1. **(c)** $\delta_{net} = 0$

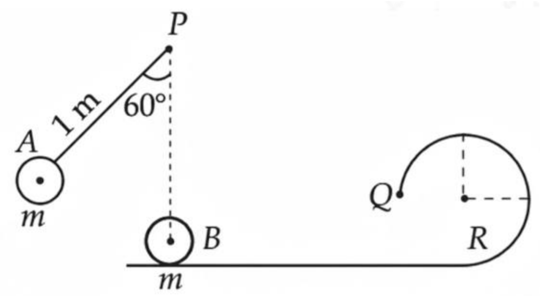
$\delta_1 + \delta_2 = 0$

$(\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0$

$A_2 = \dfrac{(\mu_1 - 1)A_1}{(\mu_2 - 1)}$

$A_2 = \dfrac{(1.72 - 1)}{(1.9 - 1)} \times 5^\circ = 4^\circ$

2. **(b)**



V_A at lowest point

$V_A = \sqrt{2 g\ell(1 - \cos\theta)}$

$V_A = \sqrt{2 \times 10 \times 1\left(1 - \frac{1}{2}\right)} = \sqrt{10}$

After collision velocity of B becomes,

$V_A = \sqrt{10} = V_B$ (Same mass)

Now to complete circular motion

$V_B = \sqrt{5gR}$

$R = \dfrac{1}{5}$

3. **(b)** A-II, B-III, C-IV, D-I

4. **(c)** $\ell_{mean} = \frac{\ell_1+\ell_2+\ell_3+\ell_4}{4}$

$\ell_{mean} = \dfrac{20.00 + 19.75 + 17.01 + 18.25}{4}$

$= 18.75$

$\Delta\ell_{mean} = \dfrac{|\Delta\ell_1| + |\Delta\ell_2| + |\Delta\ell_3| + |\Delta\ell_4|}{4}$

$= \dfrac{1.25 + 1 + 1.74 + 0.5}{4} = 1.12$

So, relative error

$= \dfrac{\Delta\ell_{mean}}{\ell_{mean}} = \dfrac{1.12}{18.75} = 0.06$

5. **(a)** $\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2\,m\left(\frac{3}{2}kT\right)}}$

$\lambda = \dfrac{h}{\sqrt{3mkT}}$

$= \dfrac{6.63 \times 10^{-34}}{\sqrt{3 \times 5.31 \times 10^{-26} \times 1.38 \times 10^{-23} \times 300}}$

$= 2.58 \times 10^{-11} = 25.8 \times 10^{-12}$

So, $x = 26$

6. **(c)** Time period becomes half

and $T \propto \sqrt{\ell}$

So, length ℓ beocmes $\frac{\ell}{4}$

So, $\frac{\ell}{4}=\frac{30}{4}=7.5$

7. **(c)** For equilibrium $kx = mg$

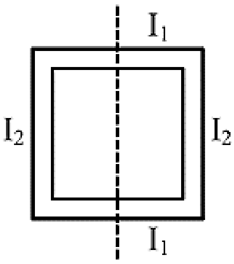
$$U=\frac{1}{2}kx^2=\frac{1}{2}\frac{m^2g^2}{k}$$

$$U \propto \frac{m^2}{k}$$

$$\frac{U_A}{U_B}=\left(\frac{m_A}{m_B}\right)^2\frac{k_B}{k_A}=\left(\frac{1}{2}\right)^2\left(\frac{4}{3}\right)=\frac{1}{3}$$

$$\frac{E}{U_B}=\frac{1}{3}\Rightarrow U_B=3E$$

8. **(d)**



$$I_{net}=2(I_1+I_2)$$

$$=2\left(\frac{M'R^2}{4}+\frac{M'\ell^2}{12}\right)+2\left(\frac{M'R^2}{2}+M'\left(\frac{\ell}{2}\right)^2\right)$$

$$=\frac{M'R^2}{2}+\frac{M'R^2}{6}+M'R^2+\frac{M'\ell^2}{2}$$

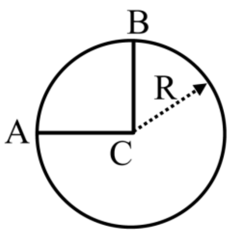
$$=\frac{3M'R^2}{2}+\frac{2M'\ell^2}{3}$$

Given masses $M'=\frac{M}{4}$

So, $I=\frac{3(M/4)R^2}{2}+2\frac{(M/4)\ell^2}{3}$

$$I=\frac{3}{8}MR^2+\frac{M\ell^2}{6}$$

9. **(c)**



$$\frac{1}{R_{AB}}=\frac{2}{\lambda\pi r}+\frac{1}{\lambda.2r}+\frac{2}{\lambda.3\pi r}$$

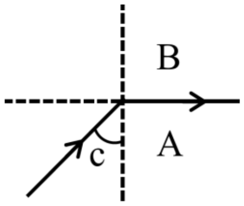
$$=\frac{1}{\lambda r}\left[\frac{2}{\pi}+\frac{1}{2}+\frac{2}{3\pi}\right]$$

$$=\frac{1}{\lambda r}\left(\frac{12+3\pi+4}{6\pi}\right)=\frac{1}{\lambda r}\cdot\left(\frac{16+3\pi}{6\pi}\right)$$

$$R_{AB}=\lambda r\left(\frac{6\pi}{16+3\pi}\right)$$

10. **(a)** $\frac{\text{Strain}}{\text{Stress}}=\frac{1}{Y}=\text{Slope}$

11. **(c)**



$\mu_A \sin c = \mu_B \sin 90$

$\Rightarrow \sin c = \frac{\mu_B}{\mu_A} = \frac{v_A}{v_B}$

$\therefore \sin c = \frac{2.4 \times 10^8}{2.7 \times 10^8} = \frac{8}{9}$

$\Rightarrow \tan c = \frac{8}{\sqrt{81-64}} = \frac{8}{\sqrt{17}}$

$c = \tan^{-1}\left(\frac{8}{\sqrt{17}}\right)$

12. (d) When the load is disconnected ($I_L = 0$), the entire current flows through the Zener diode. To protect the diode, we must limit the current to its maximum allowable value:

