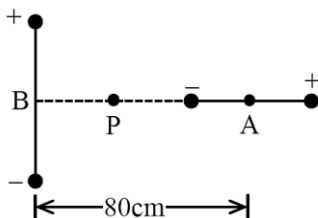


## PHYSICS

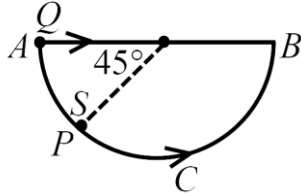
### SECTION-A

1. The internal energy of a monoatomic gas is  $3 nRT$ . One mole of helium is kept in a cylinder having internal cross section area of  $17 \text{ cm}^2$  and fitted with a light movable frictionless piston. The gas is heated slowly by supplying  $126 \text{ J}$  heat. If the temperature rises by  $4^\circ\text{C}$ , then the piston will move \_\_\_\_ cm. (atmospheric pressure =  $10^5 \text{ Pa}$ )
  - (a) 14.5
  - (b) 1.55
  - (c) 15.5
  - (d) 1.45
2. A body of mass  $14 \text{ kg}$  initially at rest explodes and breaks into three fragments of masses in the ratio  $2: 2: 3$ . The two pieces of equal masses fly off perpendicular to each other with a speed of  $18 \text{ m/s}$  each. The velocity of the heavier fragment is \_\_\_\_ m/s.
  - (a)  $10\sqrt{2}$
  - (b)  $12\sqrt{2}$
  - (c) 12
  - (d)  $24\sqrt{2}$
3. A block is sliding down on an inclined plane of slope  $\theta$  and at an instant  $t = 0$  this block is given an upward momentum so that it starts moving up on the inclined surface with velocity  $u$ . The distance ( $S$ ) travelled by the block before its velocity become zero, is \_\_\_\_ . ( $g$  = gravitational acceleration)
  - (a)  $\frac{u^2}{4g\sin\theta}$
  - (b)  $\frac{2u^2}{g\cos\theta}$
  - (c)  $\frac{u^2}{\sqrt{2}g\cos\theta}$
  - (d)  $\frac{u^2}{2g\cos\theta}$
4. The current passing through a conducting loop in the form of equilateral triangle of side  $4\sqrt{3} \text{ cm}$  is  $2\text{A}$ . The magnetic field at its centroid is  $\alpha \times 10^{-5} \text{ T}$ . The value of  $\alpha$  is \_\_\_\_ .  
(Given :  $\mu_0 = 4\pi \times 10^{-7} \text{ SI units}$ )
  - (a)  $2\sqrt{3}$
  - (b)  $\sqrt{3}$
  - (c)  $3\sqrt{3}$
  - (d)  $\frac{\sqrt{3}}{2}$
5. A paratrooper jumps from an aeroplane and opens a parachute after  $2 \text{ s}$  of free fall and starts deaccelerating with  $3 \text{ m/s}^2$ . At  $10 \text{ m}$  height from ground, while descending with the help of parachute, the speed of paratrooper is  $5 \text{ m/s}$ . The initial height of the aeroplane is \_\_\_\_ m.  
( $g = 10 \text{ m/s}^2$ )
  - (a) 62.5
  - (b) 92.5
  - (c) 20
  - (d) 82.5
6. Two short dipoles (A, B), A having charges  $\pm 2\mu\text{C}$  and length  $1 \text{ cm}$  and B having charges  $\pm 4\mu\text{C}$  and length  $1 \text{ cm}$  are placed with their centres  $80 \text{ cm}$  apart as shown in the figure. The electric field at a point P, equidistant from the centres of both dipoles is \_\_\_\_ N/C.

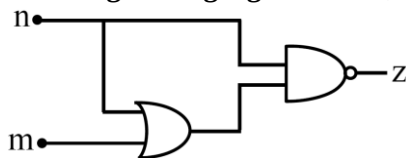


- (a)  $\frac{9}{16}\sqrt{2} \times 10^5$
  - (b)  $4.5\sqrt{2} \times 10^4$
  - (c)  $9\sqrt{2} \times 10^4$
  - (d)  $\frac{9}{16}\sqrt{2} \times 10^4$
7. Two charges  $7\mu\text{C}$  and  $-2\mu\text{C}$  are placed at  $(-9, 0, 0)\text{cm}$  and  $(9, 0, 0)\text{cm}$  respectively in an external field  $\mathbf{E} = \frac{A}{r^2} \hat{r}$ , where  $A = 9 \times 10^5 \text{ N/C} \cdot \text{m}^2$ . Considering the potential at infinity is 0, the electrostatic energy of the configuration is \_\_\_\_ J.
  - (a) 1.4
  - (b) -90.7
  - (c) 49.3
  - (d) 24.3

