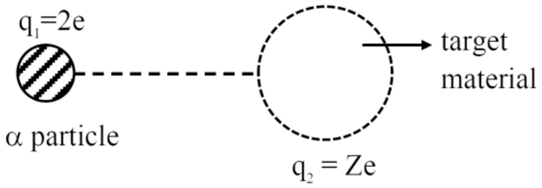
1. **(d)** $I = \frac{7}{5}[m_1R_1^2 + m_2R_2^2]$

$= \frac{7}{5}[5(10)^2 + 10 \times (20)^2] \times 10^{-4}$

$I = 63 \times 10^{-2}\ \text{kg m}^2$

$I = 0.63\ \text{kg m}^2$

2. **(a)** By mechanical energy conservation



$(Me)_i = (Me)_f$

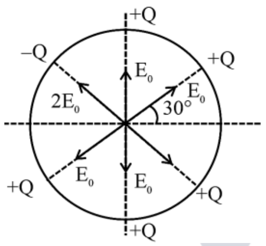
$PE_i + KE_i = PE_f + KE_f$

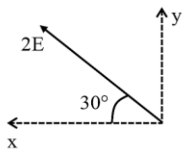
$0 + 7.9 \times 10^6 \times 1.6 \times 10^{-19} = \frac{k(2e)(Ze)}{r} + 0$

$r = \frac{9 \times 10^9 \times 2 \times (1.6 \times 10^{-19})^2 \times 79}{7.9 \times 10^6 \times 1.6 \times 10^{-19}} = 2.88 \times 10^{-14}\ \text{m}$

For diameter ⇒ $D = 2r = 5.76 \times 10^{-14}$ m

3. **(b)** Let $\frac{KQ}{r^2} = E_0$



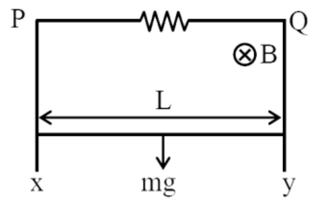


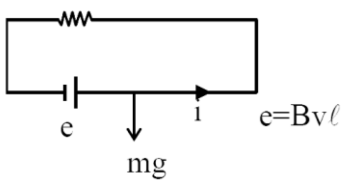
$\vec{E}_{net} = 2E_0\cos30°(-\hat{i}) + 2E_0\sin30°(\hat{j})$

$= \frac{2kQ}{r^2}\left[\frac{\sqrt{3}}{2}(-\hat{i}) + \frac{1}{2}\hat{j}\right]$

$= \frac{-1Q}{4\pi\varepsilon_0 r^2}(\sqrt{3}\hat{i} - \hat{j})$

4. **(d)**





at equilibrium (Or for terminal velocity)

$mg = iB\ell \Rightarrow mg = \left(\frac{Bv\ell}{R}\right)B\ell$

$V = \frac{mgR}{B^2\ell^2}$

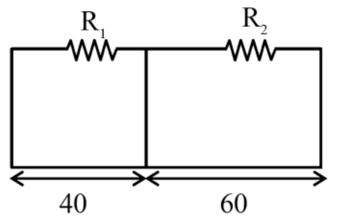
5. **(c)** $V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\times\rho\times\frac{4\pi R^3}{3}}{R}} \Rightarrow V_e \propto \sqrt{\rho}\times R$

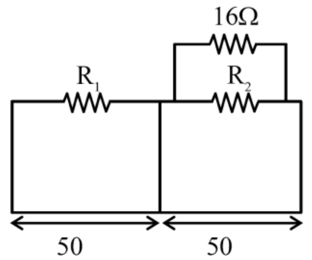
$$\frac{(V_e)_B}{(V_e)_A} = \sqrt{\frac{\rho_B}{\rho_A}}\times\frac{R_B}{R_A} = \sqrt{\frac{0.1\rho_A}{\rho_A}}\times\left(\frac{0.1R_A}{R_A}\right)$$

$$\frac{(V_e)_B}{(V_e)_A} = \frac{1}{10}\times\frac{1}{\sqrt{10}}$$

$$(V_e)_B = \frac{10\times1000}{10\sqrt{10}} = 100\sqrt{10}\ \text{m/sec}$$

6. **(c)**





$\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3}$ .... (i)

$\frac{R_1}{\left(\frac{R_2\times16}{R_2+16}\right)} = \frac{50}{50} \Rightarrow R_1 = \frac{16R_2}{16+R_2}$ .... (ii)

$$\frac{2}{3}R_2 = \frac{16R_2}{16+R_2}$$

$$\frac{32}{3}+\frac{2R_2}{3} = 16$$

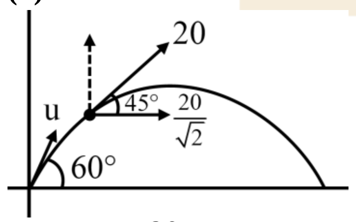
$$\frac{2R_2}{3} = 16-\frac{32}{3} = \frac{16}{3}$$

