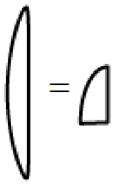
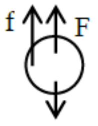
Physics
Section - A

1. (d)  $\frac{1}{f_1} = (\mu - 1)\frac{2}{R} = P = 4D$

 $= \frac{1}{f_2} = (\mu - 1)\frac{1}{R} = \frac{P}{2} = 2D$

2. (b)



$a = 0$

$$mg - F_B - f = 0$$
$$\Rightarrow mg - \frac{mg}{2} - f = 0$$
$$\therefore f = \frac{mg}{2}$$

3. (b)

$$d = \frac{m}{vol.} = \frac{m}{\pi R^2 \ell} \Rightarrow \frac{d\rho}{\rho} = \frac{dm}{m} + \frac{2dR}{R} + \frac{d\ell}{\ell}$$
$$\Rightarrow \frac{d\rho}{\rho} = \left(\frac{0.003}{0.6} + \frac{2 \times 0.01}{0.5} + \frac{0.05}{10}\right)100 = 5\%$$

4. (b)

Initially, $I_0 = \frac{\varepsilon_m}{R}$

Finally, $I_0^1 = \frac{\varepsilon_m}{2R} = \frac{I_0}{2}$

5. (c)

$$E_A = \frac{2kP}{r^3} = E_0 \,\&\, V_A = \frac{kP}{r^2} = v_0$$
$$E_B = \frac{kP}{(2r)^3} = \frac{E_0}{16} \,\&\, V_B = \frac{k\vec{P} \cdot \hat{r}}{r^2} = 0$$

6. (d)

$$C = \frac{q}{V} = \frac{q \cdot q}{V \cdot q} = \frac{q^2}{WD} = \frac{C^2}{ML^2\,T^{-2}} = C^2M^{-1}\,L^{-2}\,T^2$$

7. (d)

$$r = \frac{mv}{eB} \,\&\, mvr = \frac{nh}{2\pi} \Rightarrow (eBr)r = \frac{nh}{2\pi}$$
$$\Rightarrow r = \sqrt{\frac{nh}{2\pi eB}}$$

first excited state : $n = 2 \therefore r = \sqrt{\frac{h}{\pi eB}}$

8. (d)

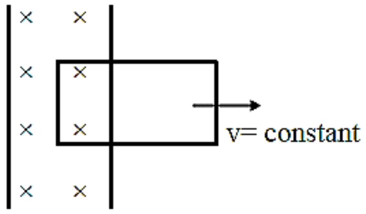
$\gamma = \frac{2}{f} + 1$

without vibration : $f = 5 : \gamma_1 = 1.4$

without vibration : $f = 7 : \gamma_2 = 1.14$

$\therefore \gamma_2 < \gamma_1$

9. (d)



B = constant

Motional emf : $\varepsilon = B\ell v$ = constant

10. (c)

$$\frac{hc}{\lambda} = \phi + eV \Rightarrow \frac{hc}{\lambda} = 1 + 2 = 3eV \ldots\ldots (1)$$

$$\frac{hc}{\lambda/2} = 6 = 1 + k_{max} \therefore k_{max} = 5eV$$

11. (a)

$$g = \frac{GM}{R^2}$$

$$g' = \frac{G(4M)}{(2R)^2} = g$$

A is correct, R is correct ; but since $T = 2\pi\sqrt{\frac{\ell}{g}}$ doesn't depend on mass ; R doesn't explain A .

12. (c) $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i} - 0\hat{j} - \hat{k}$

13. (a)

14. (b)

$$WD = \vec{F} \cdot \vec{S} = 2 - 2\,b - 1 = 0$$

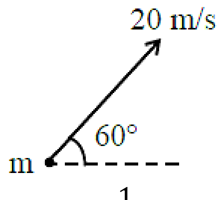
$$\therefore\ b = \frac{1}{2}$$

15. (b)

$$\beta = \frac{\lambda D}{d} \& \lambda_R > \lambda_b$$

$$\therefore \beta_R > \beta_b$$

16. (b)

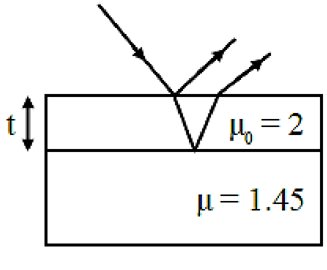


$$k_i = \frac{1}{2}mvv^2$$

$$k_f = \frac{1}{2}\ m(v\cos 60°)^2 = \frac{1}{8}mv^2$$

$$\Delta k = k_i - k_f = \frac{3}{8}mv^2 = \frac{3}{8} \times 0.1 \times 400 = 15\ J$$