8. A bead P sliding on a frictionless semi-circular string (ACB) and it is at point S at  $t = 0$  and at this instant the horizontal component of its velocity is  $v$ . Another bead Q of the same mass as P is ejected from point A at  $t = 0$  along the horizontal string AB, with the speed  $v$ , friction between the beads and the respective strings may be neglected in both cases. Let  $t_p$  and  $t_Q$  be the respective times taken by beads P and Q to reach the point B, then the relation between  $t_p$  and  $t_Q$  is



- (a)  $t_p > t_Q$  (b)  $t_p < t_Q$   
(c)  $t_p > 1.25t_Q$  (d)  $t_p = t_Q$
9. A parallel plate capacitor with plate separation 5 mm is charged by a battery. On introducing a mica sheet of 2 mm and maintaining the connections of the plates with the terminals of the battery, it is found that it draws 25% more charge from the battery. The dielectric constant of mica is \_\_\_\_.
- (a) 2.5 (b) 2.0  
(c) 1.5 (d) 1.0
10. To compare EMF of two cells using potentiometer the balancing lengths obtained are 200 cm and 150 cm. The least count of scale is 1 cm. The percentage error in the ratio of EMFs is \_\_\_\_
- (a) 1.45 (b) 1.16  
(c) 1.75 (d) 1.55
11. An air bubble of volume  $2.9 \text{ cm}^3$  rises from the bottom of a swimming pool of 5 m deep. At the bottom of the pool water temperature is  $17^\circ\text{C}$ . The volume of the bubble when it reaches the surface, where the water temperature is  $27^\circ\text{C}$ , is \_\_\_\_  $\text{cm}^3$ . ( $g = 10 \text{ m/s}^2$ , density of water =  $10^3 \text{ kg/m}^3$ , and 1 atm pressure is  $10^5 \text{ Pa}$ )
- (a) 4.2 (b) 2.0  
(c) 3.0 (d) 4.5
12. Which of the following pair of nuclei are isobars of the element?
- (a)  ${}^2_1\text{H}$  and  ${}^3_1\text{H}$   
(b)  ${}^{236}_{92}\text{U}$  and  ${}^{238}_{92}\text{U}$   
(c)  ${}^{198}_{80}\text{Hg}$  and  ${}^{197}_{79}\text{Au}$   
(d)  ${}^3_1\text{H}$  and  ${}^3_2\text{H}$
13. For the given logic gate circuit, which of the following is the correct truth table?



(a)

| n | m | z |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |
| 1 | 0 | 0 |

(b)

| n | m | z |
|---|---|---|
|---|---|---|

|   |   |   |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |
| 1 | 0 | 1 |

(c)

|   |   |   |
|---|---|---|
| n | m | z |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 1 | 0 | 0 |

(d)

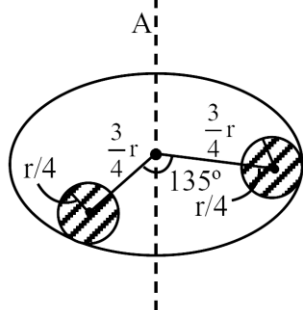
|   |   |   |
|---|---|---|
| n | m | z |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |
| 1 | 0 | 0 |

14. One mole of an ideal diatomic gas expands from volume  $V$  to  $2V$  isothermally at a temperature  $27^\circ\text{C}$  and does  $W$  joule of work. If the gas undergoes same magnitude of expansion adiabatically from  $27^\circ\text{C}$  doing the same amount of work  $W$ , then its final temperature will be (close to) \_\_\_\_  $^\circ\text{C}$ .
- (a) -189                      (b) -56  
(c) -30                      (d) -117
15. A prism of angle  $75^\circ$  and refractive index  $\sqrt{3}$  is coated with thin film of refractive index 1.5 only at the back exit surface. To have total internal reflection at the back exit surface the incident angle must be \_\_\_\_ .  
( $\sin 15^\circ = 0.25$  and  $\sin 25^\circ = 0.43$ )
- (a) between  $15^\circ$  and  $20^\circ$       (b)  $15^\circ$   
(c)  $< 25^\circ$                       (d)  $< 15^\circ$
16. A circular loop of radius 7 cm is placed in uniform magnetic field of 0.2 T directed perpendicular to plane of loop. The loop is converted into a square loop in 0.5 s . The EMF induced in the loop is \_\_\_\_ mV.
- (a) 6.6                      (b) 13.2  
(c) 8.25                      (d) 1.32
17. Suppose a long solenoid of 100 cm length, radius 2 cm having 500 turns per unit length, carries a current  $I = 10\sin(\omega t)\text{A}$ , where  $\omega = 1000\text{rad./s}$ . A circular conducting loop (B) of radius 1 cm coaxially slid through the solenoid at a speed  $v = 1\text{ cm/s}$ . The r.m.s. current through the loop when the coil B is inserted 10 cm inside the solenoid is  $\alpha/\sqrt{2}\mu\text{A}$ . The value of  $\alpha$  is \_\_\_\_ . [Resistance of the loop =  $10\Omega$  ]
- (a) 197                      (b) 80  
(c) 280                      (d) 100
18. The ratio of speeds of electromagnetic waves in vacuum and a medium, having dielectric constant  $k = 3$  and permeability of  $\mu = 2\mu_0$ , is (  $\mu_0$  = permeability of vacuum)
- (a) 36: 1                      (b) 3: 2  
(c) 6: 1                      (d)  $\sqrt{6}$ : 1

