}$ (d) $\sqrt{\frac{g}{\mu R}}$

19. A body of mass 2 kg is moving along x-direction such that its displacement as function of time is given by $x(t) = \alpha t^2 + \beta t + \gamma$ m, where $\alpha = 1 \text{ m/s}^2$, $\beta = 1 \text{ m/s}$ and $\gamma = 1 \text{ m}$. The work done on the body during the time interval $t = 2 \text{ s}$ to $t = 3 \text{ s}$, is ___ J.

- (a) 49 (b) 42
(c) 24 (d) 12

20. As shown in the diagram, when the incident ray is parallel to base of the prism, the emergent ray grazes along the second surface.



If refractive index of the material of prism is $\sqrt{2}$, the angle θ of prism is.

- (a) 60° (b) 75°
(c) 90° (d) 45°

SECTION-B

21. An electromagnetic wave of frequency 100 MHz propagates through a medium of conductivity, $\sigma = 10 \text{ mho/m}$.

The ratio of maximum conducting current density to maximum displacement current density is ____.

[Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$]

22. The terminal velocity of a metallic ball of radius 6 mm in a viscous fluid is 20 cm/s. The terminal velocity of another ball of same material and having radius 3 mm in the same fluid will be ____ cm/s.

23. A particle having electric charge $3 \times 10^{-19} \text{ C}$ and mass $6 \times 10^{-27} \text{ kg}$ is accelerated by applying an electric potential of 1.21 V. Wavelength of the matter wave associated with the particle is $\alpha \times 10^{-12} \text{ m}$. The value of α is ____.

(Take Planck's constant = $6.6 \times 10^{-34} \text{ J.s}$)

24. In a Young's double slit experiment set up, the two slits are kept 0.4 mm apart and screen is placed at 1 m from slits. If a thin transparent sheet of thickness $20 \mu\text{m}$ is introduced in front of one of the slits then centre fringe shifts by 20 mm on the screen. The refractive index of transparent sheet is given by $\frac{\alpha}{10}$, where α is ____.

25. A diatomic gas ($\gamma = 1.4$) does 100 J of work when it is expanded isobarically. Then the heat given to the gas ____ J.

CHEMISTRY

SECTION-A

26. Consider the following spectral lines for atomic hydrogen:

- (A) First line of Paschen series
(B) Second line of Balmer series
(C) Third line of Paschen series
(D) Fourth line of Bracket series

The correct arrangement of the above lines in ascending order of energy is:

- (a) $D < C < A < B$ (b) $A < B < C < D$
(c) $C < D < B < A$ (d) $D < A < C < B$

27. Match List-I with List-II.

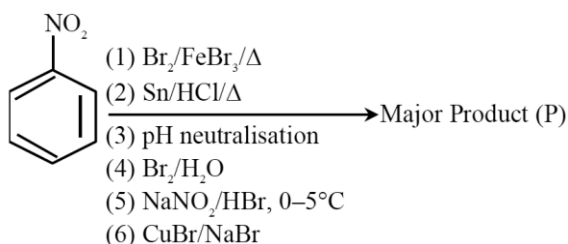
List-I Pair of Compounds		List-II Type of Isomers	
(A)	2-Methylpropene and but-1-ene	(I)	Stereoisomers
(B)	Cis-but-2-ene and trans-but-2-ene	(II)	Position isomers

(C)	2-Butanol and diethyl ether	(III)	Chain isomers
(D)	But-1-ene and but-2-ene	(IV)	Functional group isomers

Choose the correct answer from the options given below:

- (a) A-III, B-I, C-IV, D-II (b) A-III, B-I, C-II, D-IV
(c) A-I, B-IV, C-III, D-II (d) A-II, B-I, C-IV, D-III

28.



