Physics
Section -A

1. (a)

$$\frac{U}{V}=\frac{1}{2}\epsilon_0 E^2$$

$$U=\frac{1}{2}\epsilon_0 E^2 V$$

$$=\frac{1}{2}\epsilon_0 E^2(Ad)$$

2. (a) When $P = 2.5D$

$$F=\frac{1}{P}=\frac{1}{2.5}$$

When $P' = 2.6D$

$$F'=\frac{1}{P'}=\frac{1}{2.6}$$

Relative decrease in focal length

$$\frac{F-F'}{F}=\frac{\frac{2}{5}-\frac{5}{13}}{\frac{2}{5}}=1-\frac{25}{26}=\frac{1}{26}=0.04$$

3. (d) T is surface tension

$$\text{P in air bubble }=P_0+\rho gh+\frac{2T}{R}$$

$$P_{in}-P_0=\rho gh+\frac{2T}{R}=2100$$

$$\frac{2T}{R}=2100-\rho gh$$

$$T=\frac{R}{2}(2100-10^3\times 10\times 0.2)$$

$$=\frac{1}{20}(2100-2000)\times 10^{-2}$$

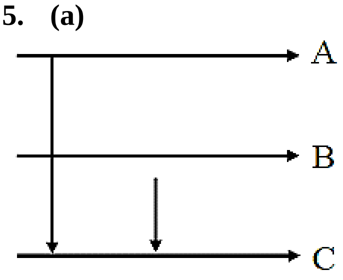
$$=0.05$$

4. (d) $y=\frac{32.3\times 1125}{27.4}=1326.186$

Last significant digits are 3 in operands so results should rounded off to 3 digits.

$\therefore y = 1330$

5. (a)



$$E_A-E_C=\frac{hc}{2000Å}\ \dots\dots\ (i)$$

$$\text{and } E_B-E_C=\frac{hc}{6000Å}\ \dots .\ (ii)$$

$$\text{Now } E_A-E_B=(E_A-E_C)-(E_B-E_C)$$

$$\frac{hc}{\lambda_{AB}}=\frac{hc}{2000}-\frac{hc}{6000}$$

$$\frac{1}{\lambda_{AB}}=\frac{1}{3000Å}$$

$$\lambda_{AB}=3000Å$$

6. (d)

7. (a) $W = \Delta U = S\Delta A$

One drop to n drop

$$\frac{4}{3}\lambda R^3 = n\frac{4}{3}\lambda r^3$$

$$r = \frac{R}{n^{\frac{1}{3}}}$$

So $W = S(n4\pi r^2 - 4\pi R^2)$

$= S4\pi R^2\left(n^{\frac{1}{3}} - 1\right)$

For on drop to 27 drops

$W = S4\pi R^2\left(27^{\frac{1}{3}} - 1\right) = 10 \ldots\ldots$ (i)

For one drop to 64 drops

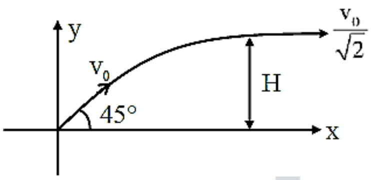
$W' = S4\pi R^2\left(64^{\frac{1}{3}} - 1\right)$ ...(ii)

(ii)/(i)

$$\frac{W'}{W} = \frac{4-1}{3-1} = \frac{3}{2}$$

$$W' = \frac{3}{2}W = 155$$

8. (d)



$$H = \frac{\left(\frac{v_0}{\sqrt{2}}\right)^2}{2g} = \frac{v_0^2}{4g}$$

$$L = mvh$$

$$L = m\frac{v_0}{\sqrt{2}}\frac{v_0^2}{4g}$$

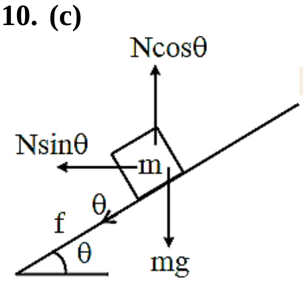
9. (d)

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

$$n480 = m600$$

$$n_{min} = 5$$

10. (c)



$$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$$

$$N\cos\theta - f\sin\theta = mg$$

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{Rg}$$

$$R\tan\theta + \mu Rg = v^2 - v^2\mu\tan\theta$$

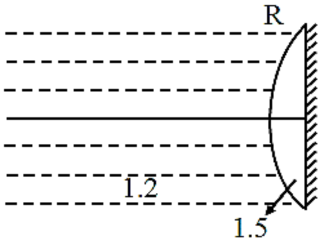
$$\mu = \frac{v^2 - Rg\tan\theta}{Rg + v^2\tan\theta}$$

11. (c)

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$$

$$a = \frac{\frac{g}{\sqrt{2}}}{1+\frac{1}{2}} = \frac{2\frac{g}{\sqrt{2}}}{3} = \frac{\sqrt{2}\,g}{3}$$

12. (b)



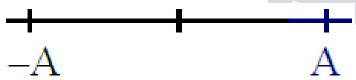
$$\frac{1.5}{v} = \frac{1.5-1.2}{R}$$

$$v = \frac{1.5R}{0.3} = 5R$$

$$\frac{1.2}{f} - \frac{1.5}{5R} = \frac{1.2-1.5}{-R}$$

$$\frac{1.2}{f} = \frac{0.3}{R} \times 2 \Rightarrow f = 2R \Rightarrow R = 0.1$$