$I_{Z(max)} = 20 \text{ mA}$

Voltage across the series resistor:

$V_R = 25 \text{ V} - 5 \text{ V} = 20 \text{ V}$

Using Ohm's law:

$R_S = \frac{20}{0.020} = 1000\Omega$

13. (b) $\frac{1}{\lambda} = R\left(1 - \frac{1}{4}\right)$

$\lambda = \frac{4}{3R}$

$\frac{1}{\lambda'} = R\left(\frac{1}{9}\right)$

$\lambda' = \frac{9}{R}$

14. (d) $(KE)_A = K = \frac{1}{2} mu^2$

$(KE)_B = \frac{K}{4} = \frac{1}{2} m \left(\frac{u}{2}\right)^2 = \frac{K}{4} \left(u_B = u\cos 60^\circ = \frac{u}{2}\right)$

$(KE)_C = K$

Ratio $= \frac{K - K/4}{K}$

$= \frac{3\,K/4}{K} = \frac{3}{4}$

15. (d) $\phi = \frac{Q_{in}}{\varepsilon_0}$

$\phi = \frac{\frac{q}{4} + \frac{q}{2}}{\varepsilon_0} = \frac{3q}{4\varepsilon_0}$

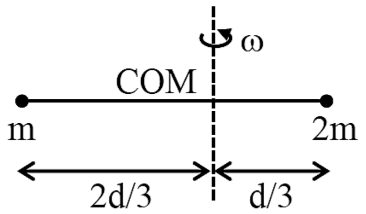
16. (b) $\varepsilon = vB\ell$

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 200} = 20\sqrt{10}$

$\varepsilon = (20\sqrt{10})(0.5 \times 10^{-4})20$

$= 20\sqrt{10} \times 10^{-3} = 20\sqrt{10}\text{mV}$

17. (a)



$L = I\omega$ and $\omega = \frac{L}{I}$

$$\omega = \frac{L}{m\left(\frac{2d}{3}\right)^2 + 2m\left(\frac{d}{3}\right)^2} = \frac{L}{\frac{4}{9}md^2 + \frac{2}{9}md^2} = \frac{L}{\frac{6md^2}{9}}$$

$$\omega = \frac{3L}{2md^2}$$

18. (b) $(K.E)_{lost} = \frac{1}{2}\mu V_{rel}^2 (1 - e^2)$

$$= \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(10 + 30)^2(1 - 0)$$

$$= \frac{1}{2}\left[\frac{(15)(25)}{40}\right][40]^2$$

$= 7500\ J$

$(K.E)_{loss} = (m_1 + m_2)(S)(\Delta T)$

$[S = 31 \times 4.2\ J/kg - {}^\circ C]$

$7500 = (40)(31)(\Delta T)$

$$\Delta T = \frac{7500}{40 \times 31 \times 4.2} = 1.44^\circ C$$

19. (c) Zero error $e = -3 \times LC = -0.03$ mm

Reading taken $= 1\ mm + 51(0.01\ mm)$

$= 1.51$ mm

So, correct reading $= 1.51 - (-0.03)$

$= 1.54$ mm

20. (b) Both (A) and (R) are true but (R) is not the correct explanation of (A)

21. (100) $\varepsilon_{max} = \frac{B\omega_{max}\ell^2}{2}$ .... (1)

Using energy conservation,

$$mg\ell(1 - \cos 60^\circ) = \frac{1}{2}(m\ell^2)\omega_m^2$$

$$\omega_m = \sqrt{\frac{g}{\ell}} = 10 rad/s$$

From eq.(1),

$$\varepsilon_{max} = \frac{2 \times 10 \times 0.01}{2} = 0.1\ V$$

$= 100mV$

22. (2) Here, $v = \frac{\omega}{K} = \frac{6.27 \times 10^3}{2.09 \times 10^{-5}}$

$= 3 \times 10^8$

So, wave moving in vacuum

Now, $I = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c = \frac{1}{2}\varepsilon_0 E_0^2 \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$= \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2 = \frac{1}{2}\frac{1}{377} \times 377$$

$$\frac{1}{d} = \frac{1}{2}$$

$d = 2$

23. **(50)** Current is maximum, so resonance

and $\omega = \frac{1}{\sqrt{LC}}$

$C = \frac{1}{\omega^2 L} = \frac{1}{2 \times 10^4}$

$= 50 \times 10^{-6} = 50\mu\ F$

24. **(4)** $\beta_1 = \beta_2$

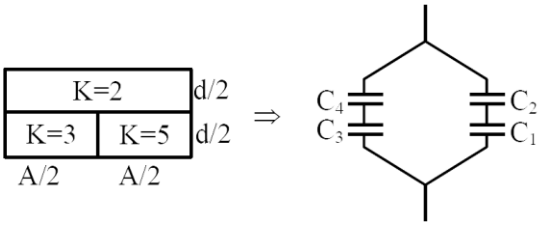
$\frac{D_1 \lambda_1}{d_1} = \frac{D_2 \lambda_2}{d_2}$

$\frac{D_1}{D_2} = \frac{\lambda_2}{\lambda_1}\left(\frac{d_1}{d_2}\right)$

$= 2 \times 2$

$\frac{D_1}{D_2} = 4$

25. **(8)**



$C_1 = \frac{5\epsilon_0\ A/2}{d/2} = \frac{5\epsilon_0\ A}{d} = 5C$

$C_2 = \frac{2\epsilon_0\ A/2}{d/2} = \frac{2\epsilon_0\ A}{d} = 2C$

$C_1 \& C_2$ in series.