Consider the above sequence of reactions. The number of bromine atom(s) in the final product (P) will be:

- (a) 1 (b) 6
(c) 5 (d) 3

29. Aqueous HCl reacts with $\text{MnO}_2(\text{s})$ to form $\text{MnCl}_2(\text{aq})$, $\text{Cl}_2(\text{g})$ and $\text{H}_2\text{O}(\text{l})$. What is the weight (in g) of Cl_2 liberated when 8.7 g of $\text{MnO}_2(\text{s})$ is reacted with excess aqueous HCl solution ? (Given Molar mass in gmol^{-1} Mn = 55, Cl = 35.5, O = 16, H = 1)

- (a) 7.1 (b) 71
(c) 21.3 (d) 14.2

30. By usual analysis, 1.00 g of compound (X) gave 1.79 g of magnesium pyrophosphate. The percentage of phosphorus in compound (X) is: (nearest integer)

(Given, molar mass in gmol^{-1} : O = 16, Mg = 24, P = 31)

- (a) 50 (b) 30
(c) 20 (d) 40

31. Consider the following data:

$$\Delta_f H^\ominus(\text{methane, g}) = -X \text{ kJmol}^{-1}$$

$$\text{Enthalpy of sublimation of graphite} = Y \text{ kJmol}^{-1}$$

$$\text{Dissociation enthalpy of } \text{H}_2 = Z \text{ kJmol}^{-1}$$

The bond enthalpy of C – H bond is given by:

- (a) $\frac{X+Y+2Z}{4}$ (b) $\frac{X+Y+4Z}{2}$
(c) $X + Y + Z$ (d) $\frac{-X+Y+Z}{4}$

32. Match List-I with List-II.

List-I Reagents		List-II Reaction Name (Involving aldehydes)	
(A)	$\text{H}_2, \text{Pd} - \text{BaSO}_4$	(I)	Etard Reaction
(B)	$\text{SnCl}_2, \text{HCl}$	(II)	Rosenmund Reduction
(C)	$\text{CrO}_2\text{Cl}_2, \text{CS}_2$	(III)	Gatterman-Koch Reaction

(D)	CO, HCl, Anhyd. AlCl ₃	(IV)	Stephen Reaction
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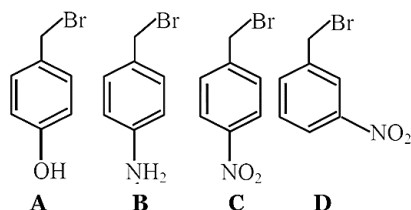
Choose the correct answer from the options given below:

- (a) A-II, B-III, C-IV, D-I (b) A-IV, B-III, C-I, D-II
(c) A-IV, B-I, C-II, D-III (d) A-II, B-IV, C-I, D-III

33. Decomposition of A is a first order reaction at T(K) and is given by $A(g) \rightarrow B(g) + C(g)$. In a closed 1 L vessel, 1 bar A(g) is allowed to decompose at T(K). After 100 minutes, the total pressure was 1.5 bar. What is the rate constant (in min^{-1}) of the reaction ? ($\log 2 = 0.3$)

- (a) 6.9×10^{-1} (b) 6.9×10^{-3}
(c) 6.9×10^{-2} (d) 6.9×10^{-4}

34. The correct order of reactivity of the following benzyl halides towards reaction with KCN is:



- (a) $A > B > C > D$ (b) $B > A > D > C$
(c) $B > A > C > D$ (d) $A > B > D > C$

35. Given below are two statements:

Statement-I: The correct order in terms of atomic/ionic radii is $Al > Mg > Mg^{2+} > Al^{3+}$.

Statement-II: The correct order in terms of the magnitude of electron gain enthalpy is $Cl > Br > S > O$.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement I and Statement II are false
(b) Statement I is false but Statement II is true
(c) Statement I is true but Statement II is false
(d) Both Statement I and Statement II are true

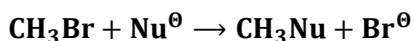
36. The correct statements are:

- (A) Activation energy for enzyme catalysed hydrolysis of sucrose is lower than that of acid catalysed hydrolysis.
(B) During denaturation, secondary and tertiary structures of a protein are destroyed but primary structure remains intact.
(C) Nucleotides are joined together by glycosidic linkage between C₁ and C₄ carbons of the pentose sugar
(D) Quaternary structure of proteins represents overall folding of the polypeptide chain.