13. (b) $A = 1$ cm



$$n = \frac{12.5}{2} = 6.25\text{cycles}$$

$$\therefore D = 4 \times 6 + 1 = 25$$

$$d = 1$$

$$\frac{D}{d} = 25$$

14. (b)

$$T^2 = KR^3$$

$$\frac{2\Delta T}{T} = \frac{3\Delta R}{R}$$

$$\frac{2\Delta T}{T} = \frac{3 \times 0.03R}{R}$$

$$\frac{\Delta T}{T} = \frac{3 \times 0.03}{2} \times 100 = 4.5\%$$

15. (b)

$$\frac{1}{f_1} = (1.5-1)\left[\frac{1}{2} - 0\right] \Rightarrow f_1 = 4 \text{ cm}$$

$$\frac{1}{f_2} = \left(\frac{1.5}{1.2} - 1\right)\left(\frac{1}{3} - 0\right)$$

$$\frac{1}{f_2} = \frac{0.3}{1.2} \times \frac{1}{3}$$

$$f_2 = 12$$

$$f_1 : f_2 = 4 : 12 = 1 : 3$$

16. (c)

$$i_{ms} = \sqrt{\frac{\int I^2 dt}{\int dt}}$$

$$\sqrt{\frac{I_A^2 + I_B^2}{2}} = i_{ms}$$

17. (a)

$$\vec{v} = v_0\hat{i} - \frac{E_0 e}{m} t\hat{k}$$

$$|\vec{v}| = \sqrt{v_0^2 + \frac{E_0^2 e^2 t^2}{m^2}}$$

$$\lambda_0 = \frac{h}{mv}$$

$$\lambda' = \frac{h}{mv_0\sqrt{1 + \frac{E_0^2 e^2 t^2}{v_0^2\, m^2}}}$$

$$\lambda' = \frac{\lambda_0}{\sqrt{1 + \frac{E_0^2 e^2 t^2}{v_0^2\, m^2}}}$$

18. (a) $C = \frac{A\epsilon_0}{d}$

A : plate area

d : distance between the plates.

Capacitance initial

$= \frac{\epsilon_0 \ell\, b}{d} = \epsilon_0$ units

Option 'C' $\ell = 6$ cm

$b = 5$ cm

$d = 3$ cm

Capacitance $= 10\epsilon_0$ units

Option 'E' $\ell = 5$ cm

$b = 2$ cm

$d = 1$ cm

Capacitance $= 10\epsilon_0$ units

19. (c) $F = \alpha + \beta x^2$

Work done $= \int F dx$

$$5 = \int (\alpha + \beta x^2) dx$$

$$5 = \alpha x + \frac{\beta x^3}{3}\Big|_0^1$$

$$5 = \alpha + \frac{\beta}{3} [\alpha = 1]$$

$$4 = \frac{\beta}{3} \Rightarrow \beta = 12 N/m^2$$

20. (b) Given that

$P = kT$

$\frac{P}{T} = $ constant

∴ Volume is constant or isochoric process.

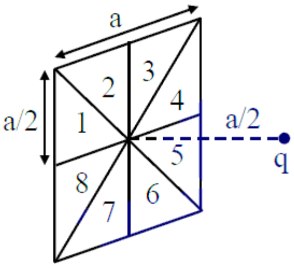
$\therefore W_D = 0$

$\therefore Q = \Delta U$

Also temperature increases hence internal energy increases.

Section – B

21. (48)



Total flux through square $= \frac{q}{\epsilon_0}\left(\frac{1}{6}\right)$

Lets divide square is 8 equal parts.

Flux is same for each part.

$\therefore$ Flux through shaded portion is $\frac{5}{8}$ (Total flux)

$= \frac{5}{8} \times \frac{q}{\epsilon_0}\frac{1}{6} = \frac{5}{48}\frac{1}{\epsilon_0}$

22. (35) Given least count of Screw Gauge $= 0.01$ mm

$$\text{L.C} = \frac{(\text{ pitch })}{\text{No. of circular turn}} = \frac{P}{N} = 0.01 \text{ mm}$$

New pitch $= \frac{P(1+0.75)}{N(1-0.5)} = \frac{P}{N}\left[\frac{1.75}{0.5}\right]$

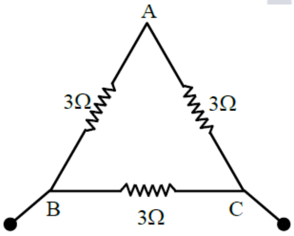
$= (0.01)3.5$

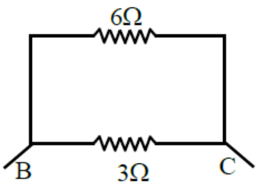
$= 0.035$ mm

$= 35 \times 10^{-3}$ mm

$\therefore$ Ans. is 35

23. (2)



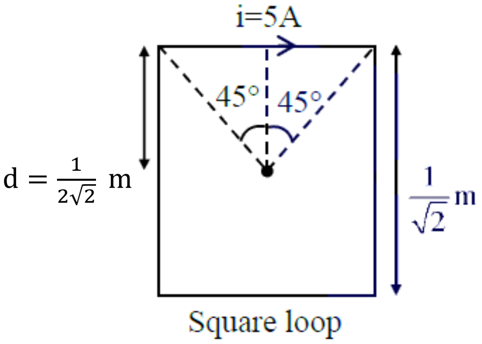


9Ω is the resistance of whole wire

$\therefore$ resistance of each wire $= 3\Omega$.