17. (c)

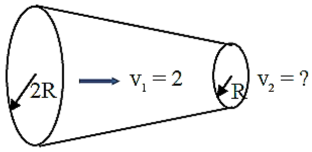


For transmitted green light to be maxima, reflected green should be minima.

$$\Delta P = 2\mu_0 t = n\lambda$$

$$\Rightarrow t = \frac{n\lambda}{2\mu_0} \therefore t_{min} = \frac{\lambda}{2\mu_0} = \frac{550}{2 \times 2} = 137.5$$

18. (d)

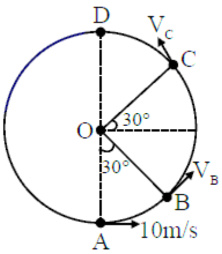


$$A_1 v_1 = A_2 V_2 \Rightarrow 2\pi(2R)^2 = V_2 \pi R^2$$

$$\therefore V_2 = 8 \text{ m/s}$$

19. (c) Extensive variables depends on size or mass of system ex : internal energy, volume, mass

20. (c)



$$\frac{1}{2} m \times 100 + 0 = \frac{1}{2} m V_B^2 + mg\left(R - \frac{R\sqrt{3}}{2}\right)$$

$$100 = V_B^2 + 2gR\left(1 - \frac{\sqrt{3}}{2}\right]$$

$$V_B^2 = 100 - 20(2 - \sqrt{3})$$

$$V_B^2 = 60 + 20\sqrt{3})$$

$$K \cdot E_B = \frac{1}{2} m V_B^2 = \frac{m}{2}(60 + 20\sqrt{3})$$

$$\frac{1}{2} m(100) = \frac{1}{2} mVV_C^2 + mg\left(\frac{3R}{2}\right)$$

$$100 = V_C^2 = 60$$

$$V_C^2 = 40$$

$$K \cdot E_C = \frac{1}{2} mVV_C^2 = \frac{1}{2} m(40)$$

$$K \cdot E_B = \frac{60 + 20\sqrt{3}}{40} = \frac{3}{2} + \frac{\sqrt{3}}{2} = \frac{3 + \sqrt{3}}{2}$$

Section - B

21. (2) For uniform speed $V = \frac{E}{B}$

$$R = \frac{mV}{eB}$$

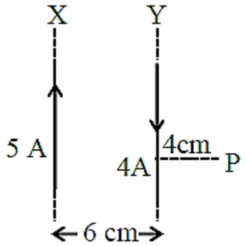
$$= \frac{mV^2}{eE}$$

$$\Rightarrow E = \frac{mV^2}{eR}$$

$$= \frac{1.6 \times 10^{-27} \times 4 \times 10^{10}}{1.6 \times 10^{-19} \times 2 \times 10^{-2}}$$

$$= 2 \times 10^4 \text{ N/C}$$

22. (1)



$$B = \frac{\mu_0(5)}{2\pi \times .01} - \frac{\mu_0 4}{2\pi \times 0.04}$$

$$= -\frac{100\mu_0}{4\pi}$$

$$= -100 \times 10^{-7}$$

$$= -1 \times 10^{-5}\ \mathrm{T}$$

23. (1200)

$$J_d = \frac{I}{A} = \frac{6}{16}$$

$\therefore$ I through small area $= J_d \times A' = \frac{6}{16} \times 3.2 =$

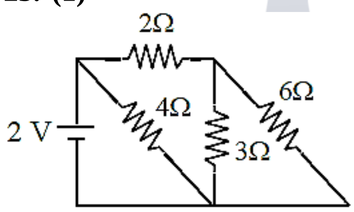
$1.2\ \mathrm{A} = 1200\ \mathrm{mA}$

24. (1)

$$F = 2M\omega^2 \frac{\ell}{2} = Mw^2\ell$$

$$\omega = \sqrt{\frac{F}{M\ell}}$$

25. (1)



$R_{eq} = 2\Omega$

$$I = \frac{2}{2} = 1\ \mathrm{A}$$

Chemistry

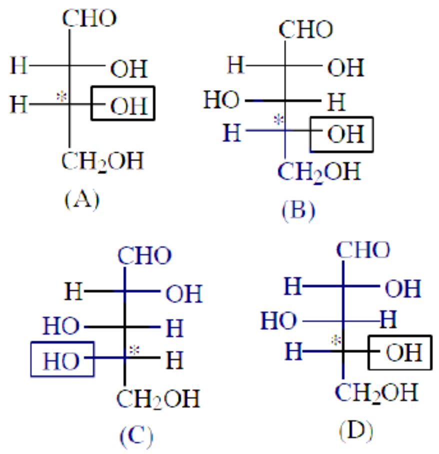
Section - A

26. (a) Increasing order of Dipole moment

$NF_3 < HBr < H_2S < CHCl_3$

$\mu = 0.24D\ 0.79D\ 0.95D\ 1.04D$

27. (a)



In A, B, D $-$ OH group in right hand side then D-configuration is assigned

28. (d) $N-N < P-P$: single (σ) bond strength

Due to L.P.-L.P. replusion

and maximum possible covalency of nitrogen is 4 .

