1. **(a)** Magnetic field at centre of coil $B_1 = \frac{\mu_0 I}{2r}$

on the axis at $x = r \Rightarrow B_2 = \frac{\mu_0 I r^2}{2(r^2+x^2)^{3/2}}$

$$B_2 = \frac{\mu_0 I r^2}{2(r^2+r^2)^{3/2}}$$

$$B_2 = \frac{\mu_0 I}{2(2\sqrt{2}r)}$$

$$\frac{B_1}{B_2} = 2\sqrt{2}$$

2. **(a)** Weight on earth surface $W = mg = 18\ \mathrm{N}$

Above earth surface $\Rightarrow W_2 = m\frac{GM}{(R+h)^2}$

$h = 3200\ \mathrm{km} = R/2$

$$W_2 = m\frac{GM}{\left(\frac{3R}{2}\right)^2} \Rightarrow W_2 = \frac{4}{9}mg$$

$$W_2 = \frac{4}{9} \times 18 \Rightarrow W_2 = 8\ \mathrm{N}$$

3. **(b)** $F = \frac{\mu_0 I_1 I_2}{2\pi r} \Rightarrow F = \frac{\mu_0 I^2 \ell}{2\pi r}$

$\ell = 10\ \mathrm{cm}(\text{ Both }) \Rightarrow F \propto \frac{I^2}{r}$

$$\frac{F_1}{F_2} = \left(\frac{I}{2I}\right)^2\left(\frac{5/2}{5}\right) \Rightarrow \frac{F_1}{F_2} = \frac{1}{8} \Rightarrow F_2 = 8\ F_1$$

4. **(3)**

Statements-1

$v_{rms} \propto \sqrt{T} \Rightarrow v_{rms_1} \propto \sqrt{273-73}$

$v_{rms_2} \propto \sqrt{273+527}$

$$\frac{v_{ms_1}}{v_{rms_2}} = \sqrt{\frac{200}{800}} \Rightarrow v_{rms_2} = 2v_{rms_1}$$

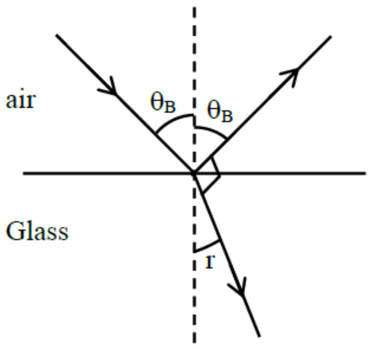
Statements-2

Translation K.E. $= \frac{3}{2}nRT = \frac{3}{2}PV$(False)

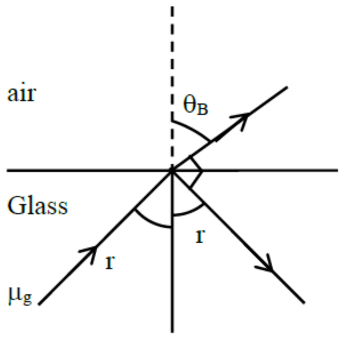
5. **(a)** Max vertical height $H = \frac{v^2}{2\,g} = 136\ \mathrm{m}$

Max horizontal distance $R = \frac{v^2}{g} \Rightarrow R = 2 \times 136 = 272\ \mathrm{m}$

6. (b)



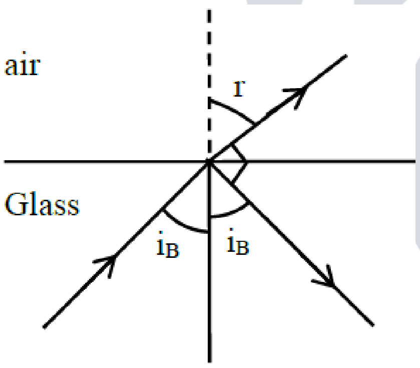
$$\theta_B + r = \frac{\pi}{2}$$



True

$$r = \frac{\pi}{2} - \theta_B$$

For glass to air



$$\mu_g \sin i_B = 1 \cdot \sin r$$

$$r + i_B = \frac{\pi}{2}$$

$$\mu_g \sin i_B = \cos i_B \Rightarrow \tan i_B = \frac{1}{\mu_g} \Rightarrow i_B = \tan^{-1}\left(\frac{1}{\mu_g}\right)$$

7. (a)

$$\text{Stress} = \text{y strain} \Rightarrow \frac{W}{A} = y\frac{\Delta\ell}{\ell}$$

$$\Delta\ell = \frac{W\ell}{yA} \Rightarrow \Delta\ell = \frac{250 \times 100}{10^{10} \times 6.25 \times 10^{-4}}$$

$$\Delta\ell = 4 \times 10^{-3}\ \text{m}$$

8. (c) For same force

$$\frac{q_1q_2}{4\pi\varepsilon_0 kd^2}=\frac{q_1q_2}{4\pi\varepsilon_0 r^2}\Rightarrow r=d\sqrt{K}$$

9. (d)

10. (c) $v_{sp}=\frac{h\nu-\phi}{e}$ (v and ϕ both)

Intensity ↑ current ↑

$kE_{max}=h\nu-\phi$

Photoelectric effect is not explained by wave theory

11. (3) Statement-I

When force balance it can move with uniform velocity (Uniform speed) True

Statement-2

Elevator going down with increasing speed means its acceleration is downwards

$mg-N=ma$ (on person)

$N=mg-ma$ (False)

12. (b)

10% of $\Delta Q=P\Delta V$(W/D by gas)

$$\frac{\Delta Q}{10}=3\times10^5(1600\times10^{-6})$$

$\Delta Q=4800\text{ J}$

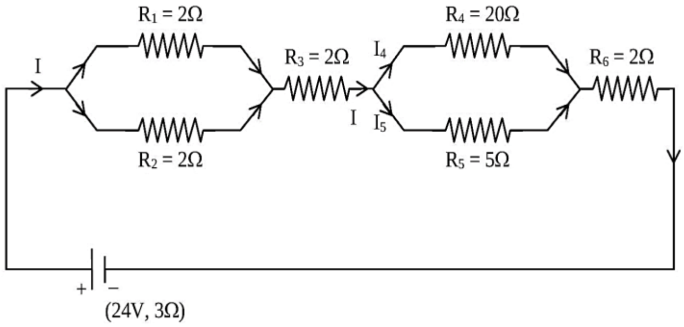
Using first law of the thermodynamics

$\Delta Q=\Delta u+W$

$$\Delta Q=\Delta u+\frac{\Delta Q}{10}\Rightarrow\Delta u=\frac{9}{10}\Delta Q$$

$$\Delta u=\frac{9}{10}\times4800\Rightarrow\Delta u=4320\text{ J}$$

13. (a)



$$R_{eq} = 3 + 1 + 2 + \frac{20 \times 5}{25} + 2 \Rightarrow R_{eq} = 12\Omega$$