Choose the correct answer from the options given below:

- (a) A, C and D Only (b) A, B and D Only
(c) A and B Only (d) B and C Only

37. The correct order of the rate of the reaction for the following reaction with respect to nucleophiles is:



- (a) $PhO^- > ^-OH > CH_3COO^- > ClO_4^-$
(b) $ClO_4^- > CH_3COO^- > ^-OH > PhO^-$
(c) $CH_3COO^- > PhO^- > ^-OH > ClO_4^-$
(d) $^-OH > PhO^- > CH_3COO^- > ClO_4^-$

38. Given below are two statements:

Statement I: Crystal Field Stabilization Energy (CFSE) of $[Cr(H_2O)_6]^{2+}$ is greater than that of $[Mn(H_2O)_6]^{2+}$.

Statement II: Potassium ferricyanide has a greater spin-only magnetic moment than sodium Ferrocyanide.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement I and Statement II are true
(b) Both Statement I and Statement II are false
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

39. The correct increasing order of C – H(A), C – O(B), C = O(C) and C $\equiv$ N(D) bonds in terms of covalent bond length is:

- (a) $A < B < C < D$ (b) $A < D < C < B$
(c) $D < C < B < A$ (d) $D < C < A < B$

40. Given below are four compounds:

- (A) n-propyl chloride (B) iso-propyl chloride
(C) sec-butyl chloride (D) neo-pentyl chloride

Percentage of carbon in the one which exhibits optical isomerism is:

- (a) 52 (b) 56
(c) 46 (d) 40

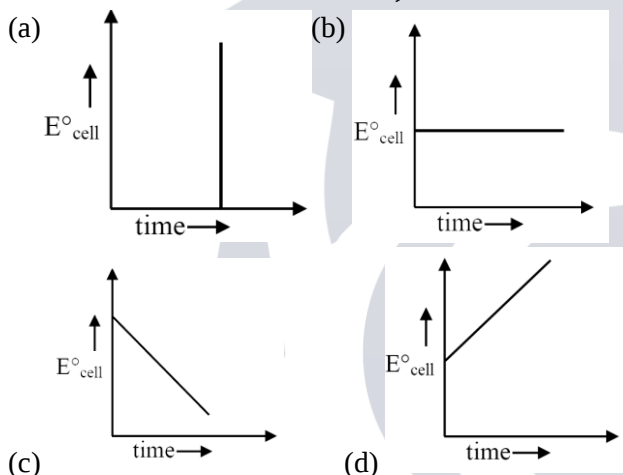
41. Given below are some of the statements about Mn and Mn_2O_7 . Identify the correct statements

- (A) Mn forms the oxide Mn_2O_7 in which Mn is in its highest oxidation state.
(B) Oxygen stabilizes the Mn in higher oxidation states by forming multiple bonds with Mn
(C) Mn_2O_7 is an ionic oxide.
(D) The structure of Mn_2O_7 consists of one bridged oxygen.

Choose the correct answer from the options given below:

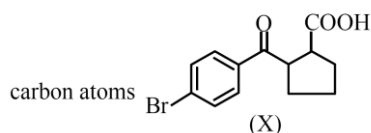
- (a) A, B, C and D
(b) A, B and D Only
(c) A, C and D Only
(d) A, B and C Only

42. For a closed circuit Daniell cell, which of the following plots is the accurate one at a given temperature?

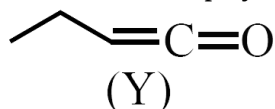


43. Given below are two statements :

Statement-I: Compound (X), shown below, dissolves in $NaHCO_3$ solution and has two chiral



Statement - II : Compound (Y), shown below, has two carbons with sp^3 hybridization, one carbon with sp^2 and one carbon with sp hybridization



In the light of the above statements, choose the correct answer from the options given below :

- (a) Statement I is true but Statement II is false
(b) Statement I is false but Statement II is true
(c) Both Statement I and Statement II are true
(d) Both Statement I and Statement II are false

44. Given below are two statements :

Statement I : The correct order in terms of bond dissociation enthalpy is $\text{Cl}_2 > \text{Br}_2 > \text{F}_2 > \text{I}_2$