$R_2 = 8\Omega$

By equation (i)

$$R_1 = \frac{2}{3}R_2 = \frac{16}{3}\Omega$$

7. **(d)**



$$u\cos60^\circ = \frac{20}{\sqrt{2}}$$

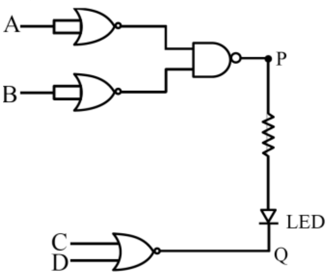
$$\frac{u}{2} = \frac{20}{\sqrt{2}}$$

$$u = \frac{40}{\sqrt{2}}$$

$$u = 20\sqrt{2}\ \text{m/s}$$

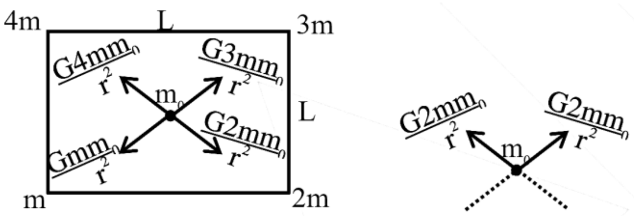
8. **(d)** According to pascal's law pressure at any point in liquid at rest is same in all direction.
It exist at every point in the liquid not just at boundaries. So statement (a) is false.
For interior molecule net cohesive forces are zero statement (b) is correct.

9. **(d)**



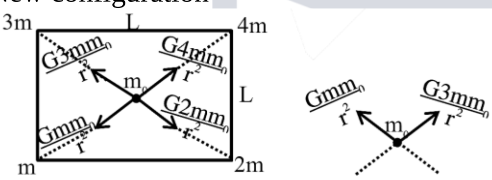
LED will glow in forward biasing :
P higher potential - 1
Q lower potential - 0

10. **(a)** Initial configuration



$F = 2\sqrt{2}\dfrac{Gmm_0}{r^2}$

New configuration

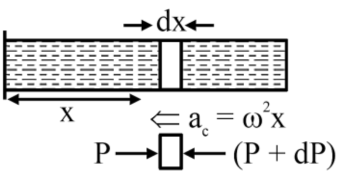


$F' = \sqrt{10}\dfrac{Gmm_0}{r^2} \Rightarrow \dfrac{F}{F'} = 2\sqrt{2}\cdot\dfrac{1}{\sqrt{10}} = \dfrac{2}{\sqrt{5}}$

$\therefore \alpha = 2$

11. **(d)** $hv = (4 \times 4.002 - 15.348) \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2 v = 14.94 \times 10^{19} kHz$

12. **(a)**



$A[(P + dP) - P] = (dm)(\omega^2 x)$

$dP = \dfrac{(dm)}{A}\omega^2 x$

$dP = \dfrac{(\rho)(A)(dx)\omega^2 x}{A}$

also $[PM = \rho RT]$

$\rho = \dfrac{PM}{RT}$

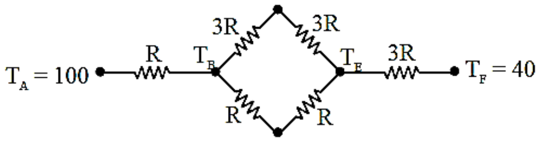
$dP = \left(\dfrac{PM}{RT}\right)\omega^2 x dx$

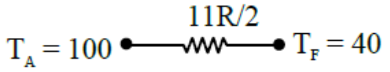
$\int_{P_A}^{P_B}\dfrac{dP}{P} = \dfrac{\omega^2 M}{RT}\int_0^{\ell} x dx$

$\ell n\left(\dfrac{P_B}{P_A}\right) = \dfrac{\omega^2\ell^2 M}{2RT}$

$P_B = P_A e^{\frac{M\omega^2\ell^2}{2RT}}$

13. (a) Let $\left[R = \frac{\ell}{3KA}\right]$





$\left[H = \frac{100-40}{\frac{11R}{2}}\right]$ .... (i)

$H = \frac{100-T_B}{R}$ .... (ii)

$H = \frac{T_E-40}{3R}$ .... (iii)

using (i) and (ii)

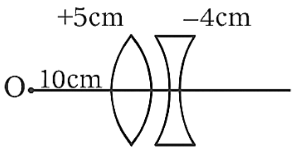
$120 = 1100 - 11\ T_A$

$T_B = 89°C$

using (i) and (iii)

$T_E = 73°C$

14. (a) Initial configuration



$\frac{1}{f} = \frac{1}{5} - \frac{1}{4} = -\frac{1}{20}$

$f = -20$ cm

$u = -10$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

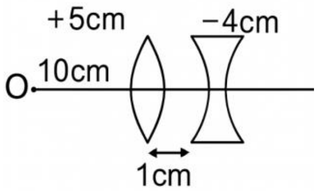
$v = \frac{uf}{u+f}$

$m_1 = \frac{v}{u} = \frac{f}{u+f}$

$= \frac{-20}{-10-20}$

$= +\frac{2}{3}$

New configuration



1^{st} refraction

$u = -10$ cm, $f = +5$ cm

$v = \frac{uf}{u+f} = +10$ cm

$m = -1$

2^{nd} refraction

$u = +9$cm$f = -4$ cm

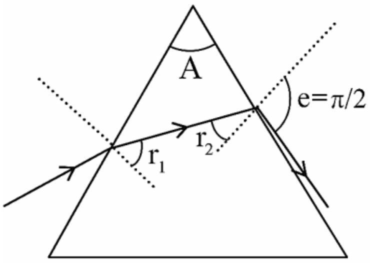
$m' = \frac{f}{u+f} = \frac{-4}{5}$

$m_2 = mm' = (-1)\left(-\frac{4}{5}\right) = \frac{4}{5}$

$\frac{m_1}{m_2} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$

15. **(a)**Equilateral prism.