19. When an unpolarized light falls at a particular angle on a glass plate (placed in air), it is observed that the reflected beam is linearly polarized. The angle of refracted beam with respect to the normal is \_\_\_\_ .  
 $(\tan^{-1}(1.52) = 57.7^\circ, \text{refractive indices of air and glass are } 1.00 \text{ and } 1.52, \text{ respectively})$   
 (a)  $39.6^\circ$  (b)  $32.3^\circ$   
 (c)  $42.6^\circ$  (d)  $36.3^\circ$
20. A small metallic sphere of diameter 2 mm and density  $10.5 \text{ g/cm}^3$  is dropped in glycerine having viscosity 10 Poise and density  $1.5 \text{ g/cm}^3$  respectively. The terminal velocity attained by the sphere is \_\_\_\_ cm/s.  
 $(\pi = \frac{22}{7} \text{ and } g = 10 \text{ m/s}^2)$   
 (a) 2.0 (b) 1.0  
 (c) 3.0 (d) 1.5

### SECTION - B

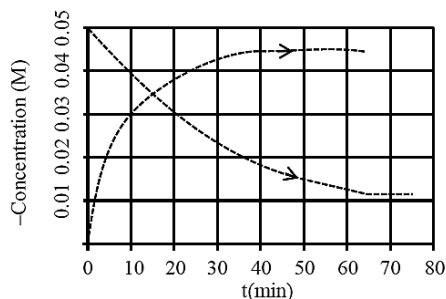
21. The average energy released per fission for the nucleus of  ${}^{235}_{92}\text{U}$  is 190 MeV. When all the atoms of 47 g pure  ${}^{235}_{92}\text{U}$  undergo fission process, the energy released is  $\alpha \times 10^{23} \text{ MeV}$ . The value of  $\alpha$  is \_\_\_\_ . (Avogadro Number =  $6 \times 10^{23}$  per mole)
22. The size of the images of an object, formed by a thin lens are equal when the object is placed at two different positions 8 cm and 24 cm from the lens. The focal length of the lens is \_\_\_\_ cm .
23. A ball of radius  $r$  and density  $\rho$  dropped through a viscous liquid of density  $\sigma$  and viscosity  $\eta$  attains its terminal velocity at time  $t$ , given by  $t = A\rho^a r^b \eta^c \sigma^d$ , where  $A$  is a constant and  $a, b, c$  and  $d$  are integers. The value of  $\frac{b+c}{a+d}$  is \_\_\_\_ .
24. Suppose there is a uniform circular disc of mass  $M$  kg and radius  $r$  m shown in figure. The shaded regions are cut out from the disc. The moment of inertia of the remainder about the axis  $A$  of the disc is given by  $\frac{x}{256} Mr^2$ . The value of  $x$  is \_\_\_\_ .



25. The velocity of sound in air is doubled when the temperature is raised from  $0^\circ\text{C}$  to  $\alpha^\circ\text{C}$ . The value of  $\alpha$  is \_\_\_\_ .

### CHEMISTRY SECTION-A

26.

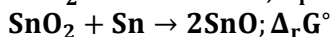
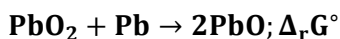


Given above is the concentration vs time plot for a dissociation reaction :  $A \rightarrow nB$ .

Based on the data of the initial phase of the reaction (initial 10 min ), the value of  $n$  is \_\_\_\_ .

- (a) 4 (b) 3  
 (c) 2 (d) 5

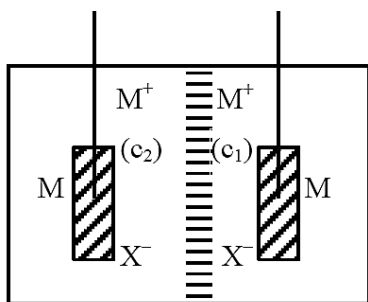
27. It is noticed that  $\text{Pb}^{2+}$  is more stable than  $\text{Pb}^{4+}$  but  $\text{Sn}^{2+}$  is less stable than  $\text{Sn}^{4+}$ . Observe the following reactions



Identify the correct set from the following.

