Physics
Section - A

1. (b) Self- inductance of coil

$L = \frac{\mu_0 N^2 A}{2\pi R}$

2. (a)

$a^2 x\rho g = ma_{net}$

$\frac{L^2 \rho g}{m} x = a_{net}$

$T = 2\pi\sqrt{\frac{m}{L^2 \rho g}}$

where $m = 10$ g, $L = 10$ cm, $\rho = 1000\ kg/m^3$

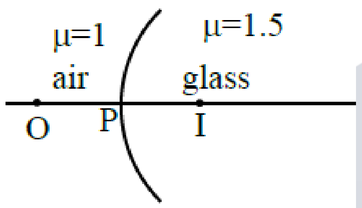
3. (b) Hot water is less viscous then cold water. Surfactant reduces surface tension.

4. (b) $\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{10^{-30}\times2.21\times10^{6}}$

$= 3\times10^{-10}$ m

Hence particle will behave as x-ray.

5. (a)



$PO = u = -x$

$PI = v = x$

$PO = PI$

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1.5}{x} + \frac{1}{x} = \frac{1}{2R}$

$\frac{5}{2x} = \frac{1}{2R}$

$X = 5R$

6. (a)

$N_2 = N_0 e^{-3\lambda t}$

$N_1 = N_0 e^{-\lambda t}$

$\frac{N_2}{N_1} = e^{-2\lambda t}$

$t_{\text{balf lifeoon } N_1}\ \ t = \frac{\ln 2}{\lambda}\ {}_n1$

$\frac{N_0}{2} = N_0 e^{-\lambda t}$

$\lambda t = \ln 2$

$t = \frac{\ln 2}{\lambda}$

$= e^{-2\lambda\frac{\ln 2}{\lambda}}$

$\frac{N_2}{N_1} = \frac{1}{4}$

7. (b) Initially capacitor behave as a short circuit.

So current will be maximum.

Charge on capacitor will be zero.
Potential difference across capacitor will be zero.

8. (c)

Dimension $[x(t)] = [L]$
$[A] = [L]$
$[B] = [L]$
$[C] = [LT^{-2}]$
$[D] = [L]$
$\left[\frac{ABC}{D}\right] = \left[\frac{L \times L \times LT^{-2}}{L}\right] = [L^2\ T^{-2}]$

9. (d)

$A \rightarrow P \propto \frac{1}{V}$
$\Rightarrow PV =$ constant
$\Rightarrow nRT =$ const. $\Rightarrow T =$ const.
Hence Isothermal III
$B \rightarrow IV$
$W \neq 0, \Delta U \neq 0, \Delta Q \neq 0$ [only isobaric]
$C \rightarrow I \Delta Q = 0$ Adiabatic
$D \rightarrow II w = 0$ Isochoric
III IV I II

10. (a)

(A) $\tau = C\theta \Rightarrow [ML^2\ T^{-2}] = [C][1]$
(B) $C \cdot S = \frac{\theta}{I} = \frac{BNA}{C}$;
V.S. $= \frac{BNA}{RC}$ [R also depends on ' N ']
(C) V.S. $\propto \frac{NAB}{CR}$ $R \rightarrow NR$
(D) False [Theory]
(E) E [False] C. S $\propto$ N
$\Rightarrow \because C.S = \frac{NAB}{C}$

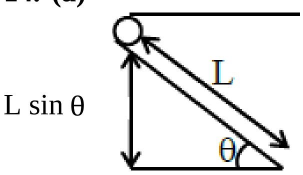
11. (d)

12. (c)

$625 = ms\Delta T + mL$
$625 = m[125 \times 300 + 2.5 \times 10^4]$
$625 = m[37500 + 25000]$
$625 = m[62500]$
$m = \frac{1}{100}$ kg
$M = 10$ grams

13. (d)

14. (a)



using WET
$W_g = k_f - k_i$
$MgL\sin\theta = k_f - k_i$
K.E. in pure rolling $\frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

$$= \frac{1}{2}mV^2 + \frac{1}{2} \times \frac{2}{5}mR^2\frac{V^2}{R^2}$$

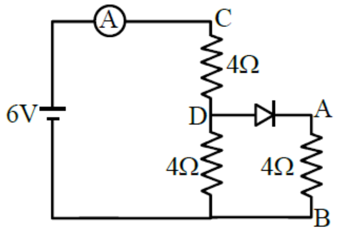
$$\frac{7}{10}mV^2$$

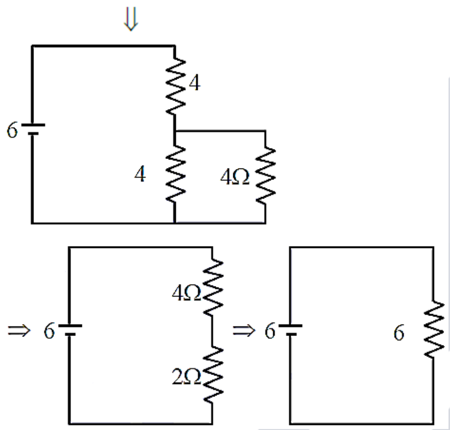
$$mgL\sin\theta = \frac{7}{10}mV_f^2 - 0$$

$$V_f^2 \propto \sin\theta$$

$$\left(\frac{V_1}{V_2}\right)^2 = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin 30^\circ}{\sin 45^\circ} = \frac{1}{\sqrt{2}}$$