Current from battery $I = \frac{24}{12} \Rightarrow I = 2A$

$$I_4 + I_5 = 2\text{ A}$$

$$I_4(20) = I_5(5) \Rightarrow I_5 = 4I_4 \Rightarrow I_4 = \frac{2}{5}\text{ A } I_5 = \frac{8}{5}\text{ A}$$

14. (b) Modulation index $\mu = \frac{A_m}{A_c}$

$A_m = 1 \& A_c = 2$

$$\mu = \frac{1}{2}$$

15. (d)

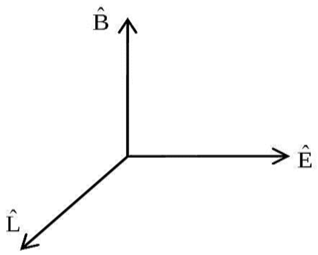
$$\text{emf} = -\frac{d\phi}{dt} \Rightarrow \varepsilon = \frac{-d(BA)}{dt}$$

$$\varepsilon = -A\frac{dB}{dt} \Rightarrow \varepsilon = -\pi R^2\left(\frac{0 - B}{\Delta t}\right)$$

$$\varepsilon = \frac{\pi R^2\ B}{\Delta t} \Rightarrow \varepsilon = \frac{\pi\left(\frac{10}{\sqrt{\pi}} \times 10^{-2}\right)^2 \times 0.5}{0.5}$$

$\varepsilon = 10^{-2}$ volt $= 10$ m volt

16. (d) $\vec{E} \& \vec{K} = \frac{W}{C}\hat{L}$



$$\hat{B} = \hat{L} \times \hat{E}$$

$$\vec{B} = B\hat{B}\left\{\frac{E}{B} = C\right\}$$

$\vec{B} = \frac{E}{C}(\hat{L} \times \hat{E})$

$\vec{B} = \frac{\omega}{C}\left(\frac{\hat{L} \times E\hat{E}}{\omega}\right) \Rightarrow \vec{B} = \frac{\vec{K} \times \vec{E}}{\omega}$

17. (d) (A) Planck's constant $h = \frac{E}{v}$

$[h] = \frac{[M^1 L^2 T^{-2}]}{[T^{-1}]} \Rightarrow [h] = [M^1 L^2 T^{-1}]$

(B) Stopping potential $V = \frac{W}{q}$

$[v] = \frac{ML^2 T^{-2}}{AT} \Rightarrow [v] = [ML^2 T^{-3} A^{-1}]$

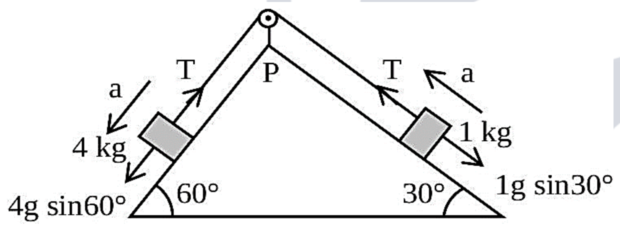
(C) Work function $= [ML^2 T^{-2}]$

(D) Momentum $[P] = [MLT^{-1}]$

18. (c) $y = 0.05\sin(8x - 4t)$

$v = \frac{\omega}{k} \Rightarrow v = \frac{4}{8} \Rightarrow v = \frac{1}{2}$ m/s

19. (b)



$4\,g\frac{\sqrt{3}}{2} - T = 4a$

$T - \frac{g}{2} = 1a$

$2\sqrt{3}\,g - T = 4\left(T - \frac{g}{2}\right) \Rightarrow 5\,T = (2\sqrt{3} + 2)g$

$T = \frac{10}{5}(2\sqrt{3} + 2) \Rightarrow T = 4(\sqrt{3} + 1)N$

20. (d) Photodiode works in reverse bias and its is used as a intensity detector. (True)

Forward bias current is more as compaired to reverse bias current (True)

21. (1)

$a\hat{i} + b\hat{j} + \hat{k}$ is $\perp$ to $(2\hat{i} - 3\hat{j} + 4\hat{k})$

$\vec{A} \cdot \vec{B} = 0 \Rightarrow\ 2a - 3\,b - 4 = 0\ 2a - 3\,b = -4$

Given $3a+2b=7$

$$\frac{2\left(\frac{a}{b}\right)-3}{3\left(\frac{a}{b}\right)+2}=\frac{-4}{7}\Rightarrow 14\frac{a}{b}-21=-12\frac{a}{b}-8$$

$$26\frac{a}{b}=13\Rightarrow\frac{a}{b}=\frac{1}{2}=\frac{x}{2}$$

$x=1$

22. (11)

$$\rho_{\text{Nucleus}}=\frac{A(m)}{\frac{4}{3}\pi R^3}\Rightarrow$$

$$\rho_N=\frac{3}{4\pi}\frac{Am}{\left(1.5\times10^{-15}\,A^{\frac{1}{3}}\right)^3}$$

$$\frac{\rho_N}{\rho_w}=\frac{3}{4\pi}\frac{(1.6)\times10^{-27}}{(1.5)^3\times10^{-45}\times10^3}$$

$$\frac{\rho_N}{\rho_w}=11\times10^{13}$$

23. (2)

$$R=\frac{\rho\ell}{A}\Rightarrow R=\frac{\rho\ell}{\pi(r_2^2-r_1^2)}$$

$$R=\frac{2.4\times10^{-8}\times3.14}{\pi(4^2-2^2)\times10^{-6}}$$

$$R=2\times10^{-3}\Omega$$

24. (2)

$$y=\frac{1}{2}at^2$$

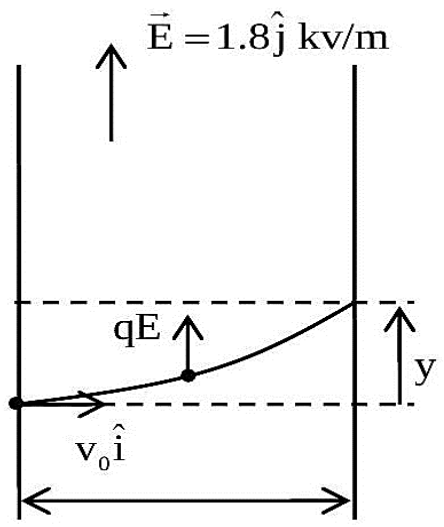
$$y=\frac{1}{2}\frac{qE}{m}t^2$$

$$\ell=v_0t$$

$$y=\frac{1}{2}\frac{qE}{m}\left(\frac{\ell}{v_0}\right)^2$$

$$y=\frac{1}{2}(2\times10^{11})(1.8\times10^3)\left(\frac{0.1}{3\times10^7}\right)^2$$

