Physics
Section - A

1. (b) Initial K.E,

$$K.E. = \frac{1}{2}mu^2$$

Speed at heighest point

$$V = u\cos 60° = \frac{u}{2}$$

$$\therefore KE_2 = \frac{1}{2} m \left(\frac{u}{2}\right)^2$$

$$= \frac{1}{4} \times \frac{1}{2} mu^2$$

$$= \frac{KE}{4}$$

2. (d) P.E. of two charges

$$u = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= 14 \text{ cm}$$

$$\therefore u = \frac{9 \times 10^9 \times 7 \times 10^{-6} \times (-4) \times 10^{-6}}{14 \times 10^{-2}}$$

$$= -1.8 \text{ J}$$

3. (b)

$$\mu = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\frac{A}{2}}$$

Given $\delta_{min} = A$

$$\sqrt{3} = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$\cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$A = 60°$$

4. (c) $y = 4\sin(20 \times 10^{-3}x + 600t)$

Here

$$\omega = 600 \text{ s}^{-1}$$

$$k = 20 \times 10^{-3} \text{ mm}^{-1}$$

$$\therefore \quad v = \frac{w}{k} = \frac{600}{20 \times 10^{-3}}$$

$$= 30 \times 10^3 \text{ mm/s}$$

$$= 30 \text{ m/s}$$

& direction is towards -ve x axis

$\therefore v = -30 \text{ m/s}$

5. (c)

$$E = \alpha^3 e^{-\beta t}$$

$$\ln E = 3\ln\alpha - \beta t$$

$$\left(\frac{dE}{E}\right)_{max} = \frac{3\, d\alpha}{\alpha} + \beta\frac{dt}{t} \times t$$

$$= 3 \times 1.2\% + (0.3 \times 1.6 \times 5)\%$$

$$= 6\%$$

6. (d)

$eV = E\phi$
$2eV = E - 2.14\ EV$
$E = 4.14 eV$
$E = \frac{hc}{\lambda}$
$\lambda = \frac{1242}{4.14} = 300\ nm$

7. (a)

$\theta = 5t^2 - 8t$
$\omega = \frac{d\theta}{dt} = 10t - 8$
$\alpha = \frac{d\omega}{dt} = 10$
$\therefore p = \tau\omega$
$= (I\alpha)\omega$
$= \left(\frac{mR^2}{2}\right)\alpha\omega$
$= \left(\frac{mR^2}{2}\right)(10)(10t - 8)$

Put $t = 2$
$p = 60mR^2$

8. (a) By Bernoulli equation

$P_1 + \frac{1}{2} \times \rho \times 0^2 = P_2 + \frac{1}{2}\rho V^2$
$v = \sqrt{2\rho(P_1 - P_2)}$

9. (d)

$B = \frac{\mu_0 I}{2\pi r}$
$\Rightarrow [\mu_0] = \left[\frac{B \times r}{I}\right] = \left[\frac{MT^{-2}\ A^{-1} \times L}{A}\right] = [MLT^{-2}\ A^{-2}]$

magnetic field $F = qvB$
$B = \left[\frac{MLT^{-2}}{AT\ L/\ T}\right] = [MT^{-2}\ A^{-1}]$
$[M] = [NTA] = [M] = [ML^2]$
$\tau = c\theta \Rightarrow c = \left[\frac{\tau}{\theta}\right] = [ML^2\ T^{-2}]$

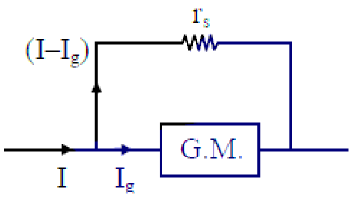
10. (a)

$T^2 \propto R^3$
$\left(\frac{T_m}{T_s}\right)^2 = \left(\frac{R}{R/9}\right)^3$
$\frac{T_m}{T_s} = (3)^3$
$\Rightarrow\ T_s = \left(\frac{27}{27}\right) = 1\ \text{day}$

11. (a)

