1. **(c)** $\Delta U = 3nR\Delta T$

$\Delta U = 3 \times 1 \times \frac{25}{3} \times 4 = 100$ Joule

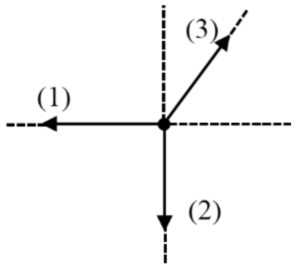
$\Delta Q = 126$

$W = 26 = P\Delta V$

$26 = 10^5 \times 17 \times 10^{-4}\Delta x$

$\Delta x = \frac{26}{170} = 15.3$

2. **(b)**



Apply C.O.M

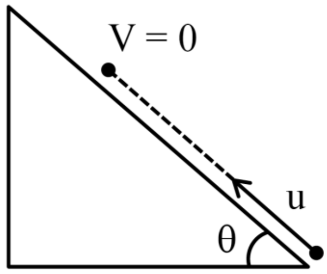
$M_1 V_1 + M_2 V_2 + M_3 V_3 = 0$

$2(-18i) + 2(-18j) + 3 V_3 = 0$

$V_3 = 12i + 12j$

$|V_3| = 12\sqrt{2}$ m/s

3. **(d)**



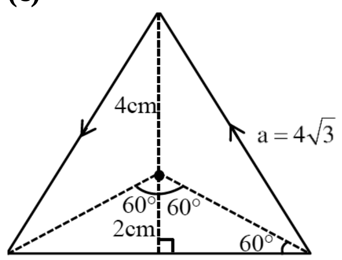
$a = -g\sin\theta$

$V^2 = U^2 + 2as$

$0 = u^2 - 2g\sin\theta \cdot s$

$s = \frac{u^2}{2g\sin\theta}$

4. **(c)**

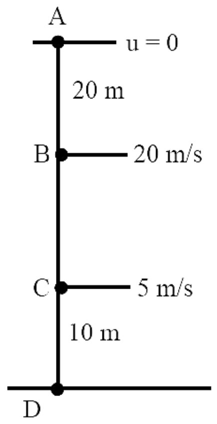


$B = \frac{\mu_0}{4\pi} \times \frac{I}{d}[\sin 60° + \sin 60°] \times 3$

$B = 10^{-7} \times \frac{2}{2 \times 10^{-2}}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) \times 3$

$= \sqrt{3} \times 10^{-5} \times 3 = 3\sqrt{3} \times 10^{-5}$

5. (b)



A to B

$x_1 = \frac{1}{2} \times 10 \times 2^2 = 20\text{ m}$

$V = 0 + 10 \times 2$

B to C

$5^2 = 20^2 - 2(3)x_2$

$x_2 = \frac{375}{6}$

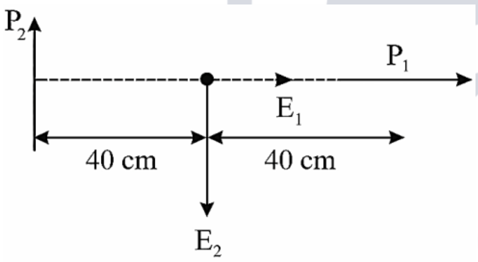
$x_2 = 62.5$

C to D

$x_3 = 10\text{ m}$

$H = x_1 + x_2 + x_3 = 92.5$

6. (d)



$\vec{E}_2 = -\frac{KP_2}{r^3}; \vec{E}_1 = -\frac{2KP_1}{r^3}$

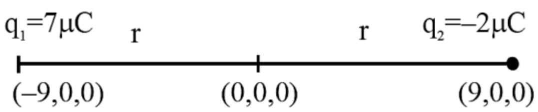
$P_1 = 2 \times 10^{-6} \times 10^{-2} = 2 \times 10^{-8}$

$P_2 = 4 \times 10^{-6} \times 10^{-2} = 4 \times 10^{-8}$

$\vec{E}_{net} \frac{2 \times 9 \times 10^9 \times 2 \times 10^{-8}}{(0.4)^3}\hat{i} - \frac{9 \times 10^9 \times 4 \times 10^{-8}}{(0.4)^3}\hat{j}$

$\vec{E}_{net} = \frac{9 \times 10^9 \times 4 \times 10^{-8}}{(0.4)^3}[\hat{i} - \hat{j}]$

$|\vec{E}_{net}| = \frac{9 \times 10^4}{16}(\sqrt{2})$



7. (c)

$dV = -\vec{E} \cdot \vec{dr}$

$\int_0^v dV = -\int_\infty^r \frac{A}{r^2} dr$

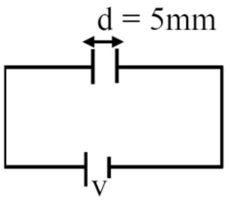
$V = -\left[\frac{-A}{r^2}\right]_\infty^r \Rightarrow V = \frac{A}{r}$

$U = U_{self} + U_{interaction}$

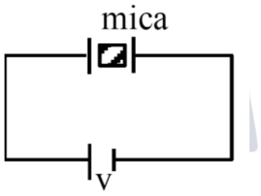
$= q_1 v_1 = q_2 v_2 + \dfrac{kq_1 q_2}{2r}$
$= 7 \times 10^{-6} \dfrac{A}{9 \times 10^{-2}} - 2 \times 10^{-6} \dfrac{A}{9 \times 10^{-2}}$
$- \dfrac{9 \times 10^{9} \times 14 \times 10^{-12}}{2 \times 9 \times 10^{-2}}$
$= \dfrac{5 \times 10^{-6} \times 9 \times 10^{5}}{9 \times 10^{-2}} - 7 \times 10^{-1}$
$= 50 - 0.7$
$= 49.3\ \mathrm{J}$

8. **(a)** Horizontal displacement of Q is more then P .
$X_Q > X_P$
Horizontal component of velocity is same
So $t_p = \dfrac{x_p}{v}$
$t_Q = \dfrac{x_Q}{v}$
$t_Q > t_p$