Statement II : The correct trend in the covalent character of the metal halides is $[\text{SnCl}_4 > \text{SnCl}_2]$, $[\text{PbCl}_4 > \text{PbCl}_2]$ and $[\text{UF}_4 > \text{UF}_6]$

In the light of the above statements, choose the correct answer from the options given below :

- (a) Statement I is true but Statement II is false
- (b) Both Statement I and Statement II are true
- (c) Statement I is false but Statement II is true
- (d) Both Statement I and Statement II are false

45. On heating a mixture of common salt and $\text{K}_2\text{Cr}_2\text{O}_7$ in equal amount along with concentrated H_2SO_4 in a test tube, a gas is evolved. Formula of the gas evolved and oxidation state of the central metal atom in the gas respectively are :

- (a) CrO_2Cl_2 and +5
- (b) CrO_2Cl_2 and +6
- (c) $\text{Cr}_2\text{O}_2\text{Cl}_2$ and +6
- (d) $\text{Cr}_2\text{O}_2\text{Cl}_2$ and +3

SECTION-B

46. The first and second ionization constants of H_2X are 2.5×10^{-8} and 1.0×10^{-13} respectively. The concentration of X^{2-} in $0.1\text{M}\text{H}_2\text{X}$ solution is $___ \times 10^{-15}\text{M}$. (Nearest Integer)

47. The osmotic pressure of a living cell in 12 atm at 300 K. The strength of sodium chloride solution that is isotonic with the living cell at this temperature is $___ \text{gL}^{-1}$. (Nearest integer)

Given : $R = 0.08 \text{ L atm K}^{-1} \text{ mol}^{-1}$

Assume complete dissociation of NaCl

(Given : Molar mass of Na and Cl are 23 and 35.5 gmol^{-1} respectively.)

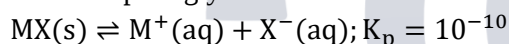
48. A substance 'X' (1.5 g) dissolved in 150 g of a solvent 'Y' (molar mass = 300 g mol^{-1}) led to an elevation of the boiling point by 0.5 K. The relative lowering in the vapour pressure of the solvent 'Y' is $___ \times 10^{-2}$. (Nearest integer)

[Given : K_b of the solvent = $5.0 \text{ K kg mol}^{-1}$]

Assume the solution to be dilute and no association or dissociation of X takes place in solution.

49. Identify the metal ions among Co^{2+} , Ni^{2+} , Fe^{2+} , V^{3+} and Ti^{2+} having a spin-only magnetic moment value more than 3.0 BM. The sum of unpaired electrons present in the high spin octahedral complexes formed by those metal ions is $___$.

50. MX is a sparingly soluble salt that follows the given solubility equilibrium at 298 K



If the standard reduction potential for $\text{M}^+(\text{aq}) \xrightarrow{+e^-} \text{M(s)}$ is $(E_{\text{M}^+/\text{M}}^\ominus) = 0.79 \text{ V}$, then the value of the standard reduction potential for the metal/metal insoluble salt electrode $E_{\text{X}^-/\text{MX(s)}/\text{M}}^\ominus$ is $___ \text{mV}$. (nearest integer) [Given:

$$\frac{2.303RT}{F} = 0.059 \text{ V}]$$

MATHEMATICS

SECTION-A

51. The positive integer n , for which the solutions of the equation $x(x+2) + (x+2)(x+4) + \dots + (x+2n-2)(x+2n) = \frac{8n}{3}$ are two consecutive even integers, is :-

- (a) 3
- (b) 6
- (c) 12
- (d) 9

52. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbb{R}$ and $f'(a-1) = 0$, where a is real number. Let $g(x) = f(\tan^2 x - 2\tan x + a)$, $0 < x < \frac{\pi}{2}$.

