1. **(c)** $\frac{\varepsilon E}{t} = \frac{\varepsilon}{t}\frac{1}{4\pi\varepsilon}\frac{q}{r^2}$

$\Rightarrow \frac{AT}{TL^2} = (AL^{-2})$

A/m^2

2. **(b)** $T = 2\pi\sqrt{\frac{\ell}{g}}$ $T^2 = \frac{4\pi^2\ell}{g}$

$\frac{1}{T^2} = \frac{g}{4\pi^2\ell}$

3. **(b)** Collision frequency $(z) = \sqrt{2}\pi d^2 N\sqrt{\frac{8RT}{\pi M}}$ Temp, N are same

$Z \propto \frac{d^2}{\sqrt{M}}$

$d_A = \frac{d_B}{2}$

$M_A = 4M_B$

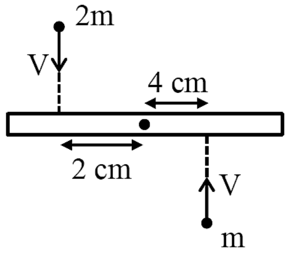
$\frac{Z_A}{Z_B} = \frac{d_A^2}{\sqrt{M_A}} \times \frac{\sqrt{M_B}}{d_B^2} = \left(\sqrt{\frac{M_B}{M_A}}\right)\left(\frac{d_A}{d_B}\right)^2$

$= \left(\sqrt{\frac{1}{4}}\right)\left(\frac{1}{2}\right)^2$

$\frac{Z_A}{Z_B} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$\Rightarrow Z_A = \frac{32 \times 10^8}{8} = 4 \times 10^8/s$

4. **(a)**



Using angular momentum conservation about COM of rod :

$L_i = L_f$

$m \times V \times 4 + 2\,m \times V \times 2 = \left(\frac{20\,m(12)^2}{12} + m \times 4^2 + 2\,m \times 2^2\right)\omega$

$8mV = (240\,m + 24\,m)\omega$

$8\,V = 264\omega$

$\frac{V}{\omega} = 33$

5. **(d)** Using volume conservation

$3\left(\frac{4}{3}\pi r^3\right) = \left(\frac{4}{3}\pi R^3\right)$

$R = 3^{1/3}r$

$\frac{V_i}{V_f} = \frac{\frac{kq}{r}}{\frac{k3q}{R}} = \frac{R}{3r} = \frac{3^{1/3}r}{3r} = \frac{1}{3^{2/3}}$

6. **(b)** Image should be real. So object should be placed beyond focus.

7. **(c)** n = 4 ___

n = 3 ___ paschen

n = 2 ___ Balmer

n = 1 ___ Lyman

Smallest wavelength of lyman

$\frac{1}{\lambda} = R\left(\frac{1}{12} - \frac{1}{\infty^2}\right)$

$R = \frac{1}{\lambda} = \frac{1}{91}\ nm^{-1}$

λ_{max} for balmer series

$n_1 = 2 \rightarrow n_2 = 3$

$\frac{1}{\lambda_B} = R\left(\frac{1}{4} - \frac{1}{9}\right)$

$\frac{1}{\lambda_B} = \frac{1}{91}\left(\frac{5}{36}\right)$

$\lambda_B = \left(\frac{91 \times 36}{5}\right) = 655.2\ nm$

λ_{max} paschen

$n_1 = 3 \rightarrow n_2 = 4$

$\frac{1}{\lambda_p} = \frac{1}{91}\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{1}{91} \times \frac{7}{144}$

$\lambda_p = \left(\frac{91 \times 144}{7}\right) = 1872\ nm$

$\Delta\lambda = \lambda_P - \lambda_B = 1872 - 655.2$

$\Delta\lambda = 1216.8$

$\Delta\lambda \approx 1217$

8. **(b)** $f = n\left(\frac{V_0}{2L}\right)$

$\frac{6V_0}{2L} - \frac{3V_0}{2L} = 2200$

$\frac{3V_0}{2L} = 2200$

$\frac{3 \times 330}{2 \times L} = 2200$

$L = \frac{3 \times 330}{2 \times 2200}$

$L = 0.225\ m$

$L = 225\ mm$

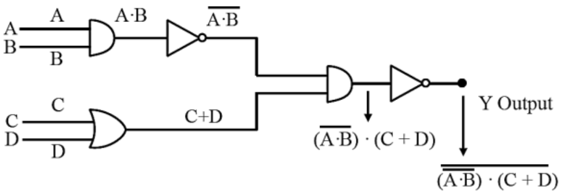
9. **(d)** $h = \frac{2T\cos\theta}{\rho gr}$

$\frac{\Delta h}{h}\% = \frac{\Delta T}{T}\% - \frac{\Delta\rho}{\rho}\% - \frac{\Delta r}{r}\%$

$\frac{\Delta h}{h}\% = 1 + 1 + 1$

$\frac{\Delta h}{h} = +1\%$

10. **(b)**



$Y = \overline{(\overline{A \cdot B}) \cdot (C + D)} = \overline{\overline{A \cdot B}} + (\overline{C + D})$

$Y = (A \cdot B) + (\overline{C + D})$

11. (a) $P_{out} = 1000\ W$

$P = VI$

$1000 = 250 \times I$

$I = 4\ A$

$P_{loss} = I^2R = (4)^2 \times 2 = 3200$

$P_{net} = 1000 + 32 = 103200$

$\eta = \left(\frac{P_{out}}{P_{net}}\right) \times 100 = \frac{1000}{1032} \times 100 = 96.9\%$

12. (c) $\frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1} \Big|\ v = f\lambda$

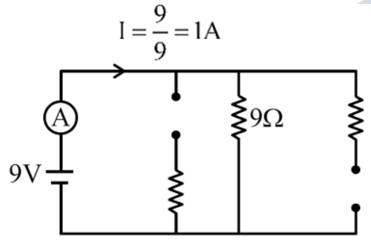
$\frac{\mu_1}{\mu_2} = \frac{\lambda_2}{\lambda_1}$

$\frac{\mu_B}{3/2} = \frac{\lambda}{540}$

$\lambda = \left(\frac{4 \times 2}{3 \times 3} \times 540\right)$

$\lambda = 480\ nm$

13. (a) Just after closing the switch, inductor will behave as open circuit,



14. (b) $T = 2\pi\sqrt{\frac{R^3}{GM}}$

$\because M = \rho \cdot \frac{4}{3}\pi R^3$

$\therefore T = 2gp\sqrt{\frac{1}{Gp\frac{4}{3}\pi}}$

Statement I is correct.