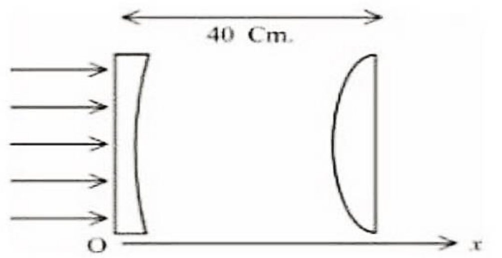
$y=2$ mm



25. (120)

Magnitude of focal length of both lens

$f = \frac{R}{\mu - 1} \Rightarrow f = \frac{30}{1.75 - 1} \Rightarrow f = 40$ cm



$f = -40$ cm $f = +40$ cm

Concave lens will form image at its focus for convex lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-80} = \frac{1}{+40}$

$V = +80$ cm

From concave lens distance of image of $d = 80 + 40$

$d = 120$ cm

26. (**110**)

$I_{PQ} = I_{cm} + md^2$

$mk^2 = \frac{2}{5}mR^2 + md^2 \Rightarrow k = \sqrt{\frac{2}{5}(5)^2 + (10)^2}$

$k = \sqrt{110}$ cm

27. (5)

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

$$T = 2\pi\sqrt{\frac{2}{2k}} \Rightarrow T = 2\pi\sqrt{\frac{1}{20}}$$

$$T = \frac{\pi}{\sqrt{5}}$$

28. (12)

$\Delta D = D \propto \Delta T$

$\Delta D = 5 \times 1.6 \times 10^{-5} \times (177 - 27)$

$\Delta D = 12 \times 10^{-3}$ cm

29. (10)

$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ & bandwidth $= \frac{R}{L}$

$$\frac{Q}{\text{Bandwidth}} = \frac{L}{R^2}\sqrt{\frac{L}{C}}$$

$$= \frac{3}{100} \times \sqrt{\frac{3}{27 \times 10^{-6}}}$$

$= 10$

30. (40)

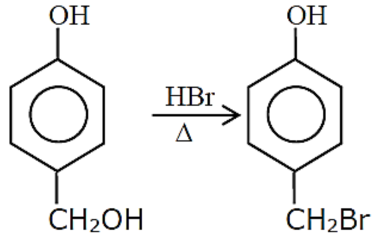
Impulse $= \Delta P$

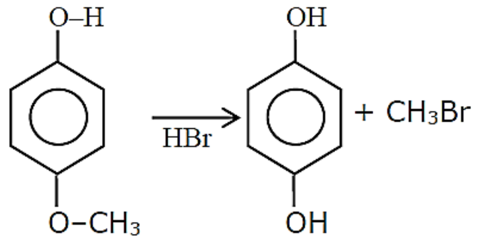
$F\Delta T = P - 0 \Rightarrow F\Delta T = \sqrt{2mk}$

$F(5) = \sqrt{2 \times 2 \times 10000}$

$F = 40$ N

31. (b)





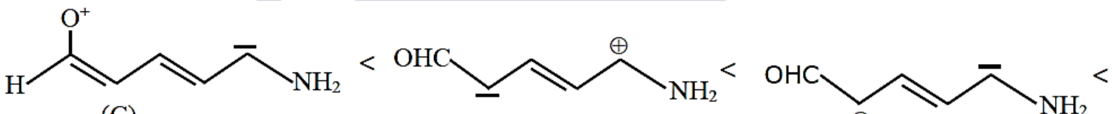
32. (a) ice > liquid water > impure water

OR

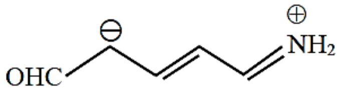
ice > H_2O liq. > impure H_2O

Hydrogen bond $\propto \frac{1}{\text{Temp}}$

33. (d)

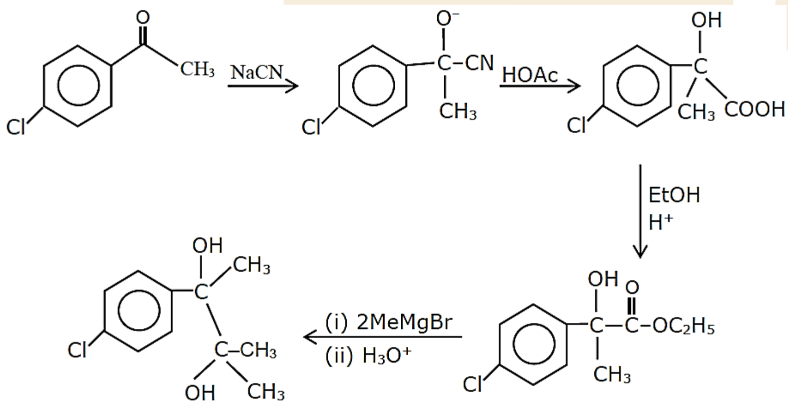


(C) (+ve on O) < (B) (Back bonding) < (A) <



(D)
more covalent bond and (+M)

34. (b)



35. (a) $[CO(NH_3)_5Cl]Cl_2$
36. (b)

$NiCl_2 + NH_4OH + dmg \rightarrow$ Rosy red ppt
$[Ni(dmg)_2]$

37. (a) $(NH_4)_2BeF_4 \xrightarrow{\Delta} BeF_2 + NH_4F$

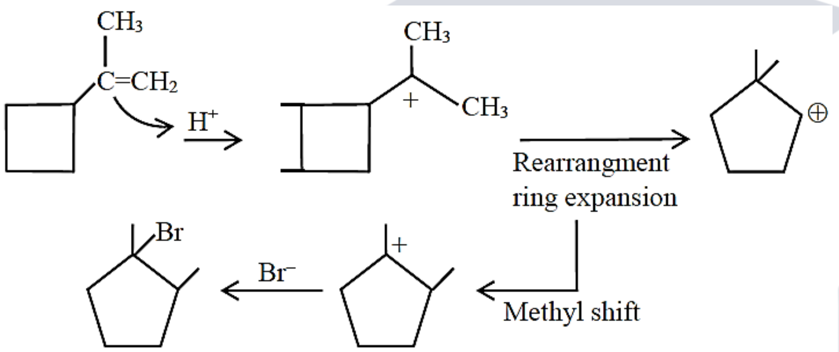
38. (a)

Reverberatory furnance → Cr
Electrolysis cell → Ar
Blast furnance → Pig iron
Zone refining furnance → silicon

39. (b)

Chlrophyl → Mg^{2+}
Sodaash → Na_2CO_3
Destistry &
ornamental work → $CaSO_4$
White washing → $Ca(OH)_2$

40. (c)