$\therefore$ Equivalent resistance $= 2\Omega$

24. (8)



Let B be the magnetic field due to single side then $B = \dfrac{\mu_0 i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

$= \dfrac{10^{-7} \times 5 \times 2}{\dfrac{1}{2\sqrt{2}}} \times \dfrac{1}{\sqrt{2}} = 2 \times 10^{-6}$

$\therefore\ B_{net}$ at centre $O = 4\,B$

$= 8 \times 10^{-6}$

$\therefore P = 8$

25. (15) Given that process is isobaric $\Delta T = 50°C$

Q in isobaric process $= nC_p\Delta T = E_1$

ΔU in isobaric process $= nC_v\Delta T = E_2$

$\therefore \dfrac{E_1}{E_2} = \dfrac{C_p}{C_v} = \gamma$

Given, gas is monoatomic

$\therefore \gamma = 1 + \dfrac{2}{f}$

$= 1 + \dfrac{2}{3}$

$= \dfrac{5}{3}$

Now, as per question.

$\dfrac{5}{3} = \dfrac{x}{9}$

$x = 15$

Chemistry
Section - A

26. (b)

$Fe^{2+}(aq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$

$$\begin{tikzcd} Fe \arrow[r, "\Delta G_1^0"] \arrow[rr, bend right, "\Delta G_3^0"'] & Fe^{+2} \arrow[r, "\Delta G_2^0"] & Fe^{+3} \end{tikzcd}$$

$\Delta G_3^0 = \Delta G_1^0 + \Delta G_2^0$

$-3\,F(-z) = -2\,F(-y) + \Delta G_2^0$

$\Delta G_2^0 = 3Fz - 2Fy$

Also $\Delta G_2^0 = -nFE^0_{Fe^{+2}/Fe^{+3}}$

$3Fz - 2Fy = -1\,F\left(E^0_{Fe^{+2}/Fe^{+3}}\right)$

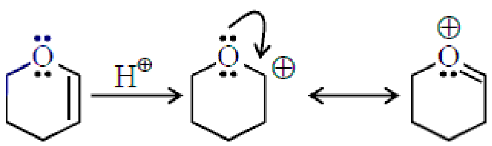
$E^0_{Fe^{+2}/Fe^{+3}} = 2y - 3z$

E^0_{Cell} for reaction will be

$E^0_{Ag^{+}/Ag} + E^0_{Fe^{+2}/Fe^{+3}}$

$= x + 2y - 3z$

27. (b) Addition of $H - Br_{(aq)}$ to alkene follows electrophilic addition mechanism. In the rate determining step a carbocation intermediate is formed. Among P, Q, R & S compound Q will form most stable carbocation intermediate since it is resonance stabilized.



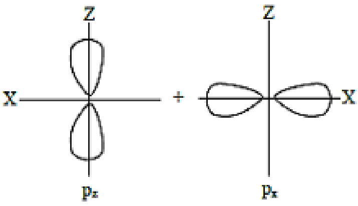
28. (a)

$[Co(NH_3)_5Cl]Cl_2 \rightarrow \underbrace{[Co(NH_3)_5Cl]^{2+}(aq.) + 2Cl^{-}(aq.)}_{\text{3 ions in water}}$

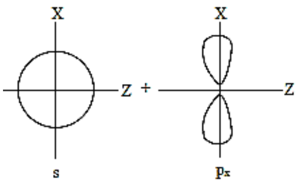
$[Co(NH_3)_5Cl]Cl_2(aq) + .2AgNO_3(aq.) \rightarrow$
$[Co(NH_3)_5Cl](NO_3)_2(aq) + .2AgCl(s)$

29. **(b)** Carbocation intermediate is stabilised by $+I, +M$ & hyperconjugation effect. Since in option 2 carbocation is in conjugation with stronger +M group $-OCH_3$ hence it will be most stable.

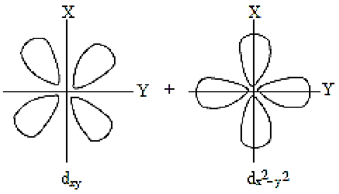
30. **(c)**

(A) 

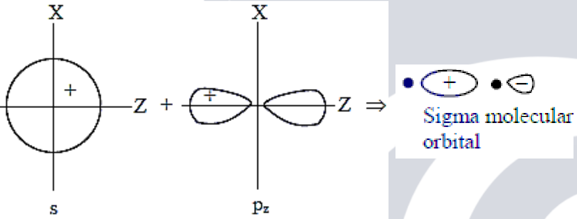
no molecular orbital

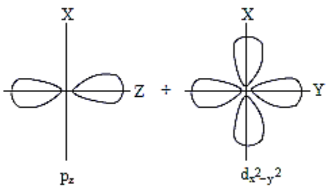
(B) 

$\Rightarrow$ no molecular orbital

(C) 

$\Rightarrow$ no molecular orbital

(D) 

(E) 