And $\therefore \frac{GM}{R^2} = g$

$\therefore T = 2\pi\sqrt{\frac{R}{g}}$

Statement II is correct

15. (a) $\beta_{cm} = \frac{2\lambda D}{a}$

(A) Correct $\beta \propto \lambda$ (B) Incorrect

(C) Correct $\beta \propto \frac{1}{d}$ (D) Incorrect

(E) Correct

Statement A, D & E are correct.

No option matching

16. (d) Statement-I : Incorrect

Correct equation is $W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$

Statement-II : Incorrect

17. **(a)** Electric potential → $V = \frac{5kq}{R}$

As regular polygon → $\vec{E} = 0$

18. **(d)** $I = \frac{1}{2} \epsilon_0 E_0^2 \cdot C$

$$\therefore E_0 = \sqrt{\frac{2I}{\epsilon_0 C}}$$

$$\& \frac{E_0}{B_0} = C$$

$$\therefore B_0 = \frac{E_0}{C} = \frac{1}{C}\sqrt{\frac{2I}{\epsilon_0 C}}$$

$$\therefore B_0 = \frac{1}{3 \times 10^8}\sqrt{\frac{2 \times 4 \times 10^{14}}{8.85 \times 10^{-12} \times 3 \times 10^8}}$$

$$B_0 = \frac{10}{3}\sqrt{\frac{8}{8.85 \times 3}}$$

$B_0 = 1.83\ T$

19. **(a)** $\phi = h\nu$

$$\nu = \frac{\phi}{h}$$

$$\omega = 2\pi\nu = \frac{2\pi\phi}{h} = \frac{2 \times 3.14 \times 110 \times 10^{-2}}{6.63 \times 10^{-34}}$$

$\omega = 1.04 \times 10^{16} \text{rad/sec}$

20. **(a)** $KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$$KE = \frac{1}{2}(m_1 + m_2)v_{cm}^2 + \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}|\vec{v}_1 - \vec{v}_2|^2$$

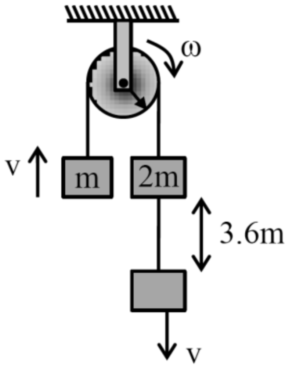
21. **(14)** $E = B\omega A \sin\omega t$

$$15.4 \times 10^{-3} = \frac{1}{2} \times 100 \times \frac{22}{7}r^2 \times \frac{1}{2}$$

$$r = \sqrt{\frac{15.4 \times 28 \times 10^{-5}}{22}}$$

$R = 14\ mm$

22. **(2)**



Using energy conservation

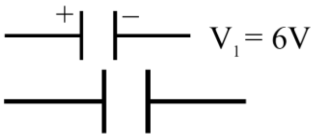
$$\frac{1}{2}mv^2 + \frac{1}{2}2mv^2 + \frac{1}{2}\frac{30mR^2}{2} \times \frac{v^2}{R^2} = mgh$$

$9mv^2 = mgh$

$v = \sqrt{\dfrac{gh}{9}} = \sqrt{\dfrac{10 \times 3.6}{9}}$

$v = \sqrt{4} = 2\ m/s$

23. (4) $C_1 = 10^{-5}\ F$



$C_2 = 2 \times 10^{-5}\ F$

$V_2 = 0$

$V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{10^{-5} \times 6 + 0}{3 \times 10^{-5}}$

$V = 2\ \text{volt}$

$Q_2 = C_2V = 2 \times 10^{-5} \times 2 = 4 \times 10^{-5} C$

24. (1) $V_\ell = \mu E = \mu \times \dfrac{V}{\ell}$

$I = neAV_d$

$V_d = \dfrac{I}{neA}$

$\mu = \dfrac{I\ell}{NneA}$

$\mu = \dfrac{1.6 \times 3}{2 \times 5 \times 10^{26} \times 1.6 \times 10^{-19} \times 2 \times 10^{-7}}$

$\mu = 1 \times 10^{-3}\ m^2/v \cdot s$

$\alpha = 1$

25. (7) $PV = nRT$

$\dfrac{P_1\ V_1}{RT_1} = \dfrac{P_2\ V_2}{RT_2}$

$\dfrac{2 \times 10^5 \times 60}{300} = \dfrac{P_2 \times 20}{350}$

$P_2 = \dfrac{2 \times 10^5 \times 7 \times 3}{6}$

$P_2 = 7 \times 10^5 = 7\ \text{atm}$

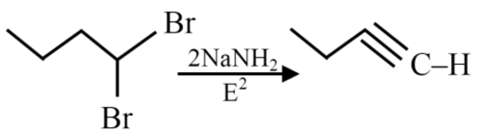
26. (d) Total weight of H_2SO_4

$= \left(100 \times \dfrac{98}{100}\right) + \left(100 \times \dfrac{49}{100}\right) = 147\text{gm}$

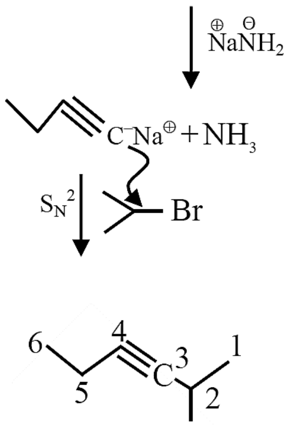
Total weight of $H_2O = 200 - 147 = 53\text{gm}$

Mole fraction of $H_2SO_4 = \dfrac{\frac{147}{98}}{\left(\frac{147}{98} + \frac{53}{18}\right)} = 0.337$

27. (c)



(x)



2–methyl hex–3–yne

28. **(c)** $A + 2\,B \longrightarrow AB_2$

Mole $\frac{36}{60}$ $\frac{56}{80}$

0.6 mole 0.7 mole

0.25 mole 0.35 mole

(A) Molecular wt. of AB_2 is

$60 + 2 \times 80 = 220$ g/mol

(B) LR is AB .