$U = -PE\cos\theta$
$w_{ext}\ = \Delta U = U_f - U_i = -PE\cos 180° + PE\cos 0°$
$w_{eext}\ = 2PE$
$= 2 \times (4 \times 10^{-6})(18) \times 10^4$
$= 144 \times 10^{-2}$
$= 14.4\ mJ$

12. (c)



$I_gR_g = (I - I_g)r_s$

$20 \times 10^{-3} \times 30 = (3 - 0.02) \times r_s$

$r_s = \left(\frac{0.6}{2.98}\right) = \frac{30}{x}$

$x = \left(\frac{2.98 \times 30}{0.6}\right) = 149$

13. (c) $I \propto (\text{ width })^2$

$\left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \frac{9}{4}$

$\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{3}{2}$

$\frac{(x+1)d}{(x-1)d} = \frac{3}{2}$

$\Rightarrow 3x - 3 = 2x + 2$

$x = 5$

14. (b)

15. (d) $dQ = msdT$

$dS = \frac{dQ}{T} = \frac{msdT}{T}$

$\Delta S = \int \frac{msdT}{T} = msln\frac{T_f}{T_i}$

$\Delta S = mln\frac{T_2}{T_1}$

16. (c) Both are forward biased

hence $R_{eq} = 10\Omega$

$i = \frac{V}{R} = \frac{5}{10} = \frac{1}{2}$ A

17. (d)

$E = BC$

$9.3 = B \times 3 \times 10^8$

$B = \frac{9.3}{3 \times 10^8} = 3.1 \times 10^{-8}$ T

18. (d)

$w_{ABCD} = w_{AB} + w_{BC} + w_{CD}$

$= P_0 V_0 + 0 + (-2P_0 \times 2 V_0)$

$= P_0 V_0 - 4P_0 V_0$

$= -3P_0 V_0$

19. (d) Focal length of mirror will not change because focal length of mirror doesn't depend on medium.

20. (c)

$kx_1 = 5$ N

$kx_2 = 7$ N

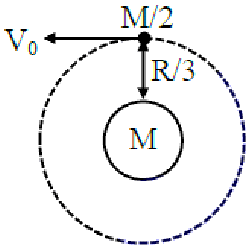
$k(5x_1 - 2x_2) = 5kx_1 - 2kx_2$

$= 5 \times 5 - 2 \times 7 = 11$ N

Section – B

21. (2190)

22. (3) (i) If earth is assumed to be stationary



orbital velocity $v_0 = \sqrt{\frac{GM}{4R/3}} = \sqrt{\frac{3GM}{4R}}$

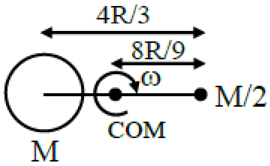
Angular momentum of satellite $= \frac{M}{2} v_0 \frac{4R}{3}$

$$= \frac{M}{2} \cdot \sqrt{\frac{3GM}{4R}} \cdot \frac{4R}{3}$$

$$= M\sqrt{\frac{GMR}{3}}$$

$x = 3$

(ii) Since mass of satellite is comparable to the mass of earth.



$$\frac{G.M \cdot \frac{M}{2}}{\left(\frac{4R}{3}\right)^2} = \frac{M}{2}\omega^2 \cdot \frac{8R}{9}$$

$$\omega = \sqrt{\frac{81GM}{128R^3}}$$

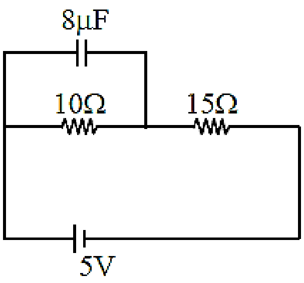
Angular momentum of satellite about common centre of mass,

$$L = \frac{M}{2} \cdot \left(\frac{8R}{9}\right)^2 \cdot \omega$$

$$L = M\sqrt{GMR\left(\frac{8}{81}\right)}$$

$$x = \frac{81}{8} \simeq 10$$

23. (16)



$$i=\left(\frac{5}{25}\right)$$
$$Q=CV$$
$$Q=(8\times10^{-6})\left(\frac{5}{25}\times10\right)$$
$$Q=\left(\frac{8\times5\times10^{-2}}{25}\right)=16\mu C$$