- (a)  $\Delta_r G^\circ(1) > 0; \Delta_r G^\circ(2) < 0$   
 (b)  $\Delta_r G^\circ(1) < 0; \Delta_r G^\circ(2) < 0$   
 (c)  $\Delta_r G^\circ(1) < 0; \Delta_r G^\circ(2) > 0$   
 (d)  $\Delta_r G^\circ(1) > 0; \Delta_r G^\circ(2) > 0$

28.



Semi permeable membrane

Consider the above electrochemical cell where a metal electrode (M) is undergoing redox reaction by forming  $\text{M}^+$  ( $\text{M} \rightarrow \text{M}^+ + \text{e}^-$ ). The cation  $\text{M}^+$  is present in two different concentrations  $c_1$  and  $c_2$  as shown above. Which of the following statement is correct for generating a positive cell potential?

- (a) If  $c_1$  is present at anode, then  $c_1 = c_2$   
 (b) If  $c_1$  is present at cathode, then  $c_1 < c_2$   
 (c) If  $c_1$  is present at cathode, then  $c_1 > c_2$   
 (d) If  $c_1$  is present at anode, then  $c_1 > c_2$
29. Both human DNA and RNA are chiral molecules. The chirality in DNA and RNA arises due to the presence of

- (a) Base unit (b) Chiral phosphate unit  
 (c) D-sugar component (d) L-sugar component

30. Identify the CORRECT set of details from the following :

- (A)  $[\text{Co}(\text{NH}_3)_6]^{3+}$  : Inner orbital complex ;  $d^2 sp^3$  hybridized  
 (B)  $[\text{MnCl}_6]^{3-}$  Outer orbital complex;  $sp^3$  hybridized  
 (C)  $[\text{CoF}_6]^{3-}$  : Outer orbital complex ;  $d^2 sp^3$  hybridized  
 (D)  $[\text{FeF}_6]^{3-}$  : Outer orbital complex:  $sp^3 d^2$  hybridized  
 (E)  $[\text{Ni}(\text{CN})_4]^{2-}$  : Inner orbital complex ;  $sp^3$  hybridized

Choose the correct answer from the options given below:

- (a) C & D only (b) A, B & D only  
 (c) A, C & E only (d) A, B, C, D & E

31. Elements X and Y belong to Group 15. The difference between the electronegativity values of 'X' and phosphorus is higher than that of the difference between phosphorus and 'Y'. 'X' & 'Y' are respectively

- (a) N & As (b) As & Bi  
 (c) Bi & N (d) As & Sb

32. Given below are two statements:

Statement I:  $(\text{CH}_3)_3\text{C}^\oplus$  is more stable than  $\text{CH}_3^\oplus$  as nine hyperconjugation interactions are possible in  $(\text{CH}_3)_3\text{C}^\oplus$ .

Statement II:  $\text{CH}_3^\oplus$  is less stable than  $(\text{CH}_3)_3\text{C}^\oplus$  as only three hyperconjugation interactions are possible in  $\text{CH}_3^\oplus$ .

In the light of the above statements, choose the correct answer from the options given below.

- (a) Statement I is true but Statement II is false  
 (b) Both Statement I and Statement II are true

- (c) Both Statement I and Statement II are false  
(d) Statement I is false but Statement II is true

**33. Iodoform test can differentiate between**

- (A) Methanol and Ethanol  
(B)  $\text{CH}_3\text{COOH}$  and  $\text{CH}_3\text{CH}_2\text{COOH}$   
(C) Cyclohexene and cyclohexanone  
(D) Diethyl ether and Pentan-3-one  
(E) Anisole and acetone

**Choose the correct answer from the options given below:**

- (a) A & E only (b) A & D only  
(c) A, B & E only (d) B, C & E only

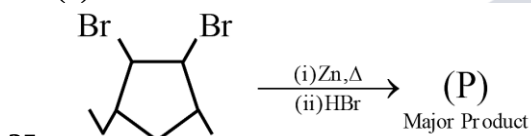
**34. Given below are two statements :**

**Statement I:** The second ionisation enthalpy of Na is larger than the corresponding ionisation enthalpy of Mg.

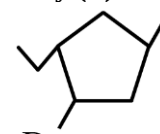
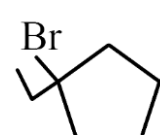
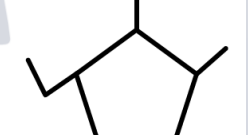
**Statement II :** The ionic radius of  $\text{O}^{2-}$  is larger than that of  $\text{F}^-$ .