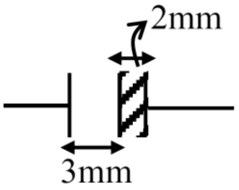
9. **(b)**



$C = \dfrac{\epsilon_0\ A}{d}$
$Q_1 = CV$



$Q_2 = (c_{eq})v$
$Q_2 = 1.25cv$



$$c_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{\epsilon_0 A}{3} \times \frac{K\epsilon_0 A}{2}}{\frac{\epsilon_0 A}{3} + \frac{K\epsilon_0 A}{2}}$$
$$c_{eq} = \frac{(\epsilon_0 A)^2 \left(\frac{K}{6}\right)}{\epsilon_0 A \left(\frac{2+3K}{6}\right)} \Rightarrow C_{eq} = \frac{K\epsilon_0 A}{2+3K}$$
$$1.25 \times \frac{\epsilon_0\ A}{5} = \frac{K\ \epsilon_0\ A}{2+3\ K} \Rightarrow 0.25(2+3\ K) = K$$
$$2 + 3\ K = 4\ K \Rightarrow\ K = 2$$

10. **(b)** $\in = \lambda \ell$
Potential gradient

$\epsilon_1 = \lambda\ell_1$
$\epsilon_2 = \lambda\ell_2$
$y = \frac{\epsilon_1}{\epsilon_2} = \frac{\ell_1}{\ell_2}$
$\frac{\Delta y}{y} = \frac{1}{200} + \frac{1}{150}$
$\frac{\Delta y}{y} \times 100 \left(\frac{1}{200} + \frac{1}{150}\right) \times 100$
$= \left(\frac{3+4}{600}\right) \times 100 \Rightarrow \frac{7}{6} = 1.16\%$

11. **(d)** For an air bubble rising in water, the no. of moles remain constant

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$
$\frac{(P_{atm} + \rho gh)2.9\ cm^3}{290\ K} = \frac{(P_{atm})V_2}{300}$
$V_2 = 4.5\ cm^3$

12. **(d)** Isobars are nuclei that have the same mass number ${}^3_1H$ & ${}^3_2He$ have same mass number.

13. **(d)** Gate 1: At bottom there is an OR gate with inputs n & m

out put $= n + m$

Gate 2 : A NAND gate, its Input are direct n & the output of OR gate ($n + m$)

out put $z = \overline{n \cdot (n+m)}$

since $n.(n+m) = (n.n) + (n.m)$

$= n + n.m = n(1+m) = n$

$\therefore$ output $z = \overline{n \cdot (n+m)} = \overline{n}$

n	m	$z = \overline{n}$
0	0	1
0	1	1
1	1	0
1	0	0

14. **(b)** For Isothermal process

$W_{isothermal} = nRT\ell n\left(\frac{V_2}{V_1}\right)$
$= 1.R300.\ell n(2)$
$= 300R(0.693)$ ………………… (1)

Now for adiabatic process,

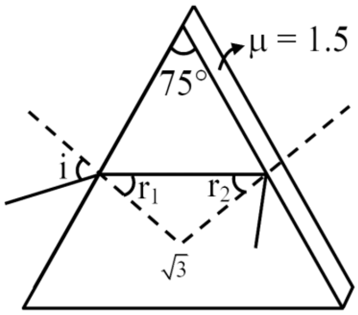
It is given work done in Isothermal = work done in adiabatic

$W_{adiabatic} = \frac{nR(T_1 - T_2)}{\gamma - 1}$ ……………. (2)

(1) = (2)

$\frac{nR(300 - T_{final})}{1.4 - 1} = 300R(0.693)$
$T_{Final} = 216.84\ K$
$= -56.3°C$

15. **(c)**



$r_1 + r_2 = 75°$

For TIR at back surface

$\sqrt{3}\sin r_2 = \frac{3}{2}\sin 90°$

$r_2 \geq 60°$

$r_1 \leq 15°$

$1\sin i = \sqrt{3}\sin 15°$

$\sin i = 1.73 \times .25$

$\sin i = .433$

$i = 25° \Rightarrow i < 25°$

16. (d) induced EMF $= \frac{d\phi}{dt}$

circumference $\simeq 14\pi$

side length of square loop $= \frac{14\pi}{4} = \frac{7\pi}{2}$

$\Delta\phi = B(A_1 - A_2)$

$= (.2)\left(\left(\frac{7\pi}{2}\right) \times \frac{7\pi}{2} - 49\pi\right) \times 10^{-4}$

$= .2\left(\frac{49\pi^2}{4} - 49\pi\right) \times 10^{-4}$

$\Delta\phi = .2 \times 33.07 \times 10^{-4}$

$\Delta\phi = 6.614 \times 10^{-4}$

$$\text{EMF} = \frac{6.614 \times 10^{-4}}{\frac{1}{2}}\text{ V} = 13.23 \times 10^{-4}\text{ V}$$

EMF $= 1.32$mV

17. (a) EMF induced $\varepsilon = A\frac{dB}{dt} = A\mu_0 n\frac{di}{dt}$

$\varepsilon = A\mu_0 n i_0 \omega\cos\omega t$

current induced $i = \frac{\varepsilon}{R} = \frac{\pi r^2\mu_0 n i_0\omega}{R}\cos\omega t$

So $i = \frac{\pi r^2\mu_0 n i_0\omega}{\sqrt{2}R}$

$$= \frac{\pi \times 10^{-4} \times 4\pi \times 10^{-7} \times 500 \times 10 \times 10^3}{\sqrt{2} \times 10}$$