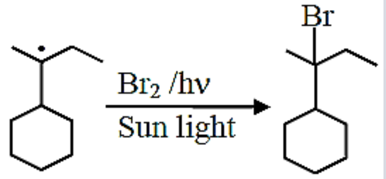
29. (c) Activation energy of forward reaction $= E_1 + E_2$

Energy of product $>$ Energy of reactant

Stability

Reactant > Product

30. (d)



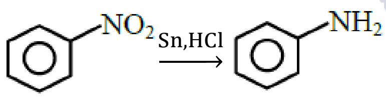
Formation of more stable free radical intermediate

31. (b) $ClO_4^- \rightarrow x + \{(-2) \times 4\} = -1 \Rightarrow x = +7$

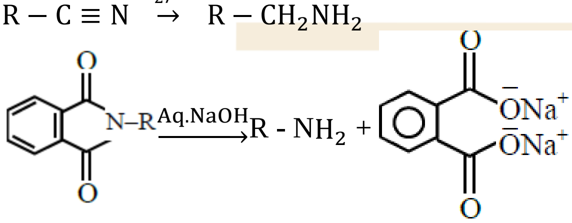
Chlorine is in its maximum oxidation state, so disproportionation not possible in ClO_4^-.

32. (d)

(A) $R-\overset{\overset{O}{\|}}{C}-NH_2 \xrightarrow[H_2O]{LiAlH_4} R-CH_2-NH_2$

(B) 

(C) $R-C\equiv N \xrightarrow{H_2/Ni} R-CH_2NH_2$

(D) 

33. (a)

34. (b) (A) $dG = VdP - SdT$

Constant pressure

$$dG = -SdT$$

$$\left(\frac{\partial G}{\partial T}\right)_P = -S$$

(B) $dH = (dq)_P = nCpdT$

$$\left(\frac{\partial H}{\partial T}\right)_P = C_P$$

(C) $dG = VdP - SdT$

At constant temperature

$$dG = VdP$$

$$\left(\frac{\partial G}{\partial P}\right)_T = V$$

(D) $dU = nC_V dT = (q)_V$

$$\left(\frac{\partial U}{\partial T}\right)_V = C_V$$

35. (b)

36. (d)

$3M\,NaCl, d_{sol} = 1.25 gm/mol$

$$\text{Molality} = \frac{M \times 1000}{1000\,d - M \times M_w}$$

$$= \frac{3000}{1250 - 175.5} = 2.79$$

37. (a)

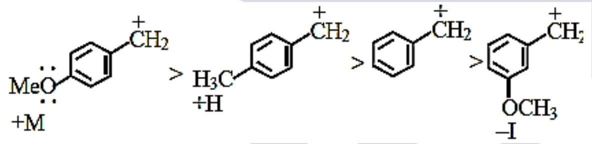
$$Zr_3(PO_4)_4(s) \rightleftharpoons 3Zr^{+4}(aq) + 4PO_4^{-3}(aq)$$

$$- \qquad\qquad 3s \qquad\qquad 4s$$

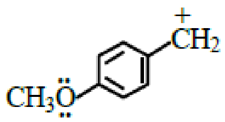
$$K_{sp} = (3s)^3(4s)^4 = 6912\,s^7$$

$$s = \left(\frac{K_{sp}}{6912}\right)^{1/7}$$

38. (a)



Due to +M effect of -OMe at para position



will be most stable among these

39. (a) Statement-I : False but Statement-II is true.

On moving left to right in periodic table nonmetallic character increases and we know that nonmetal oxides are acidc in nature.