**In the light of the above statements, choose the correct answer from the options given below.**

- (a) Both statement I and statement II are true  
(b) Both statement I and statement II are false  
(c) Statement I is false but statement II is true  
(d) Statement I is true but statement II is false



**Identify (P)**

- (a)  (b)   
(c)  (d) 

**36. Observe the following reactions at T(K)**

(I)  $\text{A} \rightarrow \text{products}$ .

(II)  $5\text{Br}^-(\text{aq}) + \text{BrO}_3^-(\text{aq}) + 6\text{H}^+(\text{aq}) \rightarrow 3\text{Br}_2(\text{aq}) + 3\text{H}_2\text{O}(\text{l})$

Both the reactions are started at 10.00 am . The rates of these reactions at 10.10 am are same. The value of  $-\frac{\Delta[\text{Br}^-]}{\Delta t}$  at 10.10 am is  $2 \times 10^{-4} \text{ mol L}^{-1} \text{Min}^{-1}$ . The concentration of A at 10.10 am is  $10^{-2} \text{ mol L}^{-1}$ . What is the first order rate constant (in  $\text{min}^{-1}$ ) of reaction I?

- (a)  $2 \times 10^{-3}$  (b)  $10^{-3}$   
(c)  $10^{-2}$  (d)  $4 \times 10^{-3}$

**37. Which of the following statements are TRUE about Haloform reaction?**

- (A) Sodium hypochlorite reacts with KI to give KOI.  
(B) KOI is a reducing agent.  
(C)  $\alpha, \beta$ -unsaturated methylketone

$\text{CH}_3\text{--CH=CH--}\overset{\text{O}}{\parallel}\text{C--CH}_3$  will give iodoform reaction.

- (D) Isopropyl alcohol will not give iodoform test.  
(E) Methanoic acid will give positive iodoform test.

Choose the correct answer from the options given below:

- (a) A, C & E only (b) A, B & C only  
(c) A & C only (d) B, D & E only

38. Which statements are NOT TRUE about  $\text{XeO}_2\text{F}_2$  ?

- (A) It has a see-saw shape.  
(B) Xe has 5 electron pairs in its valence shell in  $\text{XeO}_2\text{F}_2$ .  
(C) The  $\text{O}-\text{Xe}-\text{O}$  bond angle is close to  $180^\circ$ .  
(D) The  $\text{F}-\text{Xe}-\text{F}$  bond angle is close to  $180^\circ$   
(E) Xe has 16 valence electrons in  $\text{XeO}_2\text{F}_2$ .

Choose the correct answer from the options given below.

- (a) B, C and E only (b) B and D only  
(c) A and D only (d) B, D and E only

39. Identify the INCORRECT statements from the following:

- (A) Notation  ${}^{24}_{12}\text{Mg}$  represents 24 protons and 12 neutrons.  
(B) Wavelength of a radiation of frequency  $4.5 \times 10^{15} \text{ s}^{-1}$  is  $6.7 \times 10^{-8} \text{ m}$ .  
(C) One radiation has wavelength  $= \lambda_1$  (900 nm) and energy  $= E_1$ . Other radiation has wavelength  $= \lambda_2$  (300 nm) and energy  $= E_2$ .  $E_1:E_2 = 3:1$ .  
(D) Number of photons of light of wavelength 2000 pm that provides 1 J of energy is  $1.006 \times 10^{16}$ .

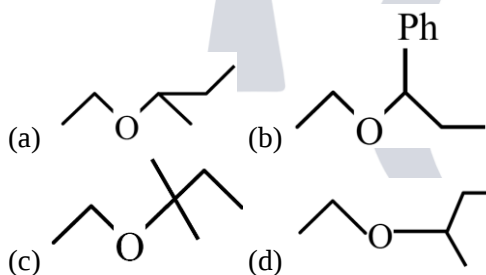
Choose the correct answer from the options given below:

- (a) A and D only (b) A and C only  
(c) A and B only (d) B and C only

40. In Carius method 0.2425 g of an organic compounds gave 0.5253 g silver chloride. The percentage of chlorine in the organic compound is

- (a) 53.58% (b) 87.65%  
(c) 37.57% (d) 34.79%

41. A mixed ether (P), when heated with excess of hot concentrated hydrogen iodide produces two different alkyl iodides which when treated with aq. NaOH give compounds (Q) and (R). Both (Q) and (R) give yellow precipitate with NaOI. Identify the mixed ether (P):