24. (100)

$$\frac{CdV}{dt}=I_d$$
$$\frac{dV}{dt}=\frac{I_d}{C}$$
$$=\frac{0.25\times10^{-3}}{2.5\times10^{-6}}=100$$

25. (2)

$$\omega=\frac{1}{\sqrt{LC}}$$
$$\omega=\frac{1}{\sqrt{25\times10^{-9}\times100\times10^{-3}}}$$
$$\omega=\frac{10^{+6}}{5\times10}=2$$

Chemistry
Section - A

26. (c) $\because \Delta G=\Delta H-T\Delta S$

For spontaneity of reaction : $\Delta G=-ve$

27. (a)

$\because \Delta G^\circ=-nFE^\circ$

Option (1)$E^\circ=0.8+0.76$
$=1.56$ V

$\therefore \Delta G^\circ=-2)\times F\times1.56$
$=-3.12$ V

Option (2) $E^\circ=-2.37+0.76$
$=-1.61$ V

$\therefore \Delta G^\circ=-2\times F\times(-1.61)$
$=+3.22$ V

Option (3) $E^\circ=-2.37-0.8$
$==-3.17$ V

$\therefore \Delta G^\circ=-2\times F\times(-3.17)$
$=+6.34$

Option (4)$E^\circ=0.8-0.34$
$=0.46$ V

$\Delta G^\circ=-2\times F\times0.46$
$=-0.92$ V

28. (c) α-helix and β-pleated sheet belongs to secondary structure of protein, which have hydrogen bonds.

29. (b)

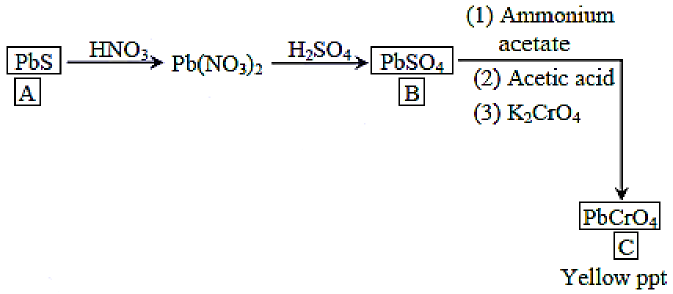
$k_{eq}=2280$ is for HCHO

$k_{eq}=2000$ is for chloral

Both data is given in clayden and warren book. $k_{eq}>1$ because HCHO and chloral are more electrophilic.

30. (b)

31. (b)



32. (d) B.P. ∝ M.W.

B.P. ∝ Inter molecular hydrogen bonding

Alcohol & Phenol have intermolecular H-bonding

33. (a)

$\because P^{\circ} - P \propto X_{solute}$

and $\because 10 \propto 0.2$

$\therefore 20 \propto 0.4$

$\therefore X_{solvent} = 1 - X_{solute}$

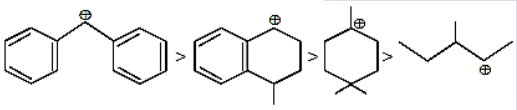
$= 1 - 0.4$

$= 0.6$

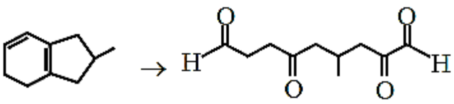
34. (c) For a shell total number of orbitals $= n^2$

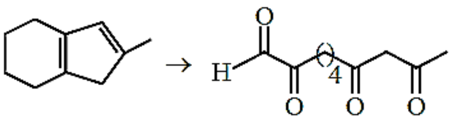
Magnetic quantum number have values $(-\ell$ to $+\ell)$ including 0 .