Consider the following two statements :

(I) g is increasing in $(0, \frac{\pi}{4})$

(II) g is decreasing in $(\frac{\pi}{4}, \frac{\pi}{2})$

Then,

- (a) Neither (I) nor (II) is True

- (b) Only (II) is True
(c) Only (I) is True
(d) Both (I) and (II) are True
53. Let $f(x) = x^3 + x^2f'(1) + 2xf''(2) + f'''(3)$, $x \in \mathbb{R}$. Then the value of $f'(5)$ is :
- (a) $\frac{62}{5}$ (b) $\frac{657}{5}$
(c) $\frac{2}{5}$ (d) $\frac{117}{5}$
54. In the line $\alpha x + 4y = \sqrt{7}$, where $\alpha \in \mathbb{R}$, touches the ellipse $3x^2 + 4y^2 = 1$ at the point P in the first quadrant, then one of the focal distances of P is :
- (a) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{11}}$ (b) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{5}}$
(c) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{5}}$ (d) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$
55. Let $y^2 = 12x$ be the parabola with its vertex at O. Let P be a point on the parabola and A be a point on the x-axis such that $\angle OPA = 90^\circ$. Then the locus of the centroid of such triangles OPA is :
- (a) $y^2 - 6x + 4 = 0$ (b) $y^2 - 9x + 6 = 0$
(c) $y^2 - 2x + 8 = 0$ (d) $y^2 - 4x + 8 = 0$
56. Let one end of a focal chord of the parabola $y^2 = 16x$ be (16, 16). If $P(\alpha, \beta)$ divides this focal chord internally in the ratio 5:2, then the minimum value of $\alpha + \beta$ is equal to :
- (a) 22 (b) 7
(c) 5 (d) 16
57. Let the line L pass through the point $(-3, 5, 2)$ and make equal angles with the positive coordinate axes. If the distance of L from the point $(-2, r, 1)$ is $\sqrt{\frac{14}{3}}$, then the sum of all possible values of r is :
- (a) 12 (b) 16
(c) 6 (d) 10
58. Let the line L_1 be parallel to the vector $-3\hat{i} + 2\hat{j} + 4\hat{k}$ and pass through the point $(2, 6, 7)$ and the line L_2 be parallel to the vector $2\hat{i} + \hat{j} + 3\hat{k}$ and pass through the point $(4, 3, 5)$. If the line L_3 is parallel to the vector $-3\hat{i} + 5\hat{j} + 16\hat{k}$ and intersects the lines L_1 and L_2 at the points C and D, respectively, then $|\overrightarrow{CD}|^2$ is equal to :
- (a) 171 (b) 290
(c) 312 (d) 89
59. Let α and β be the roots of equation $x^2 + 2ax + (3a + 10) = 0$ such that $\alpha < 1 < \beta$. Then the set of all possible values of a is :
- (a) $(-\infty, \frac{-11}{5}) \cup (5, \infty)$ (b) $(-\infty, -2) \cup (5, \infty)$
(c) $(-\infty, -3)$ (d) $(-\infty, \frac{-11}{5})$
60. A random variable X takes values 0, 1, 2, 3 with probabilities $\frac{2a+1}{30}, \frac{8a-1}{30}, \frac{4a+1}{30}, b$ respectively, where $a, b \in \mathbb{R}$. Let μ and σ respectively be the mean and standard deviation of X such that $\sigma^2 + \mu^2 = 2$. Then $\frac{a}{b}$ is equal to :
- (a) 30 (b) 3
(c) 60 (d) 12
61. If the area of the region $\{(x, y): 1 - 2x \leq y \leq 4 - x^2, x \geq 0, y \geq 0\}$ is $\frac{\alpha}{\beta}$, $\alpha, \beta \in \mathbb{N}, \gcd(\alpha, \beta) = 1$, then the value of $(\alpha + \beta)$ is :
- (a) 73 (b) 85
(c) 91 (d) 67