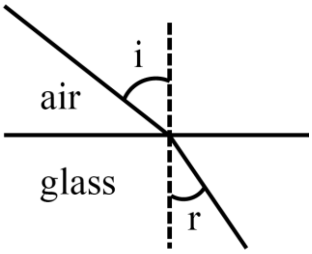
$$= \frac{20\pi^2}{\sqrt{2}} \times 10^{-6}$$

$$\simeq \frac{197}{\sqrt{2}}\mu\text{ A}$$

18. (d) $\frac{C}{V} = \mu = \sqrt{\varepsilon_r\mu_r}$

$$\frac{C}{V} = \sqrt{\frac{3 \times 2}{1}} = \frac{\sqrt{6}}{1}$$

19. (b)



$\tan i = \frac{\mu_2}{\mu_1} = \frac{\mu_g}{\mu_a}$

$\tan i = 1.52$

$i = 57.7^\circ$

$r = 90^\circ - i$

$r = 32.3^\circ$

20. (a) $V_T = \frac{2r^2 g}{9\eta}(\rho_b - \rho_\ell)$

$V_T = \frac{2}{9}\frac{(.1)^2 \times 10}{(10)}(10.5 - 1.5)$

$V_T = 2\ \text{cm/sec.}$

21. (228) Total numbers of U-235 atom is

$47\ \text{g} = \frac{47}{235}\ \text{moles} = \frac{1}{5}\ \text{moles}$

$\therefore$ Total energy released $= \frac{1}{5} \times 6 \times 10^{23} \times 190\text{MeV}$

$= 228 \times 10^{23}\text{MeV}$

22. (16) $m = \frac{f}{f+u}$

$m_1 = -m_2$

$\frac{f}{f-8} = -\frac{f}{f-24}$

$f - 8 = 24 - f$

$2f = 32$

$f = 16\ \text{cm}$

23. (1) $T = (ML^{-3})^a L^b (ML^{-1}\ T^{-1})^c (ML^{-3})^d$

$T = M^{a+c+d} L^{-3a-c-sd+b} T^{-c}$

on comparing

$c = -1; a + c + d = 0; -3a - c - 3\ d + b = 0$

$b = 2; a + d = 1$

$b + c = 1$

24. (109) $M = \sigma\pi R^2$

$\sigma\pi R^2 = 16\ m$

$m = \frac{\sigma\pi R^2}{16}$

$I_{\text{system}} = \frac{MR^2}{2} - 2\left(\frac{mR^2}{2 \times 16} + \frac{9mR^2}{16}\right)$

$= \frac{MR^2}{2} - 2 \times \frac{19mR^2}{32}$

$= \frac{MR^2}{2} - \frac{19}{16} mR^2$

$= \frac{MR^2}{2} - \frac{19}{256} MR^2$ becoz $m = \frac{M}{16}$

$= \frac{(128-19)(MR^2)}{256}$

$= \frac{109MR^2}{256}$

25. (819) $V = \sqrt{\frac{\gamma RT}{M}}$

$\frac{V_1}{V_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$

$\frac{V_0}{2\ V_0} = \sqrt{\frac{273}{T_2}}$

$\frac{1}{4} = \frac{273}{T_2}$

$T_2 = 4 \times 273 = \alpha + 273$

$\alpha = 3 \times 273$

$\alpha = 819^\circ C$

26. (b) $A \rightarrow nB$

$0.05\ 0$

$0.04\ 0.01 \times n$

$0.01 \times n = 0.03$

$n = 3$

27. (c) $PbO_2 + Pb \rightarrow 2PbO$

Pb^{2+} is more stable hence reaction will be spontaneous. So $\Delta G^\circ{}_r(1)$ is negative.

$SnO_2 + Sn \rightarrow 2SnO$

Sn^{+2} is less stable, so reaction will be nonspontaneous hance $\Delta G^\circ{}_r(2)$ is positive.

28. (c) (a) If C_1 is at anode $\Rightarrow$ cell reaction

$M^+(C_2) \rightarrow M^+(C_1)$

$E_{cell} = -0.059 \log \frac{C_1}{C_2}$

$\therefore E_{cell} > 0 \Rightarrow C_1 < C_2$

(b) If C_1 is at cathode

$M^+(C_1) \rightarrow M^+(C_2)$

$E_{cell} = -0.059 \log \frac{C_2}{C_1} > 0$

$C_2 < C_1$

29. (c) DNA & RNA are chiral molecules due to presence of chiral 2-deoxy-Ribose & Ribose sugar unit respectively.

30. (b)(A) $Co^{3+}: -3\ d^6$

$NH_3 \Rightarrow$ S.F.L

Hybridisation $\Rightarrow d^2sp^3$, Inner orbital complex

(B) $Mn^{3+}: -3\ d^4$

$Cl^- \Rightarrow$ W.F.L

Hybridisation $\Rightarrow sp^3\ d^2$, Outer orbital complex

(C) $Co^{3+}: -3\ d^6$

$F^- \Rightarrow$ W.F.L

Hybridisation $\Rightarrow sp^3\ d^2$, Outer orbital complex

(D) $Fe^{3+}: -3\ d^5$

$F^- \Rightarrow$ W.F.L

Hybridisation $\Rightarrow sp^3\ d^2$, Outer orbital complex

(E) $Ni^{2+}: -3d^8$

$CN^- \Rightarrow$ S.F.L

Hybridisation $\Rightarrow dsp^2$, Inner orbital complex

31. (a)

Element	EN
N	3.0
P	2.1
As	2.0
Sb	1.9
Bi	1.9

X = Nitrogen (N)
Y = Arsenic (As)

32. (a) S-1: $(CH_3)_3C^+ > \overset{+}{C}H_3$; due to hyperconjugation interaction

interaction $CH_3 - \overset{\oplus}{C}(CH_3) - CH_3$

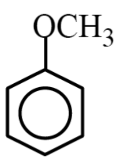
S-2 : False

33. (a)

$R - \overset{O}{\overset{\|}{C}} - CH_3$ or $R - \underset{OH}{CH} - CH_3$

$\xrightarrow[+I_2]{NaOH} R - CO\overset{\ominus}{O}\overset{\oplus}{Na} + \underset{\text{Yellow ppt}}{CHI_3} \downarrow$