41. (d) $\sqrt{v} \propto z$

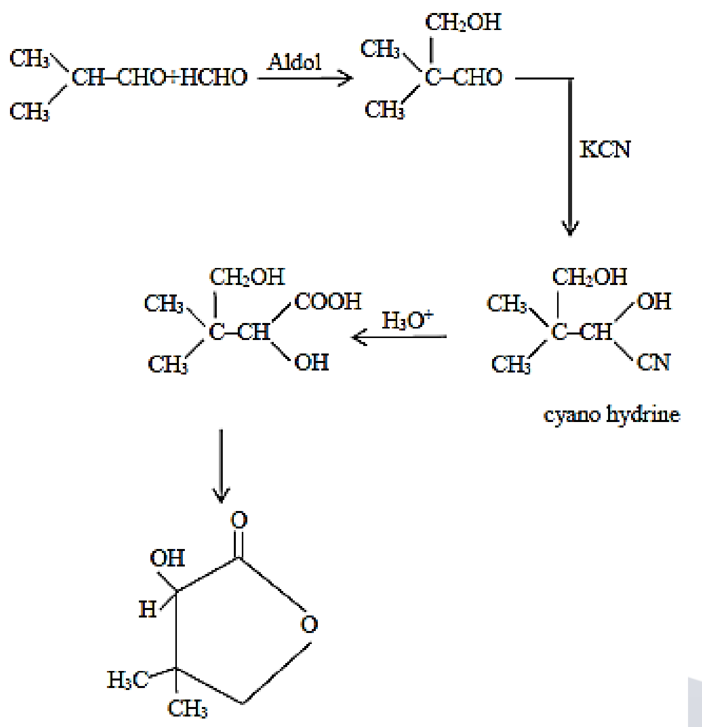
$v^n \propto z$

$n = 1/2$

42. (a)

43. (c) Silver mirror test can gives by p^{+3}, p^{+1} ox acid $H_4\overset{+3}{P}O_5 + Ag_2O \rightarrow Ag$
Silver mirror

44. (d)



45. (b)

$$\left.\begin{array}{l}\text{Small size of + ve ion}\\ \text{longer size of – ve ion}\end{array}\right\rangle \text{move covalent according to fajan's rule}$$

46. (c) Freons → chlorofluorocarbon compounds

47. (c) On adding non-volatile solute to pure solvent, depression in freezing point and lowering in vapour pressure occurs

48. (b) These layers should be of opposite charges

49. (c) The rate of hydrolysis of alkyl chloride improves because of better Nucleophilicity of I^-.

50. (c)

$\sqrt{n(n+2)} = 3.87$

$n = 3 = \text{no. of unpaired } e^-$

$Cr^{2+} = [Ar]3\,d^4$

$Tr^{-2+} = [Ar]3\,d^2$

$V^{2+} = [Ar]3\,d^3$

$Mn^{2+} = [Ar]3\,d^5$

51. (4)

$Fe_{0.93}O$
$2x + (0.93 - x)3 = 2$
$-x + 3 \times 0.93 = 2$
$x = 0.79$
$0.79 = \text{ no. of } Fe^{2+} \text{ ion}$
$0.14 = \text{ no. of } Fe^{3+} \text{ ion}$
$nf = 0.79$

$$\text{Equivalent wt } = \frac{\text{Molecular weight}}{0.79}$$

Due to presence of Fe^{3+} in FeO lattice, Metal deficiency occurs.

% Composition :- Fe^{2+} ions $= \dfrac{0.79}{0.93} \times 100$

85%

Fe^{3+} ion $= \dfrac{0.14}{0.93} \times 100$

15%

52. (4)

$K = Ae^{-Ea/RT}$

Here, Ea ↑ K ↓

$\ln K = \ln A - Ea/RT$

slope of $\ln K$ vs $1/T = -Ea/R$

The higher is the activation energy, higher is the value of the temperature coefficient.

53. (2) For process A

$\Delta G = -25 \times 10^3 - 300(-80)$

$= -25000 + 24000$

$= -1000 \Rightarrow \Delta G < 0$ spontaneous

For process B

$\Delta G = -22 \times 10^3 - 300(40)$

$= -22000 - 12000 \Rightarrow \Delta G < 0$ spontaneous

For process C

$\Delta G = 25 \times 10^3 - 300(-50)$

$= 25000 + 15000 = 40000 \text{ J}$

$\Delta G > 0 \Rightarrow$ Non-spontaneous

For process D

$\Delta G = 22 \times 10^3 - 300(20)$

$\Delta G > 0 \Rightarrow$ Non-spontaneous.

54. (180)

Molarity of stock solution

$= \frac{5/40}{450} \times 1000$

$= \frac{50}{4 \times 45} = \frac{10}{36} \text{M}$

$M_1 V_1 = M_2 V_2$

$\frac{10}{36} \times V = 0.1 \times 500$

$V = \frac{50 \times 36}{10} = 180 \text{ml}$

55. (492)

Paschen series: -

$z = 1$

I^{st} line: $-4 \rightarrow 3$

$\frac{1}{\lambda} = R \times (1)^2 \left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

$\frac{1}{720} = R\left(\frac{7}{144}\right)$

II^{nd} line $\rightarrow 5 \rightarrow 3$

$\frac{1}{\lambda} = R \times (a)\left(\frac{1}{3^2} - \frac{1}{5^2}\right)$

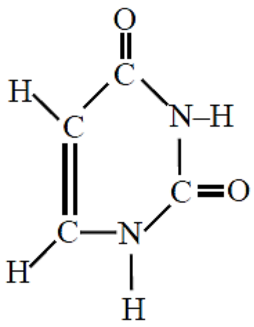
$\frac{1}{\lambda} = R\left(\frac{16}{225}\right)$

Equation (b) ÷ equation (a)

$\frac{\lambda}{720} = \frac{7}{144} \times \frac{225}{16}$

$\lambda = 492.18$

56. (25)



Moleculer formula of uracil

$= C_4\ N_2H_4O_2$

% of N $= \frac{28}{112} \times 100 = 25\%$

57. (10) $K_a = x \times 10^{-5}$

$CH_3COOH \rightarrow 0.02M\&25ml$

$CH_3COONa \rightarrow 0.2M$ and 25ml

$$pH = p^{K_a} + \log\frac{[\text{ salt }]}{[\text{ acid }]}$$

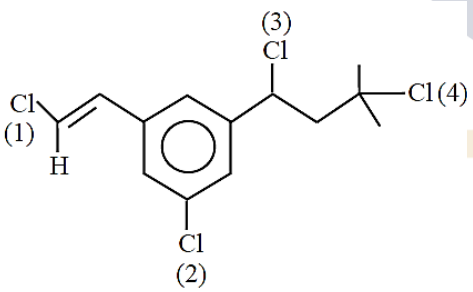
$$5 = p^{K_a} + \log\frac{0.2 \times 25}{0.02 \times 25} = p^{K_a} + \log 10$$

$p^{K_a} = 4$

$K_a = 10^{-4} = 10 \times 10^{-5}$

Hence $x = 10$

58. (2)



After treat with $AgNO_3, Cl^-$remove and possible carbocation will form

Position 1: Vinyllic carbocation forms, unstable, So not possible

Position 2: sp^2 hybridised carbocation is unstable, so not possible

Position 3: Forms 2° carbocation which will be in conjugation with ring

Position 4: 3° stable carbocation will form.