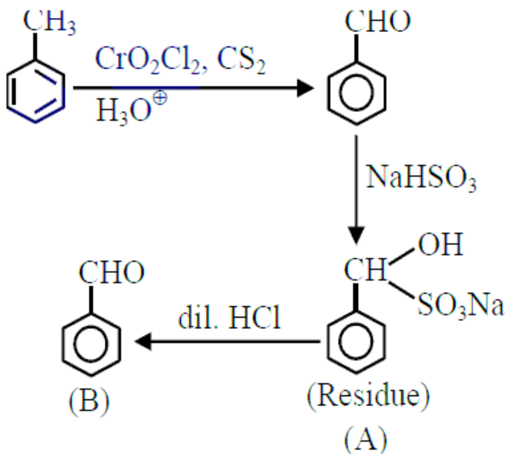
Non -metallic character ↑ Acidic strength of oxide ↑

40. (d) No spectral line will be observed for a $2p_x \rightarrow 2p_y$ transition because $2p_x$ and $2p_y$ orbitals are degenerate orbitals.

41. (c) The elements present in the compound are converted from covalent form into ionic form by fusing the compound with sodium in Lassigne's test

42. (c)

43. (d)

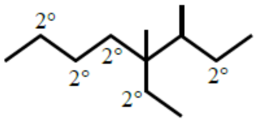
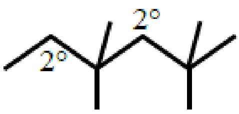
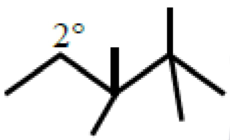
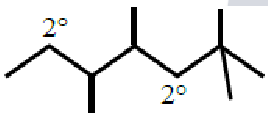


44. (d) Statement I : Corrosion is an example of electrochemical phenomenon

In which pure metal act as anode and impure metal (rusted metal) act as cathode.

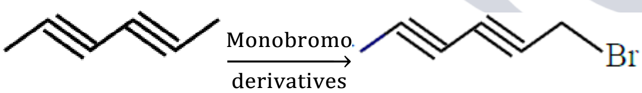
Statement II : Corrosion is more favourable in acid medium than alkaline, so rate of corrosion is high is acid medium then alkaline.

45. (c)

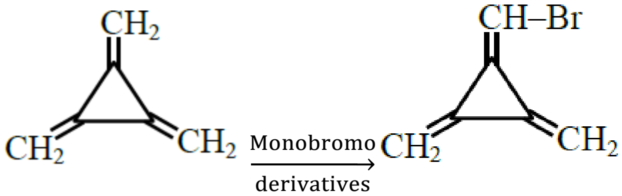
Alkane	2°H
1. 	10
2. 	4
3. 	2
4. 	4

Section – B

46. (8 & 6)



No. of $\pi e^- = 8$



No. of $\pi e^- = 6$

47. (11) $Z = 41 \rightarrow$ Nb (Niobium) : $[Kr]_{36} 4d^4 5s^1$

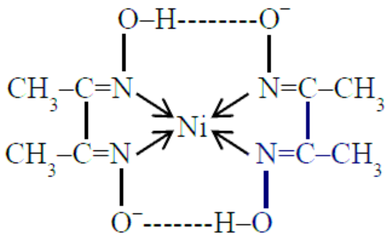
Number of electron in 4 d = 4 = x

$Z = 44 \rightarrow$ Ru (Ruthenium) $[Kr]_{36} 4d^7 5s^1$

Number of electron in 4 d = 7 = y

$x + y = 11$

48. (14) $[Ni(dmg)_2]$



Number of H – atom = 14

49. (95)

$2C_{(graphite)} + 3H_2(g) \rightarrow C_2H_6(g)\ \Delta H_f = ?$

$C_2H_6(g) + \frac{7}{2}O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$

$\Delta H_1 = -1550$

$C_{(graphite)} + O_2(g) \rightarrow CO_2(g) \Delta H_2 = -393.5$

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(/) \Delta H_3 = -286$

$\Delta H_f = 2\Delta H_2 + 3\Delta H_3 - \Delta H_1$

$= 95\ \mathrm{kJ/mole}$

50. (57)

$M_F = \frac{M_1V_1 + M_2V_2}{V_1 + V_2}$

$= \frac{2 \times 20 + 0.5 \times 400}{420} = 0.571M$

$= 57.1 \times 10^{-2}M$

$= 57$

Mathematics
Section - A

51. (a)

$\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$

$= 2\{{}^5C_0 \cdot x^5 + {}^5C_2 \cdot x^3(x^3 - 1) + {}^5C_4 \cdot x(x^3 - 1)^2\}$

$= 2\{5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x\}$

$\Rightarrow \alpha = 10, \beta = 2, \gamma = -20, \delta = 10$

Now, $10u + 2v = 18$

$-20u + 10v = 20$

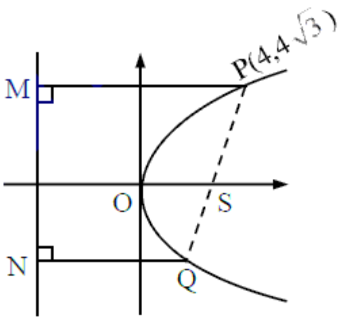
$\Rightarrow u = 1, v = 4$

$u + v = 5$

52. (a) Total - when B_1 and B_2 are together

$= 2!(3!4!) - 2!(3!(3!2!)) = 144$

53. (c)