62. Let $a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \dots, \frac{a_{10}}{2^9}$ be a G.P. of common ratio $\frac{1}{\sqrt{2}}$. If $a_1 + a_2 + \dots + a_{10} = 62$, then a_1 is equal to :
- (a) $2(\sqrt{2} - 1)$ (b) $2 - \sqrt{2}$
(c) $\sqrt{2} - 1$ (d) $2(2 - \sqrt{2})$
63. Let $A = \{x: |x^2 - 10| \leq 6\}$ and $B = \{x: |x - 2| > 1\}$.
Then
(a) $A \cup B = (-\infty, 1] \cup (2, \infty)$
(b) $A - B = [2, 3)$
(c) $A \cap B = [-4, -2] \cup [3, 4]$
(d) $B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$
64. For the matrices $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -29 & 49 \\ -13 & 18 \end{bmatrix}$, if $(A^{15} + B) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then among the following which one is true?
- (a) $x = 5, y = 7$ (b) $x = 18, y = 11$
(c) $x = 11, y = 2$ (d) $x = 16, y = 3$
65. For a triangle ABC, let $\vec{p} = \overrightarrow{BC}$, $\vec{q} = \overrightarrow{CA}$ and $\vec{r} = \overrightarrow{BA}$. If $|\vec{p}| = 2\sqrt{3}$, $|\vec{q}| = 2$ and $\cos \theta = \frac{1}{\sqrt{3}}$, where θ is the angle between $\vec{p}$ and $\vec{q}$, then $|\vec{p} \times (\vec{q} - 3\vec{r})|^2 + 3|\vec{r}|^2$ is equal to:
- (a) 340 (b) 220
(c) 410 (d) 200
66. Let $y = y(x)$ be the solution of the differential equation $\sec x \frac{dy}{dx} - 2y = 2 + 3\sin x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $y(0) = -\frac{7}{4}$. Then $y\left(\frac{\pi}{6}\right)$ is equal to:
- (a) $-\frac{5}{2}$ (b) $-\frac{5}{4}$
(c) $-3\sqrt{3} - 7$ (d) $-3\sqrt{2} - 7$
67. Let $A = \{2, 3, 5, 7, 9\}$. Let R be the relation on A defined by $x R y$ if and only if $2x \leq 3y$. Let ℓ be the number of elements in R, and m be the minimum number of elements required to be added in R to make it a symmetric relation. Then $\ell + m$ is equal to:
- (a) 23 (b) 25
(c) 21 (d) 27
68. If the system of equations
 $3x + y + 4z = 3$
 $2x + \alpha y - z = -3$
 $x + 2y + z = 4$
has no solution, then the value of α is equal to:
- (a) 19 (b) 4
(c) 13 (d) 23
69. Let z be the complex number satisfying $|z - 5| \leq 3$ and having maximum positive principal argument. Then $34 \left| \frac{5z-12}{5iz+16} \right|^2$ is equal to:
- (a) 16 (b) 12
(c) 26 (d) 20
70. The largest $n \in \mathbb{N}$, for which 7^n divides $101!$, is :
- (a) 16 (b) 18
(c) 15 (d) 19

SECTION-B

71. Let $[\cdot]$ denote the greatest integer function and $f(x) = \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n \left[\frac{k^2}{3^x} \right]$. Then $12 \sum_{j=1}^{\infty} f(j)$ is equal to ____ .
72. If $\int_0^1 4 \cot^{-1}(1 - 2x + 4x^2) dx = a \tan^{-1}(2) - b \log_e(5)$, where $a, b \in \mathbb{N}$, then $(2a + b)$ is equal to ____ .

73. Let the maximum value of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ for $x \in \left[-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right]$ be $\frac{m}{n}\pi^2$, where $\gcd(m, n) = 1$. Then $m + n$ is equal to ____ .
74. If $\left(\frac{1}{{}^{15}C_0} + \frac{1}{{}^{15}C_1}\right)\left(\frac{1}{{}^{15}C_1} + \frac{1}{{}^{15}C_2}\right) \dots \left(\frac{1}{{}^{15}C_{12}} + \frac{1}{{}^{15}C_{13}}\right) = \frac{\alpha^{13}}{{}^{14}C_0 {}^{14}C_1 \dots {}^{14}C_{12}}$, then 30α is equal to ____ .
75. If P is a point on the circle $x^2 + y^2 = 4$, Q is a point on the straight line $5x + y + 2 = 0$ and $x - y + 1 = 0$ is the perpendicular bisector of PQ, then 13 times the sum of abscissa of all such point P is ____ .