(A) CH_3–OH & $CH_3 - \underset{OH}{CH_2}$

 & $CH_3-\overset{O}{\overset{\|}{C}}-CH_3$

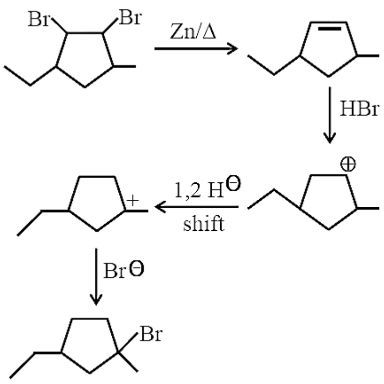
34. (a) Na^+:- $1s^2 2s^2 2p^6$ (fully filled electronic configuration)

$Mg^+: -1s^2 2s^2 2p^6 3s^1$

IE_2 of Na > IE_2 of Mg

Size of $O^{2-} > F^-$

35. (b)



36. **(d)** At t = 10 minutes

Rate of reaction $= -\frac{1}{5}\frac{\Delta[Br^-]}{\Delta t} = \frac{1}{5} \times (2 \times 10^{-4}) = 4 \times 10^{-5}$

For reaction $A \rightarrow P$

at t = 10 minutes

Rate of reaction $= 4 \times 10^{-5} = k[A]$

$k = 4 \times 10^{-3}\ \text{min}^{-1}$.

37. **(c)**(A) $NaOCl + KI \rightarrow NaCl + KOI$

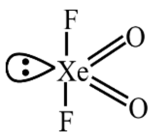
(B) Incorrect statement

(C) $CH_3-CH=CH-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$ gives iodoform reaction.

(D) Incorrect statement

(E) Incorrect statement

38. **(a)**



(A) See-saw

(B) Xe has 7 electron pair in its valence shell

(C) $O-Xe-O$ Bond angle is close to 120°

(D) F-Xe-F Bond angle is close to 180°

(E) Xe has 14 valence electron in XeO_2F_2

39. **(b)** (A) $_{12}Mg^{24}$ represents 12 protons and 12 neutrons.

(B) $\lambda = \frac{C}{v} = \frac{3\times10^8}{4.5\times10^{15}} = 6.67 \times 10^{-8}$ m.

(C) $\frac{E_1}{E_2} = \frac{hC/\lambda_1}{hC/\lambda_2} = \frac{\lambda_2}{\lambda_1}$

$\frac{E_1}{E_2} = \frac{300}{900} = \frac{1}{3}$ (false)

(D) No. of photons $= \frac{\text{Energy}}{hC/\lambda} = 10^{16}$ (True)

40. **(a)** Organic compound : 0.2425 gm

AgCl obtained : 0.5253 gm

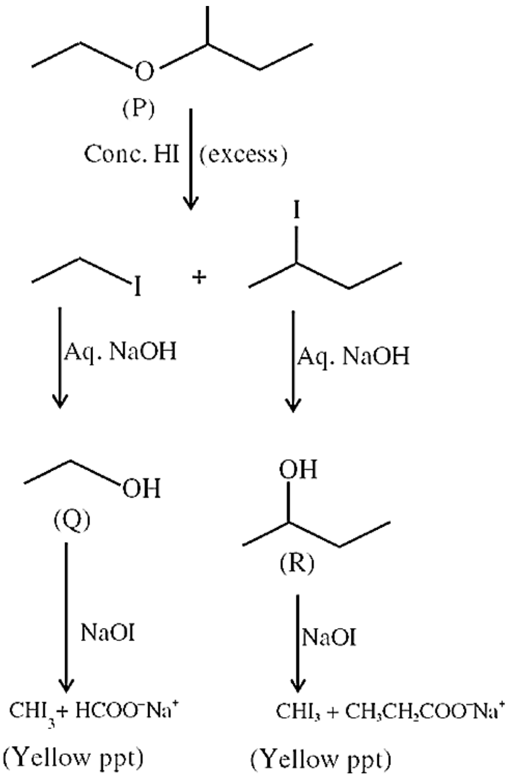
In carius method for estimation of halogen amount of AgCl obtained is 0.5253 gm from 0.2425 gm of organic compound.

Hence percentage of Cl.

Percentage of Cl $= \frac{35.5}{143.5} \times \frac{0.5253}{0.2425} \times 100$

$= 53.58\%$

41. **(a)**



42. (b) $K_2Cr_2O_7 + KI \xrightarrow{H^+} I_2 + Cr^{3+}$

43. (b) $KE_{max} = E - \phi$

$(KE_{max})_1 = 6 - \phi_1$ .... (1)

$(KE_{max})_2 = 6 - \phi_2$ .... (2)

By eq. (1) divide eq. (2)

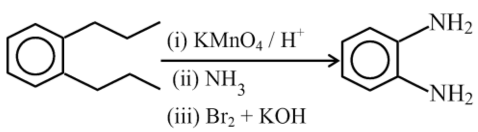
$\frac{(KE_{max})_1}{(KE_{max})_2} = \frac{2.642}{1} = \frac{6-\phi_1}{6-\phi_2}$

$\frac{2.642}{1} = \frac{6-\phi_1}{6-2\phi_1}$

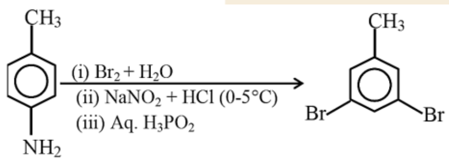
$\phi_1 = 2.3eV$

$\phi_2 = 4.6eV.$

44. (c) Statement-I

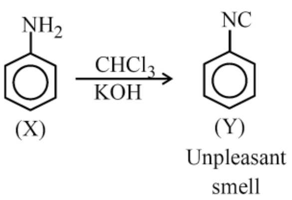


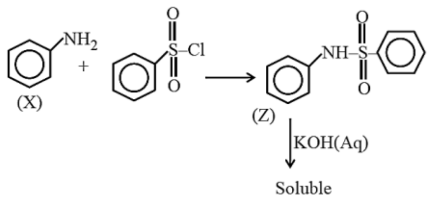
Statement-II

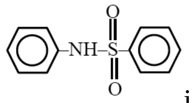


Both statement (I) and (II) are true.