$(4,4\sqrt{3})$ lies on $y^2 = 4ax$

$\Rightarrow 48 = 4a \cdot 4$

$4a = 12$

$\Rightarrow y^2=12x$ is equation of parabola

Now, parameter of P is $t_1=\frac{2}{\sqrt{3}} \Rightarrow$ Parameters of Q is $t_2=-\frac{\sqrt{3}}{2} \Rightarrow Q\left(\frac{9}{4},-3\sqrt{3}\right)$

Area of trapezium PQNM

$$=\frac{1}{2}MN\cdot(PM+QN)$$

$$=\frac{1}{2}MN\cdot(PS+QS)$$

$$=\frac{1}{2}MN\cdot PQ$$

$$=\frac{1}{2}7\sqrt{3}\cdot\frac{49}{4}=(343)\frac{\sqrt{3}}{8}=3$$

54. (d)

$$|A|=\frac{1}{2}, \operatorname{trace}(A)=3, B=\operatorname{adj}(\operatorname{adj}(2A))=|2A|^{n-2}(2A)$$

$$n=3, B=|2A|(2A)=2^3\cdot|A|(2A)=8A$$

$$|B|=|8A|=8^3\cdot|A|=2^8=256$$

$$\operatorname{trace}(B)=8\operatorname{trace}(A)=24$$

$$|B|+\operatorname{trace}(B)=280$$

55. (a)

$$a_1, a_2, a_3, \ldots\ldots, a_{2k}$$

$$\sum_{r=1}^{k} a_{2r-1}=40, \sum_{r=1}^{k} a_{2r}=55, a_{2k}-a_1=27$$

$$\frac{k}{2}[2a_1+(k-1)2d]=40, \frac{k}{2}[2a_2+(k-1)2d]=55,$$

$$d=\frac{27}{2k-1}$$

$$a_1=\frac{40}{k}-(k-1)d=\frac{55}{k}-kd$$

$$d=\frac{15}{k} \Rightarrow \frac{27}{2k-1}=\frac{15}{k} \Rightarrow 9k=10k-5$$

$$\therefore k=5$$

56. (a)

$\overrightarrow{PQ}$ parallel to $3\hat{i}+2\hat{j}+2\hat{k}, R(1,3,3)$

$$\Rightarrow Q(3\lambda-2, 2\lambda-1, 2\lambda+3), \lambda \in R-\{0\}$$

$$|\overrightarrow{QR}|=5=\sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2}$$

$$\therefore 17\lambda^2-34\lambda+25=25 \Rightarrow \lambda=2(\because \lambda \neq 0)$$

$$\therefore Q(4,3,7), P(-2,-1,3), R(1,3,3)$$

Area of $\Delta PQR=[PQR]=\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|$

$$[PQR]=\frac{1}{2}\left\|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 6 & \hat{4} & \hat{4}\\ 3 & 4 & 0\end{vmatrix}\right\|=\left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & \hat{2} & \hat{2}\\ 3 & 4 & 0\end{vmatrix}\right|$$

$$[PQR]=|-8\hat{i}+6\hat{j}+6\hat{k}|=\sqrt{136}$$

$$\therefore [PQR]^2=136$$

57. (a)

$$\alpha = \lim_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)\right)^{x} (1^{\infty} \text{ form })$$

$\therefore \alpha = e^{L}$

Where $L = \lim_{x\to\infty} x\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)-1\right)$

$\Rightarrow L = \lim_{x\to\infty}\left(\frac{e}{1-e}\right)x\left(\frac{1}{e}-\frac{x}{1+x}-\left(\frac{1-e}{e}\right)\right)$

$\Rightarrow L = \frac{e}{1-e^{x}\to\infty}\lim_{x}\left(1-\frac{x}{1+x}\right)$

$\Rightarrow L = \frac{e}{1-e}\lim_{x\to\infty}\frac{x}{x+1}$

$\Rightarrow \ L = \frac{e}{1-e}\cdot 1$

$\Rightarrow \ L = \frac{e}{1-e}$

$\therefore \alpha = e^{\frac{e}{1-e}} \Rightarrow \log\alpha = \frac{e}{1-e}$

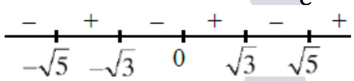
$\therefore$ Required value $= \dfrac{\frac{e}{1-e}}{1+\frac{e}{1-e}} = e$