$A = 60^\circ$



$\mu \mathrm{Sin} r_2 = 1.\mathrm{sin} e = 1$

$\mathrm{Sin} r_2 = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$

$r_2 = 45^\circ$

$\therefore r_1 = A - r_2 = 15^\circ$

16. **(d)**PV^γ = constant

$TV^{\gamma-1}$ = constant

$TV^{\gamma-1} = \left(\frac{T}{4}\right)(8\ V)^{(\gamma-1)}$

$4 = 8^{(\gamma-1)}$

$2^2 = 2^{3\gamma-3}$

$2 = 3(\gamma - 1)$

$\gamma = \frac{5}{3}$

Gas is a monoatomic gas

17. **(c)** $\vec{E} = 10x\hat{i} + 5y\hat{j}$

$V_{\text{at }(10,20)} = 500\ V$

$\Delta V - \int \vec{E} \cdot d\vec{r}$

$500 - V_0 = -\int_{(0,0)}^{(10,20)} (10x\hat{i} + 5y\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$

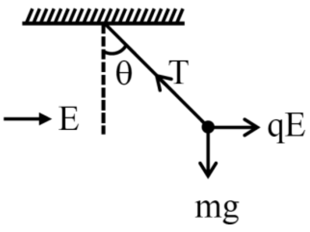
$500 - V_0 = -\left[5x^2 + \frac{5y^2}{2}\right]_{(0,0)}^{(10,20)}$

$V_0 - 500 = \left(500 + 5 \times \frac{400}{2}\right) - (0 - 0)$

$V_0 - 500 = 500 + 1000$

$V_0 = 2000\ V$

18. **(c)**



$T = \sqrt{(qE)^2 + (mg)^2}$

19. **(d)** (A) $F = Kx$

$[MLT^{-2}] = [K][L]$

$[K] = ML^0\ T^{-2}$

(B) Thermal conductivity

$$\frac{dQ}{dt} = \frac{kA}{\ell}\Delta T$$

$$ML^2\,T^{-3} = \frac{[k]L^2\,K}{L}$$

$$[K^1] = MLT^{-3}\,K^{-1}$$

(C) Boltzman constant

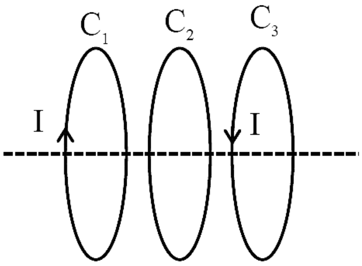
$$[K] = ML^2\,T^{-2}\,K^{-1}$$

(D) Inductive reactance

$$\frac{[V]}{[I]} = \frac{ML^2\,T^{-3}\,A^{-1}}{A}$$

$$= ML^2\,T^{-3}\,A^{-2}$$

20. (b)



Magnetic field through the coil is

$$\vec{B} = \left(B_{C_2} - B_{C_1}\right)\hat{i}$$

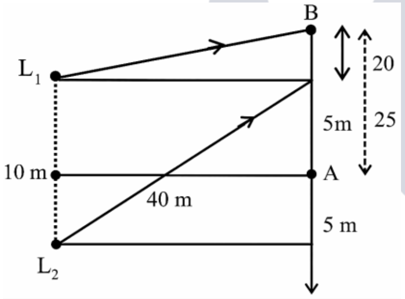
$$\phi = \left(B_{C_2} - B_{C_1}\right)A$$

$$\varepsilon = \frac{-d\phi}{dt}$$

Find the direction according to Lenz's law

If coil move away then magnetic field decreases & vice versa

21. (600)



Point B will 10^{th} maxima

$$\Delta x = L_2\,B - L_1\,B$$

$$L_1\,B = \sqrt{20^2 + 40^2} = 20\sqrt{5}\text{ m} = 44.6\text{ m}$$

$$L_2\,B = \sqrt{40^2 + 30^2} = 50\text{ m}$$

$$\Delta x = 50 - 44.6 = 5.4\text{ m}$$

$$\Delta x = n\lambda$$

$$5.4 = 10 \times \lambda$$

$$\lambda = 0.54\text{ m}$$

$$V = f\lambda$$

$$f = \frac{324}{0.54} = 600\text{ Hz}$$

22. (4) $n = \frac{c}{v}$

$$V = \frac{\omega}{k} = \frac{4.5 \times 10^{14}}{3 \times 10^6} = \frac{3}{2} \times 10^8$$

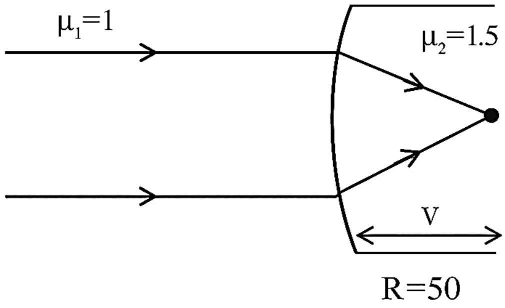
$n = 2$

$n = \sqrt{\mu_r \varepsilon_r}\ (\mu_r = 1)$

$2 = \sqrt{\varepsilon_r}$

$\varepsilon_r = 4$

23. (100)