(C) Wt. of A remaining $= 0.25 \times 60 = 15$ g

(D) wt. of AB_2 formed $= 0.35 \times 220 = 77$gm.

29. **(b)** $Cr = (Ar)3\,d^5 4\,s^1$

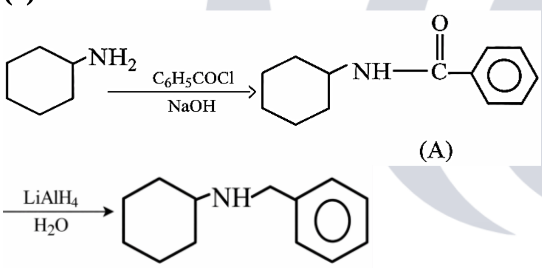
$Mn = (Ar)3\,d^5 4\,s^2$

$IE_1(Cr) < IE_1(Mn)$

$IE_2(Cr) > IE_2(Mn)$

$IE_3(Cr) < IE_3(Mn)$

30. **(c)**



31. **(c)** eq. of H_2SO_4 = eq. of Ammonia

$\Rightarrow \frac{15 \times 1 \times 2}{1000}$ = moles of ammonia $\times 1$

$\Rightarrow$ Moles of ammonia = moles of ' N '

$\Rightarrow$ Weight of nitrogen $= \frac{15 \times 1 \times 2}{1000} \times 14 = 0.42$

% weight of ' N' $= \frac{0.42}{1} \times 100 = 42\%$

32. **(d)** $K = Ae^{-Ea/RT}$

$$\log K = \log A - \frac{Ea}{2.303RT}$$

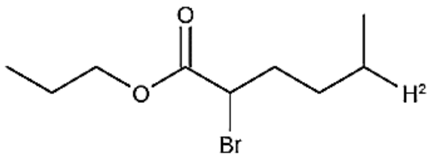
Plot of log K vs $\frac{1}{T}$ has

$$\Rightarrow \text{ slope } = -\frac{Ea}{2.303R}$$

$Ea \downarrow \Rightarrow -\frac{Ea}{RT} \uparrow \Rightarrow k \uparrow$ (rate increases)

B, C are correct.

33. (a)



Propyl-2-Bromo-5-Methylheptanoate

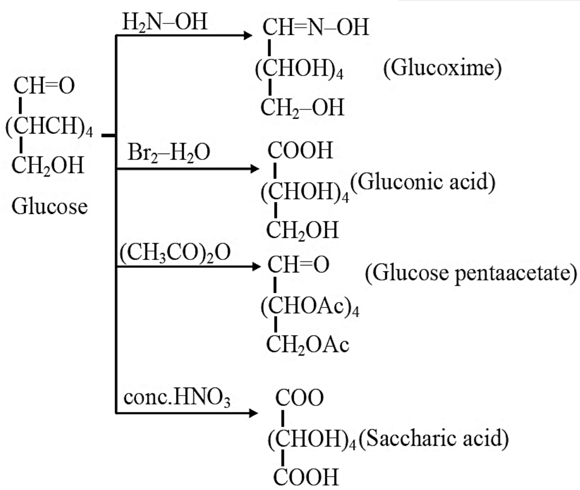
34. (b)

Electronegativity order : N > P > As > Sb

Most electronegative (N) Least electronegative (Sb)

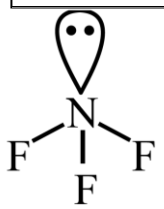
$X = N\ X_2O_3 = N_2O_3$ (Acidic)
$Y = Sb\ Y_2O_3 = Sb_2O_3$ (Amphoteric)
Statement-I is true
$BCl_3 + 3H_2O \rightarrow B(OH)_3 + 3HCl$
Statement-II is false

35. (b)



36. (c)

Molecule	Dipole moment
H_2S	0.95
H_2O	1.85
NF_3	0.23 (minimum)
NH_3	1.47
$CHCl_3$	1.04



Number of lone pair on central atom = 1

37. **(b)** $C < O < N < F$

$\xrightarrow[Z_{eff}\uparrow \text{I. E.}\uparrow]{2p^2 2p^4 2p^3 2p^5}$

Statement-I is correct

$S > Se > Te > Po > O$

$|\Delta_{eg} H| 200 \quad 195 \quad 190 \quad 174 \quad 141$(in kJ/mol)

Statement-II is correct.

38. **(b)** $pOH = pK_b + \log\frac{\text{Salt}}{\text{Base}}$

$4.75 = 4.75 + \log\frac{\text{Salt}}{\text{Base}}$

Milimoles of [Salt] = milimoles of [Base]

Option (D) :

$NH_4OH + HCl \rightarrow NH_4Cl + H_2O$

0.2M, 0.5 L 0.1M, 0.5 L

100 mmole 50 mmole

50 mmole - 50 mmole

Milimoles of NH_4OH = milimoles of NH_4Cl

39. **(d)** Transition of first Balmer line

$n_1 = 2; n_2 = 3$

$\Delta E = x = 13.6(1)^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$ ………………… (i)

Transition of 2 nd Balmer line

$n_1 = 2; n_2 = 4$

$\Delta E = 13.6(1)^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$ ……………. (ii)

Divide Eq. (ii) by eq. (i)

$$\frac{\Delta E}{x} = \frac{\frac{1}{4} - \frac{1}{16}}{\frac{1}{4} - \frac{1}{9}}$$

$$\frac{\Delta E}{x} = \frac{\frac{3}{16}}{\frac{5}{36}}$$

$$\frac{\Delta E}{x} = \frac{27}{20}$$

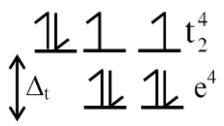
$\Delta E = 1.35x$.