$\Rightarrow$ no molecular orbital

31. **(c)** Tb^{4+} is strongest oxidising agent

32. **(a)** $Cr(OH)_{3(s)} \rightleftharpoons Cr^{+3}_{(aq)} + 3OH^{-}_{(aq)}$

At eq : $\quad s \quad 3s$

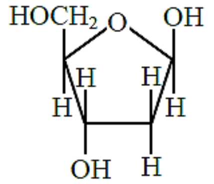
$$K_{sp} = (s) \cdot (3s)^3 = 27s^4$$
$$27s^4 = 1.6 \times 10^{-30}$$
$$s = \left(\frac{1.6}{27} \times 10^{-30}\right)^{1/4}$$

33. **(a)** Reaction is spontaneous at relatively high temperature and non-spontaneous at low temperature $\Delta G = \Delta H - T\Delta S$ It is only possible when ΔH and ΔS both are positive.

34. **(d)** In Dumas method nitrogen present in organic compound is converted into N_2 gas whose volumetric analysis gives the percentage of nitrogen atom in the organic compound.

35. **(a)** Properties of elements are periodic function of their atomic number. Elements having similar outer electronic configuration are arranged in same group. Number of elements in a period is not equal to number of atomic orbitals available in energy level that is being filled.
Hence, A, C & E are incorrect

36. (a) In Ribose carbohydrate present in DNA is $\beta-2-$ Deoxy-D-Ribose whose structure is



which is a reducing D-sugar in β anomeric form & it is a pentose sugar.

37. (d) $MnO_2 \xrightarrow[KNO_3 \Delta]{KOH} K_2MnO_4$

38. (b) Oxygen exists as O_2 (Atomicity $= 2$)

Sulphur exists as S_8(Atomicity $= 8$)

Hence, Melting point & Boiling point of sulphur are significantly large compared to oxygen.

39. (d)

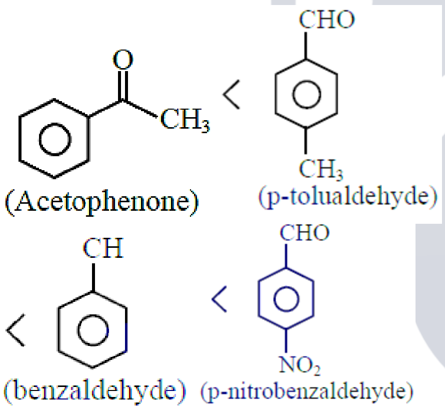
$$N_2O_{5(g)} \longrightarrow 2NO_{2(g)} + \frac{1}{2}O_{2(g)}$$

$$\begin{array}{llll} t=0 & P_0 & - & - \\ t=t & P_0-x & 2x & \frac{x}{2} \end{array}$$

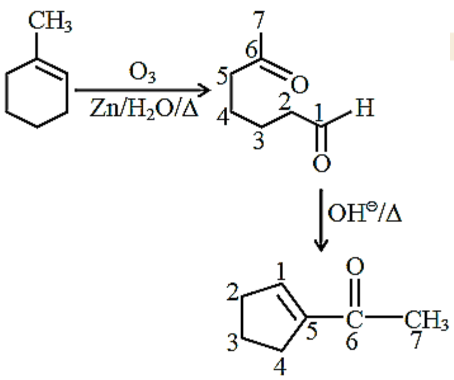
$$x = \frac{P_0}{2}$$

$$P_{total} = P_0 - \frac{P_0}{2} + P_0 + \frac{P_0}{4} = \frac{7}{4}P_0$$

40. (c) The rate of nucleophilic addition decreased due to steric crowding around carbonyl carbon & increased by electron withdrawing group if the steric crowding is same hence the reactivity towards nucleophilic addition will be



41. (a)

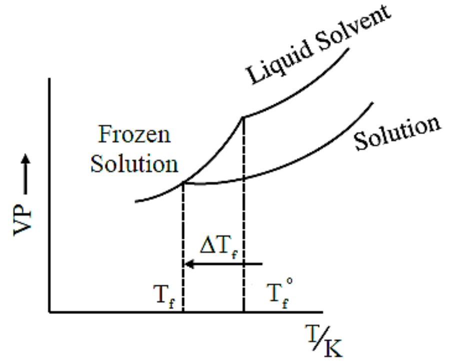


42. (c) On adding non-volatile solute in a solvent, the freezing point of solution decreases.

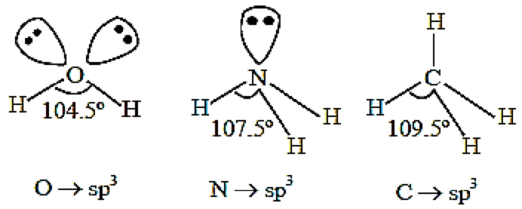
$T_f < T_f^0$

F.P. of solution < F.P. of pure solvent

Also V.P. of solution decreases on adding nonvolatile solute in a solvent.