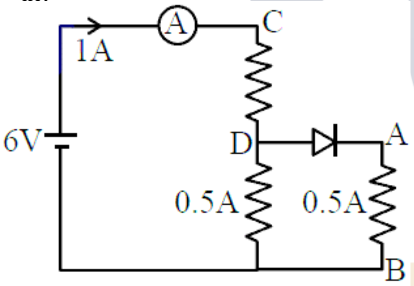
15. (a)





Current through ammeter = 1 A

$R_{net} = 6\Omega$



$V_{AB} = 0.5 \times 4 = 2\text{volt}$

$V_{CD} = 1 \times 4 = 4 \text{ volt}$

A, B & D are correct

16. (c)

$$\phi = \alpha\sigma + \beta\lambda$$

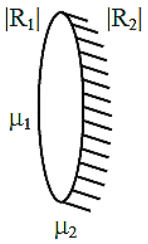
$$[\phi] = [\alpha\sigma] = [\beta\lambda]$$

$$[\alpha] = \frac{[\phi]}{[\sigma]} \quad \left[\frac{\alpha}{\beta}\right] = \frac{[\lambda]}{[\sigma]}$$

$$[\beta] = \frac{[\phi]}{[\lambda]} = \frac{[Q/L]}{[Q/\text{ Area }]} = \left[\frac{\text{Area}}{\text{Length}}\right]$$

$$\left[\frac{\alpha}{\beta}\right] = L$$

17. (b)



$$\frac{1}{f_{eq}}=\frac{2}{f_L}-\frac{1}{f_m}$$

$$f_m=-\frac{|R_2|}{2}$$

$$\frac{1}{f_L}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\frac{1}{f_{eq}}=2\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left(\frac{R_1+R_2}{R_1R_2}\right)+\frac{2}{R_2}$$

$$=\frac{2}{R_2}\left[\frac{(\mu_2-\mu_1)(R_1+R_2)+\mu_1R_1}{\mu_1R_1}\right]$$

$$=\frac{2}{R_2}\left[\frac{\mu_2R_1+\mu_2R_2-\mu_1R_1-\mu_1R_2+\mu_1R_1}{\mu_1R_1}\right]$$

$$\frac{1}{f_{eq}}=\frac{2[\mu_2R_1+\mu_2R_2-\mu_1R_2]}{\mu_1R_1R_2}$$

For same size of image

$$u=2f$$

$$u=\frac{\mu_1R_1R_2}{\mu_2R_1+\mu_2R_2-\mu_1R_2}$$

18. (c)

$$\vec{K}=3\hat{i}+4\hat{j}$$

$$\hat{K}=\frac{3\hat{i}+4\hat{j}}{5}$$

$$\hat{E}=\frac{4\hat{i}-3\hat{j}}{5}$$

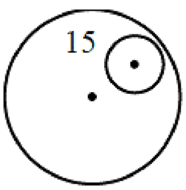
$$\hat{B}=\hat{K}\times\hat{E}$$

$$\hat{B}=-\hat{Z}$$

$$B_0=\frac{E_0}{C}=\frac{57}{3\times10^8}$$

19. (d) Total Area under curve.

20. (d)



mass of disc $=m$

mass of cut part $=\frac{m}{16}$

$$X_{com}=\frac{m\times0-\frac{m}{16}\times15}{m-\frac{m}{16}}$$

$=1$ cm.

Section - B

21.

$$\frac{9\times10^{9}\times6.67\times10^{-19}\times9.6\times10^{-10}}{6.67\times10^{-11}\times19.2\times10^{-27}\times9\times10^{-27}}$$

$$\frac{1}{2}\times10^{45}$$

Charge is not integral multiple of electron.

22. (3)

$\vec{A}\cdot\vec{B}=0$

$4-6n+8p=0$

$|\vec{A}|=|\vec{B}|$

$4+9n^2+4=4+4+16p^2$

$9n^2=16p^2$

$P=+\frac{3}{4}n$

$4-6n\pm6n=0$

$12n=4$

$n=\frac{1}{3}$

23. (273)

$\gamma=\frac{3}{2}$

$Tv^{\gamma-1}=C$

$273\ V_0^{0.5}=T\left(\frac{V_0}{4}\right)^{0.5}$

$T=273\times2=546$

$\Delta T=273$

24. (152)

$\int_0^4 x^2(10-x)dx+\int_0^2 y^2dy$

$=\left[\frac{10x^3}{3}-\frac{x^4}{4}\right]_0^4+\left[\frac{y^3}{3}\right]_0^2=\frac{640}{3}-64+\frac{8}{3}=152$

25. (3)

$\varepsilon-\frac{LdI}{dt}-IR=0$

$12-3\times(-8)-I\times12=0$

$I=3$

Chemistry

Section - A

26. (a) Palladium ⇒ 5^{th} period

Iridium, Osmium, Platinum ⇒ 6^{th} Period

27. (a)

$\lambda=900\text{ nm}$ $\quad$ $H-\text{atom}(Z=1)$

$=9\times10^{-5}\text{ cm}$

$R_H=10^5\text{ cm}^{-1}$

Ryderg eq. $=\frac{1}{\lambda}=R_HZ^2\times\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

$\Rightarrow\frac{1}{\lambda\times R_H}=\frac{1}{n_1^2}-\frac{1}{n_2^2}$

$\Rightarrow\frac{1}{9\times10^{-5}\text{ cm}\times10^5\text{ cm}^{-1}}=\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

$\Rightarrow\frac{1}{n_1^2}-\frac{1}{n_2^2}=\frac{1}{9}$

It is possible when $n_1 = 3, n_2 = \infty$

Possible series : $\infty \rightarrow 3$

28. (d) SO_2 can oxidise as well as reduce.