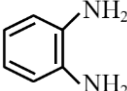
42. The oxidation state of chromium in the final product formed in the reaction between KI and acidified  $\text{K}_2\text{Cr}_2\text{O}_7$  solution is:

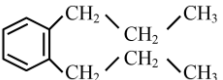
- (a) +4 (b) +3  
(c) +2 (d) +6

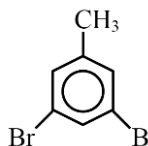
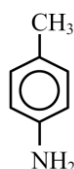
43. The work functions of two metals ( $M_A$  and  $M_B$ ) are in the 1: 2 ratio. When these metals are exposed to photons of energy 6 eV, the kinetic energy of liberated electrons of  $M_A:M_B$  is in the ratio of 2.642 : 1. The work functions (in eV) of  $M_A$  and  $M_B$  are respectively.

- (a) 3.1, 6.2 (b) 2.3, 4.6  
(c) 1.4, 2.8 (d) 1.5, 3.0

44. Given below are two statements:

**Statement I:**  can be synthesized from using simpler reagents in the order (i) Acidic  $\text{KMnO}_4$ , (ii) Ammonia, (iii) Bromine and alkali





**Statement II :** can be converted into using reagents in the order (i) Bromine-  $\text{H}_2\text{O}$  (ii)  $\text{NaNO}_2/\text{HCl}(0 - 5^\circ\text{C})$  (iii) aq.  $\text{H}_3\text{PO}_2$ .

**In the light of the above statements, choose the correct answer from the options given below**

- (a) Both Statement I and Statement II are false  
(b) Statement I is true but Statement II is false  
(c) Both Statement I and Statement II are true  
(d) Statement I is false but Statement II is true

45. A student has been given a compound "x" of molecular formula  $\text{C}_6\text{H}_7\text{N}$ . 'x' is sparingly soluble in water. However, on addition of dilute mineral acid, 'x' becomes soluble in water. 'x' when treated with  $\text{CHCl}_3$  and  $\text{KOH}$  (alc.) 'y' is produced. 'y' has a specific unpleasant smell. On treatment with benzenesulphonyl chloride, 'x' gives a compound 'z' which is soluble in alkali. The number of different "H" atoms present in 'z' is:-

- (a) 5 (b) 8  
(c) 4 (d) 7

## SECTION-B

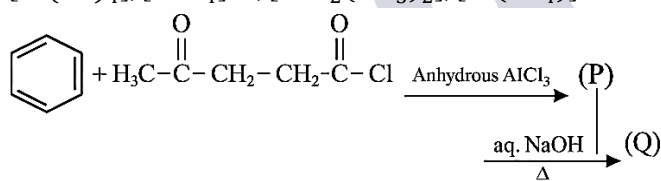
46.  $\text{X}_2(\text{g}) + \text{Y}_2(\text{g}) \rightleftharpoons 2\text{Z}(\text{g})$

$\text{X}_2(\text{g})$  and  $\text{Y}_2(\text{g})$  are added to a 1 L flask and it is found that the system attains the above equilibrium at T(K) with the number of moles of  $\text{X}_2(\text{g})$ ,  $\text{Y}_2(\text{g})$  and  $\text{Z}(\text{g})$  being 3, 3 and 9 mol respectively (equilibrium moles). Under this conditions of equilibrium, 10 mol of  $\text{Z}(\text{g})$  is added to the flask and the temperature is maintained at T(K). Then the number of moles of  $\text{Z}(\text{g})$  in the flask when the new equilibrium is established is \_\_\_\_\_. (Nearest integer).

47. Two liquids A and B form an ideal solution. At 320 K, the vapour pressure of the solution, containing 3 mol of A and 1 mol of B is 500 mm Hg. At the same temperature, if 1 mol of A is further added to this solution, vapour pressure of the solution increases by 20 mm Hg. Vapour pressure (in mm Hg) of B in the pure state is \_\_\_\_\_. (Nearest integer)

48. 200 cc of  $x \times 10^{-3}\text{M}$  potassium dichromate is required to oxidise 750 cc of 0.6 M Mohr's salt solution in acidic medium. Here x =

49. Total number of unpaired electrons present in the central metal atoms/ions of  $[\text{Ni}(\text{CO})_4]$ ,  $[\text{NiCl}_4]^{2-}$ ,  $[\text{PtCl}_2(\text{NH}_3)_2]$ ,  $[\text{Ni}(\text{CN})_4]^{2-}$  and  $[\text{Pt}(\text{CN})_4]^{2-}$  is \_\_\_\_\_.