$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5 - 1}{50}$

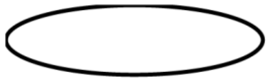
$V = 150$ cm

$x \to$ measure from center

$x = V - R$

$= 150 - 50 = 100$ cm

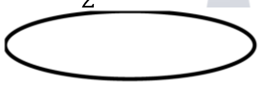
24. (16)



(T_1, R_1)

$m_1 = \pi R_1^2\ T_1 \rho$

$I_1 = \frac{m_1 R_1^2}{2}$



(T_2, R_2)

$m_2 = \pi R_2^2\ T_2 \rho$

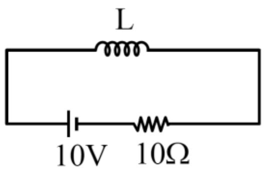
$I_2 = \frac{m_2 R_2^2}{2}$

$I_1 = I_2$

$\frac{\pi R_1^2\ T_1 \rho R_1^2}{2} = \frac{\pi R_2^2\ T_2 \rho R_2^2}{2} \Rightarrow \frac{T_1}{T_2} = \frac{1}{16}$

25. (20) $L = 10 \times 10^{-3}$H

$N = 10^4$



$I_0 = \frac{10}{10} = 1$ A (max current)

$I = \frac{1}{e}$

$E_d = \frac{B^2}{2\mu_0}$

$B = \mu_0 n I$

$L = \mu_0 n^2 \pi R^2 \ell$

$$E_d = \frac{\mu_0 n^2 I^2}{2}$$
$$= \frac{4\pi \times 10^{-7} \times 10^8 \times \frac{1}{e^2}}{2}$$
$$= \frac{20\pi}{e^2}$$

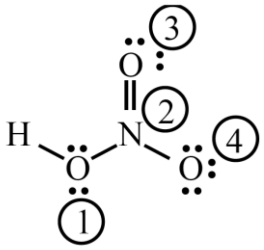
26. (d) $Co^{3+} \longrightarrow 3\,d^6 \Rightarrow t_{2\,g}^{2,2,2} e_g^{0,0}$, unpaired electron $= 0$

$Fe^{3+} \rightarrow 3\,d^5 \Rightarrow t_{2\,g}^{2,2,1} e_g^{0,0}$ unpaired electron $= 1$

$Cr^{3+} \rightarrow 3\,d^3 \Rightarrow t_{2\,g}^{1,1,1} e_g^{0,0}$ unpaired electron $= 3$

$Mn^{3+} \rightarrow 3\,d^4 \Rightarrow t_{2\,g}^{2,1,1} e_g^{0,0}$ unpaired electron $= 2$

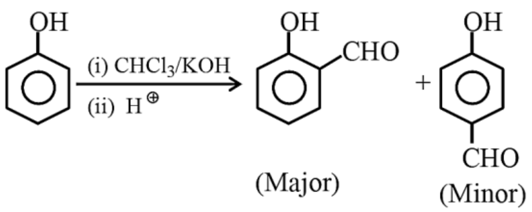
27. (c) Consider the structure of HNO_3



1 :- 0
2 :- (+1)
3 :- 0
4 :- (-1)

Formal charge $=$ valence e's - non bonding e's
$$-\left(\frac{\text{bonding electrons}}{2}\right)$$

28. (d)



(can be separated by steam distillation)

29. (b) $E_n = -R_H \times \frac{Z^2}{n^2}$

$$\Delta E = 2.18 \times 10^{-11} \times 10^{-7} \times 1^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$
$$= 1.635 \times 10^{-18} \text{ Joule/atom}$$
$$= 1.635 \times 10^{-18} \times 6.02 \times 10^{23} \text{ Joule / mole}$$
$$= 9.835 \times 10^5 \text{ Joule / mole}$$

30. (c) Element E is N , the species is NH_4^+, among B, C, N and O, N has highest first ionization energy.

31. (a) $2Al(s) + 6HCl(aq) \rightarrow 2Al^{3+}(aq) + 6Cl^-(aq) + 3H_2(g)$

Mole of H_2 produced
$= 2 \times$ mole of HCl used
$= \frac{2}{3} \times$ mole of Al used

32. From Henry's law :

$P(g) = K_H \cdot X(g)$
$\log P(g) = \log K_H + \log X(g)$

33. **(d)** $Cu \xrightarrow{\text{dil. HCl}}$ no reaction

$CuSO_4 \xrightarrow{KCN} \underset{\text{perfect complex}}{[Cu(CN)_4]^{3-}} \xrightarrow{H_2S}$ no sulphide ppt

34. **(b)**Order of nucleophilicity :

$I^{\ominus} > C_2H_5O^{\ominus} > PhO^{\ominus} > F^{\ominus}$

35. **(b)**Option (A) $W = -nRT \ln \frac{V_2}{V_1}$

$= \frac{-2 \times 8.314 \times 300}{1000} \times \ln\left(\frac{20}{2}\right) kJ$

$= -11.5\ kJ$

Option (B)

$W = -P_{ext}[V_2 - V_1]$

$= -3[3 - 1]$

$= -6\ kJ$

Option (C)

$\Delta U = nC_v \Delta T$

$= 1 \times \frac{3}{2} \times \frac{8.314 \times 320}{1000} 100\ kJ$

$= 3.99$

Option (D)

$\Delta H = nC_p \Delta T$

$= 1 \times \frac{5}{2} \times \frac{8.314 \times 337}{1000}\ kJ = 7\ kJ$

36. **(c)** Nitrobenzene does not give Fridel-Craft acylation since it is highly deactivated ring.