45. (d)





Number of different H atom in  is 7 .

46. **(15)** $X_2(g) + Y_2(g) \rightleftharpoons 2Z(g)$

$K_C = \frac{(9)^2}{3 \times 3} = 9$

Now 10 moles of Z are added then reaction will move in backward direction.

$X_2(g) + Y_2(g) \rightleftharpoons 2Z(g)$

$3 + X \; 3 + X \; 19 - 2X$

$K_C = \frac{(19 - 2X)^2}{(3 + X)(3 + X)} = 9$

$\frac{19 - 2X}{3 + X} = 3$

$19 - 2X = 9 + 3X$

$10 = 5X$

$X = 2$

At equilibrium $\Rightarrow$ moles of $Z = 19 - 2 \times 2$

$= 15$ moles

47. **(200)** $X_A = \frac{3}{4}, X_B = \frac{1}{4}$

$P_S = P_A^o X_A + P_B^o X_B$

$500 = P_A^o \times \frac{3}{4} + P_B^o \times \frac{1}{4}$

$3P_A^o + P_B^o = 2000$ .... (i)

Now 1 moles of A is further added so $n_A = 4$ mole, $n_B = 1$ mole

$X_A' = \frac{4}{5}, X_B' = \frac{1}{5}$

$P_s = 520 = P_A^o \times \frac{4}{5} + P_B^o \times \frac{1}{5}$

$4P_A^o + P_B^o = 2600$ .... (ii)

By equation (ii) - equation (i)

$P_A^o = 600$ mmHg

$P_B^o = 200$ mmHg

48. **(375)** Fe^{2+} moles $= 0.6 \times 0.75 = 0.45$

$Cr_2O_7^{2-} : Fe^{2+} = 1 : 6$

Moles of $K_2Cr_2O_7 = \frac{0.45}{6} = 0.075$

$(x \times 10^{-3}) \times 0.2 = 0.075$

$x = 375$

49. **(2)** In $[Ni(CO)_4]$, $Ni^0 : 3\,d^8 4\,s^2$

Hybridisation state: sp^3

Unpaired electron $= 0$

In $[NiCl_4]^{2-}$, $Ni^{2+} : 3\,d^8$

Hybridisation state: sp^3

Unpaired electron $= 2$

In $[PtCl_4]^{2-}$, $Pt^{2+} : 5\,d^8$

Hybridisation state: dsp^2

Unpaired electron $= 0$

In $[Ni(CN)_4]^{2-}, Ni^{2+}: 3\ d^8$

Hybridisation state: dsp^2

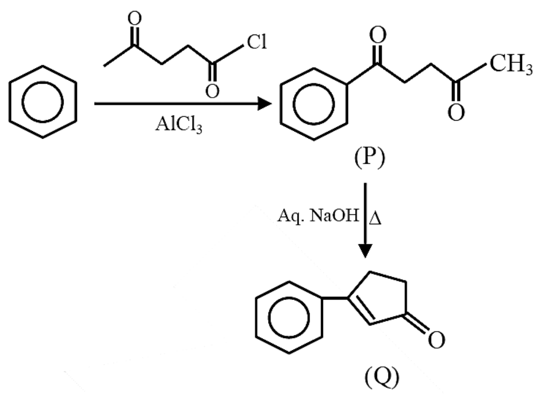
Unpaired electron $= 0$

In $[Pt(NH_3)_2Cl_2], Pt^{2+}: 5\ d^8$

Hybridisation state: dsp^2

Unpaired electron $= 0$

50.



Molecular mass of Q is $= 157$

% of oxygen in product ' Q ' is $= \frac{16}{157} \times 100 = 10.19\%$

51. $f(x) = \begin{cases} \frac{a|x|+x^2-2\sin|x|\cos|x|}{x} & ; x \neq 0 \\ b & ; x = 0 \end{cases}$

for continuity

$\lim_{x\to0^-} f(x) = \lim_{x\to0^+} f(x) = f(0)$

$\lim_{x\to0^-} \dfrac{ah + h^2 - 2(\sin h)\cos h}{-h}$

$= \lim_{x\to0^+} \dfrac{ah + h^2 - 2(\sin h)\cos h}{h}$

or $-a + 2 = a - 2 = b$

$2a = 4$

$a = 2, b = 0$

$\therefore a + b = 2$

52. (a) $\frac{\pi}{2} < \theta < \pi$ and $\cot\theta = -\frac{1}{2\sqrt{2}}$

$\Rightarrow \sin\left(\frac{15\theta}{2}\right)(\cos8\theta + \sin8\theta) + \cos\left(\frac{15\theta}{2}\right)(\cos8\theta - \sin8\theta)$

$\Rightarrow \sin\left(\dfrac{15\theta}{2}\right)\cos8\theta - \cos\left(\dfrac{15\theta}{2}\right)\sin8\theta + \sin\dfrac{15\theta}{2}\sin8\theta + \cos\dfrac{15\theta}{2}\cos8\theta$

$\Rightarrow \sin\left(\dfrac{15\theta}{2} - 8\theta\right) + \cos\left(\dfrac{15\theta}{2} - 8\theta\right)$

$\Rightarrow \cos\dfrac{\theta}{2} - \sin\dfrac{\theta}{2} = -\sqrt{1 - \sin\theta}\left(\dfrac{\pi}{4} < \dfrac{\theta}{2} < \dfrac{\pi}{2}\right)$

$\Rightarrow$ given $\cot\theta = -\frac{1}{2\sqrt{2}}, \sin\theta = \frac{2\sqrt{2}}{3}$