Hence it can act as both oxidising and reducing agent.

29. (c) Given : $\Delta T_f = 0.558^\circ C$

$$k_f = 1.86 \frac{K \times kg}{mol}$$

0.1 m aq. sol.

$\Rightarrow \quad \Delta T_f = i \times k_f \times m$

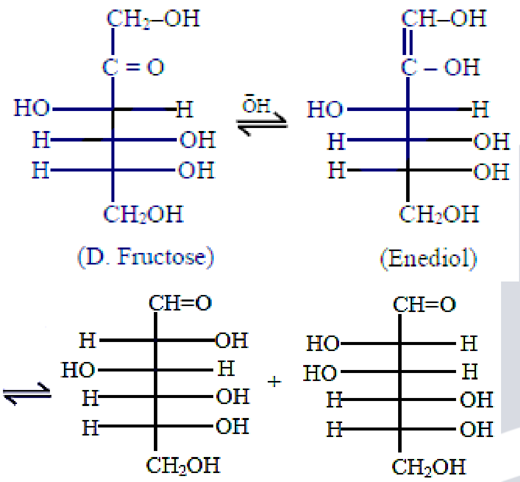
$\Rightarrow \quad 0.558 = i \times 1.86 \times 0.1$

$\Rightarrow \quad i = 3$

30. (a)

31. (c)

32. (b)



(D. Fructose) (Enediol)

D-Glucose D-Mannose

33. (a)

$$(\text{ moles })_{\text{initial}} = \frac{x \times 10^{-3}}{44}$$

$$(\text{moles})_{\text{removal}} = \frac{10^{21}}{6.02 \times 10^{23}}$$

$$(\text{ moles })_{\text{left}} = (\text{moles})_{\text{initial}} - (\text{moles})_{\text{removed}}$$

$$2.8 \times 10^{-3} = \frac{x \times 10^{-3}}{44} - \frac{10^{21}}{6.02 \times 10^{23}}$$

$\Rightarrow x = 196.2\text{mg}$

34. (b)

Ice $\rightarrow$ Ice $\rightleftharpoons$ Water $\rightarrow$ Water $\rightleftharpoons$ $\underset{\text{vapour}}{\text{Water}}$ $\rightarrow$ $\underset{\text{vapour}}{\text{Water}}$

$\underbrace{268\text{ K} \quad 273\text{ K}}_{(1)}$ $\underbrace{273\text{ K} \quad 273\text{ K}}_{(2)}$ $\underbrace{273\text{ K} \quad 273\text{ K}}_{(3)}$ $\underbrace{273\text{ K} \quad 373\text{ K}}_{(4)}$ $\underbrace{373\text{ K} \quad 383\text{ K}}_{(5)}$

$$\Delta S_{\text{overall}} = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 + \Delta S_5$$

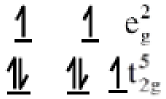
$$\Delta S_2 = \frac{\Delta H_{\text{m fusion}}}{273} \quad T_f = 273\text{ K}'$$

$$\Delta S_3 = \int_{273}^{373} \frac{C_{p,m} dT}{T}$$

$$\Delta S_4 = \frac{\Delta H_{\text{m vaporisation}}}{373} \quad T_b = 373'K'$$

$$\Delta S_5 = \int_{373}^{383} \frac{C_{p,m} dT}{T}$$

35. (c) $Co^{+2} = (Ar)_{18} 3d^7\ 4s^\circ$



36. (a) $Ma_3\ b_3$ type complexes show Facial - Meridional isomerism

(i) $[Co(NH_3)_3Cl_3] \Rightarrow Ma_3\ b_3$

(ii) $[Co(NH_3)_4Cl_2]^+ \Rightarrow Ma_4\ b_2$

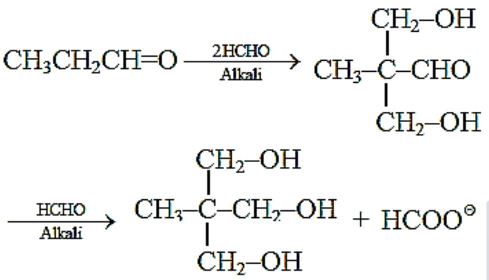
(iii) $[Co(en)_3]^{3+} \Rightarrow M(AA)_3$