50. In compound (Q), the percentage of oxygen is \_\_\_\_\_. (Nearest integer)

## MATHEMATICS

### SECTION-A

51. If  $f(x) = \begin{cases} \frac{a|x|+x^2-2(\sin|x|)(\cos|x|)}{x} & , x \neq 0 \\ b & , x = 0 \end{cases}$  is continuous at  $x = 0$ , then  $a + b$  is equal to :

- (a) 1 (b) 2  
(c) 0 (d) 4

52. Let  $\frac{\pi}{2} < \theta < \pi$  and  $\cot \theta = -\frac{1}{2\sqrt{2}}$ . Then the value of  $\sin\left(\frac{15\theta}{2}\right)(\cos 8\theta + \sin 8\theta) + \cos\left(\frac{15\theta}{2}\right)(\cos 8\theta - \sin 8\theta)$  is equal to :

- (a)  $\frac{1-\sqrt{2}}{\sqrt{3}}$  (b)  $-\frac{\sqrt{2}}{\sqrt{3}}$   
(c)  $\frac{\sqrt{2}-1}{\sqrt{3}}$  (d)  $\frac{\sqrt{2}}{\sqrt{3}}$

53. Let  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ ,  $\vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$  and  $\vec{v} = \vec{a} \times \vec{b}$ . If  $\vec{v} \cdot \vec{c} = 11$  and the length of the projection of  $\vec{b}$  on  $\vec{c}$  is  $p$ , then  $9p^2$  is equal to :
- (a) 9 (b) 6  
(c) 4 (d) 12
54. Let  $A(1, 2)$  and  $C(-3, -6)$  be two diagonally opposite vertices of a rhombus, whose sides  $AD$  and  $BC$  are parallel to the line  $7x - y = 14$ . If  $B(\alpha, \beta)$  and  $D(\gamma, \delta)$  are the other two vertices, then  $|\alpha + \beta + \gamma + \delta|$  is equal to :
- (a) 9 (b) 3  
(c) 6 (d) 1
55. Consider two sets  $A = \{x \in \mathbb{Z} : (|x - 3| - 3) \leq 1\}$  and  $B = \left\{x \in \mathbb{R} - \{1, 2\} : \frac{(x-2)(x-4)}{x-1} \log_e(|x - 2|) = 0\right\}$ . Then the number of onto functions  $f: A \rightarrow B$  is equal to :
- (a) 62 (b) 79  
(c) 32 (d) 81
56. The system of linear equations  
 $x + y + z = 6$   
 $2x + 5y + az = 36$   
 $x + 2y + 3z = b$   
 has
- (a) unique solution for  $a = 8$  and  $b = 16$   
 (b) infinitely many solutions for  $a = 8$  and  $b = 14$   
 (c) infinitely many solutions for  $a = 8$  and  $b = 16$   
 (d) unique solution for  $a = 8$  and  $b = 14$
57. The sum of all the real solutions of the equation  $\log_{(x+3)}(6x^2 + 28x + 30) = 5 - 2\log_{(6x+10)}(x^2 + 6x + 9)$  is equal to :
- (a) 2 (b) 1  
(c) 0 (d) 4
58. If the points of intersection of the ellipses  $x^2 + 2y^2 - 6x - 12y + 23 = 0$  and  $4x + 2y^2 - 20x - 12y + 35 = 0$  lie on a circle of radius  $r$  and centre  $(a, b)$ , then the value of  $ab + 18r^2$  is
- (a) 53 (b) 51  
(c) 52 (d) 55
59. Let  $PQ$  be a chord of the hyperbola  $\frac{x^2}{4} - \frac{y^2}{b^2} = 1$ , perpendicular to the  $x$ -axis such that  $OPQ$  is an equilateral triangle,  $O$  being the centre of the hyperbola. If the eccentricity of the hyperbola is  $\sqrt{3}$ , then the area of the triangle  $OPQ$  is :
- (a)  $2\sqrt{3}$  (b)  $\frac{8\sqrt{3}}{5}$   
(c)  $\frac{11}{5}$  (d)  $\frac{9}{5}$
60. Let  $\vec{a}, \vec{b}, \vec{c}$  be three vectors such that  $\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{c})$ . If  $|\vec{a}| = 1$ ,  $|\vec{b}| = 4$ ,  $|\vec{c}| = 2$ , and the angle between  $\vec{b}$  and  $\vec{c}$  is  $60^\circ$ , then  $|\vec{a} \cdot \vec{c}|$  is :
- (a) 2 (b) 4  
(c) 0 (d) 1
61. Bag A contains 9 white and 8 black balls, while bag B contains 6 white and 4 black balls. One ball is randomly picked up from the bag B and mixed up with the balls in the bag A. Then a ball is randomly drawn from the bag A. If the probability, that the ball drawn is white, is  $\frac{p}{q}$ ,  $\gcd(p, q) = 1$ , then  $p + q$  is equal to
- (a) 22 (b) 23  
(c) 24 (d) 21