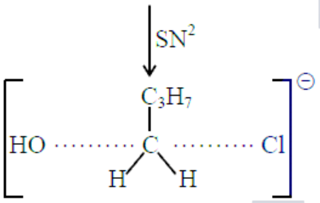


43. (a)

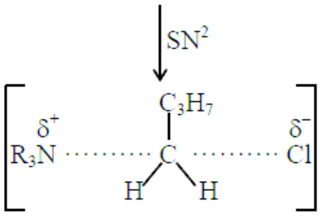


Dipole moment $H_2O > NH_3 > CH_4$
H_2O & NH_3 are Lewis Bases
NH_3 act as Lowry- Bronsted base
Hence, A, B & C are correct

44. (a) $CH_3 - CH_2 - CH_2 - CH_2 - Cl + \overset{\ominus}{O}H$
Reactant (higher charge density)

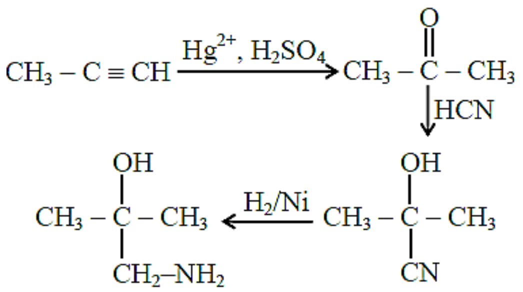


Transition state (less charge density)
⇒ This reaction will proceed faster in less polar medium which will not increase the activation energy value.
$CH_3 - CH_2 - CH_2 - CH_2 - Cl + R_3\,N$
Reactant (low charge density)



Transition state (Higher charge density)
⇒ This reaction will proceed faster in more polar medium which will decrease the activation energy value.

45. (b)



Section – B

46. (962)

Initial pressure of N_2O_5

$$= \frac{\frac{37.8}{108} \times 0.082 \times 500}{1} = 14.35\text{bar}$$

$$2\,N_2O_5 \rightleftharpoons 2\,N_2O_4 + O_2$$

t = 0 $\quad 14.35$

t = eq $\quad 14.35 - 2P \quad 2P \quad P$

P_{Total} at eqb $= 14.35 + P = 18.65$

$P = 4.3$

$P_{N_2O_5} = 5.75\text{bar}$

$P_{N_2O_4} = 8.6\text{bar}$

$P_{O_2} = 4.3\text{bar}$

$$k_p = \frac{(8.6)^2 \times (4.3)}{(5.75)^2} = 9.619 = x \times 10^{-2}$$

$x = 961.9 \approx 962$

47. (700)

$$\frac{1}{2}X_2 + \frac{5}{2}Y_2 \rightleftharpoons XY_5$$

$$\Delta S^0_{Rxn} = 110 - \left[\left(\frac{1}{2} \times 70\right) + \left(\frac{5}{2} \times 50\right)\right]$$

$= 110 - 160 = -50\text{JK}^{-1}\ \text{mol}^{-1}$

$\Delta G^0 = 0$ at eqb

$\Delta G^0 = \Delta H^0 - T\Delta S^0$

$0 = -35000 - T(-50)$

$T = 700\text{Kelvin}$

48. (61)

$C_6H_5COOH + NaHCO_3 \rightarrow C_6H_5COONa^+$
$+ H_2O + CO_2$

x gm $\qquad\qquad$ 11.2 L

mole of $C_6H_5COOH =$ mole of $CO_2 = \frac{11.2}{22.4} = 0.5$

mass of $C_6H_5COOH = x = 0.5 \times 122 = 61\text{gm}$

49. (4)

50. (420) moles of $Fe_3O_4 = \frac{2.32 \times 10^3 \times 10^3}{232} = 10000$ mol

moles of $CO = \frac{2.8 \times 10^2 \times 10^3}{28} = 10000$ mol

$Fe_3O_4 + 4CO \longrightarrow 3Fe + 4CO_2$

10^4 mol 10^4 mol

CO is L.R.

mole of $Fe = \frac{3}{4} \times 10^4$

mass of $Fe = \frac{3}{4} \times \frac{10^4 \times 56}{1000}$ kg $= 420$ kg

Mathematics

Section - A

51. (d)

$\vec{c} = \lambda(\vec{b} \times (\vec{a} \times \vec{b}))$

$= \lambda((\vec{b} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{b})\vec{b})$

$= \lambda(11\vec{a} - 2\vec{b}) = \lambda(11i + 22j + 33k - 6i - 2j + 2k)$

$= \lambda(5i + 20j + 35k)$

$= 5\lambda(5i + 4j + 7k)$

$=$ Given $\vec{c} \cdot \vec{a} = 5$

$= 5\lambda(1 + 8 + 21) = 5 \;= \lambda = \frac{1}{30}$

$\Rightarrow \vec{c} = \frac{1}{6}(i + 4j + 7k)$

$\vec{c} = \frac{\sqrt{1 + 16 + 49}}{6} = \sqrt{\frac{11}{6}}$

52. (d) $I(m, m) = \int_0^1 x^{m-1}(1 - x)^{n-1}dx$

Let $x = \sin^2\theta\ dx = 2\sin\theta\cos\theta d\theta$

$I(m, n) = 2\int_0^{\pi/2} (\sin\theta)^{2m-1}(\cos\theta)^{2n-1}\, d\theta$

$I(9,14) + I(10,13) = 2\int_0^{\pi/2} (\sin\theta)^{17}(\cos\theta)^{27}\, d\theta$

$+2\int_0^{\pi/2} (\sin\theta)^{19}(\cos\theta)^{25}\, d\theta$

$= 2\int_0^{\pi/2} (\sin\theta)^{17}(\cos\theta)^{25}\, d\theta$

$= I(9,13)$

53. (c)