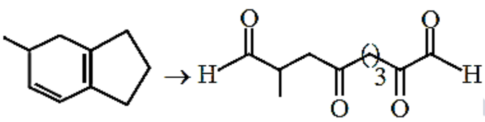
35. (a) Solvolysis or $S_N1 \propto$ stability of carboccation Stability order

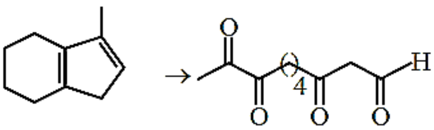


36. (b) Ozonolysis product

(A) 

(B) 

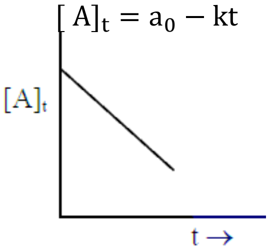
(C) 

(D) 

37. (b) For zero order reaction : A → P

Rate $= k$

$[A]_t = a_0 - kt$



38. (b)

Bronze $\rightarrow$ Cu, Sn
Brass $\rightarrow$ Cu, Zn
UK silver coin $\rightarrow$ Cu, Ni
Stainless steel $\rightarrow$ Fe, Cr, Ni, C

39. (c)
$Fe^{+6} = [Ar]3\,d^2$
$Mn^{+3} = [Ar]3\,d^4$
$Fe^{+3} = [Ar]3\,d^5$
$Cr^{+2} = [Ar]3\,d^4$
$Ni^{+4} = [Ar]3\,d^6$

40. (d) Order of M.P. of group 14: C > Si > Ge > Pb > Sn

element	M.P.(°C)
Z = 6 = C	3730
Z = 14 = Si	1410
Z = 32 = Ge	937
Z = 50 = Sn	232
Z = 82 = Pb	327

41. (a) With increase in temperature, K_w of water increases So, degree of dissociation of water increase ∴ pH as well as pOH of water decrease.

42. (c) A is phenol and B is para benzoquinone.

43. (a) ∵ For liquid solution of two liquids ' 1 ' and ' 2 '

$$P_1 = P_T y_1 = P_1^\circ x_1$$

$$\therefore \frac{P_T}{x_1} = \frac{P_1^\circ}{y_1}$$

$$\therefore \frac{P_2^\circ + x_1(P_1^\circ - P_2^\circ)}{x_1} = \frac{P_1^\circ}{y_1}$$

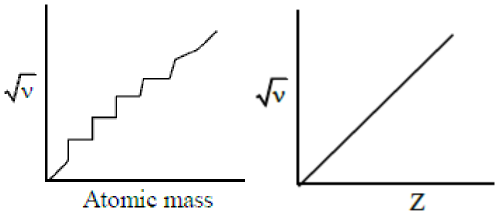
$$\therefore \frac{P_2^\circ}{x_1} + (P_1^\circ - P_2^\circ) = \frac{P_1^\circ}{y_1}$$

$$\therefore \frac{1}{x_1} = \left(\frac{P_1^\circ}{P_2^\circ}\right)\left(\frac{1}{y_1}\right) + \left(\frac{P_2^\circ - P_1^\circ}{P_2^\circ}\right)$$

$$\therefore \text{ Slope } = \left(\frac{P_1^\circ}{P_2^\circ}\right)$$

$$\therefore \text{ Intercept } = \left(\frac{P_2^\circ - P_1^\circ}{P_2^\circ}\right)$$

44. (c)



45. (c)

$$\underset{}{K_2Cr_2O_7} \xrightarrow[-H_2O]{KOH} \underset{[A]}{K_2CrO_4} \xrightarrow[-H_2O]{H_2SO_4} \underset{[B]}{K_2Cr_2O_7} + K_2SO_4$$

Section - B

46. (100)

Organic compound $\xrightarrow{\text{combustion}} H_2O$

0.9gm

$\therefore$ mole of $H_2O = \frac{0.9}{18} = 0.05$ mole

$\therefore$ mole of H in $H_2O = 0.05 \times 2 = 0.1$ mole

= mole of H in 0.01 mole

Organic compound

$\therefore$ wt of H atom in 0.01 mole compound $= 0.1 \times 1$

$= 0.1$gm

$\therefore$ wt of H atom in one mole compound

$= \frac{0.1}{0.01} = 10$gm

$\because$ wt. % of H $= \frac{\text{wt. of H in one mole compound}}{\text{Molar mass of compound}} \times 1$

$10 = \frac{10}{M} \times 100$

$\therefore M = 100$

47. (4)

48. (153)