(iv) $[Co(en)_2Cl_2]^+ \Rightarrow M(AA)_2\ b_2$

$a, b, = NH_3, Cl^-$

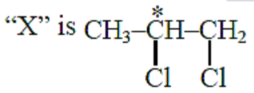
$AA = en$

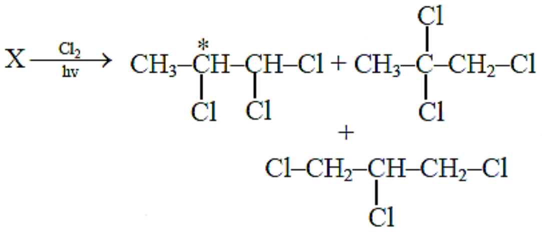
37. (c) This is an example of Tollen's reaction i.e. multiple cross aldol followed by cross Cannizaro reaction



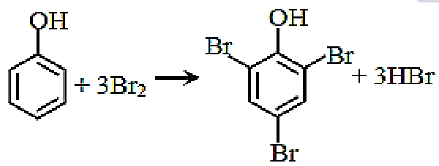
38. (a) q is aromatic r is nonaromatic p is antiaromatic

39. (d)





40. (a)



$$\text{Moles of phenol } = \frac{2}{94} = 0.021$$

$\therefore$ Moles of bromine $= 0.021 \times 3 = 0.064$

$\therefore$ Mass of bromine $= 0.064 \times 160 = 10.22$ g

41. (a)

(a) V^{2+} (Violet), Cr^{3+} (Violet), Mn^{3+}(Violet)

(b) Zn^{2+} (Colourless), V^{3+} (Green), Fe^{3+} (Yellow)

(c) Ti^{4+} (Colourless), V^{4+} (Blue), Mn^{2+} (Pink)

(d) Sc^{3+} (Colourless), Ti^{3+} (Purple), Cr^{2+} (Blue)

42. (d) The sodium fusion extract of organic compound having N& S gives blood red colour with $FeSO_4$ and $Na_4[Fe(CN)_6]$

43. (a) Condition for precipitation $Q_{ip} > K_{sp}$

For $[A(OH)_2]$

$[A^{2+}][OH^-]^2 > 9 \times 10^{-10}$

$[A^{+2}] = 1M$

$\Rightarrow [OH^-] > 3 \times 10^{-5}M$

For $[B(OH)_3]$

$[B^{3+}][OH^-]^3 > 27 \times 10^{-18}$

$[B^{3+}] = 1M$

$\Rightarrow [OH^-] > 3 \times 10^{-6}M$

So, $B(OH)_3$ will precipitate before $A(OH)_2$

44. (a)

(A) A → IV

(B) B → II

(C) C → I

(D) D → III

45. (d) B and D are 3° amine which does not have replaceable H on N , So does not react.

Section – B

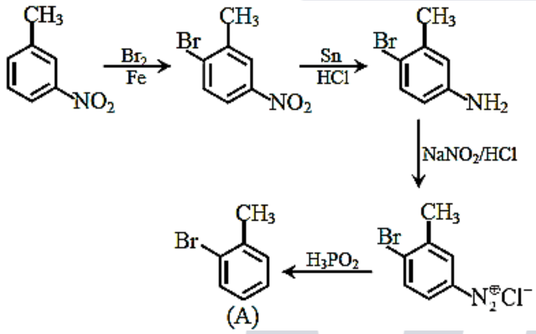
46. (7)

47. (40)

$$\text{Millimoles of } BaSO_4 = \frac{466}{233} = 2 \text{ m mol}$$

$$\% \ S = \frac{\frac{466}{233} \times 32}{160} \times 100 = 40\%$$

48. (171)



Molar mass of product $(C_7H_7Br)(A)$ is 171 g mol^{-1}

49. (900)

50. (2850)

$\Delta H^\circ_{rxn} = 55 \text{ kJ/mol}, \ T = 298 \text{ K}$

$\Delta S^\circ_{rxn} = 175 \text{ J/mol}$

$\Delta G^\circ_{rxn} = \Delta H^\circ_{rxn} - T\Delta S^\circ_{rxn}$

$\Rightarrow \Delta G^\circ_{rxn} = 55000 \text{ J/mol} - 298 \times 175 \text{ J/mol}$

$\Rightarrow \Delta G^\circ_{xxn} = 55000 - 52150$

$\Rightarrow \Delta G^\circ_{rxn} = 2850 \text{ J/mol}$

Mathematics

Section - A

51. (c) Let $\ln x = t \Rightarrow \frac{dx}{x} = dt$

$$I=\int_2^4 \frac{e^{\frac{1}{1+t^2}}}{\frac{1}{e^{1+t^2}}+e^{\frac{1}{1+(6-t)^2}}}dt$$

$$I=\int_2^4 \frac{\frac{1}{e^{1+(6-t)^2}}}{\frac{1}{e^{1+(6-t)^2}}+e^{\frac{1}{1+t^2}}}dt$$

$$2I=\int_2^4 dt=(t)_2^4=4-2=2$$

$$I=1$$

52. (b)

$$I(x)=\int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$$

$$\text{Put } \frac{x-11}{x+15}=t \Rightarrow \frac{26}{(x+5)^2}dx=dt$$

$$I(x)=\frac{1}{26}\int \frac{dt}{t^{11/13}}=\frac{1}{26}\cdot\frac{t^{2/13}}{2/13}$$

$$I(x)=\frac{1}{4}\left(\frac{x-11}{x+15}\right)^{2/13}+C$$

$$I(37)-I(24)=\frac{1}{4}\left(\frac{26}{52}\right)^{2/13}-\frac{1}{4}\left(\frac{13}{39}\right)^{2/13}$$