59. (7)

60. (917)

$14H^+ + Cr_2O_7^{2-} + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

$$E = E^\circ - \frac{2.303RT}{6\ F}\log\frac{[Cr^{3+}]^2}{[Cr_2O_7^{2-}][H^+]^{14}}$$

$pH = 3$

$[H^+] = 10^{-3}$

$$E = 1.330 - \frac{0.059}{6}\log\frac{10^{-2}}{10^{-2}(10^{-42})}$$

$E = 0.917$

$= 917 \times 10^{-3}$

$x = 917$

61. (c)

$$\begin{aligned}
&\bar{v}\cdot\bar{w} = \bar{u} + \lambda\bar{v}\\
&\bar{v}\cdot\bar{w}\cdot\bar{v} = \bar{u}\cdot\bar{v} + \lambda\bar{v}\cdot\bar{v}\\
&0 = 2 - 1 + 2 + \lambda 4 + 1 + 1\\
&\lambda = \frac{-3}{6} \Rightarrow \lambda = \frac{-1}{2}
\end{aligned}$$

Now

$$\begin{aligned}
&\bar{v}\times\bar{w} = \bar{u} + \lambda\bar{v}\\
&\bar{v}\times\bar{w}\cdot\bar{w} = \bar{u}\cdot\bar{w} + \lambda\bar{v}\cdot\bar{w}\\
&0 = \bar{u}\cdot\bar{w} + \lambda 2\\
&\bar{u}\cdot\bar{w} = -2\lambda = 1
\end{aligned}$$

62. (c)

$$\begin{aligned}
&\mathrm{lt}_{t\to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \cdots \ldots. + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}\\
&= \mathrm{lt}_{t\to 0}\, n\left(\left(\frac{1}{n}\right)^{\frac{1}{\sin^2 t}} + \left(\frac{2}{n}\right)^{\frac{1}{\sin^2 t}} + \cdots \ldots. + \left(\frac{n-1}{n}\right)^{\frac{1}{\sin^2 t}} + 1\right)^{\sin^2 t}\\
&= n\cdot[0 + 0 + \cdots + 1]^0\\
&= n
\end{aligned}$$

63. (c) $(a-c)x^2 + (b-a)x + (c-b) = 0$

$x = 1$ is one root & other root is $\frac{c-b}{a-c}$ ... (a)

now $\begin{vmatrix} \alpha^2 & \alpha & 1\\ 1 & 1 & 1\\ a & b & c\end{vmatrix}$ is singular

$\Rightarrow \begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix} = 0 \Rightarrow \alpha^2(c-b) - \alpha(c-a) + (b-a) = 0$

$\Rightarrow \alpha^2(c-b) + \alpha(a-c) + (b-a) = 0$

satisfied by $\alpha = 1$ or $\alpha = \frac{b-a}{c-b}$

Now, if $\alpha = 1$ then $\forall a \neq b \neq c$

$$\sum \frac{(a-c)^2}{(b-a)(c-b)} = \frac{\sum (a-c)^3}{(a-b)(b-c)(c-a)}$$

$$= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}$$

$= 3$

[if $A + B + C = 0 \Rightarrow A^3 + B^3 + C^3 = 3ABC$]

64. (a)

$y^2 + 4x = 4$

$P: y^2 = 4(1-x)$

Now

$y^2 + 4\left(\frac{y}{2} - 1\right) = 4$

$y^2 + 2y - 8 = 0$

$(y+4)(y-2) = 0$

required Area

$$A = \int_{-4}^{2} \left[\left(\frac{4-y^2}{4}\right) - \left(\frac{y-2}{2}\right)\right] dy$$

$$A = \int_{-4}^{2} \left(2 - \frac{y^2}{4} - \frac{y}{2}\right) dy$$

$$= \left[2y - \frac{y^3}{12} - \frac{y^2}{4}\right]_{-4}^{2}$$

$$= \left(4 - \frac{8}{12} - 1\right) - \left(-8 + \frac{64}{12} - 4\right)$$

$A = 9$

65. (b)

$(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$

$\Rightarrow 2^{200} \text{cis}\left(\frac{-\pi}{3}\right)^{200} = 2^{199}(p + iq)$

$\Rightarrow 2^{200}\left(\operatorname{cis}\left(-\frac{200\pi}{3}\right)\right) = 2^{199}(p + iq)$

$\Rightarrow 2\left(\operatorname{cis}\left(-66\pi - \frac{2\pi}{3}\right)\right) = (p + iq)$

$\Rightarrow 2\left[\operatorname{cis}\left(\frac{-2\pi}{3}\right)\right] = (p + iq)$

$\Rightarrow 2\left[\frac{-1}{2} - \frac{\sqrt{3}i}{2}\right] = (p + iq)$

$\Rightarrow p = -1, q = -\sqrt{3}$

Now

$\alpha = p + q + q^2 = 2 - \sqrt{3}$

$\beta = p - q + q^2 = 2 + \sqrt{3}$

req. quad is $x^2 - 4x + 1 = 0$

66. (c) for unique solu.

$\Delta \neq 0$

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix} \neq 0$

$\Rightarrow (N^2 - 6) - (2N - 6) + (6 - 3N) \neq 0$

$\Rightarrow N^2 - 5N + 6 \neq 0$

$\Rightarrow N \neq 3 \;\&\; N \neq 2$

Hence N can be $\{1,4,5,6\}$ Fav case : $\frac{4}{6} = \frac{k}{6} \Rightarrow k = 4$ Sum $= 20$

67. (d) $x^{pq^2} = y^{qr} = z^{p^2r}$
&
$r = pq + 1$

$3, 3\log_y^x, 3\log_z^y, 7\log_x^z$ are in A.P.