$F(x) - 6f(1/x) = \frac{35}{3x} - \frac{5}{2}$ ….. (1)

Replace $x \to \frac{1}{x}$

$F(1/x) - 6(x) = \frac{35x}{3} - \frac{5}{2}$ ….. (2)

Using (1) & (2)

$f(x) = -2x - \frac{1}{3x} + \frac{1}{2}$

$B = \lim_{x\to 0}\left(\frac{1}{\alpha x} + f(x)\right)$

$= \lim_{x\to 0}\left(\frac{1}{\alpha x} - 2x - \frac{1}{3x} + \frac{1}{2}\right)$

$\alpha = 3,\ B = 1/2$

So, $\alpha + 2B = 3 + 1 = 4$

54. (a)

$Sn = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20}$ ... N terms

$S_{2025} = \sum_{n=1}^{2025} \frac{1}{n(n+1)} = \sum_{n=1}^{2025} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) \dots\dots \left(\frac{1}{2025} - \frac{1}{2026}\right)$

$= \frac{2025}{2026}$

$\sqrt{2026 \cdot S_{2025}} = \sqrt{2025} = 45$

Given : $\frac{6}{2}[-2p + (6-1)p] = 45$

$9p = 45$

$p = 5$

$|A_{20} - A_{15}| = |-5 + 19 \times 5| - [-5 + 14 \times 5]$

$= |90 - 65|$

$= 25$

55. (a) $f(x) = \frac{42^x + 16}{2.2^{2x} + 16.2^x + 32}$

$f(x) = \frac{2(2^x + 4)}{2^{2x} + 8.2^x + 16}$

$f(x) = \frac{2}{2^x + 4}$

$f(4-x) = \frac{2^x}{2(2^x + 4)}$

$f(x) + f(4-x) = \frac{1}{2}$

So,$f\left(\frac{1}{15}\right) + f\left(\frac{59}{15}\right) = \frac{1}{2}$

Similarly $= f\left(\frac{29}{15}\right) + f\left(\frac{31}{15}\right) = \frac{1}{2}$

$f\left(\frac{30}{15}\right) = f(2) = \frac{2}{2^2 + 4} = \frac{2}{8} = \frac{1}{4}$

$\Rightarrow 8\left(29 \times \frac{1}{2} + \frac{1}{4}\right)$

Ans. 118

56. (d) $2z^2 - 32 - 2i = 0$

$2\left(z - \frac{i}{z}\right) = 3$

$\alpha - \frac{i}{\alpha} = \frac{3}{2}$

$\Rightarrow \quad \alpha^2 - \frac{1}{\alpha^2} - 2i = \frac{9}{4}$

$\Rightarrow \quad \alpha^2 - \frac{1}{\alpha^2} - 2i = \frac{9}{4}$

$\Rightarrow \quad \frac{9}{4} + 2i = \alpha^2 - \frac{1}{\alpha^2}$

$\Rightarrow \quad \frac{81}{16} - 4 + 9i = \alpha^4 + \frac{1}{\alpha^4} - 2$

$\Rightarrow \quad \frac{49}{16} + 9i = \alpha^4 + \frac{1}{\alpha^4}$

Similarly

$\Rightarrow \frac{49}{16} + 9i = \beta^4 + \frac{1}{\beta^4}$

$$\Rightarrow \frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}=\frac{\alpha^{15}\left(\alpha^4+\frac{1}{\alpha^4}\right)+\beta^{15}\left(\beta^4+\frac{1}{\beta^4}\right)}{\alpha^{15}+\beta^{15}}$$

$$=\frac{(\alpha^{15}+\beta^{15})\left(\frac{49}{16}+9i\right)}{(\alpha^{15}+\beta^{15})}$$

$$\text{Real } = \frac{49}{16}$$

$$\text{Im } = 9$$

Ans 441

57. (d)

$$\lim_{x\to 0} \operatorname{cosec} x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$$

$$\lim_{x\to 0} \frac{\operatorname{cosec} x(\cos^2 x+3\cos x-\sin x-4)}{\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0} \frac{1}{\sin x}\frac{(\cos^2 x+3\cos x-4)-\sin x}{\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

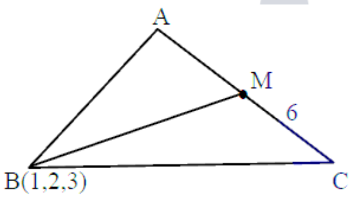
$$\lim_{x\to 0} \frac{(\cos+4)(\cos x-1)-\sin x}{\sin x\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0} \frac{-2\sin^2\frac{x}{2}(\cos x+4)-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}\left(\sqrt{2\cos 2x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0} \frac{-\left(\sin\frac{x}{2}(\cos x+4)+\cos\frac{x}{2}\right)}{\cos\frac{x}{2}\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$-\frac{1}{2\sqrt{5}}$$

58. (b)



Let $M(3\lambda+6, 2\lambda+7, -2\lambda+7)$

$\overrightarrow{BM}=(3\lambda+5)\hat{i}+(2\lambda+5)\hat{j}+(-2\lambda+4)\hat{k}$

$\overrightarrow{AC}\cdot\overrightarrow{BM}=0=3(3\lambda+5)+2(2\lambda+5)-2(-2\lambda+4)$