$$=\frac{1}{4}\left(\frac{1}{2^{2/13}}-\frac{1}{3^{2/13}}\right)$$

$$=\frac{1}{4}\left(\frac{1}{4^{1/13}}-\frac{1}{9^{1/13}}\right)$$

$$\therefore b=4, c=9$$

$$3(b+c)=39$$

53. (d)

$$\lim_{x\to 0^-}\frac{2}{x}\{\sin(k_1+1)x+\sin(k_2-1)x\}=4$$

$$\Rightarrow 2(k_1+1)+2(k_2-1)=4$$

$$\Rightarrow k_1+k_2=2$$

$$\Rightarrow \lim_{x\to 0^+}\frac{2}{x}\ln\left(\frac{2+k_1x}{2+k_2x}\right)=4$$

$$\Rightarrow \lim_{x\to 0^+}\frac{1}{x}\ln\left(1+\frac{(k_1-k_2)x}{2+k_2x}\right)=2$$

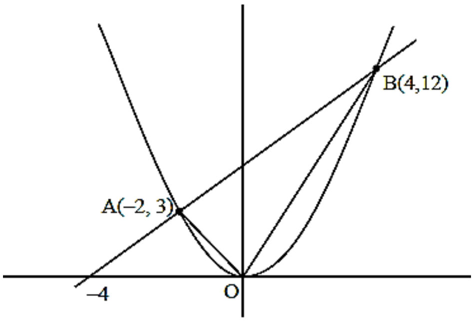
$$\Rightarrow \frac{k_1-k_2}{2}=2$$

$$\Rightarrow k_1-k_2=4$$

$$\therefore k_1=3, k_2=-1$$

$$k_1^2+k_2^2=9+1=10$$

54. (d)



$3x - 2y + 12 = 0$

$4y = 3x^2$

$\therefore 2(3x + 12) = 3x^2$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2, 4$

$m_{OA} = -3/2, \ m_{OB} = 3$

$$\tan\theta = \left(\frac{\frac{-3}{2} - 3}{1 - \frac{9}{2}}\right) = \frac{9}{7}$$

$\theta = \tan^{-1}\left(\frac{9}{7}\right)$ (angle will be acute)

55. (b)

$$\frac{dy}{dx} = \frac{2(3+y)\cdot e^{2x}}{7 + e^{2x}}$$

$$\frac{dy}{dx} - \frac{2y\cdot e^{2x}}{7 + e^{2x}} = \frac{6\cdot e^{2x}}{7 + e^{2x}}$$

$$\text{I.F.} = e^{-\int \frac{2e^{2x}\,dx}{7+e^{2x}dx}} = \frac{1}{7 + e^{2x}}$$

$$\therefore y \cdot \frac{1}{7 + e^{2x}} = \int \frac{6e^{2x}}{(7 + 3^{2x})^2} dx$$

$$\frac{y}{7 + e^{2x}} = \frac{-3}{7 + e^{2x}} + C$$

$$(0,5) \Rightarrow \frac{5}{8} = \frac{-3}{8} + C \Rightarrow C = 1$$

$\therefore y = -3 + 7 + e^{2x}$

$y = e^{2x} + 4$

$\therefore k = 8$

56. (a) $f(x) = \ln x$

$$g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$$

$D_g \in R$

$D_f \in (0, \infty)$

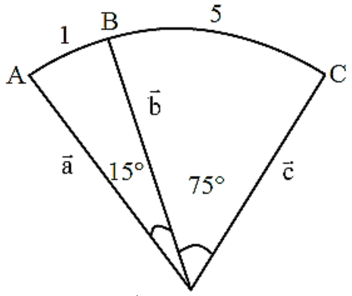
For $D_{fog} \Rightarrow g(x) > 0$

$$\frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1} > 0$$

$\Rightarrow x^4 - 2x^3 + 3x^2 - 2x + 2 > 0$

Clearly $x < 0$ satisfies which are included in option (a) only.

57. (a)



$\vec{c} = \alpha\vec{a} + \beta\vec{b}$ ....(1)

$\vec{a}\cdot\vec{c} = \alpha\vec{a}\cdot\vec{a} + \beta\vec{b}\cdot\vec{a}$

$0 = \alpha + \beta\cos 15°$ .....(2)

$(1) \Rightarrow \vec{b}\cdot\vec{c} = \alpha\vec{a}\cdot\vec{b} + \beta\vec{b}\cdot\vec{b}$

$\Rightarrow \cos 75° = \alpha\cos 15° + \beta$ .....(3)

(2) & (3) $\Rightarrow \cos 75° = -\beta\cos^2 15° + \beta$

$$\beta = \frac{\cos 75°}{\sin^2 15°} = \frac{1}{\sin 15°} = \frac{2\sqrt{2}}{\sqrt{3}-1}$$

$$(2) \Rightarrow \alpha = \frac{-\cos 15°}{\sin 15°} = \frac{-(\sqrt{3}+1)}{(\sqrt{3}-1)}$$

$$\therefore \vec{c} = \frac{-(\sqrt{3}+1)}{(\sqrt{3}-1)}\vec{a} + \left(\frac{2\sqrt{2}}{\sqrt{3}-1}\right)\vec{b}$$