Now

$3\log_y^x = 3 + \frac{1}{2} = \frac{7}{2} \Rightarrow \log_y^x = \frac{7}{6}$

$x^6 = y^7$

$3\log_z^y = 3 + 1 = 4 \Rightarrow \log_z^y = \frac{4}{3}$

$y^3 = z^4$

$7\log_x^z = 3 + \frac{3}{2} = \frac{9}{2} \Rightarrow \log_x^z = \frac{19}{14}$

$z^{14} = x^9$

Now

$x^{pq^2} = x^{\frac{6}{7}qr} = x^{\frac{9p^2r}{14}}$

$pq^2 = \frac{6}{7}qr = \frac{9}{14}p^2r$

$pq = \frac{6}{7}r\,q^2 = \frac{9}{14}pr$

$r = pq + 1 \;\Rightarrow q^3 = \frac{9}{14}\frac{6}{7}r \cdot r$

$\Rightarrow r = \frac{6}{7}r + 1$

$\Rightarrow r = 7 \;\Rightarrow q = 3$

Now

$r - p - q$

$= 7 - 2 - 3$

$= 2$

68. (d) $\gcd(a, b) = 1,\; 2a \neq b$

reflexive $\gcd(a, a) = a$

Not possible

symmetric $\gcd(b, a) = 1 \& 2a \neq b$

Not possible

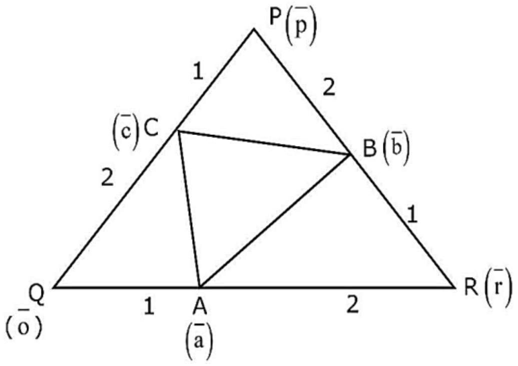
$a = 2, b = 1$

transitive

$$\begin{array}{lll} (a, b) = (2,3) & \gcd\{a, b\} = 1, & 2a \neq b \\ (b, c) = (3,4) & \gcd\{c, d\} = 1, & 2a \neq c \\ (a, c) = (2,4) & \gcd\{2,4\} = 2, & 2a = c \end{array}$$

Not possible

69. (b)



$\bar{a} = \frac{\bar{r}}{3}$

$\bar{b} = \frac{\bar{b} + 2\bar{r}}{3}$

$\bar{c} = \frac{2\bar{p}}{3}$

$\Delta PQR = \frac{1}{2}|\bar{r} \times \bar{p}|$

$\Delta PQR = \frac{1}{2}|\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|$

$= \frac{1}{2}\left|\frac{\bar{r} \times \bar{p}}{9} + \frac{4(\bar{r} \times \bar{p})}{9} + \frac{2}{9}\bar{p} \times \bar{r}\right|$

$= \frac{1}{18}|3(\bar{r} \times \bar{p})|$

Hence $\frac{|\Delta PQR|}{|\triangle ABC|} = 3$

70. (a) $x^3 dy + (xy - 1)dx = 0$

$x^3 \frac{dy}{dx} = 1 - xy$

$x^3 \frac{dy}{dx} + xy = 1$

$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}\Big|_{LDE}$

$I.F = e^{\int \frac{1}{x^2}dx} = e^{-\frac{1}{x}}$

$y \cdot e^{\frac{-1}{x}} = \int e^{\frac{-1}{x}} \cdot \frac{1}{x^3}dx$

$\frac{-1}{x} = t$

$= -\int e^t t dt$

$y \cdot e^{\frac{-1}{x}} = -e^t(t - 1) + k$

$y \cdot e^{\frac{-1}{x}} = -e^{\frac{-1}{x}}\left(\frac{-1}{x} - 1\right) + k$

$y = \left(\frac{1}{x} + 1\right) + ke^{\frac{1}{x}}$

put $x = \frac{1}{2} \Rightarrow 3 - e = (2 + 1) + ke^2$

$k = -\frac{1}{e}$

Now $y(a) = 2 + \frac{-1}{e}e$

$= 1$

71. (d) $A^2 + B = A^2 B$

$A^2 - A^2B + B = 0$

$A^2 - A^2B - (I - B) = -I$

$(I - A^2)(I - B) = I$

So,

$(I - A^2)\&(I - B)$ are inverses of each other

So

$(I - B)(I - A^2) = I$

$I - B - A^2 + BA^2 = I$

$BA^2 = B + A^2$

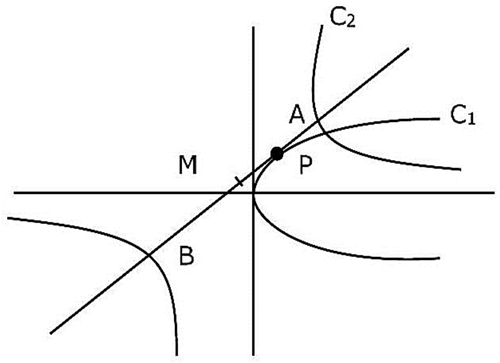
So, from (a) & (b)

$A^2 B = BA^2$

72. (d)

$x^2 - 4x + [x] + 3 = x[x]$
$x^2 - 4x + 3 = (x - 1)[x]$
$(x - 1)(x - 3) = (x - 1)[x]$
$x = 1$ or $x - 3 = [x]$
$x - [x] = 3$
$\{x\} = 3$

73. (d) $c_1 : y^2 = 24x$ & $c_2 : xy = 2$



AB : [Tangent to parabola at $p(t)$]

$ty = x + 6t^2$

AB : [chord with given mid point of hyperbola]

$T = S_1$

$\frac{x}{h} + \frac{y}{k} = 2$

from (1) & (2)

$\frac{-1}{\frac{1}{h}} = \frac{t}{\frac{1}{k}} = \frac{6t^2}{2}$

$-h = kt = 3t^2$

$h = -3^{t^2}$ & $k = 3^t$

$h = -3\frac{k^2}{9} \Rightarrow y^2 = -3x$

$\ell(LR) = 3$ & directrix is $x = \frac{3}{4}$

74. (d) Let $\Omega = [0,1]$

Let $A \rightarrow$ selecting $\frac{1}{2}$

$A = \left\{\frac{1}{2}\right\}$

then, $P(A) = 0$ but $A \neq \phi$

$B = A^c = [0,1] - \left\{\frac{1}{2}\right\}$

$P(B) = 1$

but $B \neq \Omega$

4

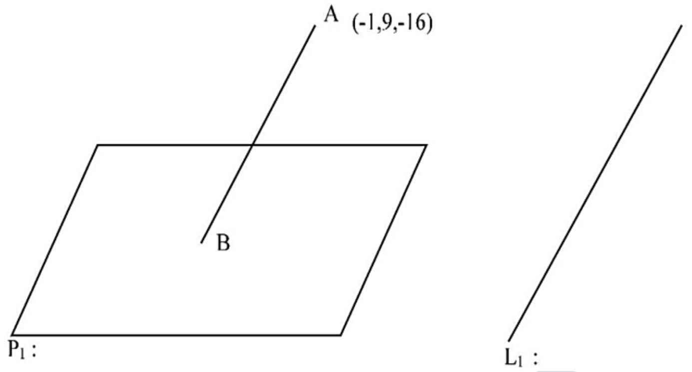
75. (b)