$4Al + 3O_2 \longrightarrow 2Al_2O_3$

$\frac{81}{27} = 3$ mole $\quad \frac{128}{32} = 4$ mole

Limiting reagent

$\therefore$ mole of Al_2O_3 formed $= \frac{1}{2} \times 3$ mole

$\therefore$ wt. of Al_2O_3 formed $= \frac{3}{2} \times 102$

$= 153$gm

49. (200)

50. (27)

Mathematics
Section - A

51. (c)

$(1+x)^p(1-x)^q = \left({}^pC_0 + {}^pC_1x + {}^PC_2x^2 + \cdots\right)\left({}^9C_0 - {}^qC_1x + {}^qC_2x^2 + \cdots\right)$

coff of $x \equiv {}^pC_0\,{}^qC_1 - {}^pC_1\,{}^qC_0 = 1$

$p - q = 1$

coff of $x^2 \equiv {}^pC_0\,{}^qC_2 - {}^pC_1\,{}^qC_1 + {}^pC_2\,{}^qC_0 = -2$

$\frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -2$

$q^2 - q - 2pq + p^2 - p = -4$

$(p-q)^2 - (p+q) = -4$

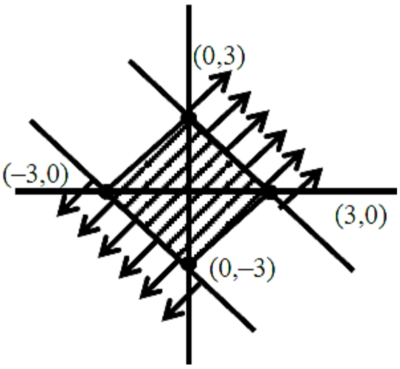
$p + q = 5$

$p = 3$

$q = 2$

so $p^2 + q^2 = 13$

52. (d)



$C = \{(3,0), (-3,0), (0,3), (0,-3)\}$

$\Sigma|x + y| = 12$

53. (a)

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 5 & \lambda \end{vmatrix} = 0$$

$\lambda = 17$

$$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 9 \\ 1 & 5 & \mu \end{vmatrix} \neq 0$$

$\mu \neq 18$

54. (a)

$$\int x^3 \sin x dx = -x^3 \cos x + \int 3x^2 \cos x dx$$

$$= -x^3 \cos x + 3x^2 \sin x - \int 6x \sin x dx$$

$$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + c$$

So $g(x) = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x$

$$g\left(\frac{\pi}{2}\right) = \frac{3\pi^2}{4} - 6$$

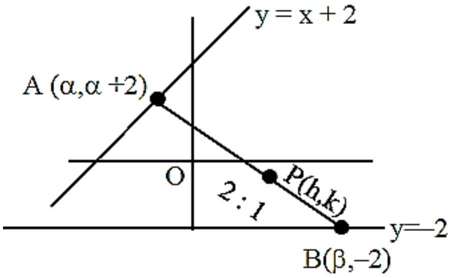
$$g'(x) = x^3 \sin x$$

$$g'\left(\frac{\pi}{2}\right) = \frac{\pi^3}{8}$$

$$8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \pi^3 + 16\pi^2 - 48$$

So $\alpha + \beta - \gamma = 55$

55. (b)



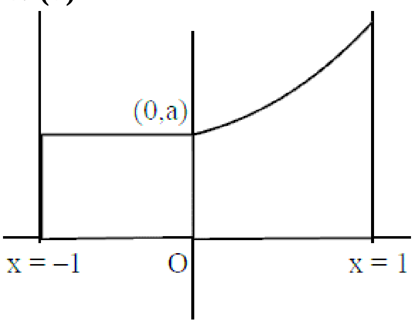
$$h=\frac{3\beta+\alpha}{3}$$
$$k=\frac{-4+\alpha+2}{3}$$
$$\alpha=3k+2$$
$$2\beta=3h-a=3h-3k-2$$
so $AB=8$
$$(\alpha-\beta)^2+(\alpha+4)^2=64$$
$$\left(3k+2-\left(\frac{3h-3k-2}{2}\right)\right)^2+(3k+2+4)^2=64$$
$$\frac{(9k-3h+6)^2}{4}+(3k+6)^2=64$$
$$9[(3k-h+2)^2+4(k+2)^2]=64\times4$$
$$9(x^2+13y^2-6xy-4x+28y)=76$$
$$\alpha-\beta-\gamma=13+6+4=23$$