37. **(b)** For I st order reaction

Rate $= k[A]$

with decrease in concentration of A , rate of reaction decreases.

38. **(a)**

List-I Reagents		List-II Name of Reaction involving carbonyl compound	
A.	$NH_2 - NH_2$, KOH	III.	Wolff-Kishner Reduction
B.	$Ag(NH_3)_2OH$	I.	Tollen's Test
C.	Aq. $CuSO_4$, Sodium Potassium tartarate, KOH	IV.	Fehling's Test
D.	$Zn - Hg$, HCl	II.	Clemmensen Reduction

39. **(c)** $HF < HCl < HBr < HI$ (bond length order)

$F < Cl < Br < I$ (radius order)

$PH_3 < AsH_3 < NH_3 < SbH_3 < BiH_3$ (Boiling point order)

Maximum possible covalency of phosphorous is 6

40. **(d)** $C_{12}H_{22}O_{11} + H_2O \xrightarrow{HCl} C_6H_{12}O_6 + C_6H_{12}O_6$

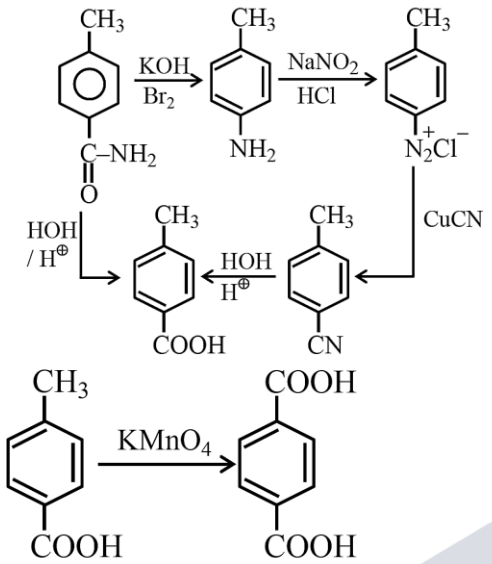
D- Glucose + D fructose

$[\alpha]_{\text{D-sucrose}} = +66.5°, [\alpha]_{\text{D-Glucose}} = +52.5°,$

$[\alpha]_{\text{D-Fructose}} = -92.4°$

⇒ Sucrose is dextrorotatory and hydrolysed product is laevorotatory.

41. (c)



42. (c) Rate of $S_N1 \propto$ Stability of $C^{\oplus}$ formed

(I) and (II) are unstable due to Bredt's rule, (I) has more +I effect.

(II) < (I) < (III) < (IV)

43. (b) $XF_3 = BF_3; sp^2$

$YF_3 = NF_3; sp^3$

44. (d) Chlorocylohexane is more polar due to -I effect of -Cl,

Whereas chlorobenzene has $-I > +M$, so it is less polar & also has partial double bond character.

45. (b) K_H depends on the nature of gas and solvent.

46. (66) $C_6H_{10} \xrightarrow{Br_2} C_6H_{10}Br_2$

Molecular mass of $C_6H_{10}Br_2$ is :

$12 \times 6 + 10 + 160$

$72 + 10 + 160 = 242$

$\% \text{ of Br} = \frac{160}{242} \times 100$

$\% \text{ of Br} = 66.11\% \approx 66\%$

47. (4) $E^{\circ}_{cell} = -0.44 - (-0.90)$

$= +0.46\ V$

Applying Nernst equation :-

$$E_{cell} = E^{\circ}_{cell} - \frac{0.06}{n} \log Q$$

$$E_{cell} = 0.46 - \frac{0.06}{6} \log 10^6$$

$E_{cell} = 4 \times 10^{-1}$

$x = 4$

48. (1303) $10^4 e^{\frac{-24000}{T}} = 10^6 e^{\frac{-30000}{T}}$

$e^{\frac{6000}{T}} = 100$

$$\frac{6000}{T} = 2\ln 10$$

$$T = \frac{6000}{2 \times 2.303}$$

$T = 1302.64\ K$

$T \approx 1303\ K$

49. (43) Iodine gives violet colour

$\% \text{ of I} = \frac{\text{Atomic weight of I}}{\text{Molecular weight of AgI}} \times \frac{m}{W} \times 100$

$= \frac{127}{235} \times \frac{0.12}{0.15} \times 100$

$\% \text{ of I} = 43.23\% \approx 43\%$

50. (33) $-1.664 \times 10^3 = -8.3 \times 300 \ln K_p$

$\ln K_P = 0.693$

$K_p = 2$

$2 = \frac{4\alpha^2 P_0}{1-\alpha^2}$

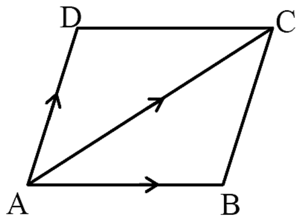
$\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \left(\frac{100}{3} \times 10^{-2}\right)^{1/2}$

$= (33.33 \times 10^{-2})^{1/2}$

51. (c) $\overrightarrow{AC} = 3\hat{i} + 6\hat{j} + (\lambda - 5)k$

$\vec{v} \cdot AC = 1 \Rightarrow 3 + 6 + \lambda - 5 = \sqrt{9 + 36 + (\lambda - 5)^2}$