$\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r$

$= \sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r$

${}^{45}C_{22} = {}^{45}C_{23}$

76. (d)



$L_{AB}: \dfrac{x+1}{3} = \dfrac{y-9}{-4} = \dfrac{z+16}{12} = t$

B : $(3t-1, 9-4t, 12t-16)$ lies on plane

$$2(3t-1) + 3(9-4t) - (12t-16) = 5$$
$$-18t - 2 + 27 + 16 = 5$$
$$-18t + 25 + 16 = 5$$
$$-18t = -36 \Rightarrow t = 2$$

$$B: (5,1,8) \ \& \ A: (-1,9,-16)$$
$$l(AB) = \sqrt{36+64+576} = \sqrt{676} = 26$$

77. (a)

$$\tan^{-1}\left(\frac{1+\sqrt{3}}{\sqrt{3}(1+\sqrt{3})}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$$
$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\sqrt{\frac{4(2+\sqrt{3})}{3(2+\sqrt{3})}}$$
$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\frac{2}{\sqrt{3}}$$
$$= \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$$

78. (d)

$$f(x) = \begin{cases} x^2 \sin\dfrac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$\rightarrow$ cont. of f(x) at x = 0

$$\left.\begin{aligned} \text{LHL} &= \text{lt}_{h\to 0}\; h^2\sin\frac{1}{(-h)} = 0 \\ \text{RHL} &= \lim_{h\to 0} h^2\sin\frac{1}{(-h)} = 0 \\ f(0) &= 0 \end{aligned}\right\} \text{continuous at } x = 0$$

$\rightarrow$ Diff. of f(x) at x = 0

$$\text{RHD} = \text{dt}_{h\to 0}\frac{h^2\sin\left(\frac{1}{h}\right) - 0}{h} = \underset{h\to 0}{\text{dt}}\, h\sin\frac{1}{h} = 0$$

$$\text{LHD} = \text{dt}_{h\to 0}\frac{h^2\sin\left(\frac{-1}{h}\right) - 0}{-h} = \underset{h\to 0}{\text{dt}} h\sin\frac{1}{h} = 0$$

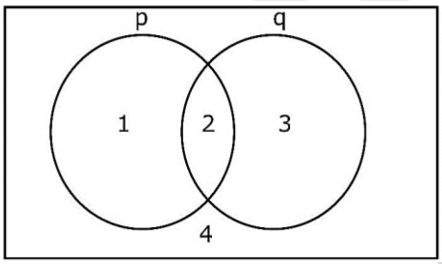
Hence f(x) is diff. at x = 0
Now diff. f(s) at x = 0

$$f'(x) = \begin{cases} 2x\sin\left(\frac{1}{x}\right) + x^2\cos\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$f'(x) = \begin{cases} 2x\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

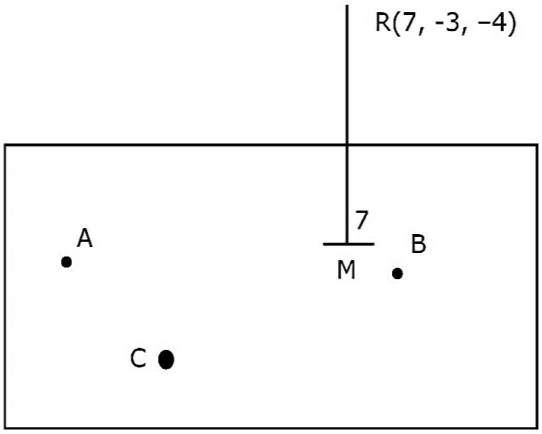
hence f′(x) limit ossicilate at x = 0
hence f′(x) is D.C. at x = 0

79. (d) $(\sim(p \wedge q) \vee (\sim p \wedge q)) \rightarrow \sim p \wedge \sim q$



$(1 + 2 + 4) + 2 \Rightarrow 4$
$(1 + 2 + 4) \Rightarrow 4 \qquad = \sim (1 + 2 + 4) + 4$
$= 3 + 4$

80. (c)



$$\bar{n}_P = \overline{AB} \times \overline{DC} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & -4 & 3 \\ -1 & 1 & -1 \end{vmatrix} = <1, 0, -1>$$

Equation of plane P : $1(x-2) + 0(y+3) - (z-1) = 0$

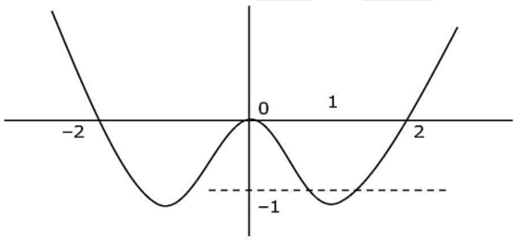
P : $x - z - 1 = 0$

$$d(RM) = \left|\frac{7+4-1}{\sqrt{2}}\right| = 5\sqrt{2}$$

81. (5)

$|x|^2 - 2|x| + |\lambda - 3| = 0$

$|x|^2 - 2|x| = -|\lambda - 3|$



$LHS = -1 \le |x|^2 - 2|x| < \infty$

RHS $-|\lambda - 3| \le 0$

Now LHS = RHS only when

$0 \le |\lambda - 3| \le 1$ & $x \in [-2,2]$

$-1 \le \lambda - 3 \le 1$

$2 \le \lambda \le 4$

82. (7)

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Elipse

Let $P = (4\cos\theta, 3\sin\theta)$

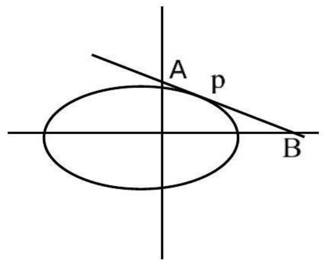
Tp: $\frac{x}{4}\cos\theta + \frac{y}{3}\sin\theta = 1$

A: $(0, 3\operatorname{cosec}\theta)$, B: $(4\sec\theta, 0)$

$\ell(AB) = \sqrt{16\sec^2\theta + 9\operatorname{cosec}^2\theta}$

$= \sqrt{16 + 9 + (4\tan\theta - 3\cot\theta)^2 + 24}$

$\ell(AB)_{\min} = 7$



83. (14)