$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{(5C)(2C)}{7C} = \frac{10}{7}C$

$C_3 = \frac{3\epsilon_0\ A/2}{d/2} = 3C$

$C_4 = \frac{2\epsilon_0\ A/2}{d/2} = 2C$

$C_4 \& C_3$ in series; $C'' = \frac{(2C)(3C)}{5C} = \frac{6}{5}C$

$C' \& C''$ in parallel;

So, $C_{eq} = C\left(\frac{6}{5} + \frac{10}{7}\right) = C\left(\frac{42+50}{35}\right) = \left(\frac{92}{35}\right)C$

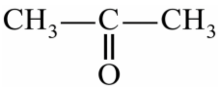
$\frac{92}{35}C = \frac{nC}{3}$

$n = \frac{92 \times 3}{35} = 7.9 \Rightarrow n \simeq 8$

26. **(d)** $E_n = -2.18 \times 10^{-18} \frac{Z^2}{n^2}$ J/ atom .

For 3 rd orbit of Li^{+2} ion

$= -2.18 \times 10^{-18} \times \frac{3^2}{3^2} = -2.18 \times 10^{-18}$ J.

27. **(a)** Mixture of CS_2 and  show positive deviation

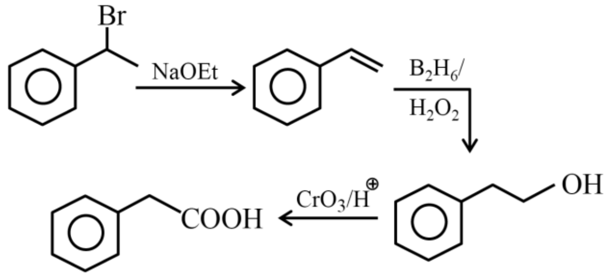
$P_{CS_2} > P^o_{CS_2} \cdot X_{CS_2}$

28. **(a)** In general on moving from left to right in a period ionization energy increases as $\mathbf{Z}_{eff}$ increases.

Al < Si < S < P < Cl

(Ionisation energy of phosphorus is more because of half filled stable configuration)

29. (d)



30. (d) $Cl^{-}_{aq} + Ag_{(S)} + Fe^{+3}_{aq} \rightarrow Fe^{+2}_{aq} + AgCl_{(S)}$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.059}{1}\log\frac{[Fe^{2+}]}{[Cl^{-}][Fe^{3+}]}$$

$$\underset{\text{Propaneitrile}}{CH_3-CH_2-C\equiv N} \xrightarrow[H_3O^{\oplus}]{SnCl_2/HCl} \underset{\text{Propanal (X)}}{CH_3-CH_2-CHO}$$

31. (d)

$$CH_3-CH=CH-CH_3 \xrightarrow{\text{Ozonolysis}} \underset{\text{Ethanal (Y)}}{-CH_3-CHO}$$

$$(X)+(Y) \xrightarrow[\Delta]{NaOH} \underset{\text{(Not formed)}}{3-\text{Methylbut}-2-\text{enal}}$$

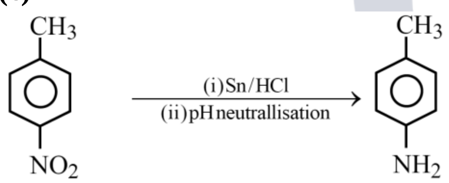
$$CH_3-CH_2-CHO + CH_3-CHO \xrightarrow[\Delta]{NaOH}$$

$$\underset{\text{2-Methylpent-2-enal}}{CH_3-CH_2-CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-CHO} + \underset{\text{But-2-enal}}{CH_3-CH=CH-CHO}$$

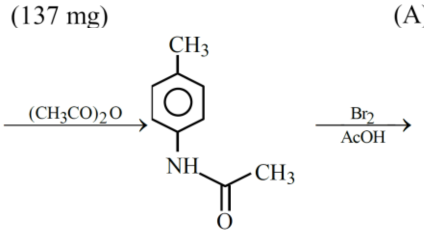
$$+ \underset{\text{2-Methylbut-2-enal}}{CH_3-CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-CHO}$$

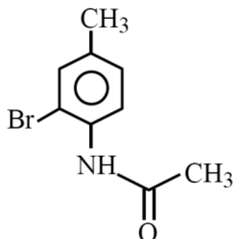
$$\underset{\text{Pent-2-enal}}{CH_3-CH_2-CH=CH-CHO}$$

32. (c)



(137 mg) (A)





(B)

(Mass of B is 228 gm)

$$\text{Mole} = \frac{137\times10^{-3}}{137} = 0.001 \text{ mole}$$

Mole of product $= 0.001$ mole
Mass of product $= 0.001 \times 228\text{gm}$
$= 0.228\text{gm} = 228\text{mg}$

33. (b) $\underset{5^\circ C}{H_2O(\ell)} \longrightarrow \underset{100^\circ C}{H_2O(\ell)} \rightleftarrows \underset{100^\circ C}{H_2O(g)}$

due to expansion

$w = -ve$

as heat is given to system so $q = +ve$ and

internal energy of gas will be more than internal energy of liquid so $\Delta U = +ve$

34. (b) Statement 1 (True)

Strength of ligand : $Cl^- > Br^-$

$\Delta_t : [CoCl_4]^{2-} > [CoBr_4]^{2-}$

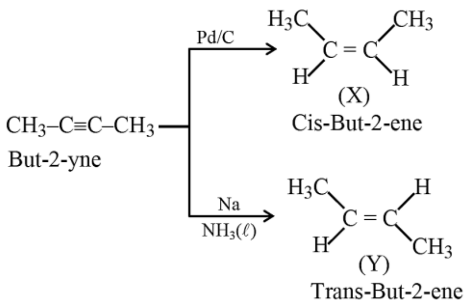
$E_{absorbed} : [CoCl_4]^{2-} > [CoBr_4]^{2-}$