$\Rightarrow -\sqrt{1 - \sin\theta} = \sqrt{\dfrac{3 - 2\sqrt{2}}{3}} = -\dfrac{(\sqrt{2} - 1)}{3}$

$= \dfrac{1 - \sqrt{2}}{\sqrt{3}}$

53. (d) $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$, and $\vec{v} = \vec{a} \times \vec{b}$. If $\vec{v} \cdot \vec{c} = 11$

$\vec{v} = (\vec{a} \times \vec{b}) = (-\hat{i} + 7\hat{j} + 5\hat{k})$

$\vec{v} \cdot \vec{c} = 11 = (-\hat{i} + 7\hat{j} + 5\hat{k}) \cdot (\lambda\hat{i} + \hat{j} + \hat{k}) = 11$

$\Rightarrow -\lambda + 7 + 5 = 11$

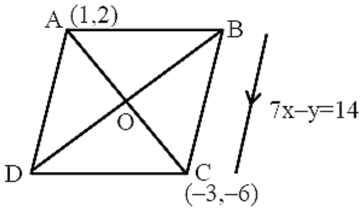
$\Rightarrow \lambda = 1$

Length of projection of $\vec{b}$ on $\vec{c} = \vec{b} \cdot \hat{c}$

$\Rightarrow \left|(2\hat{i} + \hat{j} - \hat{k}) \cdot \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}\right| = \frac{2 + 1 - 1}{\sqrt{3}} = P = \frac{2}{\sqrt{3}}$

$\Rightarrow 9p^2 = 9\left(\frac{4}{3}\right) = 12$

54. (c)



Given the points of B and D are (α, β) and (γ, δ) and mid point of A and C is $(-1, -2)$

So $\frac{\alpha+\gamma}{2} = -1$ and $\frac{\beta+\delta}{2} = -2$

$|\alpha + \gamma + \beta + \delta| = 6$

55. (a) A : $||x - 3| - 3| \le 1$

$\Rightarrow -1 \le |x - 3| - 3 \le 1$

$2 \le |x - 3| \le 4$

$2 \le (x - 3) \le 4$ or $-4 \le (x - 3) \le -2$

$5 \le x \le 7$ or $-1 \le x \le 1$

$A = \{-1,0,1,5,6,7\}$

$B \Rightarrow x = 4, |x - 2| = 1 \Rightarrow x = 3$ or 1 (reject) +

$B = \{3,4\}$

Number of onto functions from A to B $= 2^6 - 2 = 62$

56. (b) If $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & a \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow a = 8$

If $D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 36 & 5 & a \\ b & 2 & 3 \end{vmatrix} = 0 \Rightarrow ab - 5b - 12a + 54 = 0$

If $D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 36 & a \\ 1 & b & 3 \end{vmatrix} = 0 \Rightarrow ab - 6a - 2b - 36 = 0$

If $D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 5 & 36 \\ 1 & 2 & b \end{vmatrix} = 0 \Rightarrow b = 14$

For $a = 8 \& b = 14 \Rightarrow D_1 \& D_2$ are also zero

For $a = 8 \& b = 14 \Rightarrow D = D_1 = D_2 = D_3 = 0$

$\Rightarrow$ infinitely many solutions.

57. (c) $\log_{x+3}[(x + 3)(6x + 10)] = 5 - 2\log_{6x+10}(x + 3)^2$

$1 + \log_{x+3}(6x + 10) = 5 - 4\log_{6x+10}(x + 3)$

Let $\log_{(x+3)}(6x + 10) = A$

$\Rightarrow A + \frac{4}{A} = 4$ or $A = 2$

$\Rightarrow \log_{(x+3)}(6x + 10) = 2$

$\Rightarrow 6x + 10 = (x+3)^2$

$\Rightarrow 6x + 10 = x^2 + 9 + 6x$

$\Rightarrow x^2 = 1, x = \pm 1$

So sum of roots $= 0$

58. **(d)** By family of curve equation of circle will be

$\Rightarrow S_1 + \lambda S_2 = 0$

$\Rightarrow (x^2 + 2y^2 - 6x - 12y + 23)$

$+\lambda(4x^2 + 2y^2 - 20x - 12y + 35) = 0$

$\Rightarrow$ for circle coeff of $x^2 =$ coeff. of y^2

$\Rightarrow \lambda = \frac{1}{2}$

So equation of circle is

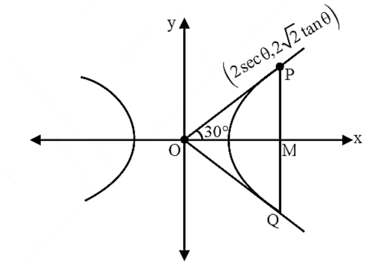
$\Rightarrow x^2 + y^2 - \frac{16}{3}x - 6y + \frac{27}{2} = 0$

Centre $\left(\frac{8}{3}, 3\right)$: Radius $r = \sqrt{\frac{47}{18}} = r$

$\therefore ab + 18r^2 = 8 + 47 = 55$

59. **(b)** $e = \sqrt{1 + \frac{b}{4}} = \sqrt{3} \Rightarrow b = 8$

$\therefore$ Hyperbola $\frac{x^2}{4} - \frac{y^2}{8} = 1$



$\frac{PM}{OM} = \tan 30°$

$\Rightarrow \frac{2\sqrt{2}\tan\theta}{2\sec\theta} = \frac{1}{\sqrt{3}} \Rightarrow \sin\theta = \frac{1}{\sqrt{6}}$

Area $= 2 \times \frac{1}{2} \times OM \times MP$

$= 2\sec\theta \times 2\sqrt{2}\tan\theta$

$= 4\sqrt{2}\frac{\sin\theta}{\cos^2\theta} = 4\sqrt{2} \times \frac{1}{\sqrt{6} \times \left(1 - \frac{1}{6}\right)} = \frac{8\sqrt{3}}{5}$