$L_1: \bar{a} = <2, -1, 6>$ $L_2: \bar{b} = <6, 1, -8>$
$\bar{p} = <3, 2, 2>$ $\bar{q} = <3, -2, 0>$

$$\bar{p} \times \bar{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix} = <4, 6, -12> \text{ (s.f.)}$$

$$b\Delta = \left|\frac{(\bar{b} - \bar{a}) \cdot |\bar{p} \times \bar{q}|}{|\bar{p} \times \bar{q}|}\right|$$

$$= \left|\frac{(4\hat{i} + 2\hat{j} - 14\hat{k}) \cdot (2\hat{i} + 3\hat{j} - 6\hat{k})}{\sqrt{4 + 9 + 36}}\right|$$

$$= \left|\frac{8 + 6 + 84}{\sqrt{49}}\right| = \left|\frac{98}{7}\right| = 14$$

84. (1012)

$$\sum_{r=0}^{n} r^{2n}C_r$$

$$= \sum_{r=0}^{n} r^2 \cdot \frac{n}{r}\, {}^{n-1}C_{r-1}$$

$$= n\sum_{r=1}^{n} \left((r-1)^{n-1}C_{r-1} + {}^{n-1}C_{r-1}\right)$$

$$= n\sum_{r=2}^{n} (n-1)^{n-2}C_{r-2} + n\sum_{r=1}^{n} {}^{n-1}C_{r-1}$$

$= n(n-1)[2^{n-2}] + n[2^{n-1}]$
$= 2023 \cdot 2022 \cdot 2^{2021} + 2023 \cdot 2^{2022}$
$= 2023 \cdot 2^{2021}[2022 + 2]$
$= 2023 \cdot 2^{2021} \cdot 2024$
$= 2023 \cdot 1012 \cdot 2^{2022} \Rightarrow \alpha = 1012$

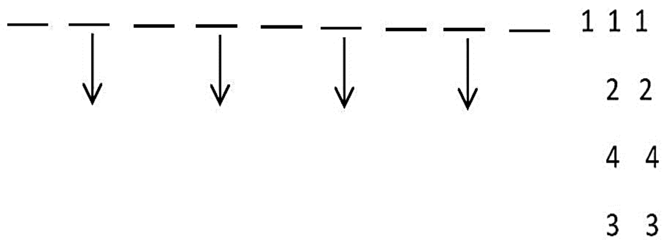
85. (2)

$$I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$$

$$2I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}1dx$$

$$2I=\frac{8}{\pi}\cdot\frac{\pi}{2}\Rightarrow I=2$$

86. (60)



4 even place can be occupied by 4 even digits

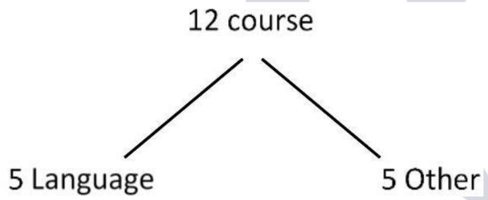
No of always $=\frac{4:}{2!2!}=6$

Odd place can be occupied by 5 odd digits

No of always $=\frac{5!}{3!2!}=10$

Total no. $=6\times10=60$

87. (546)



(0 language +5 other) + (1 Language +4 other) + (2 Language +3 other)

$=5_{C_0}\cdot7_{C_5}+5_{C_1}\cdot7_{C_4}+5_{C_2}\cdot7_{C_3}$

$=21+175+350$

$=546$

88. (12) $T_4=500\Rightarrow a\left(\frac{1}{m}\right)^3=500\Rightarrow a=500\ m^3$

Now $S_n-S_{n-1}=a\left(\frac{1-r^n}{1-r}\right)-a\left(\frac{1-r^{n-1}}{1-r}\right)$

$=\frac{a}{1-r}[r^{n-1}(1-r)]$

$=ar^{n-1}$

$= 500\ m^3\left(\frac{1}{m}\right)^{n-1}$

$S_n - S_{n-1} = 500\ m^{4-n}$

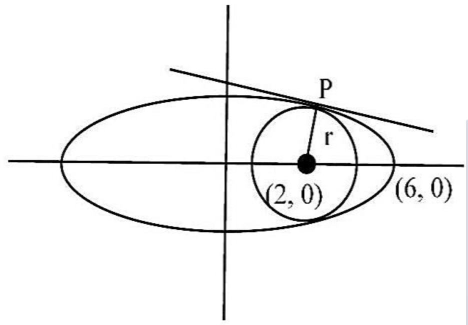
Now $S_6 - S_5 > 1 \Rightarrow 500\ m^{-2} > 1$

& $S_7 - S_6 < \frac{1}{2} \Rightarrow 500\ m^{-3} < \frac{1}{2}$.

$$\left.\begin{array}{ll} \text{from (a)} & m^2 < 500 \\ \text{from (b)} & m^3 > 1000 \end{array}\right] 10 < m \le 22$$

Number of possible values of m is = 12

89. (118)



E: $\frac{x^2}{36} + \frac{y^2}{16} = 1$&C: $(x-2)^2 + y^2 = r^2$

For largest circle r is maximum

$P(6\cos\theta, 4\sin\theta)$

$N_P: 6x\sec\theta - 4\ y\ \text{cosec}\,\theta = 20$ pass (2,0)

$12\sec\theta = 20 \Rightarrow \cos\theta = \frac{3}{5}$

Now P: $\left(6 \times \frac{3}{5}, 4 \times \frac{4}{5}\right) \Rightarrow$ P: $\left(\frac{18}{5}, \frac{16}{5}\right)$

$$r = \sqrt{\left(2 - \frac{18}{5}\right)^2 + \left(\frac{16}{5}\right)^2}$$

$$r = \frac{\sqrt{64+256}}{5} = \frac{8\sqrt{5}}{5} = \frac{8}{\sqrt{5}}$$

$$C: = (x-2)^2 + y^2 = \frac{64}{5}$$

Now $(1, \alpha)$ lies on C

$$\Rightarrow (1-2)^2 + \alpha^2 = \frac{64}{5}$$

$$\alpha^2 = \frac{64}{5} - 1$$

$\alpha^2 = \frac{59}{5} \Rightarrow 10\alpha^2 = 118$

90. (22) $I = 12\int_0^3 |x^2 - 3x + 2|dx$

$I = 12\int_0^3 |(x-2)(x-1)|dx$

$= 12\left[\int_0^1 (x^2 - 3x + 2) + \int_1^2 -(x^2 - 3x + 2) + \int_2^3 (x^2 - 3x + 2)\right]$

$= 12\left[\left(\frac{1}{3} - \frac{3}{2} + 2\right) - \left(\frac{7}{3} - \frac{9}{2} + 2\right) + \left(\frac{19}{3} - \frac{15}{2} + 2\right)\right]$

$= 12\left[\frac{5}{6} + \frac{1}{6} + \frac{5}{6}\right]$

$= \frac{12 \cdot 11}{6}$

$= 22$