58. (a)

$$f'(x) = \left(\frac{x^4-8x^2+15}{e^{x^2}}\right)(2x)$$

$$= \frac{(x^2-3)(x^2-5)(2x)}{e^{x^2}}$$

$$= \frac{(x-\sqrt{3})(x+\sqrt{3})(x-\sqrt{5})(x+\sqrt{5})2x}{e^{x^2}}$$

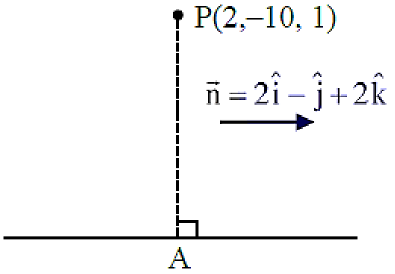


Maxima at $x \in \{-\sqrt{3}, \sqrt{3}\}$

Minima at $x \in \{-\sqrt{5}, 0, \sqrt{5}\}$

2 points of maxima and 3 points of minima.

59. (c)



$$\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2} = \lambda(\text{ let })$$

$(2\lambda+1, -\lambda-2, 2\lambda-3)$

$\because \overrightarrow{PA}\cdot\vec{n} = 0$

$\Rightarrow (2\lambda-1)2 + (-\lambda+8)(-1) + (2\lambda-4)2 = 0$

$\Rightarrow 4\lambda - 2 + \lambda - 8 + 4\lambda - 8 = 0$

$\Rightarrow 9\lambda - 18 = 0 \Rightarrow \lambda = 2$

$\therefore \ A(5,-4,1)$

$\therefore AP = \sqrt{3^2+6^2+0^2} = \sqrt{45} = 3\sqrt{5}$

60. (d)

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{2e^{\tan^{-1}y}}{1+y^2}$$

I.F. $=e^{\tan^{-1}y}$

$$xe^{\tan^{-1}y}=\int\frac{2\left(e^{\tan^{-1}y}\right)^2dy}{1+y^2}$$

Put $\tan^{-1}y=t,\frac{dy}{1+y^2}=dt$

$$xe^{\tan^{-1}y}=\int 2e^{2t}dt$$

$$xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+c$$

$$x=e^{\tan^{-1}y}+ce^{-\tan^{-1}y}$$

$\because y=0,x=1$

$1=1+c\Rightarrow c=0$

$y=\frac{1}{\sqrt{3}},x=e^{\pi/6}$

61. (c)

$$\because\frac{d}{dx}\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right)=\frac{\sin^{-1}x}{(1-x^2)^{3/2}}+\frac{x}{1-x^2}$$

$$\Rightarrow\int e^x\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\frac{\sin^{-1}x}{(1-x^2)^{3/2}}+\frac{x}{1-x^2}\right)dx$$

$$=e^x\cdot\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+c=g(x)+C$$

Note : assuming $g(x)=\frac{xe^x\sin^{-1}x}{\sqrt{1-x^2}}$

$$g(1/2)=\frac{e^{1/2}}{2}\cdot\frac{\frac{\pi}{6}\times 2}{\sqrt{3}}=\frac{\pi}{6}\sqrt{\frac{e}{3}}$$

62. (b)

$(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$

$[(\alpha+\beta)^2-2\alpha\beta]^2-2(\alpha\beta)^2$

$$\left[\frac{\cos^2\theta}{4}+1\right]^2-2\cdot\frac{1}{4}$$

$$\left(\frac{\cos^2\theta}{4}+1\right)^2-\frac{1}{2}$$

$$M=\frac{25}{16}-\frac{1}{2}=\frac{17}{16}$$

$m=\frac{1}{2},16(M+m)=25$

63. (b)

Total $=4^4$

One-one $=4!$

Many-one $=256-24=232$

Many-one which $1\notin f(A)$

$=3.3\cdot 3\cdot 3=81$

$232-81=151$

64. (c)

$$\Delta=\begin{vmatrix}1 & 1 & 2\\ 2 & 3 & a\\ -1 & -3 & b\end{vmatrix}=0$$

$\Rightarrow 2a+b-6=0 \quad \ldots\ldots(i)$

$$\Delta_1=\begin{vmatrix}1 & 1 & 6\\ 2 & 3 & a+1\\ -1 & -3 & 2b\end{vmatrix}=0$$

$\Rightarrow a+b-8=0 \quad \ldots\ldots(2)$

Solving (1) +(2)

$a=-2, b=10$

$\Rightarrow 7a+3b=16$

65. (d)