60. **(d)** $\vec{a} \times \vec{b} - 2(\vec{a} \times \vec{c}) = 0$

$\vec{a} \times (\vec{b} - 2\vec{c}) = 0 \Rightarrow \vec{b} - 2\vec{c} = \lambda\vec{a}$ .... (i)

$|\lambda a|^2 = |\vec{b} - 2\vec{c}|^2 \Rightarrow \lambda^2|\vec{a}|^2 = b^2 + 4c^2 - 4\,\vec{b} \cdot \vec{c}$

$\lambda^2 = 16 + 16 - 4.4.2 \cdot \frac{1}{2}$

$\lambda^2 = 16$

$\lambda = \pm 4$

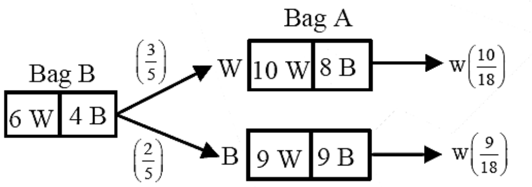
$\because \vec{b} - 2\vec{c} = \pm 4\vec{a}$

Dot with $\vec{c} \Rightarrow \vec{b} \cdot \vec{c} - 2|\vec{c}|^2 = \pm 4(\vec{a} \cdot \vec{c})$

$4.2.\frac{1}{2} - 2.4 = \pm 4(\vec{a}.\vec{c})$

$|\vec{a}.\vec{c}| = 1$

61. (b)

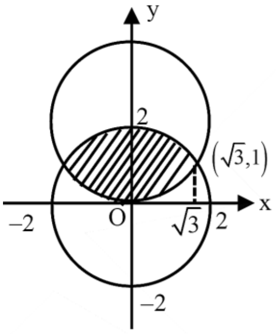


$\therefore$ P(Drawn ball is white) $= \frac{3}{5} \times \frac{10}{18} + \frac{2}{5} \times \frac{9}{18}$

$= \frac{48}{90} = \frac{8}{15} = \frac{p}{q}$

$\therefore p + q = 23$

62. (d)



$$A = 2\int_0^{\sqrt{3}} \left[\sqrt{4-x^2} - \left(2 - \sqrt{4-x^2}\right)\right] dx$$

$$= 2\int_0^{\sqrt{3}} \left(2\sqrt{4-x^2} - 2\right) dx$$

$$= 4\int_0^{\sqrt{3}} \left(\sqrt{4-x^2} - 1\right) dx$$

$$= \left[4\left[\frac{1}{2}\left(x\sqrt{4-x^2} + 4\sin^{-1}\frac{x}{2}\right) - x\right]\right]_0^{\sqrt{3}}$$

$$= 4\left[\frac{1}{2}\left(\sqrt{3} + 4 \times \frac{\pi}{3}\right) - \sqrt{3}\right] = 4\left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right]$$

$$= \frac{8\pi}{3} - 2\sqrt{3} \text{ (Sq. units)}$$

63. (c) $f(\theta) = \frac{1+\cos 2\theta}{2} - 3\sin 2\theta + 3\left(\frac{1-\cos 2\theta}{2}\right) + 2$

$f(\theta) = 4 - 3\sin 2\theta - \cos 2\theta$

$f(\theta) \in [4 - \sqrt{10}, 4 + \sqrt{10}]$

64. (c) Let $4x + 6 = \frac{1}{t} \Rightarrow x = \frac{\frac{1}{t} - 6}{4}$

$$4dx = -\frac{dt}{t^2} \left\{ \frac{x + 1 = \frac{1}{t} - 2}{4} \right.$$

$$\int \frac{3dx}{(4x+6)\sqrt{4(x+1)^2 - 1}}$$

$$\int \frac{3(-dt)}{4t^2 \times \frac{1}{t}\sqrt{\left(\frac{\frac{1}{t} - 2}{4}\right)^2 - 1}}$$

$-\frac{3}{4}\int \frac{dt}{t\sqrt{\frac{(1-2t)^2}{4t^2}-1}}$

$-\frac{3}{4}\int \frac{dt(2t)}{t\sqrt{1-4t}}$

$-\frac{3}{2}\int \frac{dt}{\sqrt{1-4t}} = -\frac{3}{2}\left(\frac{\sqrt{1-4t}}{\frac{1}{2}\times -4}\right) + c$

$= \frac{3}{4}\sqrt{1-4t} + c \ \because t = \frac{1}{4x+6}$

$= \frac{3}{4}\sqrt{1-4\left(\frac{1}{4x+6}\right)} + c$

$= \frac{3}{4}\sqrt{\frac{4x+6-4}{4x+6}} + c$

$I(x) = \frac{3}{4}\sqrt{\frac{4x+2}{4x+6}} + c$

$I(0) = \frac{3}{4}\sqrt{\frac{2}{6}} + c$

$I(0) = \frac{\sqrt{3}}{4} + c \Rightarrow c = 20$

Hence $I(x) = \frac{3}{4}\sqrt{\frac{4x+2}{4x+6}} + 20$

$I\left(\frac{1}{2}\right) = \frac{3}{4}\sqrt{\frac{4}{8}} + 20$

$= \frac{3}{4\sqrt{2}} + 20$

$= \frac{3\sqrt{2}}{8} + 20$

$a + b + c = 3 + 8 + 20 = 31$

65. **(d)**Let oranges are identical then

$x_1 + x_2 + x_3 + x_4 = 16$

and $x_1, x_2, x_3, x_4 \geq 1$

or $x_1' + x_2' + x_3' + x_4' = 12$

so total number of solutions are

$= {}^{12+3}C_3 = {}^{15}C_3 = 455$

66. **(b)**Number of form 3 K = 4

Number of form 3 K + 1 = 3

Number of form 3 K + 2 = 4

$4 \times 4 + 3 \times 3 + 3 \times 3 = 34$ relations

$\Rightarrow xRy \Rightarrow yRx$

$\Rightarrow (x-y) = 3\lambda, (y-z) = 3\mu$

$\Rightarrow (x-z) = 3(\lambda + \mu)$

R is reflexive, symmetric and transitive S_2 is ture

S_1 is false but S_2 is ture

67. (d) $z = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$z^{201} = \cos\left(201\frac{\pi}{6}\right) + i\sin\left(201\frac{\pi}{6}\right) = -i$

$(z^{201} - i)^8 = (-2i)^8 = 256$

68. (d) $\mu = \frac{\sum f_i x_i}{\sum f_i} = \frac{18+10\lambda+56+126}{14+\lambda}$

$= \frac{200+10\lambda}{\lambda+14} = 10 + \left(\frac{60}{\lambda+14}\right)$

$\lambda + 14$ is multiply of 60 $\Rightarrow \lambda = 1$ or 6 or 16 .