40. **(b)** Primary standard must be soluble for standard solution formation.

41. **(b)** $[Ni(PPh_3)_2Cl_2]$

Given : Paramagnetic complex hence it must be tetrahedral so

Crystal field splitting :



$Ni^{+2} = (Ar)\ 3d^8$

(A) Tetrahedral complex not show geometrical isomerism.

(B) Complex is Blue colour

(C) Calculated spin only magnetic moment of the complex is 2.84 B.M.

(D) C.F.S.E $= -0.6\Delta_t(4) + 0.4\Delta_t(4)$

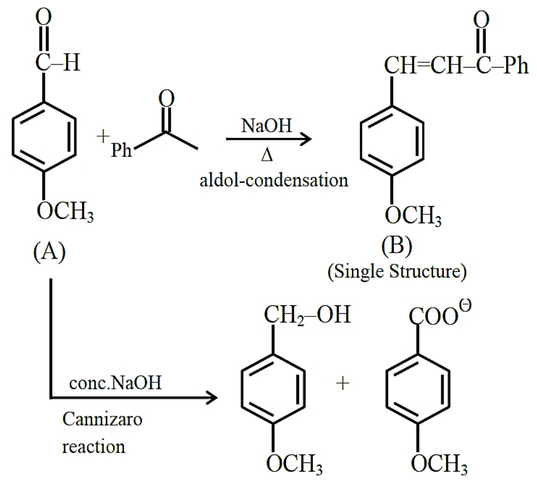
$= -0.8\Delta_t(\text{not} - 0.8\Delta_0)$

(E) $Ni(CO)_4$ also tetrahedral

Hence only A, B, D correct.

42. **(a)** Reducing power $\propto \frac{1}{\text{Reduction potential}}$

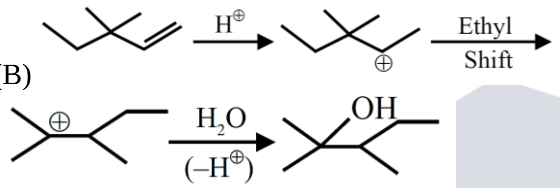
43. **(b)**



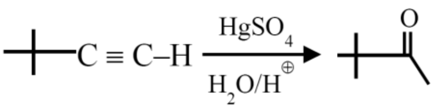
(On cross aldol reaction, a single structure is obtained but it can show geometrical isomerism).

44. (b)

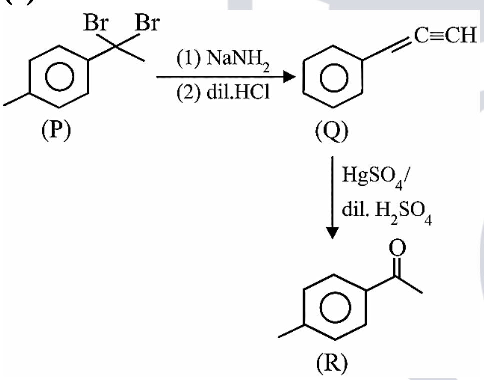
(B)



(E)



45. (c)



46. (4) For spontaneity $E_{cell} > 0$

At limiting condition :

E_{Oxi} (anode) $= -E_{Red}$ (cathode)

$H_2O \rightarrow 2H^+ + \frac{1}{2}O_2 + 2e^-$

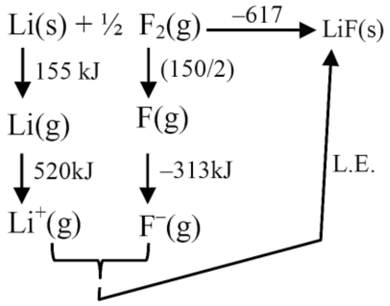
$$E = E^\circ - \frac{0.059}{2}\log\left[\frac{[H^+]^2 \times P_{O_2}^{1/2}}{1}\right]$$

$-0.997 = -1.23 + 0.059 \times pH$

$pH = 3.94$

$pH \simeq 4$

47. (1031)



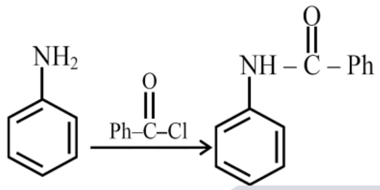
$-594 = 155 + 520 + \frac{150}{2} - 313 + (\text{ L.E.})$

$\text{L.E.} = -1031\ \text{kJ/mol}$

48. (3)

$\left.\begin{array}{l} Mn_3O_4 = MnO \cdot Mn_2O_3 \\ Fe_3O_4 = FeO \cdot Fe_2O_3 \\ Co_3O_4 = CoO \cdot Co_2O_3 \end{array}\right]$ Only three mixed oxides

49. (10)



5.8gm $\quad n = 0.0623 \times \frac{82}{100}$

$n = \frac{5.8}{93}$ $\quad \text{mass } = 0.051 \times 197$

$n = 0.0623$ $\text{mass } = 10.047$

50. (57) $\frac{E_{a_1}}{2.303R} = 1.5 \times 10^4$

$E_{a_1} = 1.5 \times 10^4 \times 2.303 \times 8.314$

$E_{a_1} = 28.7207 \times 10^4\ \text{J}$

$E_{a_1} = 287.207\ \text{kJ}$

$E_{a_2} = \frac{E_{a_1}}{5} = \frac{287.207}{5} = 57.44\ \text{kJ}$

51. (d) $x - ny + z = 6$

$x + (n-2)y + (n+1)z = 8$

$(n-1)y + z = 1$

$$\begin{vmatrix} 1 & -n & 1 \\ 1 & (n-2) & n+1 \\ 0 & n-1 & 1 \end{vmatrix} \neq 0$$