$\hat{a}\cdot\hat{b}=\frac{1}{2}$

Now $(\lambda\hat{a}+2\hat{b})\cdot(3\hat{a}-\lambda\hat{b})=0$

$3\lambda a\hat{a}\cdot\hat{a}-\lambda^2\hat{a}\cdot\hat{b}+6\hat{a}\cdot\hat{b}-2\lambda\hat{b}\cdot\hat{b}=0$

$3\lambda-\frac{\lambda^2}{2}+3-2\lambda=0$

$\lambda^2-2\lambda-6=0$

$\lambda=1\pm\sqrt{7}$

$\Rightarrow$ number of values $=0$

66. (c)

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ foci are $(ae,0)$ and $(-ae,0)$

$\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$ foci are $(Ae',0)$ and $(-Ae',0)$

$\Rightarrow 2ae=2\sqrt{3}\Rightarrow ae=\sqrt{3}$

and $2Ae'=2\sqrt{3}\Rightarrow Ae'=\sqrt{3}$

$\Rightarrow ae=Ae'\Rightarrow\frac{e}{e'}=\frac{A}{a}$

$\Rightarrow\frac{1}{3}=\frac{A}{a}\Rightarrow a=3A$

Now $a-A=2\Rightarrow a-\frac{a}{3}-2\Rightarrow a=3$ and $A=1$ $Ae=\sqrt{3}\Rightarrow e=\frac{1}{\sqrt{3}}$ and $e'=\sqrt{3}$

$b^2=a^2(1-e^2)$

$b^2=6$ and $B^2=A^2((e')^2-1)=(2)\Rightarrow B^2=2$

sum of LR $=\frac{2b^2}{a}+\frac{2B^2}{A}=8$

67. (c)

$12x^2-7x+1=0$

$x=\frac{1}{3},\frac{1}{4}$

Let $P\left(\frac{A}{B}\right)=\frac{1}{3}\&P\left(\frac{B}{A}\right)=\frac{1}{4}$

$\frac{P(A\cap B)}{P(B)}=\frac{1}{3}\&\frac{P(A\cap B)}{P(A)}=\frac{1}{4}$

$\Rightarrow P(B)=0.3$

$\&P(A)=0.4$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.3+0.4-0.1=0.6$

Now $\frac{P(\bar{A}\cup\bar{B})}{P(\bar{A}\cap\bar{B})}=\frac{P(\overline{A\cap B})}{P(\overline{A\cup B})}$

$=\frac{1-P(A\cap B)}{1-P(A\cup B)}=\frac{1-0.1}{1-0.6}=\frac{9}{4}$

68. (d)

$2\sin^2\theta = \cos 2\theta$

$2\sin^2\theta = 1 - 2\sin^2\theta$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$2\cos^2\theta = 3\sin\theta$

$2 - 2\sin^2\theta + 3\sin\theta - 2 = 0$

$(2\sin\theta - 1)(2\sin\theta - 2) = 0$

$\sin\theta = \frac{1}{2}$

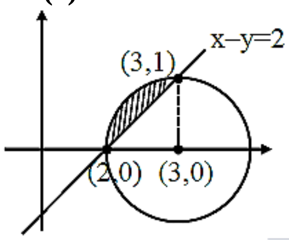
so common equation which satisfy both equations

is $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $(\theta \in [0,2\pi])$

Sum $= \pi$

69. (a)



Let $z = x + iy$

$(x + iy)(1 + i) + (x - iy)(1 - i) = 4$

$x + ix + iy - y + x - ix - iy - y = 4$

$2x - 2y = 4$

$x - y = 2$

$|z - 3| \le 1$

$(x - 3)^2 + y^2 \le 1$

Area of shaded region $= \frac{\pi\cdot 1^2}{4} - \frac{1}{2}\cdot 1\cdot 1 = \frac{\pi}{4} - \frac{1}{2}$

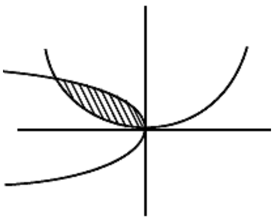
Area of unshaded region inside the circle

$= \frac{3}{4}\pi\cdot 1^2 + \frac{1}{2}\cdot 1\cdot 1 = \frac{3\pi}{4} + \frac{1}{2}$

$\therefore$ difference of area $= \left(\frac{3\pi}{4} + \frac{1}{2}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi}{2} + 1$

70. (a)



$y = (x - 2)^2, y^2 = 8(x - 2)$

$y = x^2, y^2 = -8x$

$= \frac{16ab}{3} = \frac{16 \times \frac{1}{4} \times 2}{3} = \frac{8}{3}$

Section - B

71. (27)