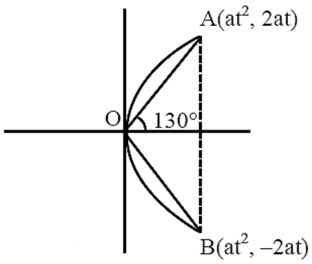
$\sigma^2 = \frac{\sum x_1^2}{\lambda+14} - (\mu)^2$

$\sigma^2 = \frac{6^2(3) + 10^2(\lambda) + 14^2(4) + (18)^2(7)}{\pi + 14} - \mu^2$

for $\lambda = 6, \mu = 10 + 3 = 13$

$\lambda + \mu = 19$

69. (a)



$M_{OA} = \frac{2t-0}{t^2-0} = \frac{2}{f}$

$\frac{2}{t} = \tan 30°$

$t = 2\sqrt{3}$

Req. Circle : $(x-12)^2 + y^2 = (4\sqrt{3})^2$

Least distance $= |CP - R|$

$=| 2 - 4\sqrt{3} = 4(3-\sqrt{3})$

70. (c) $a_n = S_n - S_{n-1}$

$= (\alpha n^2 + \beta n) - (\alpha(n-1)^2 + \beta(n-1))$

(1) $a_{59} \Rightarrow 19\alpha + \beta = 59$

(2) $a_6 = 7a_1 \Rightarrow 11\alpha + \beta = 7(\alpha+\beta)$

$\Rightarrow 2\alpha = 3\beta$

$a = 3, \beta = 2$

$\alpha + \beta = 5$

71. (16) $(x^2-4)y^1 - 2xy = -2x(x^2-4)^2$

$d\left(\frac{y}{x^2-4}\right) = -2x$

$y = (-x^2 + C)(x^2-4)$

for $x = 3y = 15 \Rightarrow C = 12$

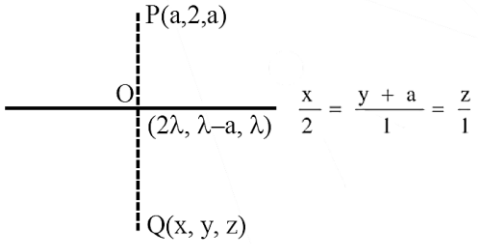
$y = (-x^2+12)(x^2-4)$

$y^1 = 0 \Rightarrow x = 2\sqrt{2}$

$y_{\text{local maxi.}} = \left((2\sqrt{2})^2 - 4\right)\left(-(2\sqrt{2})^2 + 12\right)$

$= 16$

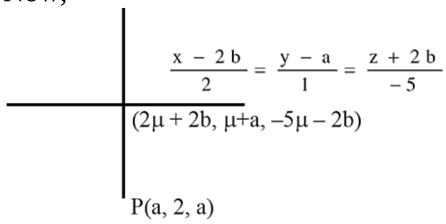
72. (3)



Now,

Q. $(4\lambda - a, 2\lambda - 2a - 2, 2\lambda - a)$

Now,



$\frac{2\lambda - a + a}{2} = -5\mu - 2b$

and $\frac{a+4\lambda-a}{2} = 2\mu + 2b$

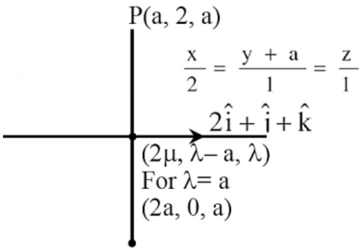
and $\frac{2\lambda-2a-2+2}{2} = \mu + a$

$\Rightarrow \lambda - \mu = b$

$\lambda - \mu = 2a$

$\lambda + 5\mu = -2b$

$\Rightarrow b = 2a$ and $\lambda = a, \mu = -a$



$(a\hat{i} - 2\hat{j} + 0\hat{k}) \cdot (2\hat{i} + \hat{j} + \hat{k}) = 0$

$2a - 2 = 0$

$a = 1$

$b = 2a = 2 \times 1 = 2$

$a + b = 1 + 2 = 3$

73. (16) $\int_0^x t^2 \sin(x - t)dt = x^2$

Use by parts

$-x^2\cos x + x^2 + 2x^2\cos x - 2x\sin x$

$-x^2\cos x + 2x\sin x + 2\cos x - 2 = x$

$\cos x = 1$

$x = 0, 2\pi, 4\pi, \dots\dots, 30\pi$

Total Elements $= 16$

74. (3) $A^T = -A$

$B = (I + A)(I - A)^{-1}$

$B^T = ((I - A)^{-1})^T (I + A)^T$

$B^T = \left(I - A^T\right)^{-1}\left(I + A^T\right)$

$B^T = (I + A)^{-1}(I + A)$

$B^T B = (I + A)^{-1}(I - A)(I + A)(I - A)^{-1}$

$= (I + A)^{-1}(I + A)(I - A)(I - A)^{-1}$

$= I$

$tr(B^T B) = 3$

75. **(260)**For $n(s) = {}^{10}C_4 = 210$

(5,4,1,1,1) (5,2,2,1,1)

For $n(p) = \frac{5!}{3!} + \frac{5!}{2!2!} = 50$

$n(s) + n(p) = 210 + 50 = 260$