$\Rightarrow n^2 - 3n + 2 \neq 0$

$n \neq 1,2$

for unique solution $n = 3,4,5,6$

Now

P (probability when system of equations has unique solution) $= \frac{4}{6}$

So $k = 4$

Now required sum $= 4 + (3 + 4 + 5 + 6) = 22$

52. (b) median $= \frac{1001k}{2} = X_M$

$\therefore$ mean deviation about median $= \frac{\sum |X_i - X_M|}{n}$

$$= \frac{2\left(\frac{k}{2} + \frac{3k}{2} + \frac{5k}{2} + \cdots 500 \text{ terms}\right)}{1000}$$

$= \frac{2 \cdot \frac{k}{2}(500)^2}{1000} = \frac{500k}{2} = 500 \text{ (given)}$

$\therefore k = 2$

$\therefore k^2 = 4$

53. (a) $4x^2 + y^2 < 52, x, y \in Z$

$\downarrow \qquad \downarrow$

$0 \quad 0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm7 \rightarrow 1 \times 15 = 15$

$\pm1 \quad 0, \pm1, \pm2, \pm3 \ldots\ldots\ldots \pm6 \rightarrow 2 \times 13 = 26$

$\pm2 \quad 0, \pm1, \pm2, \pm3 \ldots\ldots\ldots \pm5 \rightarrow 2 \times 11 = 22$

$\pm3 \quad 0, \pm1, \pm2 \pm 3 \rightarrow 2 \times 7 = 14$

Number of elements $= 77$

54. (a) $4z^2 + \bar{z} = 0$

let $z = x + iy$

$4(x + iy)^2 + x - iy = 0$

$4x^2 - 4y^2 + 8xyi + x - iy = 0$

$4x^2 - 4y^2 + x = 0 \& y(8x - 1) = 0$

$\Rightarrow y = 0 \text{ or } x = \frac{1}{8}$

If $y = 0, 4x^2 + x = 0$

$x = 0, \frac{-1}{4}$

$\therefore z_1 = 0 + 0.i \; |z_1|^2 = 0$

$z_2 = 0 - \frac{1}{4} i \; |z_2|^2 = \frac{1}{16}$

If $x = \frac{1}{8}$,

$4 \times \frac{1}{64} - 4y^2 + \frac{1}{8} = 0$

$\Rightarrow 4y^2 = \frac{3}{16} \Rightarrow y = \pm\frac{\sqrt{3}}{8}$

$\therefore z_3 = \frac{1}{8} + \frac{\sqrt{3}}{8} i$

$|z_3|^2 = \frac{1}{64} + \frac{3}{64} = \frac{1}{16}$

$z_4 = \frac{1}{8} - \frac{\sqrt{3}}{8} i$

$|z_4|^2 = \frac{1}{64} + \frac{3}{64} = \frac{1}{16}$

$\therefore \sum_{i=1}^{n} |z_i|^2 = 0 + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$

55. (c) $\lim_{x\to0} \frac{\left(1+(a-1)x+\frac{(a-1)^2x^2}{2!}\right)+2\left(1-\frac{b^2x^2}{2!}\right)+(c-2)\left(1-x+\frac{x^2}{2!}\right)}{x\left(1-\frac{x^2}{2!}\right)-\left(x-\frac{x^2}{2}\ldots\right)} = 2$

$\lim_{x\to0} \frac{(1 + 2 + c - 2) + x(a - 1 - c + 2) + x^2\left(\frac{(a-1)^2}{2} - b^2 + \left(\frac{c-2}{2}\right)\right)}{\frac{x^2}{2} - \frac{x^3}{2!} + \cdots} = 2$

For which

$\because c + 1 = 0 \Rightarrow c = -1$

$\because a - c = -1 \Rightarrow a = -2$

$\because \frac{(a-1)^2}{2} - b^2 + \left(\frac{c-2}{2}\right) = 1$

$\frac{9}{2} - b^2 - \frac{3}{2} = 1 \Rightarrow b^2 = 2$

$a^2 + b^2 + c^2 = 4 + 2 + 1 = 7$

56. (a) $\int \frac{\cos y}{1+2\sin y} dy = \int \frac{dx}{16(\sqrt{9\sqrt{x}+x})(4+\sqrt{9+\sqrt{x}})}$

$4 + \sqrt{9+\sqrt{x}} = t$

$\frac{1}{2\sqrt{9+\sqrt{x}}} \times \frac{dx}{2\sqrt{x}} = 1dx$

$\frac{1}{2}\ell n|1+2\sin y| = \int \frac{4dt}{16t} + C$

$\frac{1}{2}\ell n|1+2\sin y| = \frac{1}{4}\ell n\left|\ 4+\sqrt{9+\sqrt{x}} + C\right.$

$\frac{1}{2}\ell n(2\sin y + 1) = \frac{1}{4}\ell n|4 + \sqrt{9+\sqrt{x}}| + C$

Substituting $\left(256, \frac{\pi}{2}\right)$

$\frac{1}{2}\ell n3 = \frac{1}{2}\ell n3 + C\ C = 0$

Substituting $(49, \alpha)$

$\frac{1}{2}\ell n(2\sin\alpha + 1) = \frac{1}{4}\ell n8$

$\ell n(2\sin\alpha + 1) = \frac{1}{2}\ell n8$

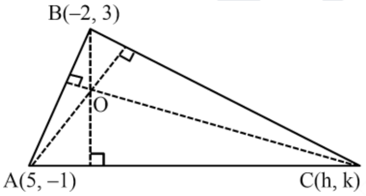
$\ell n(2\sin\alpha + 1) = \ell n2\sqrt{2}$

$2\sin\alpha + 1 = 2\sqrt{2}$

$2\sin\alpha = 2\sqrt{2} - 1$

57. (d) Solution of statement-1

$m_{AO} \cdot m_{BC} = -1$



$\Rightarrow 5h - k + 13 = 0$ .... (i)

& $m_{AB} \cdot m_{OC} = -1$

$\Rightarrow 4k = 7h$ .... (ii)

$\Rightarrow$ third vertex is $(-4, -7)$

$\therefore$ Statement 1 is correct.