Now

$$\alpha + \sqrt{2}(\sqrt{3}-1)\beta = \frac{-(\sqrt{3}+1)}{(\sqrt{3}-1)} + \frac{\sqrt{2}(\sqrt{3}-1)\cdot 2\sqrt{2}}{\sqrt{3}-1}$$

$$= \frac{-(\sqrt{3}+1)^2}{2} + 4$$

$$= \frac{-3-1-2\sqrt{3}+8}{2}$$

$= 2-\sqrt{3}$

58. (b)

$a = 3$

$$S_4 = \frac{1}{5}(S_8 - S_4)$$

$\Rightarrow 5S_4 = S_8 - S_4$

$\Rightarrow 6S_4 = S_8$

$$\Rightarrow 6\cdot\frac{4}{2}[2\times 3 + (4-1)\times d]$$

$$= \frac{8}{2}[2\times 3 + (8-1)d]$$

$\Rightarrow 12(6+3d) = 4(6+7d)$

$\Rightarrow 18 + 9d = 6 + 7d$

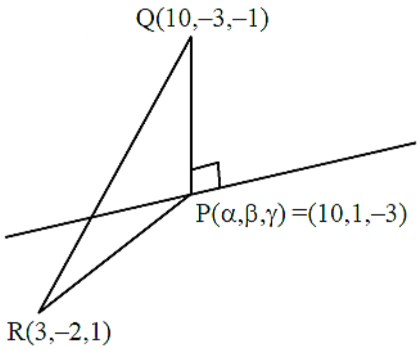
$\Rightarrow d = -6$

$$S_{20} = \frac{20}{2}[2\times 3 + (20-1)(-6)]$$

$= 10[6-114]$

$= -1080$

59. (d)



$$\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}=\lambda$$

$\Rightarrow 7\lambda+3,-\lambda+2,-2\lambda-1$

dr's of QP $\Rightarrow$

$7\lambda-7,-\lambda+5,-2\lambda$

Now

$(7\lambda-7)\cdot 7-(-\lambda+5)+(2\lambda)\cdot 2=0$

$54\lambda-54=0\Rightarrow\lambda=1$

$\therefore P=(10,1,-3)$

$\overrightarrow{PQ}=-4\hat{j}+2\hat{k}$

$\overrightarrow{PR}=-7\hat{i}-3\hat{j}+4\hat{k}$

$$\text{Area }=\left|\frac{1}{2}\right|\begin{vmatrix} i & j & k\\ 0 & -4 & 2\\ -7 & -3 & 4\end{vmatrix}=3\sqrt{30}$$

60. (a)

$$\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}$$

$$\left|\frac{\bar{z}-i}{\bar{z}+\frac{i}{2}}\right|=\frac{2}{3}$$

$3|x-iy-i|=2\left|x-iy+\frac{i}{2}\right|$

$9(x^2+(y+1)^2)=4(x^2+(y-1/3)^2)$

$9x^2+9y^2+18y+9=4x^2+4y^2-4y+1$

$5x^2+5y^2+22y+8=0$

$x^2+y^2+\frac{22}{5}y+\frac{8}{5}=0$

centre $\Rightarrow\left(0,-\frac{11}{5}\right)$

$$\left|\left|\frac{1}{2}\right|\begin{matrix}0 & 0 & 1\\ 0 & -11/5 & 1\\ \alpha & 0 & 1\end{matrix}\right|=11$$

$\Rightarrow\left(-\frac{11}{5}\alpha\right)^2=(11\times 2)^2$

$\Rightarrow\alpha^2=100$

61. (d) $A=\{1,2,3,4\}$

For relation to be reflexive

$R=\{(1,2),(2,3),(3,3)\}$

Minimum elements added will be

$(1,1),(2,2),(4,4)(2,1)(3,2)(3,2)(3,1)(1,3)$

$\therefore$ Minimum number of elements $=7$

62. (c) DAUGHTER

Total words $= 8\,!$

Total words in which vowels are together $= 6! \times 3!$

words in which all vowels are not together

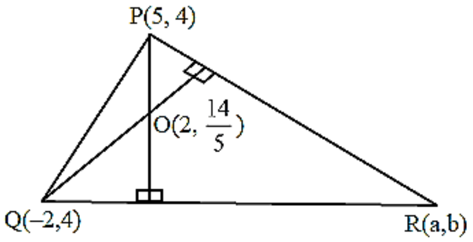
$= 8! - 6! \times 3!$

$= 6!\,[56 - 6]$

$= 720 \times 50$

$= 36000$

63. (b)



Equation of lines QR $= 5x + 2y + 2 = 0$

Equation of lines PR $= 10x - 3y - 38 = 0$

$\therefore$ Point $R(2, -6)$

Centroid $= \left(\frac{5 - 2 + 2}{3}, \frac{4 + 4 - 6}{3}\right)$

$= \left(\frac{5}{3}, \frac{2}{3}\right)$

$c + 2d = \frac{5}{3} + \frac{4}{3} = 3$

64. (b)