$\Rightarrow \lambda^2 + 8\lambda + 16 = \lambda^2 - 10\lambda + 70$

$\Rightarrow \lambda = \frac{54}{18} = 3$

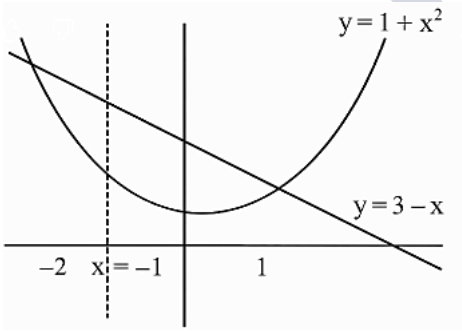
$\therefore$ Quadratic : $9x^2 - 18x + 5 = 0 \Rightarrow x = \frac{1}{3}, \frac{5}{3}$

$\therefore 2\alpha - \beta = \frac{10-1}{3} = 3$

52. (b) $R = \{(3,3), (7,8), (11,13)\}$

to make it symmetric $(8,7), (13,11)$ must be added.

53. (d)



$\frac{m}{n} = \frac{\int_{-1}^{1} [(3-x) - (1+x^2)]dx}{\int_{-2}^{1} [(3-x) - (1+x^2)]dx} = \frac{20}{7}$

$\therefore m + n = 20 + 7$

$= 27$

54. (b) Req. probability $= 1$ - (product not divisible by 3)

Multiple of $3 = 16$

Not multiple of $3 = 34$

$= 1 - \frac{^{34}C_2}{^{50}C_2} = \frac{664}{1225}$

55. **(a)** $f(x) = x^{2025} - x^{2000}$

$f'(x) = 0 \Rightarrow x = \left(\frac{2000}{2025}\right)^{1/25} = \alpha(\text{ say })$

$f(0) = 0, f(1) = 0, f(\alpha) = \left(\frac{80}{81}\right)^{80} \cdot \frac{-1}{81} = 80^{80} \cdot (-81)^{-81}$

56. **(b)** Let $P(2\lambda + 1, -3\lambda - 1, \lambda)$

Then $4\lambda^2 + 9\lambda^2 + \lambda^2 = 16 \cdot 14 \Rightarrow \lambda = \pm 4 \Rightarrow -4$ (nearer to origin)

$P(-7, 11, -4)$

Shortest distance $= \frac{\begin{vmatrix} 2 & -1 & 7 \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}}$

$= \frac{28}{\sqrt{1 + 25 + 9}} = \frac{4\sqrt{7}}{\sqrt{5}}$

57. **(d)** $\sum P(x_i) = 1$

$\Rightarrow 9k + 10k^2 = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow k = \frac{1}{10}$

$P(3 < x \leq 6) = 3k + 3k^2$

$= \frac{3}{10} + \frac{3}{100} = 0.33$

$= 0.33$

58. **(b)** Case I $x < -4$

$x(-(x + 4)) + 3(-(x + 2)) + 10 = 0$

$x^2 + 7x - 4 = 0$

$\Rightarrow x = -\frac{7+\sqrt{65}}{2}$ or $-\frac{7-\sqrt{65}}{2}$

Reject Acept

Case II $-4 \leq x < -2$

$x(x + 4) + 3(-(x + 2)) + 10 = 0$

$x^2 + x + 4 = 0$

$D < 0$ No solution

Case III $x \geq -1$

$x(x + 4) + 3(x + 2) + 10 = 0$

$x^2 + 7x + 16 = 0$

$D < 0$ No solution

$\Rightarrow$ No. of solution = 1.

59. **(d)** $6\int_1^x f(t)dt = 3xf(x) + x^3 - 4$

Diff. both side

$6f(x) = 3xf'(x) + 3f(x) + 3x^2$

$3f(x) = 3xf'(x) + 3x^2$

$x\frac{dy}{dx} - y = -x^2$

$\frac{x\frac{dy}{dx} - 9}{x^2} = -1$

$\Rightarrow \frac{d}{dx}\left(\frac{y}{x}\right) = -1$

$\frac{y}{x} = -x + C$

$\Rightarrow f(x) = -x^2 + Cx$

at $x = 1, y = 1 \Rightarrow C = 2$

$f(x) = -x^2 + 2x$

$f(2) - f(3) = 3$

60. **(b)** $y = \frac{1-\alpha x}{2}$

Put this in equation of hyperbola

$\therefore x^2 - 9\left(\frac{1-\alpha x}{2}\right)^2 = 9$

$(4 - 9\alpha^2)x^2 + 18\alpha x - 45 = 0$

$\because$ line does not intersect hyperbola

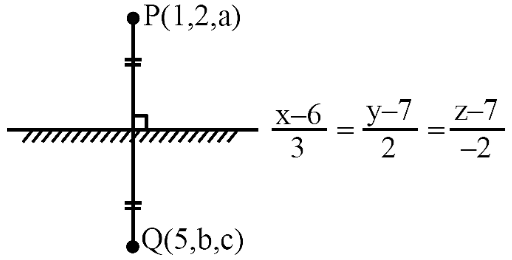
$\therefore D < 0$

$\Rightarrow \alpha^2 - \frac{5}{9} > 0$

$\Rightarrow \alpha \in \left(-\infty, -\frac{\sqrt{5}}{3}\right) \cup \left(\frac{\sqrt{5}}{3}, \infty\right)$

Here $\frac{\sqrt{5}}{3} \simeq 0.74$

61. **(c)**



Point $M \equiv \left(3, \frac{b}{2} + 1, \frac{c+a}{2}\right)$ satisfies the line