Solution of statement-2

$2a, b, c \rightarrow$ A.P.

$b = \frac{2a + c}{2}$

$\Rightarrow 2a - 2b + c = 0$

$\because$ lines $ax + by + c = 0$ are concurrent then

$\frac{x}{2} = \frac{y}{-2} = \frac{1}{1}$

$x = 2$ and $y = -2$

$\therefore$ Point of concurrency is $(2, -2)$

Statement 2 is correct.

58. (c) $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{b} = \lambda\hat{j} + 2\hat{k}; \lambda \in Z$

$\vec{c} = \vec{a} \times \vec{b} = (-2-\lambda)\hat{i} - 4\hat{j} + 2\lambda\hat{k}$

$|\vec{c}| = \sqrt{53}$

$\Rightarrow 5\lambda^2 + 4\lambda - 33 = 0$

$\lambda = 2.2$ or -3

$\Rightarrow \lambda = -3$

$\vec{c} = \hat{i} - 4\hat{j} - 6\hat{k}$

let $\vec{d} = y\hat{j} + z\hat{k}$

$|\vec{d}| = 2$

$\Rightarrow y^2 + z^2 = 4$

$(\vec{c} \cdot \vec{d})^2 = (4y + 6z)^2 \leq \left(\sqrt{4^2 + 6^2} \times \sqrt{y^2 + z^2}\right)^2 \leq 208$

59. **(d)** $X = A^{-1}B = \left(\frac{\text{adj}A}{|A|}\right) B$

$= \pm \frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$

$= \pm \frac{1}{10}\begin{pmatrix} 20 \\ -10 \\ 10 \end{pmatrix} = \pm\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$\therefore |x + y + z| = 2$

60. **(a)**M is the point of intersection of $L_1 \& L_2$

$\Rightarrow 2\lambda - 1 = 2\mu - 1, 3\lambda - 1 = 3\mu - 1, 6\lambda - 3 = 9$

$\Rightarrow \lambda = 2 = \mu$

$\Rightarrow M(3,5,9)$

Now let point P be $(2K - 1, 3K - 1, 6K - 3)$ on L_2 such that $PM = 7$

$\Rightarrow \sqrt{(2K - 4)^2 + (3K - 6)^2 + (6K - 12)^2} = 7$

$\Rightarrow 49K^2 + 196 - 196K = 49$

$\Rightarrow K^2 + 4 - 4K = 1$

$\Rightarrow K^2 - 4K + 3 = 0$

$\Rightarrow K = 1,3$

So points P&Q are $(1,2,3) \& (5,8,15)$

So sum of all co-ordinates of P & Q = 34

61. **(d)** $P(10, 2\sqrt{15})$ lies on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore \frac{100}{a^2} - \frac{60}{b^2} = 1$ .... (i)

$\because$ length of latus rectum $= 8$

$\frac{2 \cdot b^2}{a} = 8 \Rightarrow \frac{b^2}{a} = 4$ .... (ii)

From (i) & (ii)

$\frac{100}{a^2} - \frac{60}{4a} = 1$

$400 - 60a = 4a^2$

$4a^2 + 60a - 400 = 0$

$a^2 + 15a - 100 = 0$

$a = 5 \& -20$ (rejected)

$\Rightarrow b = \sqrt{20}$

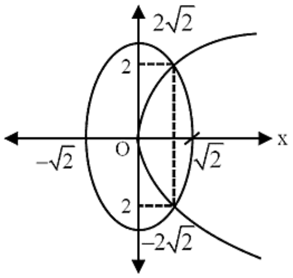
$\therefore$ Hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1$

$\therefore$ Focal length $S_1 S_2 = 2ae = 2.5 \cdot \left(\sqrt{1 + \frac{4}{5}}\right) = 6\sqrt{5}$

$\therefore$ Area of $\triangle PS_1 S_2 = \frac{1}{2} \cdot 6\sqrt{5} \cdot 2\sqrt{15} = 30\sqrt{3} = A$

$\therefore A^2 = 2700$

62. (b)



$$A=\int_0^2 2\sqrt{x}dx+2\int_1^{\sqrt{2}}\sqrt{8-4x^2}dx$$

$$=\frac{8}{3}\left(x^{\frac{3}{2}}\right)\int_0^1+4\int_1^{\sqrt{2}}\sqrt{2-x^2}dx$$

$$=\frac{8}{3}+4\times\frac{1}{2}\left[x\sqrt{2-x^2}+2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]\Bigg|_1^{\sqrt{2}}$$

$$=\frac{8}{3}+2\left[2\times\frac{\pi}{2}-1-2\times\frac{\pi}{4}\right]$$

$$=\frac{8}{3}+2\pi-2-\pi=\pi+\frac{2}{3}\text{ sq. units}$$

63. (c) $\frac{1}{2}\le|\alpha-\beta|\le\frac{3}{2}$

$$\frac{1}{4}\le|\alpha-\beta|^2\le\frac{9}{4}$$

$$\frac{1}{4}\le(\alpha+\beta)^2-4\alpha\beta\le\frac{9}{4}$$

$$\frac{1}{4}\le\frac{25}{9}-4\times\frac{\lambda}{4}\le\frac{9}{4}$$

$$-\frac{91}{36}\le-\lambda\le\frac{-19}{36}$$

$$\frac{19}{36}\le\lambda\le\frac{91}{36}$$

$\lambda=1,2$

Sum $=3$

64. (c) Let $x^2-10x+85=\lambda$

$\therefore$ Domain for first term

$\lambda>0$ …………… (i)

$7-\log_2\lambda>0\Rightarrow\lambda<2^7$ ………… (ii)

$\log_5(7-\log_2\lambda)>0\Rightarrow\lambda<2^6$ ……………. (iii)

$\therefore$ from (i), (ii) & (iii)

$0<\lambda<2^6$

$0<x^2-10x+85<64$

$\Rightarrow x\in(3,7)$ ……………… (A)