$\frac{\pi}{2} \leq x \leq \frac{3\pi}{4}$

$\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{12}\sin x\right)$

$\cos^{-1}(\cos x \cos\alpha + \sin x \sin\alpha)$

$\cos^{-1}(\cos(x - \alpha))$

$\Rightarrow x - \alpha$ because $x - \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow x - \tan^{-1}\frac{5}{12}.$

65. (a)

$\sin 70^\circ(\cot 10^\circ \cot 70^\circ - 1)$

$\Rightarrow \frac{\cos(80^\circ)}{\sin 10} = 1$

66. (b)

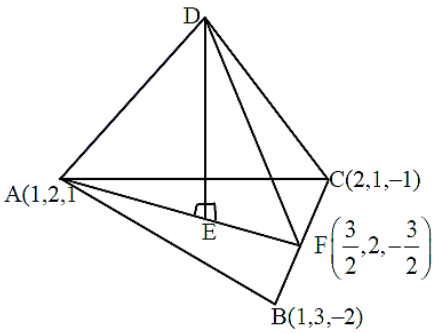
medain $= \ell + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$

$= 12 + \left(\frac{\frac{N}{2} - 18}{12}\right) \times 6 = 14$

$\Rightarrow \left(\frac{\frac{N}{2} - 18}{12}\right) \times 6 = 2$

$\frac{N}{2} - 18 = 4 \Rightarrow N = 44$

67. (d)



Area of $\triangle ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{1}{2}|5\hat{i} + 3\hat{j} + \hat{k}| = \frac{1}{2}\sqrt{35}$

volume of tetrahedron

$= \frac{1}{3} \times$ Base area $\times h = \frac{\sqrt{805}}{6\sqrt{2}}$

$\frac{1}{3} \times \frac{1}{2}\sqrt{35} \times h = \frac{\sqrt{805}}{6\sqrt{2}}$

$h = \sqrt{\frac{23}{2}}$

$AE^2 = AD^2 - DE^2 = \frac{13}{18} \therefore AE = \sqrt{\frac{13}{18}}$

$\overrightarrow{AE} = |AE| \cdot \left(\frac{\hat{i} - 5\hat{k}}{\sqrt{26}}\right)$

$= \sqrt{\frac{13}{18}} \cdot \left(\frac{\hat{i} - 5\hat{k}}{\sqrt{26}}\right)$

$= \sqrt{\frac{13}{18}} \cdot \left(\frac{\hat{i} - 5\hat{k}}{\sqrt{26}}\right) = \frac{\hat{i} - 5\hat{k}}{6}$

P.V. of $E = \frac{\hat{i} - 5\hat{k}}{6} + \hat{i} + 2\hat{j} + \hat{k} = \frac{1}{6}(7\hat{i} + 12\hat{j} + \hat{k})$

68. (c)

$\left[A\left(\operatorname{adj}(A^{-1}) + \operatorname{adj}(B^{-1})\right)^{-1} \cdot B\right]^{-1}$

$B^{-1} \cdot \left(\operatorname{adj}(A^{-1}) + \operatorname{adj}(B^{-1})\right) \cdot A^{-1}$

$B^{-1}\operatorname{adj}(A^{-1})A^{-1} + B^{-1}\left(\operatorname{adj}(B^{-1})\right) \cdot A^{-1}$

$B^{-1}|A^{-1}|I + |B^{-1}|IA^{-1}$

$\frac{B^{-1}}{|A|} + \frac{A^{-1}}{|B|}$

$\Rightarrow \frac{\operatorname{adj}B}{|B||A|} + \frac{\operatorname{adj}A}{|A||B|}$

$= \frac{1}{|A||B|}(\operatorname{adj}B + \operatorname{adj}A)$

69. (b)

$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$

$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$

$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$

For infinitely many solutions

$$D = \begin{vmatrix} \lambda-1 & \lambda-4 & \lambda \\ \lambda & \lambda-1 & \lambda-4 \\ \lambda+1 & \lambda+2 & -(\lambda+2) \end{vmatrix} = 0$$

$(\lambda-3)(2\lambda+1) = 0$

$$D_x = \begin{vmatrix} 5 & \lambda-4 & \lambda \\ 7 & \lambda-1 & \lambda-4 \\ 9 & \lambda+2 & -(\lambda+2) \end{vmatrix} = 0$$

$2(3-\lambda)(23-2\lambda) = 0$

$\lambda = 3$

$\therefore \lambda^2 + \lambda = 9 + 3 = 12$

70. (a) a = number or dice 1

b = number on dice 2

$(a, b) = (1,3), (3,1), (2,2), (2,3), (3,2), (1,4), (4,1)$

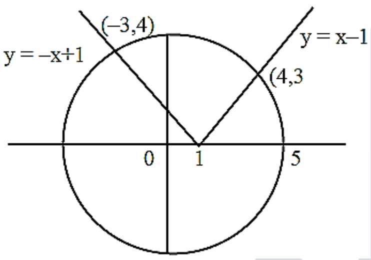
Required probability

$$= \frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{2}{6}$$

$$= \frac{18}{36} = \frac{1}{2}$$

Section – B

71. (77)