56. (b) Let the parallel line is
$$\frac{x-1}{1}=\frac{y-4}{2}=\frac{z-0}{3}$$
so their point of intersection is
$$(\lambda+1,2\lambda+43\lambda)=(2t+2,3t+6,4t+3)$$
$$\lambda=2t+1$$
$$2\lambda+4=3t+6\Rightarrow t=0$$
so POI is $(2,6,3)$

so distance $=\sqrt{(2-1)^2+(6-4)^2+(3-0)^2}=\sqrt{14}$

57. (d)
$$A\equiv\left(\frac{5r-1}{r+1},\frac{5r-1}{r+1},\frac{10r+2}{r+1}\right)$$
$$(\overrightarrow{OQ}\cdot\overrightarrow{OA})-\frac{|\overrightarrow{OP}\times\overrightarrow{OA}|^2}{5}=10\ \ldots\ldots(1)$$
$$\overrightarrow{OQ}\cdot\overrightarrow{OA}=\frac{10}{r+1}(15r+1)$$
$$|\overrightarrow{OP}\times\overrightarrow{OA}|^2=\frac{r^2}{(r+1)^2}(800)$$
so by equation (1)
$$\frac{10}{r+1}(15r+1)-\frac{1}{5}\frac{r^2(800)}{(r+1)^2}=10$$
$$2r^2-14r=0$$
$$r=7,r\neq0$$

58. (d)



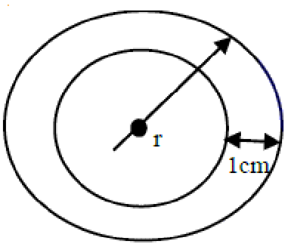
required area is $a+\int_0^1(a+e^x-e^{-x})dx$

$a + [a + e^x + e^{-x}]_0^1$

$2a + e - 1 + e^{-1} - 1 = e + 8 + \frac{1}{e}$

$2a = 10 \Rightarrow a = 5$

59. (d)



$v = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$81 = 4\pi r^2 \times \frac{1}{4\pi}$

$r^2 = 81$

$r = 9$

surface area of chocolate $= 4\pi(r-1)^2 = 256\pi$

60. (a) Total ways for selecting any two squares $= {}^{16}C_2$

$= 120$

Total ways for selecting common side squares

$= \underbrace{3 \times 4}_{\text{Horizontal side}} + \underbrace{3 \times 4}_{\text{vertical side}}$

$= 24$

so required probability

$= 1 - \frac{24}{120}$

$= \frac{4}{5}$

61. (b)

$ydy = (xdy - ydx)\sin\left(\frac{x}{y}\right)$

$\frac{dy}{y} = \left(\frac{xdy - ydx}{y^2}\right)\sin\left(\frac{x}{y}\right)$

$\frac{dy}{y} = \sin\left(\frac{x}{y}\right) d\left(-\frac{x}{y}\right)$

$€ny = \cos\frac{x}{y} + C$

$x(1) = \frac{\pi}{2} \Rightarrow 0 = \cos\frac{\pi}{2} + C \Rightarrow C = 0$

$\ln y = \cos\frac{x}{y}$

but $y = 2 \Rightarrow \cos\frac{x}{2} = \ell\ln 2$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$= 2(\ell n2)^2 - 1$

62. (a)

$f(x) = 6 + 16\left(\frac{1}{4}\cos 3x\right)\sin 3x \cdot \cos 6x$

$= 6 + 4\cos 3x \sin 3x \cos 6x$

$= 6 + \sin 12x$

Range of f(x) is [5,7]

$(\alpha, \beta) \equiv (5,7)$

$$\text{distance } = \left|\frac{15+28+12}{5}\right| = 11$$

63. (a)

64. (b) Statement - I :