I.F. $e^{-\frac{1}{2}\int\frac{2x}{1-x^2}dx} = e^{-\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$

$y \times \sqrt{1-x^2} = \int (x^6 + 4x)dx = \frac{x^7}{7} + 2x^2 + c$

Given $y(0) = 0 \Rightarrow c = 0$

$y = \dfrac{\frac{x^7}{7} + 2x^2}{\sqrt{1-x^2}}$

Now, $6\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\frac{x^7}{7}+2x^2}{\sqrt{1-x^2}}dx = 6\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{2x^2}{\sqrt{1-x^2}}dx$

$= 24\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$

Put $x = \sin\theta$

$dx = \cos\theta d\theta$

$= 24\int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos\theta}\cos\theta d\theta$

$= 24\int_0^{\frac{\pi}{6}} \left(\frac{1-\cos 2\theta}{2}\right)d\theta = 12\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{6}}$

$= 12\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

$= 2\pi - 3\sqrt{3}$

$\alpha^2 = (3\sqrt{3})^2 = 27$

72. (145) All the three points A, B, C lie on the circle $x^2 + y^2 = 100$ so circumcentre is (0,0)

$O(0,0) \overset{1}{\text{———}} G(h,k) \overset{2}{\text{———}} L(a,9)$

$\frac{a+0}{3} = h \Rightarrow a = 3h$

and $\frac{9+0}{3} = k \Rightarrow k = 3$

also centroid $\frac{6+10\cos\alpha-10\sin\alpha}{3} = h$

$\Rightarrow 10(\cos\alpha - \sin\alpha) = 3h - 6$ ..... (i)

and $\frac{8+10\cos\alpha-10\sin\alpha}{3} = k$

$\Rightarrow 10(\cos\alpha - \sin\alpha) = 3k - 8 = 9 - 8 = 1$ .... (ii)

on squaring $100(1 - \sin 2\alpha) = 1$

$\Rightarrow 100\sin 2\alpha = 99$

from equ. (i) and (ii) we get $h = \frac{7}{3}$

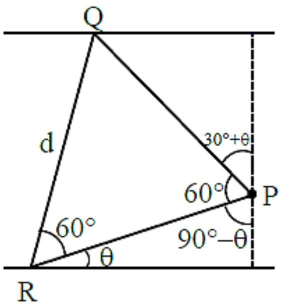
Now $5a - 3h + 6k + 100\sin 2\alpha$

$= 15h - 3h + 6k + 100\sin 2\alpha$

$= 12 \times \frac{7}{3} + 18 + 99$

$= 145$

73. (28)



$PR = \operatorname{cosec}\theta, PQ = 4\sec(30+\theta)$

For equilateral

$d = PR = PQ$

$\Rightarrow \cos(\theta + 30^\circ) = 4\sin\theta$

$\Rightarrow \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta = 4\sin\theta$

$\Rightarrow \tan\theta = \frac{1}{3\sqrt{3}}$

$QR^2 = d^2 = \operatorname{cosec}^2\theta = 28$

74. (465)

$\sum_{r=1}^{30} \frac{r^2\left({}^{30}C_r\right)^2}{{}^{30}C_{r-1}}$

$= \sum_{r=1}^{30} r^2\left(\frac{31-r}{r}\right)\cdot\frac{30!}{r!\,(30-r)!}$

$\left(\because \frac{{}^{30}C_r}{{}^{30}C_{r-1}} = \frac{30-r+1}{r} = \frac{31-r}{r}\right)$

$= \sum_{r=1}^{30} \frac{(31-r)30!}{(r-1)!\,(30-r)!}$

$= 30\sum_{r=1}^{30} \frac{(31-r)29!}{(r-1)!\,(30-r)!}$

$= 30\sum_{r-1}^{30}(30-r+1)^{29}C_{30-r}$

$= 30\left(\sum_{r=1}^{30}(31-r)^{29}C_{30-r} + \sum_{r=1}^{30}{}^{29}C_{30-r}\right)$

$= 30(29\times 2^{28} + 2^{29}) = 30(29+2)2^{28}$

$= 15\times 31\times 2^{29}$

$= 465(2^{29})$

$\alpha = 465$

75. (3) Transitivity

$(1,2) \in R, (2,3) \in R \Rightarrow (1,3) \in R$

For reflexive $(1,1),(2,2)(3,3) \in R$

Now $(2,1),(3,2),(3,1)$

$(3,1)$ cannot be taken

(1) $(2,1)$ taken and $(3,2)$ not taken

(2)$(3,2)$ taken and $(2,1)$ not taken

(3) Both not taken

therefore 3 relations are possible.