$x^2 + y^2 = 5$

$x^2 + (x-1)^2 = 25 \Rightarrow x = 4$

$x^2 + (-x+1)^2 = 5 \Rightarrow x = -3$

$$A = 25\pi - \int_{-3}^{4}\sqrt{25-x^2}\,dx + \frac{1}{2}\times 4\times 4 + \frac{1}{2}\times 3\times 3$$

$$A = 25\pi + \frac{25}{2} - \left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-3}^{4}$$

$$A = 25\pi + \frac{25}{2} - \left[6 + \frac{25}{2}\sin^{-1}\frac{4}{5} + 6 + \frac{25}{2}\sin^{-1}\frac{3}{5}\right]$$

$$A = 25\pi + \frac{1}{2} - \frac{25}{2}\cdot\frac{\pi}{2}$$

$$A = \frac{75\pi}{4} + \frac{1}{2}$$

$$A = \frac{1}{4}(75\pi + 2)$$

$b = 75, c = 2$

$b + c = 75 + 2 = 77$

72. (612)

$$\left(1 + 2^{\frac{1}{3}} + 3^{\frac{1}{2}}\right)^6$$

$$= \frac{\lfloor}{|r_1|r_2\ |\ r_3}(1)^{r_1}(2)^{\frac{r_2}{3}}(3)\frac{r_3}{2}$$

r_1	r_2	r_3
6	0	0

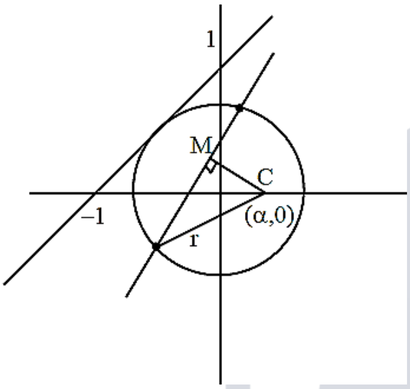
4	0	2
2	0	4
0	0	6
3	3	0
1	3	2
0	6	0

$$\text{sum} = \frac{6}{6|0|0} + \frac{6}{\lfloor 4|0|2}(3) + \frac{6}{2\lfloor 0|4}(3)^2 + \frac{6}{0|0|6}(3)^3$$

$$+ \frac{6}{\lfloor 3|3|0}(2) + \frac{6}{\lfloor 1|3|2}(2)^1(3)^1 + \frac{6}{0|6|0}(2)^2$$

$$= 1 + 45 + 135 + 27 + 40 + 360 + 4 = 612$$

73. (19)



$$x - y + 1 = 0$$

$$p = r$$

$$\left|\frac{\alpha - 0 + 1}{\sqrt{2}}\right| = r \Rightarrow (\alpha + 1)^2 = 2r^2 \ldots.(1)$$

$$\text{now} \left(\frac{-3\alpha + 0 - 1}{\sqrt{9 + 4}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2 = r^2$$

$$\Rightarrow (3\alpha + 1)^2 + 4 = 13r^2 \ldots\ldots(2)$$

$$(1)\ \&\ (2) \Rightarrow (3\alpha + 1)^2 + 4 = 13\frac{(\alpha + 1)^2}{2}$$

$$\Rightarrow 18\alpha^2 + 12\alpha + 2 + 8 = 13\alpha^2 + 26\alpha + 13$$

$$\Rightarrow 5\alpha^2 - 14\alpha - 3 = 0$$

$$\Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0$$

$$\Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0$$

$$\Rightarrow$$

$$\Rightarrow \alpha = \frac{-1}{5}, 3$$

$$\therefore r = 2\sqrt{2}$$

How $\alpha e = 3$ and $2\alpha = 4\sqrt{2}$

$$\alpha^2 e^2 = 9 \Rightarrow \alpha = 2\sqrt{2} \Rightarrow \alpha^2 = 8$$

$$\alpha^2\left(1 + \frac{\beta^2}{\alpha^2}\right) = 9$$

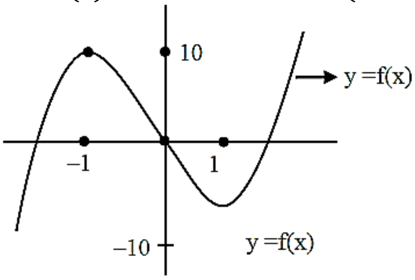
$$\alpha^2 + \beta^2 = 9$$

$$\therefore \beta^2 = 1$$

$$\therefore 2\alpha^2 + 3\beta^2 = 2(8) + 3(1) = 19$$

74. (30)

$5x^3 - 15x - a = 0$
$f(x) = 5x^3 - 15x$
$f(x) = 15x^2 - 15 = 15(x-1)(x+1)$



$a \in (-10, 10)$
$\alpha = -10, \beta = 10$
$\beta - 2\alpha = 10 + 20 = 30$

75. (117)

$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$
$x = 1$ is root ∴ other root is 1

$$\alpha + \beta = -\frac{b(c-a)}{a(b-c)} = 2$$

$\Rightarrow -bc + ab = 2ab - 2ac$
$\Rightarrow 2ac = ab + bc$
$\Rightarrow 2ac = b(a+c)$
$\Rightarrow 2ac = 15\,b \ldots (1)$

$$\Rightarrow 2ac = 15\left(\frac{36}{5}\right) = 108$$

$\Rightarrow ac = 54$
$a + c = 15$
$a^2 + c^2 + 2ac = 225$
$a^2 + c^2 = 225 - 108 = 117$