$\overrightarrow{BM}=2\hat{i}+3\hat{j}+6\hat{k}$

$|\overrightarrow{BM}=7$

$\text{Area } = \frac{1}{2}\times 6\times 7=21$

59. (a)

$$(1+x^2)\frac{dy}{dx}+xy=5x^1\sqrt{1+x^2}$$

$$\frac{dy}{dx}+\frac{xy}{1+x^2}=\frac{5x^2}{\sqrt{1+x^2}}$$

$$\therefore \text{ I.F. } =e^{\int \frac{x}{1+x^2}dx}=e^{\frac{\ln(1+x^2)}{2}}=\sqrt{1+x^2}$$

$$\therefore y\sqrt{1+x^2}=\int \frac{5x^2}{\sqrt{1+x^2}}\cdot\sqrt{1+x^2}dx$$

$$\therefore y\sqrt{1+x^2}=\int \frac{5x^2}{\sqrt{1+x^2}}\cdot\sqrt{1+x^2}dx$$

$$y\sqrt{1+x^2}=\frac{5x^3}{3}+C$$

$$\because y(0)=0\Rightarrow 0=0+C\Rightarrow C=0$$

$$\therefore y=\frac{5x^3}{3\sqrt{1+x^2}}$$

$$y(\sqrt{3})=\frac{15\sqrt{3}}{32}=\frac{5\sqrt{3}}{2}$$

60. (c)

Product of focal distances $=(a+ex_1)(a-ex_1)$

$$=a^2-e^2x_1^2=a^2-e^2(3)$$

$$=a^2-3e^2=\frac{7}{4}\Rightarrow a^2=\frac{7}{4}+3e^2$$

$$\Rightarrow 4a^2=7+12e^2$$

& $\left(\sqrt{3},\frac{1}{2}\right)$ lines on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\therefore \frac{3}{a^2}+\frac{1}{4\,b^2}=1$$

$$\frac{3}{a^2}+\frac{1}{4(a^2)(1-e^2)}=1$$

$$12(1-e^2)+1=4a^2(1-e^2)$$

$$13-12e^2=(7+12e^2)(1-e^2)$$

$$\Rightarrow 13-12e^2=7-7e^2+12e^2-12e^4$$

$$\Rightarrow 12e^4-17e^2+6=0$$

$$\therefore e^2=\frac{17\pm\sqrt{289-288}}{24}=\frac{17\pm 1}{24}=\frac{3}{4}\&\frac{2}{3}$$

$$\therefore e=\frac{\sqrt{3}}{2}\&\sqrt{\frac{2}{3}}$$

$$\therefore \text{ difference } ==\frac{\sqrt{3}}{2}-\sqrt{\frac{2}{3}}=\frac{3-2\sqrt{2}}{2\sqrt{3}}$$

61. (b)

$$p(S_5)=\frac{1}{9}$$

$$p(S_5)=\frac{5}{36}$$

required prob $=\frac{1}{9}+\frac{8}{9}\cdot\frac{31}{36}\cdot\frac{1}{9}+\left(\frac{8}{9}\cdot\frac{31}{36}\right)^2\cdot\frac{1}{9}+\cdots\infty$

$$=\frac{\frac{1}{9}}{1-\frac{62}{81}}=\frac{9}{19}$$

62. (c)

63. (c) $x^2+4x+2 \le y \le |x+2|$

The area bounded between

$y = x^2+4x+2 = (x+2)^2-2$

and $y = |x+2|$ is same as area bounded between $y = x^2-2$ and $y=|x|$

For P.O.I $|x|^2-2=|x|$

$\Rightarrow |x| = 2 \Rightarrow x = \pm 2$

$\therefore$ Required area $= -\int_{-2}^{2} (x^2-2)dx + \int_{-2}^{2} |x|dx$

$= -2\int_0^2 (x^2-2)dx + 2\int_0^2 x \cdot dx$

$= -2\left[\frac{x^3}{3} - 2x\right]_0^2 + 2\left[\frac{x^2}{2}\right]_0^2$

$= --2\left[\frac{8}{3}-4\right] + 2\left[\frac{4}{2}\right]$

$= -2 \times \left(\frac{-4}{3}\right) + 4$

$= \frac{20}{3}$

64. (a) Mean $\overline{x} = 5.5$

$= \sum_{i=1}^{10} x_i = 5.5 \times 10 = 55$

$= \sum_{i=1}^{10} x_i^2 = 371$

$\left(\sum x_i\right)_{new} = 55 - (4+5) + (6+8) = 60$

$\left(\sum x_i\right)_{new} = 371 - (4^2+5^2) + (6^2+8^2) = 430$

Variance $\sigma^2 = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10}\right)^2$

$\sigma^2 = \frac{430}{10} - \left(\frac{60}{10}\right)^2$

$\sigma^2 = 43 - 36$

$\sigma^2 = 7$

65. (d)

66. (b)

$(1+x)^{n+4}$

${}^{n+4}C_4\ {}^{n+4}C_5\ {}^{n+4}C_6, \rightarrow$ A.P.