Reflexive : $(a_1, b_1)R(a_1, b_1) \Rightarrow b_1 = b_1$ True

$$\text{Symmetric : } \left.\begin{array}{l}(a_1, b_1)R(a_2, b_2) \Rightarrow b_1 = b_2 \\ (a_2, b_2)R(a_1, b_1) \Rightarrow b_2 = b_1\end{array}\right\} \text{True}$$

$$\text{Transitive: } \left.\begin{array}{l}(a_1, b_1)R(a_2, b_2) \Rightarrow b_1 = b_2 \\ \&(a_2, b_2)R(a_3, b_3)b_2 = b_3 \\ \Rightarrow (a_1, b_1)R(a_3, b_3) \Rightarrow \text{ True}\end{array}\right\} b_1 = b_3$$

Hence Relation R is an equivence relation Statement-I is true.

For statement - II $\Rightarrow y = b$ so False

65. (a)

$T = S_1$

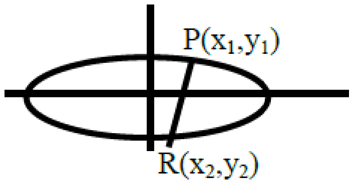
$$\frac{x \cdot 1}{4} + \frac{y \cdot 1/2}{2} = \frac{1}{4} + \frac{1}{8}$$

$$x + y = \frac{3}{2}$$

solve with ellipse

$$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{2}|x_2 - x_1|$$



$$y_2 = \frac{3}{2} - x_2$$

$$y_1 = \frac{3}{2} - x_1$$

$y_2 - y_1 = x_2 - x_1$

$x^2 + 2y^2 = 4$

$$x^2 + 2\left(\frac{3}{2} - x\right)^2 = 4$$

$6x^2 - 12x + 1 = 0$

$x_1 + x_2 = 2$

$x_1 x_2 = 1/6$

$$|x_2 - x_1| = \sqrt{(x_2 + x_1)^2 - 4x_1 x_2}$$

$$= \sqrt{4 - 4/6}$$

$$PR = \sqrt{2} \cdot 2 \cdot \frac{\sqrt{5}}{\sqrt{2}\sqrt{3}} = \frac{2}{3}\sqrt{15}$$

66. (a)

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$A\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix} \Rightarrow \begin{bmatrix}a_{12}\\a_{22}\\a_{32}\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix} \Rightarrow \begin{array}{c} a_{22} = 0; a_{12} = 0 \\ a_{32} = 1 \end{array}$$

$$A\begin{bmatrix}4\\1\\3\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix} \Rightarrow \begin{array}{c} 4a_{11} + a_{12} + 3a_{13} = 0 \\ 4a_{21} + a_{22} + 3a_{23} = 1 \Rightarrow 4a_{21} + 3a_{23} = 1 \\ 4a_{31} + a_{32} + 3a_{33} = 0 \end{array}$$

$$A\begin{bmatrix}2\\1\\2\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix} \Rightarrow \begin{array}{c} 2a_{11} + a_{12} + 2a_{13} = 1 \\ 2a_{21} + a_{22} + 2a_{23} = 0 \Rightarrow a_{21} + a_{23} = 0 \\ 2a_{31} + a_{32} + 2a_{33} = 0 \end{array}$$

$-4a_{23} + 3a_{23} = 1 \Rightarrow a_{23} = -1$

67. (d)

$z = e^{i\theta}$

$\frac{z}{\bar{z}} = e^{i2\theta}$

$\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1 \Rightarrow \left|e^{i2\theta} + e^{-i2\theta}\right| = 1 \Rightarrow |\cos 2\theta| = \frac{1}{2}$

8 solution in $[0,2\pi)$

68. (b)