Statement 2 (False)

Strength of ligand : $I^- < Cl^-$

$\Delta_t : [CoI_4]^{2-} < [CoCl_4]^{2-}$

35. (b)



Dipole moment $x \neq 0$

$\underset{(\text{Cis / Trans})}{\text{But} - 2 - \text{ene}} \xrightarrow[\text{(ii) } Zn/H_2O]{\text{(i) } O_3} \underset{(\text{Acetaldehyde})}{2CH_3 - CHO}$

36. (d)(A) $n = 5$

$l = 0 \ \ m_l = 0$

$l = 1 \ \ m_l = -1,0,1 \Rightarrow 2$ electrons

$l = 2 \ \ m_l = -2,-1,0,1,2 \Rightarrow 2$ electrons

$l = 3 \ \ m_l = -3,-2,-1,0,1,2,3 \Rightarrow 2$ electrons

$l = 4 \ \ m_l = -4,-3,-2,-1,0,1,2,3,4 \Rightarrow 2$ electrons

Total number of electrons $= 8$

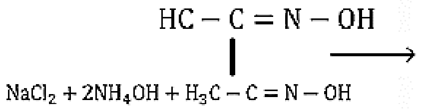
(B) $n = 3, l = 2, \ m_l = -1, \ m_s = +1/2 \Rightarrow$ only 1 electron is possible

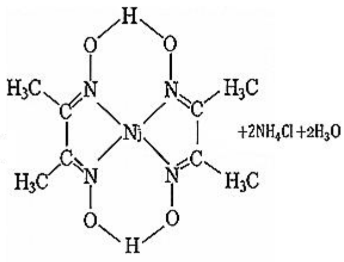
37. (d) In $Ph - OMe$, $-OMe$ is a electron donor group $(+M)$.

$Ph - NO_2, -NO_2$ is a strong withdrawing group $(-M)$.

$Ph - Cl, -Cl$ is a electron withdrawing group.

38. (c)





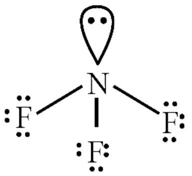
Complex of red colour
(Stable form of complex)

In the above complex, Ni is present in +2 oxidation number.

(A) It is rosy red ppt
(B) It is precipitated in basic medium
(C) $Ni^{+2}: 3d^8$
Hybridisation : dsp^2
Unpaired $e^- = 0$
Geometry : Square planar
(D) $N - Ni - N$ Bond angle is close to 90°
(E) 2 five membered metal containing rings are formed.

39. (d) Compound in option (B) and (C) are acetals (i.e. not having anomeric -OH). Hence they do not give Tollen's test.

40. (c)

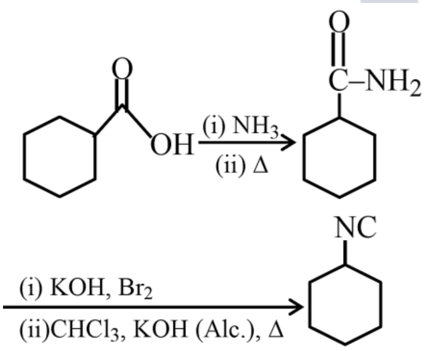
NF_3  , hybridization : sp^3

Number of lone pair in $NF_3 = 10$
Bond angle in $NF_3 \approx 102^\circ$

41. (c) (A) - IV, (B)-I, (C)-II, (D)-III

42. (d) Sublimation is used to for purifying volatile compound from non volatile compound. Boiling point increases as the external pressure is increased.

43. (b)



44. (c) (A) $B^{+3} < Al^{+3} < Ga^{+3} < In^{+3} < Tl^{+3}$: ionic size
(B) $B > Tl > In > Ga > Al$: EN
(C) $B > Tl > Ga > Al > In$: IE
(D) Trichlorides and triiodides of group 13 th elements are covalent in nature

45. (d) For reaction (I) : $K_p > K_C$
$\Delta n_g > 0$
$y - x > 0.$
$85.87 = 2.586(0.0821 \times 400)^{y-x}$
Solving $y - x \simeq 1$
For reaction (II) : $K_p < K_C$

$y - x < 0.$

46. (10) $[AB]_0 - [AB]_t = kt$

$\mathbf{0.60} - 0.55 = k(100)$

$k = 5 \times 10^{-4}$

Half life $\left(t_{1/2}\right) = \frac{[AB]_0}{2k}$

$= \frac{0.60}{2 \times 5 \times 10^{-4}}$

$= 600\text{sec}$

$= 10 \text{ min}$

47. (125) $NH_{3(g)} \rightleftharpoons \frac{1}{2} N_{2(g)} + \frac{3}{2} H_{2(g)}$

$t = 0$	1 mole	-	-
$t = t_{eq}$	$1 - \alpha$	$\alpha/2$	$3\alpha/2$

$k_P = \frac{\left(\frac{\alpha}{2}\right)^{1/2}\left(\frac{3\alpha}{2}\right)^{3/2}}{(1-\alpha)}\left[\frac{P_T}{1+\alpha}\right]^1 \quad [\because P_T = \sqrt{3}\text{ atm}]$

$9 = \frac{\left(\frac{\alpha}{2}\right)^{1/2}\left(\frac{3\alpha}{2}\right)^{3/2}}{(1-\alpha)} \times \frac{(3)^{1/2}}{1+\alpha}$

$9 = \frac{9\left(\frac{\alpha}{2}\right)^2}{1-\alpha^2}$

$1 - \alpha^2 = \frac{\alpha^2}{4}$

$\frac{5\alpha^2}{4} = 1$

$\alpha^2 = 0.8$

$\alpha = (0.8)^{1/2}$

$\alpha = \left[\frac{1}{0.8}\right]^{-1/2}$

$\alpha = [125 \times 10^{-2}]^{-1/2}$

$x = 125.$

48. (1825) $Ba(OH)_{2(aq)} + 2HCl_{(aq)} \rightarrow BaCl_{2(aq)} + 2H_2O_{(\ell)}$