$\frac{3-6}{3} = \frac{\frac{b}{2} + 1 - 7}{2} = \frac{\frac{c+a}{2} - 7}{-2}$

$-1 = \frac{b-12}{4} = \frac{c+a-14}{-4}$

$\Rightarrow b = 8$ ... (i) & $c + a = 18$ .... (ii)

Now $PQ \perp L$

$\Rightarrow (4i + (b-2)j + (c-a)k) \cdot (3i + 2j - 2k) = 0$

$\Rightarrow 12 + 2(b-2) - 2(c-a) = 0$

$\Rightarrow 6 + (b-2) - (c-a) = 0$

$\Rightarrow b - c + a + 4 = 0$

$\Rightarrow 8 - c + a + 4 = 0$

$\Rightarrow c + a = 12$ .... (iii)

From (ii) & (iii)

$c = 15 \& a = 3$

So $a^2 + b^2 + c^2 = 9 + 64 + 225 = 298$

62. **(a)** $(x-2)^2 + (y-1)^2 = 3^2 \& (x+1)^2 + (y+4)^2 = r^2$

$|r_1 - r_2| < c_1c_2 < r_1 + r_2$

$|r - 3| < \sqrt{(2+1)^2 + (1+4)^2} < r + 3$

$|r - 3| < \sqrt{34} \& r + 3 > \sqrt{34}$

$-\sqrt{34} < r - 3 < \sqrt{34} \& r > \sqrt{34} - 3$

i.e. $r = (3 - \sqrt{34}, 3 + \sqrt{34}) \cap (\sqrt{34} - 3, \infty)$

i.e. $r \in (\sqrt{34} - 3, \sqrt{34} + 3)$

$\therefore \alpha\beta = (\sqrt{34} - 3)(\sqrt{34} + 3)$

$= 34 - 9 = 25$

63. (d) $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 13 & 21 \\ 21 & 34 \end{bmatrix}$

$|A^{2025} - 3A^{2024} + A^{2023}|$

$= |A^{2023}(A^2 - 3A + I)|$

$= |A|^{2023} |A^2 - 3A + I|$

$= 1.\begin{vmatrix} 8 & 12 \\ 12 & 20 \end{vmatrix} = 160 - 144 = 16$

64. (a) $-1 \le \frac{5-x}{2x+3} \le \&10 - x > 0, 10 - x \ne 1$

$\left|\frac{5-x}{2x+3}\right| \ge 1\&x < 10\&x \ne 9$

$(5-x)^2 - (2x+3)^2 \le 0\&x < 10\&4x \ne 9$

$(x+8)(3x-2) \ge 0\&x < 10\&x \ne 9$

$\Rightarrow (-\infty, -8] \cup \left[\frac{2}{3}, 10\right) - \{9\}$

$\Rightarrow (\alpha + \beta + \gamma + \delta) = 6\left(-8 + \frac{2}{3} + 10 + 9\right)$

$= 70$

65. (c) $I = \int_{-\pi/2}^{\pi/2} \frac{1}{[x]+4} dx$

$I = \int_{-\pi/2}^{-1} \frac{dx}{2} + \int_{-1}^{0} \frac{dx}{3} + \int_{1}^{0} \frac{dx}{4} + \int_{1}^{\pi/2} \frac{dx}{5}$

$= \frac{1}{2}\left(-1 + \frac{\pi}{2}\right) + \frac{1}{3}(1) + \frac{1}{4}(1) + \left(\frac{\pi}{2} - 1\right)\frac{1}{5}$

$= \frac{7\pi}{20} - \frac{7}{60} = \frac{7}{60}(3\pi - 1)$

66. (d) Let $1 + x = r$

$\therefore S = 1.r + 2.r^2 + 3.r^3 + \cdots ... + 100r^{100}$ ... . (i) (AGP)

$rS = 1.r^2 + 2.r^3 + \cdots .. + 99r^{100} + 100r^{101}$ …………… (ii)

(i) - (ii) gives

$S = -\frac{(1+x)^{101}}{x^2} + \frac{1}{x^2} + \frac{100(1+x)^{101}}{x}$

$\therefore$ coefficient x^{48} in S

$= -$ coefficient of x^{48} in $\frac{(1+x)^{101}}{x^2} + 100.$

Coefficient of x^{48} in $\frac{(1+x)^{101}}{x}$

$= 100\,{}^{101}C_{49} - {}^{101}C_{50}$

67. (a) $(x_1y_1) = (3t_1^2, 6t_1)\&(x_2y_2) = (3t_2^2, 6t_2)$

$t_1t_2 = -4$

$x_1x_2 = 9(t_1t_2)^2, y_1y_2 = 36t_1t_2$

$x_1x_2 - y_1y_2 = 9(16) - 36(-4)$

$= 144 + 144$

$= 288$

68. (c) $\tan^{-1}4x + \tan^{-1}6x = \frac{\pi}{6}$

$\Rightarrow \tan^{-1}\left(\frac{4x+6x}{1+24x^2}\right) = \frac{\pi}{6}$

$\Rightarrow \frac{10x}{1-24x^2} = \frac{1}{\sqrt{3}}$

$\Rightarrow 24x^2 + 10\sqrt{3}x - 1 = 0$

$x = \frac{-10\sqrt{3} \pm \sqrt{300+96}}{48}$

$x = \dfrac{\sqrt{396} - 10\sqrt{3}}{48}$

Only 1 solution in $\left(-\frac{1}{2\sqrt{6}}, \frac{1}{2\sqrt{6}}\right)$

69. (a) $\frac{xdy-ydx}{x^2} = \frac{\sqrt{x^2+y^2}}{x^2}dx$

$d\left(\frac{y}{x}\right) = \sqrt{1+\frac{y^2}{x^2}} \cdot \frac{1}{x}dx$

$\int \dfrac{d\left(\frac{y}{x}\right)}{\sqrt{1+\left(\frac{y}{x}\right)^2}} = \int \frac{1}{x}dx$