62. The area of the region enclosed between the circles  $x^2 + y^2 = 4$  and  $x^2 + (y - 2)^2 = 4$  is :
- (a)  $\frac{2}{3}(2\pi - 3\sqrt{3})$  (b)  $\frac{4}{3}(2\pi - 3\sqrt{3})$   
(c)  $\frac{4}{3}(2\pi - \sqrt{3})$  (d)  $\frac{2}{3}(4\pi - 3\sqrt{3})$
63. The least value of  $(\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2)$  is
- (a) -1 (b)  $4 + \sqrt{10}$   
(c)  $4 - \sqrt{10}$  (d) 1
64. Let  $I(x) = \int \frac{3dx}{(4x+6)(\sqrt{4x^2+8x+3})}$  and  $I(0) = \frac{\sqrt{3}}{4} + 20$ . If  $I\left(\frac{1}{2}\right) = \frac{a\sqrt{2}}{b} + c$ , where  $a, b, c \in \mathbb{N}$ ,  $\gcd(a, b) = 1$ , then  $a + b + c$  is equal to :
- (a) 29 (b) 28  
(c) 31 (d) 30
65. The number of ways, in which 16 oranges can be distributed to four children such that each child gets at least one orange, is
- (a) 429 (b) 384  
(c) 403 (d) 455
66. Let  $A = \{0, 1, 2, \dots, 9\}$ . Let  $R$  be a relation on  $A$  defined by  $(x, y) \in R$  if and only if  $|x - y|$  is a multiple of 3. Given below are two statements:  
Statement I:  $n(R) = 36$   
Statement II:  $R$  is an equivalence relation.  
In the light of the above statements, choose the correct answer from the options given below
- (a) Both Statement I and Statement II are correct  
(b) Statement I is incorrect but Statement II is correct  
(c) Statement I is correct but Statement II is incorrect  
(d) Both Statement I and Statement II are incorrect
67. If  $z = \frac{\sqrt{3}}{2} + \frac{i}{2}$ ,  $i = \sqrt{-1}$ , then  $(z^{201} - i)^8$  is equal to
- (a) -1 (b) 0  
(c) 1 (d) 256
68. If the mean and the variance of the data
- |           |       |           |         |         |
|-----------|-------|-----------|---------|---------|
| Class     | 4 – 8 | 8 – 12    | 12 – 16 | 16 – 20 |
| Frequency | 3     | $\lambda$ | 4       | 7       |
- are  $\mu$  and 19 respectively, then the value of  $\lambda + \mu$  is
- (a) 18 (b) 21  
(c) 20 (d) 19
69. An equilateral triangle  $OAB$  is inscribed in the parabola  $y^2 = 4x$  with the vertex  $O$  at the vertex of the parabola. Then the minimum distance of the circle having  $AB$  as a diameter from the origin is
- (a)  $4(3 - \sqrt{3})$  (b)  $2(8 - 3\sqrt{3})$   
(c)  $4(6 + \sqrt{3})$  (d)  $2(3 + \sqrt{3})$
70. Let  $\sum_{k=1}^n a_k = \alpha n^2 + \beta n$ . If  $a_{10} = 59$  and  $a_6 = 7a_1$  then  $\alpha + \beta$  is equal to
- (a) 12 (b) 3  
(c) 5 (d) 7

### SECTION-B

71. If the solution curve  $y = f(x)$  of the differential equation  $(x^2 - 4)y' - 2xy + 2x(4 - x^2)^2 = 0$ ,  $x > 2$ , passes through the point  $(3, 15)$ , then the local maximum value of  $f$  is \_\_\_\_
72. If the image of the point  $P(a, 2, a)$  in the line  $\frac{x}{2} = \frac{y+a}{1} = \frac{z}{1}$  is  $Q$  and the image of  $Q$  in the line  $\frac{x-2b}{2} = \frac{y-a}{1} = \frac{z+2b}{-5}$  is  $P$ , then  $a + b$  is equal to \_\_\_\_
73. The number of elements in the set  $S = \{x: x \in [0, 100] \text{ and } \int_0^x t^2 \sin(x-t) dt = x^2\}$  is.....

74. Let  $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$  and B be a matrix such that  $B(I - A) = I + A$ . Then the sum of the diagonal elements of  $B^T B$  is equal to

75. Let S denote the set of 4-digit numbers abcd such that  $a > b > c > d$  and P denote the set of 5-digit numbers having product of its digits equal to 20. Then  $n(S) + n(P)$  is equal to..... .