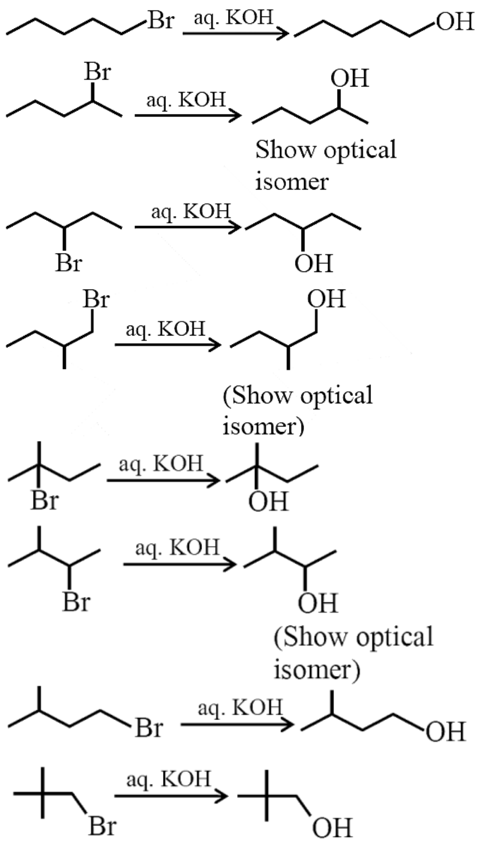
2.5 mmole 5 mmole

wt of HCl $= 5 \times 36.5$ (milligram)

$= 182.5$ (milligram)

Hence $x = 1825.$

49. (3 or 6) $C_5H_{11}Br$



As per the language given and considering the condition we are going with answer 3 and considering both active isomers we will be giving 6 too.

50. **(2)** Pairing energy neglected w.r.t. Δ_0

$[Co(ox)_3]^{3-} : Co^{+3} : d^6 ; t_2\, g^{2,2,2} eg^{0,0}$

$CFSE = 6 \times (-0.4\Delta_0) = -2.4\Delta_0$

$[Cr(ox_3)]^{3-} : Cr^{+3} : d^3 ; t_2\, g^{1,1,1} eg^{0,0}$

$CFSE = 3 \times (-0.4\Delta_0) = -1.2\Delta_0$

$$\frac{(CFSE)_{Co^{+3}}}{(CFSE)_{Cr^{+3}}} = 2$$

51. **(d)** $\log_5\left(\log_7(9x - x^2 - 13)\right) > 0$

$\Rightarrow 9x - x^2 - 13 > 7$

$x^2 - 9x + 20 < 0 \Rightarrow 4 < x < 5$

$m = 4, n = 5$

$$\Rightarrow e = \sqrt{1 + \frac{b^2}{a^2}} = \frac{5}{3} \Rightarrow \frac{b^2}{a^2} = \frac{25}{9} - 1 = \frac{16}{9}$$

$$\frac{b}{a} = \frac{4}{3}$$

$$\Rightarrow \frac{2b^2}{a} = \frac{8m}{3} \Rightarrow \frac{2b^2}{a} = \frac{32}{3}$$

$$\Rightarrow 2b^2 = \frac{32}{3} \times \frac{3b}{4} \Rightarrow b = 4, a = 3$$

$b^2 - a^2 = 16 - 9 = 7$

52. **(a)** $\int e^x \left(\frac{(1-x^2)+1}{\sqrt{1+x}\cdot(1-x)^{3/2}}\right) dx$

$$\int e^x \left(\frac{(1-x^2)}{\sqrt{1+x}\cdot(1-x)^{3/2}} + \frac{1}{\sqrt{1+x}\cdot(1-x)^{3/2}}\right) dx$$

$$\int e^x\left(\sqrt{\frac{1+x}{1-x}}+\frac{1}{\sqrt{1+x}\cdot(1-x)^{3/2}}\right)dx$$

$$= e^x\sqrt{\frac{1+x}{1-x}}+C$$

$$f(x) = e^x\sqrt{\frac{1+x}{1-x}}-1$$

$$f\left(\frac{1}{2}\right) = \sqrt{3}e-1$$

53. (c) $\vec{d} = (\vec{a}\times\vec{b})\times\vec{a}$

$\vec{d} = (a^2)\vec{b}-(\vec{a}\cdot\vec{b})\vec{a}$

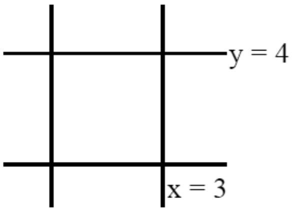
$\vec{d} = 6\vec{b}+8\vec{a}$

$(\vec{a}-\vec{b})\cdot\vec{d} -= (\vec{a}-\vec{b})\cdot(6\vec{b}+8\vec{a})$

$= 8a^2-6b^2-2\vec{a}\cdot\vec{b}$

$= 48-66+16 = -2$

54. (d)



Line is $y = \frac{x}{3}+C$

Line passes thru $\left(\frac{3}{2},2\right)$

$2 = \frac{1}{2}+C \Rightarrow C = \frac{3}{2}$

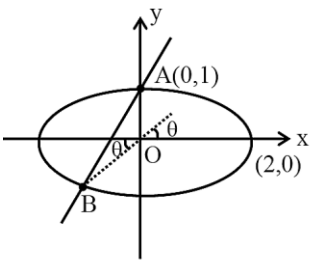
$y = \frac{x}{3}+\frac{3}{2}$

$\Rightarrow 6y = 2x+9$

Line is $2x-6y+9 = 0$ &

Dist $= \left|\frac{1+30+9}{\sqrt{40}}\right| = \sqrt{40} = 2\sqrt{10}$

55. (b)