$\vec{a} = (2,1,-3)$

$\vec{b} = (-1,-3,-5)$

$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$

$= 2\hat{i} - \hat{j}$

$\vec{b} - \vec{a} = -3\hat{i} - 4\hat{j} - 2\hat{k}$

$S_d = \frac{|(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|} -$

$= \frac{2}{\sqrt{5}}$

$(S_d)^2 = \frac{4}{5}$

$m = 4, n = 5 \Rightarrow m + n = 9$

69. (a) For I

Apply king (P-5) and add

$2I = \int_0^{\pi/2} dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

$I_2 = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

Apply king and add

$I_2 = \frac{\pi}{4}\int_0^{\pi/2} \frac{\tan x \sec^2 x dx}{\tan^4 x + 1}$

put $\tan^2 x = t$

$\frac{\pi}{8}\int_0^{\infty} \frac{dt}{t^2 + 1}$

$= \frac{\pi}{8} \cdot \frac{\pi}{2} = \frac{\pi^2}{16}$

70. (d)

$$\lim_{x\to\infty} \frac{\left(2-\frac{3}{x}+\frac{5}{x^2}\right)\left(1-\frac{1}{3x}\right)^{x/2}}{\left(3+\frac{5}{x}+\frac{4}{x^2}\right)\left(1+\frac{2}{3x}\right)^{x/2}}$$

$$= \lim_{x\to\infty} \frac{2}{3} \cdot \frac{e^{\frac{x}{2}\left(1-\frac{1}{3x}-1\right)}}{e^{\frac{x}{2}\left(1+\frac{2}{3x}-1\right)}}$$

$$= \frac{2}{3} \cdot \frac{e^{-\frac{1}{6}} e^{1/3}}{e^x} = \frac{2}{3} e^{-\frac{1}{2}}$$

Section -B

71. (17280)

(A) number of ways that all boys sit together $= 5! \times 5!$

(B) number of ways if no 2 boys sit together $= 4! \times 5!$

$A \cap B = \phi$

Required no. of ways $= 5! \times 5! + 4! \times 5! = 17280$

72. (31)

$\alpha + \beta = a \; \alpha\beta = -b$

$P_6 = aP_5 + bP_4$

$45\sqrt{7}i = a \times 11\sqrt{7}i + b(-3\sqrt{7})i$

$45 = 11a - 3b \quad \ldots..(1)$

and

$P_5 = aP_4 + bP_3$

$11\sqrt{7}i = a(-3\sqrt{7}i) + b(-5\sqrt{7}i)$

$11 = -3a - 5b \quad \ldots\ldots(2)$

$a = 3, b = -4$

$|\alpha^4 + \beta^4| = \sqrt{(\alpha^4 - \beta^4)^2 + 4\alpha^4\beta^4}$

$= \sqrt{-63 + 4.4^4}$

$= \sqrt{-63 + 1024} = \sqrt{961} = 31$

73. (15) $y^2 = 4(x+4)$

Equation of circle

$(x+3)^2 + y^2 = 25$

Passes through the point of intersection of two lines $3x - y = 0$ and $x + \lambda y = 4$ which is $\left(\frac{4}{3\lambda+1}, \frac{12}{3\lambda+1}\right)$, after solving with circle, we get

$\lambda = -\frac{7}{6}, 1$

$12\lambda_1 + 29\lambda_2$

$-14 + 29 = 15$

74. (8788)

$\text{Var}(8,21,34,47,\ldots\ldots,320)$

$\text{Var}(0,13,26,39,\ldots\ldots,312)$

$13^2 \cdot \text{Var}(0,1,2,\ldots\ldots,24)$

$13^2 \cdot \text{Var}(1,2,3,\ldots\ldots,25)$

So, $\sigma^2 = 13^2 \times \left(\frac{25^2-1}{12}\right) = 8788$

75. (474)

$S_{11} = \frac{11}{2}(2a + 10d) = 88$

$a + 5d = 8$

$a = 8 - 5 \times \frac{3}{2} = \frac{1}{2}$

Roots are

$$T_{10} = a + 9d = \frac{1}{2} + 9 \times \frac{3}{2} = 14$$

$$T_{11} = a + 10d = \frac{1}{2} + 10 \times \frac{3}{2} = \frac{31}{2}$$

$$\frac{p}{3} = T_{10} + T_{11} = 14 + \frac{31}{2} = \frac{59}{2}$$

$$p = \frac{177}{2}$$

$$\frac{q}{3} = T_{10} \times T_{11} = 7 \times 31 = 217$$

$$q = 651$$

$$q - 2p$$

$$= 651 - 177$$

$$= 474$$