& domain for second term $-1\le\frac{3x-7}{x-17}\le1$

$\Rightarrow x\in[-5,6]$ ……… (B)

From (A) & (B), domain of function will be (3,6]

$\Rightarrow\alpha=3,\beta=6$

$\Rightarrow\alpha+\beta=9$

65. (c) $g(x)=|x|[x^2]$

points of discontinuity of $g(x)$ in $(-2,2)$ are

$(\pm1,\pm\sqrt{2},\pm\sqrt{3})$

$\therefore S=\{-1,1,-\sqrt{2},\sqrt{2},-\sqrt{3},\sqrt{3}\}$

$\because f(x) = \min\{\sqrt{2}x, x^2\}$

$\therefore \sum_{x \in S} f(x) = -\sqrt{2} + 1 - 2 + 2 - \sqrt{6} + \sqrt{6}$

$= 1 - \sqrt{2}$

66. (d) P lies on ellipse $\Rightarrow \frac{\alpha^2}{25} + \frac{\beta^2}{9} = 1$

$\because PS + PS' = 2a \Rightarrow PS + PS' = 10$

$\therefore (PS)^2 + (PS')^2 - PS'PS' = 37$

$(PS + PS')^2 - 3PS.PS' = 37$

100 - 3 PS.PS' = 37

3 PS.PS' = 63 $\Rightarrow$ PS.PS = 21

$\because$ PS&PS′ are $\left(5 \pm \frac{4}{5} \cdot \alpha\right)$

$\therefore$ PS.PS' $= 25 - \frac{16}{25}\alpha^2 = 21$

$\frac{16}{25}\alpha^2 = 4$

$\alpha = \frac{5}{2} \Rightarrow \alpha^2 = \frac{25}{4}$

$\therefore \beta^2 = \frac{27}{4}$

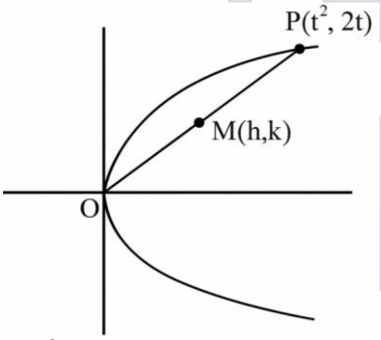
$\therefore \alpha^2 + \beta^2 = \frac{52}{4} = 13$

67. (b) $y^2 = 4x$

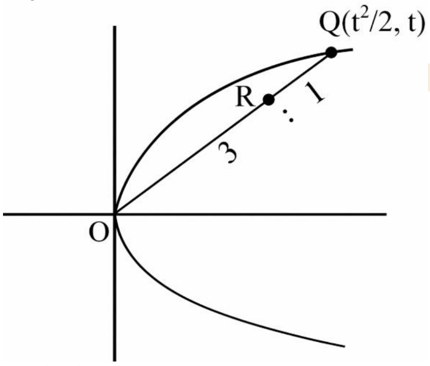
Locus of mid point of OP

$M(h, k) \Rightarrow h = \frac{t^2}{2}, k = t$

$\Rightarrow k^2 = 2\,h \Rightarrow y^2 = 2x$



$S: y^2 = 2x$



R (h, k)

$\Rightarrow h = \frac{\frac{3t^2}{2}}{4}, k = \frac{3t}{4}$

$t^2 = \frac{8\,h}{3}, t = \frac{4k}{3}$

$\Rightarrow \frac{16k^2}{9} = \frac{8\text{ h}}{3} \Rightarrow 2k^2 = 3\text{ h}$

Locus of R: $2y^2 = 3x$

68. **(b)** $f(x) = [x]^2 - [x] - 6 = ([x]+2)([x]-3)$

(a) $f(x) > 0 \Rightarrow [x] \in (-\infty, -2) \cup (3, \infty)$

$\Rightarrow x \in (-\infty, -2) \cup [4, \infty)$

(b) $f(x) < 0 \Rightarrow [x] \in (-2, 3)$

$\Rightarrow x \in [-1, 3)$

option (b) is correct

(c) $\int_0^2 f(x)dx = \int_0^1 (0-0-6)dx + \int_1^2 (1-1-6)dx$

$= -6 - 6$

$= -12$

(d) $f(x) = 0 \Rightarrow [x] = 3$ or $[x] = -2$

infinitely many solutions

69. **(b)** $f(x+y) = f(x) \cdot f(y) \Rightarrow f(x) = a^x$

$(\because f(1) = 7 \Rightarrow = a^1 = 7)$

So $f(x) = 7^x$

Now

$g(x+y) = g(xy)$ (put $y = 1$)

$\Rightarrow g(x+1) = g(x)$

so $g(1) = g(2) = g(3) = \cdots = g(n) = 1$

Given $\sum_{x=1}^{n} \frac{f(x)}{g(x)} = 19607$

$\sum_{x=1}^{n} \frac{7^x}{1} = 19607$

$\Rightarrow 7\left(\frac{7^n - 1}{7-1}\right) = 19607$

$7^n - 1 = \frac{6}{7} \times 19607$

$7^n = 16807 \Rightarrow n = 5$

70. **(d)** $P_n = \sum_{r=0}^{n} \frac{{}^nC_r(-2)^r}{r+1} = \sum_{r=0}^{n} \frac{1}{(n+1)} \, {}^{n+1}C_{r+1}(-2)^r$

$= \frac{-1}{2(n+1)} \sum_{r=0}^{n} {}^{n+1}C_{r+1}(-2)^{r+1}$

$= \frac{-1}{2(n+1)}[(1-2)^{n+1} - 1]$

$P_n = \frac{1}{2(n+1)}[1 - (-1)^{n+1}]$

$P_{2n} = \frac{1}{2(2n+1)}[1 - (-1)^{2n+1}]$

$P_{2n} = \frac{1}{2n+1}$

$\sum_{n=1}^{25} \frac{1}{P_{2n}} = \sum_{n=1}^{25} (2n+1)$

$= 3 + 5 + \cdots + 51$

$= \frac{25}{2}[51 + 3]$

$= 25 \times 27 = 675$

71. **(5)** $\frac{\vec{a} \cdot \hat{k}}{|\vec{a}|} = \cos\theta \Rightarrow \frac{\lambda}{\sqrt{3+\lambda^2}} = \cos\theta$