By solving line & equation of ellipse we get $x = 0$

& $x = -\frac{4}{3}$

$\therefore B\left(-\frac{4}{3},-\frac{1}{3}\right)$

$m_{OB} = \tan\theta = \frac{1}{4}$

$\because \angle AOB = \frac{\pi}{2}+\theta = \frac{\pi}{2}+\tan^{-1}\frac{1}{4}$

56. (c) $R\{(-2,a),(-1,b),(0,c),(1,d),(2,e)\}$

$a=\{-2,-1,0,1,2,3,4\}; b=\{-2,-1,0,1,2,3,4\}$

$c=\{-2,-1,0,1,2\}; d=\{-2,-1,0\}$

$e=\{-2\}$

$\therefore$ No. of elements in R

$=7+7+5+3+1=23=\ell$

Minimum number of element to be added to make it reflexive $=m=4 \Rightarrow \{(1,1),(2,2),(3,3),(4,4)\}$ minimum number of element to be added to make it symmetric $=n=6$

for ' n '

$\Rightarrow R=\{(3,-2),(4,-2),(2,-1),(2,0),(3,-1),(4,-1)\}$

$\therefore \ell+m+n=23+4+6=33$

57. (a) $(x^4dy+4x^3ydx)=-2\sin x dx$

$\Rightarrow \int d(x^4y)=\int -2\sin x dx$

$\Rightarrow x^4y=2\cos x+c$

$\Rightarrow x^4f(x)=2\cos x+c$

As $f\left(\frac{\pi}{2}\right)=0$

So, $c=0$

$\left(\frac{\pi}{3}\right)^4 f\left(\frac{\pi}{3}\right)=2\cos\frac{\pi}{3}$

$\pi^4 f\left(\frac{\pi}{3}\right)=81$

58. (c) $(x-6\sqrt{x}+9)-(2-\sqrt{3})|\sqrt{x}-3|-2\sqrt{3}=0$

$\Rightarrow |\sqrt{x}-3|^2-(2-\sqrt{3})|\sqrt{x}-3|-2\sqrt{3}=0$

$\Rightarrow |\sqrt{x}-3|=2$ or $|\sqrt{x}-3|=-\sqrt{3}$ (not possible)

$\Rightarrow \sqrt{x}=1$ or 5

$\Rightarrow x=1$ or 5

$\Rightarrow \alpha=1$ and $\beta=25$

59. (b) $f(x)=\begin{cases}\frac{ax^2+2ax+3}{(2x-1)(2x+3)} & ; x\neq\frac{-3}{2},\frac{1}{2}\\ b & ; x=\frac{-3}{2},\frac{1}{2}\end{cases}$

for continuous at $x=\frac{-3}{2}$

LHL = RHL

$\Rightarrow \lim_{x\to\frac{-3}{2}}\frac{(ax^2+2ax+3)}{(2x-1)(2x+3)}$

at $x=\frac{-3}{2}\Rightarrow$ Numerator $=0$

$a\left(\frac{-3}{2}\right)^2+2a\left(\frac{-3}{2}\right)+3=0$

$\frac{9}{4}a-3a+3=0$

$\frac{3a}{4}=3\Rightarrow a=4$

$\therefore f(x)=\begin{cases}\frac{4x^2+8x+3}{(2x-1)(2x+3)} & ; x\neq\frac{-3}{2},\frac{1}{2}\\ b & ; x=\frac{-3}{2},\frac{1}{2}\end{cases}$

$$f(x)=\begin{cases}\dfrac{(2x+1)(2x+3)}{(2x-1)(2x+3)} & ;\ x\neq \dfrac{-3}{2},\dfrac{1}{2}\\ b & ;\ x=\dfrac{-3}{2},\dfrac{1}{2}\end{cases}$$

$$f\circ f(x)=f\left(\frac{2x+1}{2x-1}\right)=\frac{2\left(\frac{2x+1}{2x-1}\right)+1}{2\left(\frac{2x+1}{2x-1}\right)-1}$$

$$=\frac{6x+1}{2x+3}=\frac{7}{5}$$

So $x=1$

60. (a) $I=\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}}\frac{dx}{1+\sqrt[3]{\tan 2x}}$ …………… (i)

Apply king

$I=\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}}\frac{dx}{\sqrt[3]{\tan 2\left(\frac{\pi}{4}-x\right)}}=\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}}\frac{dx}{1+\sqrt[3]{\cot 2x}}$ …………… (ii)

Add (i) + (ii) $\Rightarrow 2I=\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}}(i)dx$

$I=\frac{1}{2}\left(\frac{\pi}{6}\right)=\frac{\pi}{12}$

61. (a) Let $\cos\alpha=x$

$\cos\beta=y$

$\cos\gamma=z$

$$\begin{vmatrix}0&x&y\\x&0&z\\y&z&0\end{vmatrix}=\begin{vmatrix}1&x&y\\x&1&z\\y&z&1\end{vmatrix}$$

Expending both sides, we get

$x^2+y^2+z^2=1$

i.e. $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

Statement 1 is false

Now,

$$\begin{vmatrix}x^2+x&1+x&x-2\\2x^2+3x-1&3x&3x-3\\x^2+2x+3&2x-1&2x-1\end{vmatrix}=px+q$$

Put $x=0$ both sides

$$q=\begin{vmatrix}0&1&-2\\-1&0&-3\\3&-1&-1\end{vmatrix}$$

$\Rightarrow q=-12$

Now put $x=1$ both sides

$$p+q=\begin{vmatrix}2&2&-1\\4&3&3\\6&1&1\end{vmatrix}=42$$

$\Rightarrow p=54$

Now $\frac{p^2}{q^2}=\left(\frac{54}{-12}\right)^2+196$

$\Rightarrow p^2\neq 196q^2$

Statement (II) is false

62. (d) $S=\sum_{r=50}^{100}\frac{{}^{100}C_r}{r+1}=\sum_{r=50}^{100}\frac{1}{r+1}\cdot\frac{r+1}{101}\cdot\ {}^{101}C_{r+1}$