$\Rightarrow \ln\left(\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}}\right) = \ln x + \ln k = \ln kx$

$\Rightarrow y + \sqrt{x^2+y^2} = kx^2$

$0 + 1 = k$

$\Rightarrow y + \sqrt{x^2+y^2} = x^2$

$y + \sqrt{9+y^2} = 9$

$y = 4$

70. (d) Sum of first 4 term $S_4 = 6$

$\frac{4}{2}(2a+3d) = 6 \Rightarrow 2a+3d = 3$ …………….. (i)

Sum of first 6 terms $S_6 = 4$

$\frac{6}{2}(2a+5d) = 4 \Rightarrow 2a+5d = \frac{4}{3}$ …………. (ii)

eq. (ii) − eq.(i)

$(2a+5d) - (2a+3d) = \frac{4}{3} - 3$

$\Rightarrow d = -\frac{5}{6}$

$\therefore 2a + 3\left(-\frac{5}{6}\right) = 3 \Rightarrow a = \frac{11}{4}$

$S_{12} = \frac{12}{2}\left\{2 \times \frac{11}{4} + (12-1)\left(-\frac{5}{6}\right)\right\}$

$S_{12} = 6\left(-\frac{22}{6}\right) = -22$

71. (49) $(9+7\omega-7\omega^2) + \omega^{20}(9+7\omega-7\omega^2)^{20} + \omega^{40}(9+7\omega-7\omega^2)^{20} + \left(14+7(\omega+\omega^2)\right)^{20}(9+7\omega-7\omega^2)^{20}(1+\omega+\omega^2) + (14-7)^{20} = 7^{20} = (49)^{10}$

Hence, $M = 49$

72. (18) $A\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}3\\3\\2\end{bmatrix}$ and $A\begin{bmatrix}3\\3\\2\end{bmatrix} = \begin{bmatrix}-3\\19\\-24\end{bmatrix}$

det. $(\text{adj.}(2\text{adj}(A+I)) = |2\text{adj}(A+I)|^2 = 64|\text{adj}(A+I)|^2 = 64|A+I|^4$

Let $A = \begin{bmatrix}0 & a & b\\-a & 0 & c\\-b & -c & 0\end{bmatrix} \Rightarrow \begin{Bmatrix}-a = 3\\-b+c = 2\\3a+2b = -3\end{Bmatrix} \Rightarrow \begin{Bmatrix}a = -3\\b = 3\\c = 5\end{Bmatrix}$

$\therefore |A+I| == \begin{vmatrix}1 & -3 & 3\\3 & 1 & 5\\-3 & -5 & 1\end{vmatrix} = 44$

73. (16) $\int (\tan x)^{-11/2} \cdot \sec^8 x dx$

$= \int (\tan x)^{-11/2}(1+\tan^2 x)3\sec^2 x dx$

Put $\tan x = t$

$\Rightarrow \int t^{-11/2}(1+t^2)^3 dx = \int t^{-11/2}(1+t^6+3t^2+3(4))dt$

$= \int \left(t^{-11/2}+t^{1/2}+3\cdot t^{-7/2}+3\cdot t^{-3/2}\right)dt$

$= -\frac{2}{9}(\cot x)^{9/2}-\frac{6}{5}(\cot)^{5/2}-6(\cot x)^{1/2}+\frac{2}{3}(\cot x)^{-3/2}+C$

$\Rightarrow p_1=2, p_2=6, p_3=6, p_4=2$

$\& q_1=9, q_2=5, q_3=1, q_4=3$

$\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4}=\frac{15\cdot2\cdot6\cdot6\cdot2}{9\cdot5\cdot1\cdot3}=16$

74. **(4)** Use $\sin(A+B)\sin(A+B)=\sin^2A-\sin^2B$

$\cos(A+B)\cos(A-B)=\cos^2A-\sin^2B$

$\frac{\cos60°\cos36°}{\sin30°\sin18°}=\frac{\sqrt{5}+1}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}=\frac{(\sqrt{5}+1)^2}{4}=\frac{3+\sqrt{5}}{2}$

$\alpha=3;\beta=1$

So, $(\alpha+\beta)=4$

75. **(660)** Case 1 :

2 from AB, 2 from BC, 1 from AC

$\binom{4}{2}\cdot\binom{5}{2}\cdot\binom{4}{1}=6\cdot10\cdot4=240$

Case 2 :

2 from AB, 1 from BC, 2 from AC

$\binom{4}{2}\cdot\binom{5}{1}\cdot\binom{4}{2}=6\cdot5\cdot6=180$

Case 3 :

1 from AB, 2 from BC, 2 from AC

$\binom{4}{1}\cdot\binom{5}{2}\cdot\binom{4}{2}=4\cdot10\cdot6=240$