$\Rightarrow 0 < \dfrac{\lambda}{\sqrt{3+\lambda^2}} < \dfrac{\sqrt{3}}{2}$

$\Rightarrow \lambda > 0 \& 4\lambda^2 < 9 + 3\lambda^2 \Rightarrow \lambda^2 < 9$

$\Rightarrow \lambda \in (0,3)$ .... (i)

$\Rightarrow \vec{a}\cdot\vec{b} < 0 \Rightarrow -\sqrt{2}\lambda^2 - 4\sqrt{2} + 4\sqrt{2}\lambda < 0$

$\Rightarrow \lambda^2 - 4\lambda + 4 > 0 \Rightarrow (\lambda-2)^2 > 0$

$\Rightarrow \lambda \neq 2$ .... (ii)

from (i) & (ii)

$\lambda \in (0,3) - \{2\}$

$\therefore \alpha = 0, \beta = 3, \gamma = 2$

$\Rightarrow \alpha + \beta + \gamma = 5$

72. (36) $\int_0^{64} x^{\frac{1}{3}}dx = \frac{3}{4}\cdot\left[x^{\frac{4}{3}}\right]_0^{64} = 192$ & $\int_0^{64}[x^{1/3}]dx = \int_0^1[x^{1/3}]dx + \int_1^8[x^{1/3}]dx + \int_8^{27}[x^{1/3}]dx + \int_{27}^{64}[x^{1/3}]dx = 156$ So $\alpha = 192 - 156 = 36$

Now $E = \frac{1}{\pi}\int_0^{36\pi}\frac{\sin^2\theta}{\sin^6\theta+\cos^6\theta}d\theta$

$= \dfrac{36}{\pi}\int_0^{\pi}\dfrac{\sin^2\theta}{\sin^6\theta+\cos^6\theta}d\theta$

$\Rightarrow E = \dfrac{36\cdot 2}{\pi}\int_0^{\pi/2}\dfrac{\sin^2\theta\, d\theta}{\sin^6\theta+\cos^6\theta}$

Let $J = \int_0^{\pi/2}\frac{\sin^2\theta}{\sin^6\theta+\cos^6\theta}\, d\theta$................. (i)

Applying King

$J = \int_0^{\pi/2}\frac{\cos^2\theta}{\sin^6\theta+\cos^6\theta}d\theta$ .... (ii)

Now $2J = \int_0^{\pi/2}\frac{1}{\sin^6\theta+\cos^6\theta}\, d\theta$ (add(1)&(2))

$= \int_0^{\pi/2}\dfrac{\sec^6\theta}{\tan^6\theta+1}\, d\theta$

$= \int_0^{\infty}\dfrac{(1+\lambda^2)}{\lambda^4-\lambda^2+1}\, d\lambda$

$= \int_0^{\infty}\dfrac{1+\frac{1}{\lambda^2}}{\lambda^2-1+\frac{1}{\lambda^2}}d\lambda$

$= \pi$

$\Rightarrow J = \dfrac{\pi}{2}$

$\Rightarrow E = \dfrac{36\cdot 2}{\pi}\times J = 36$

73. (20) $\tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)]$

$\tan 2\alpha = \dfrac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}$

$\tan 2\alpha = \dfrac{\left(-\sqrt{99}+\frac{3}{\sqrt{55}}\right)}{1-(\sqrt{99})\left(\frac{3}{\sqrt{55}}\right)}$

$\tan 2\alpha = \dfrac{-3\sqrt{11}+\frac{3}{\sqrt{5}\times\sqrt{11}}}{1+\frac{9\sqrt{11}}{\sqrt{5}\times\sqrt{11}}}$

$\tan 2\alpha = \dfrac{3(1-11\sqrt{5})}{\sqrt{11}(9+\sqrt{5})}$

$r = 11, s = 9$

$r + s = 20$

74. (9) $a = b - d, c = b + d, \Rightarrow b = \frac{1}{3}$

$\Rightarrow 4\,b^4 = a^2c^2$

$\Rightarrow 4\,b^4 = [(b-d)(b+d)]^2$

$\Rightarrow \frac{4}{81} = \left(\frac{1}{9} - d^2\right)^2$

$\Rightarrow \left(\frac{1}{9} - d^2\right) = \pm\frac{2}{9}$

$d^2 = 1/3 \Rightarrow d = +\frac{1}{\sqrt{3}}(as\,a < b < c)$

$\therefore 9(a^2 + b^2 + c^2)$

$= 9\left[\left(\frac{1}{3} - \frac{1}{\sqrt{3}}^2\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3} + \frac{1}{\sqrt{3}}\right)^2\right]$

$= 9\left[\frac{1}{3} + \frac{2}{3}\right] = 3 + 6 = 9$

75. (1979) A $\{1,2,3 \ldots..11\}$

$\therefore$ $n(B) \geq 2$ & product of all elements in B is even

Case (i) $n(B) = 2 \Rightarrow {}^{11}C_2 - {}^{6}C_2$

$n(B) = 3 \Rightarrow {}^{11}C_3 - {}^{6}C_3$

$n(B) = 4 \Rightarrow {}^{11}C_4 - {}^{6}C_4$

$n(B) = 5 \Rightarrow {}^{11}C_5 - {}^{6}C_5$

$n(B) = 6 \Rightarrow {}^{11}C_6 - {}^{6}C_6$

$n(B) = 7 \Rightarrow {}^{11}C_7$

:

$n(B) = 11 \Rightarrow {}^{11}C_{11}$

$\therefore$ number of set B $\Rightarrow \sum_{r=2}^{11} {}^{11}C_r - \sum_{r=2}^{6} {}^{6}C_r$

$= 2^{11} - (12) - (2^6 - 7)$

$= 2048 - 64 - 5$

$= 1979$