$S=\frac{1}{101}\sum_{r=50}^{100}\ {}^{101}C_{r+1}$

$=\frac{1}{101}\times\frac{2^{101}}{2}=\frac{2^{100}}{101}$

63. (a) Mean $=\frac{-18+x+y+2+9+16}{8}=\frac{7}{3}$

$= \frac{x+y+9}{8} = \frac{7}{2} \Rightarrow x + y + 9 = 28$ .... (i)

Variance $= \frac{\sum z_i^2}{8} - (\mu)^2 = \frac{293}{4}$

$\Rightarrow \frac{10^2+7^2+1^2+x^2+y^2+2^2+9^2+16^2}{8} - \left(\frac{7}{2}\right)^2 = \frac{293}{4}$ .... (ii)

Solving (i) & (ii) $\Rightarrow x = 12, y = 7$

Mean of $(1 + x + y), x, y, |y - x|$ is

$$\Rightarrow \frac{20 + 12 + 7 + 5}{4} = \frac{44}{4} = 11$$

64. (d) $(x^4 + 2x^2 + 1)({}^nC_0x^0 + {}^nC_1x^1 + {}^nC_2x^2 + {}^nC_3x^3 + \cdots)$

Coefficient x $\Rightarrow {}^nC_1$,

coeff. of $x^2 \Rightarrow 2 + {}^nC_2$

$$2 + \frac{n(n-1)}{2}$$

Coeff. of $x^3 = 2.\ {}^nC_1 + {}^nC_3$

$= 2n + \frac{n(n-1)(n-2)}{6}$ (if $x \geq 3$)

Now according to question

$$n + 2n + \frac{n(n-1)(n-2)}{6} = 2\left[2 + \frac{n(n-1)}{2}\right]$$

$$3n + \frac{n(n-1)(n-2)}{6} = 4 + n(n-1)$$

$\Rightarrow n^3 - 9n^2 + 26n - 24 = 0$

$\Rightarrow n = 2,3,4 \ \Rightarrow n = 3,4$

Now checking for $n = 2$

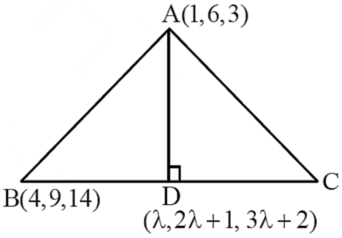
$$\left.\begin{array}{l}\text{Coeff. of } x = 2\\ \text{Coeff. of } x^2 = 3\\ \text{Coeff. of } x^3 = 4\end{array}\right\} \Rightarrow \text{are in A.P.}$$

$\Rightarrow n = 2$ is also the correct choice

Required sum of values of ' n '

$= 2 + 3 + 4 = 9$

65. (a) $\frac{4}{1} = \frac{9-1}{2} = \frac{\alpha-2}{3} \Rightarrow \alpha = 14$



$\overrightarrow{AD} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 0$

$(\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k} = \overrightarrow{AD}$

$\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$\Rightarrow 14\lambda = 14 \Rightarrow \lambda = 1$

$D = (1,3,5)$

$AD = \sqrt{3^2 + 2^2} = \sqrt{13}$

$$\text{Ar}(\triangle \text{ABC}) = \frac{1}{2} \times \sqrt{13} \times 10 = 5\sqrt{13}$$

66. (b) $\sqrt{3}(2\cos^2\theta - 1) + 8\cos\theta + 3\sqrt{3} = 0$

$2\sqrt{3}\cos^2\theta + 8\cos\theta + 2\sqrt{3} = 0$

$(\sqrt{3}\cos + 1)(\cos\theta + \sqrt{3}) = 0$

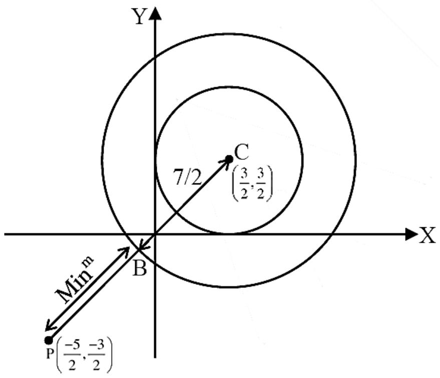
$$\cos\theta = -\frac{1}{\sqrt{3}}$$

as $-\sqrt{3}$ (reject)

$\therefore \theta =$ will have 5 value in $[-3\pi, 2\pi]$

$= 5$

67. (b) $\frac{3}{2} \le \left|z - \frac{3}{2}(1+i)\right| \le \frac{7}{2}$



$\underset{z\in s}{\text{Min}}\left|z - \left(\frac{-5}{2} - \frac{3}{2}i\right)\right| = PB$

$PB = PC - \frac{7}{2} \Rightarrow 5 - \frac{7}{2} \Rightarrow \frac{3}{2}$

68. (b) $f(\theta) = 4\left(\sin^4\left(\frac{7\pi}{2} - \theta\right) + \sin^4(11\pi + \theta)\right) - 2\left(\sin^6\left(\frac{3\pi}{2} - \theta\right) + \sin^6(9\pi - \theta)\right)$

$f(\theta) = 4(\cos^4(\theta) + \sin^4(\theta)) - 2(\cos^6\theta + \sin^6\theta)$

$f(\theta) = 4(1 - 2\sin^2\theta\cos^2\theta) - 2(1 - 3\sin^2\theta\cos^2\theta)$

$f(\theta) = 2 - 2\sin^2\theta\cos^2\theta$

$f(\theta) = 2 - \frac{\sin^2(2\theta)}{2}$

$\alpha = f(\theta)_{max} = 2$

$\beta = f(\theta)_{min} = \frac{3}{2}$

$\Rightarrow \alpha + 2\beta = 5$

69. (d) Let time taken by mason a alone to complete the work be in x days so, mason B along take $x + 24$ days

work done by A in 1 day $= \frac{1}{x}$

work done by B in 1 day $= \frac{1}{x+24}$

so work done by A + B in 1 day $= \frac{1}{22.5}$

So, $\frac{1}{x} + \frac{1}{x+24} = \frac{2}{45}$

$x^2 - 21x - 540 = 0$

$x = 36\&$

$x = -15$(rejected)

$= 36$

70. (d) Direction cosines of two lines satisfy the equation

$\Rightarrow 4\ell + m - n = 0$……………….. (i)

$2mn + 10n\ell + 3\ell m = 0$ ………………… (ii)

& we know

$\Rightarrow \ell^2 + m^2 - n^2 = 1$ ……………… (iii)

$\Rightarrow n = 4\ell + m$ putting in eqn.

$\Rightarrow n(2m + 10\ell) + 3\ell m = 0$

$\Rightarrow (4\ell + m)(2m + 10\ell) + 3\ell m = 0$

$\Rightarrow 8\ell m + 40\ell^2 + 2m^2 + 10\ell m + 3\ell m = 0$

$\Rightarrow 40\ell^2 + 21\ell m + 2m^2 = 0$

$\Rightarrow (8\ell + m)(5\ell + 2m) = 0$

Case 1: $8\ell + m = 0 \Rightarrow m = -8\ell$

Case 2 : $5\ell + 2m = 0 \Rightarrow m = \frac{-5}{2}\ell$

So direction ratio of L_1 is $\ell, -8\ell, -4\ell$

& direction ratio of L_2 is $\ell, \frac{-5\ell}{2}, \frac{3\ell}{2}$

$$\cos\theta = \left|\frac{\ell^2 + 20\ell^2 - 6\ell^2}{\sqrt{\ell^2 + 64\ell^2 + 16\ell^2}\sqrt{\ell^2 + \frac{25\ell^2}{4} + \frac{9\ell^2}{4}}}\right|$$

$$= \frac{15\ell^2}{(9\ell)\frac{\sqrt{38}\ell}{2}} = \frac{10}{3\sqrt{38}}$$

71. **(62)** $\text{adj}2A = 2^2\text{adj}A \because \text{adj}kA = k^{n-1}(\text{adj}A)$

$= 4\text{adj}A$

Now $A^2(\text{adj}2A) = 4A(\text{adj}A)$

$= 4A|A|I_3$

$= 24A$

Now $3\text{adj}(A^2(\text{adj}2A)) = 3\text{adj}(24A)$

$= 3.(24)^2\text{adj}A$

Now $|\text{adj}(3\text{adj}(A^2(\text{adj}2A)))|$

$= |\text{adj}(3.(24)^2\text{adj}A)|$

$= |(3.(24)^2)^2(\text{adj adj}A)|$

$= |3^6.2^{12}\text{adj adj}A|$

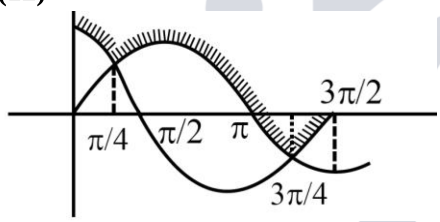
$= (3^6.2^{12})^3|\text{adj adj}A|$

$= 3^{18}.2^{36}.(A)^4$

$3^{22}.2^{40}$

$\therefore m + n = 62$

72. **(12)**



$A = \int_0^{\pi/4}\cos x dx + \int_{\pi/4}^{\pi}\sin x dx + \int_{\pi}^{5\pi/4} -\sin x dx + \int_{5\pi/4}^{3\pi/2} -\cos x dx$

$A = (\sin x)_0^{\pi/4} + (\cos x)_{\pi}^{\pi/4} + (\cos x)_{\pi}^{5\pi/4} + (\sin x)_{3\pi/2}^{5\pi/4}$

$A = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 1 + 1 - \frac{1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} = 3$

$A^2 + A = 12$

73. **(1565)** $2f(x)f'(x) = f^2(x) + (f'(x))^2$

$\Rightarrow (f(x) - f'(x))^2 = 0$

$\Rightarrow f(x) = f'(x)$

$\Rightarrow \ell n(f(x)) = x + c \Rightarrow f(x) = c'e^x$

$f(0) = 5 \Rightarrow f(x) = 5e^x$

Mean $= \frac{f(\ell n1) + f(\ell n2) + \cdots \ldots\ldots + f(\ell n625)}{625}$

$= \frac{5[1 + 2 + \cdots \ldots \ldots \ldots + 625]}{625} = 1565$

74. **(311)** $a - b \geq 10$

Total cases $= 100 \times 99$

Fav. Cases $= 1 + 2 + 3 + \cdots 90$

Req. Prob $= \frac{1 + 2 + \cdots + 90}{100 \times 99}$

$\frac{m}{n} = \frac{90\left(\frac{91}{2}\right)}{100(99)} = \frac{91}{220}$

$m + n = 311$

75. (1422) $P \to 3, Q \to 2, R \to 2, S, T, U, V$

Case I 3 alike, 1 different

${}^{1}C_1 \times {}^{6}C_1 \times \frac{4!}{3!} = 24$

Case II 2 alike, 2 alike

${}^{3}C_2 \times \frac{4!}{2!\,2!} = 18$

Case III 2 alike, 2 different

${}^{3}C_1 \times {}^{6}C_2 \times \frac{4!}{2!} = 540$

Case IV All 4 different

${}^{7}C_4 \times 4! = 840$

Total words $= 1422$