$\Rightarrow 2 \times {}^{n+4}C_5 = {}^{n+4}C_4 + {}^{n+4}C_6$

$\Rightarrow 4 \times {}^{n+4}C_5 = ({}^{n+4}C_4 + {}^{x+4}C_5) + ({}^{n+4}C_5 + {}^{n+4}C_6)$

$\Rightarrow 4 \times {}^{n+4}C_5 = {}^{n+5}C_5 + {}^{n+5}C_6$

$\Rightarrow 4 \times \frac{(n+4)!}{5!.(n-1)!} = \frac{(n+6)!}{6!\,n!}$

$\Rightarrow 4 = \frac{(n+6)(n+5)}{6n}$

$\Rightarrow n^2 + 11n + 30 = 24n$

$\Rightarrow n^2 - 13n + 30 = 0$

$\Rightarrow n = 3, 10(\text{rejected})$

$\because n \ne 10$

$\therefore$ Largest binomial coefficient in expansion of $(1+x)^7$

$(\because n+4=7)$

is coeff. of middle term

$\Rightarrow {}^7C_4 = {}^7C_3 = 35$

67. (a) $(x^2 - 9x + 11)^2 - (x^2 - 9x + 20) = 3$

Let

$\Rightarrow x^2 - 9x = t$

$\Rightarrow t^2 + 22t + 121 - t - 20 - 3 = 0$

$\Rightarrow t^2 + 21t + 98 = 0$

$\Rightarrow (t + 14)(t + 7) = 0$

$\Rightarrow t = -7, -14$

So,

$x^2 - 9x = -7, -14$

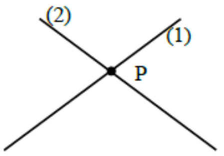
$x^2 - 9x + 7 = 0$ or $x^2 - 9x + 14 = 0$

$x = \dfrac{9 \pm \sqrt{81 - 4(7)}}{2 \times 1}$ $\qquad x = \dfrac{9 \pm \sqrt{81 - 4(14)}}{2}$

$= \dfrac{9 \pm \sqrt{53}}{2}$ $\qquad = \dfrac{9 \pm 5}{2}$

Product of all rational roots $= 7 \times 2 = 14$

68. (d) Equation of line through point $(-1,2,1)$ is $\rightarrow$



$\Rightarrow \dfrac{x+1}{2} = \dfrac{y-2}{3} = \dfrac{z-1}{4} - (2) = \lambda$

So, $\begin{bmatrix} x = 2\lambda - 1 \\ y = 3\lambda + 2 \\ z = 4\lambda + 1 \end{bmatrix}$

By(1) $\rightarrow \frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1} = \mu$(Let)

So, $\begin{bmatrix} x = 3\mu - 2 \\ y = 2\mu + 3 \\ z = \mu + 4 \end{bmatrix}$

For intersection point ' P '

$x = 2\lambda - 1 = 3\mu - 2$

$y = 3\lambda + 2 = 2\mu + 3$ $\begin{bmatrix} \lambda = 1 \\ \mu = 1 \end{bmatrix}$

$z = 4\lambda + 1 = \mu + 4$

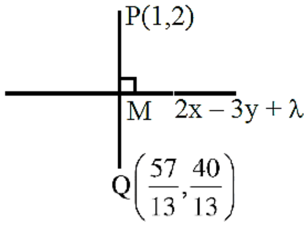
So, point $P(x, y, z) = (1,5,5)$

$\&Q(4, -5,1)$

$\therefore PQ = \sqrt{9 + 100 + 16}$

$= \sqrt{125} = 5\sqrt{5}$

69. (b)



$\because PM = QM$

So, $M\left(\dfrac{\frac{57}{13}+1}{2}, \dfrac{\frac{-40}{13}+2}{2}\right)$

$= \left(\frac{35}{13}, \frac{-7}{13}\right)$

$\because$ M lies on the time

$2x - 3y + \lambda = 0$

$2\left(\frac{35}{13}\right) - 3\left(\frac{-7}{13}\right) + \lambda = 0$

$\lambda = -\frac{70}{13} + \frac{21}{13}$

$= \frac{-91}{13} = -7$

$\begin{vmatrix} 3 & -4 & -\alpha \\ 8 & -11 & -33 \\ 2 & 3 & \lambda \end{vmatrix} = 0$

$\Rightarrow 3(-11\lambda - 99) + 4(8\lambda + 66) - \alpha(-24 + 22) = 0$

$\Rightarrow 33\lambda - 297 + 32\lambda + 264 + 24\alpha - 22\alpha = 0$

$\Rightarrow -\lambda + 2\alpha - 33 = 0 \quad \ldots..(1)$

$\therefore \lambda = -7$

$-(-7) + 2\alpha - 33 = 0$

$2\alpha = 26$

$\alpha = 13$

$\therefore |\alpha\lambda| = |13 \times (-7)|$

$= 91$

70. (d)

$\Delta = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ 5 & \lambda & 3 \\ 100 & -47 & \mu \end{vmatrix} = 0$

$2(\lambda\mu + 141) + (5\mu - 300) - 235 - 100\lambda = 0 \ldots$

$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 4 \\ 5 & \lambda & 12 \\ 100 & -47 & 212 \end{vmatrix} = 0$

$6\lambda = -12 \Rightarrow \lambda = -2$

Put $\lambda = 2$ in (1)

$2(-2\mu + 141) + 5\mu - 300 - 235 + 200 = 0$

$\mu = 53$

$\therefore 57$

Section - B

71. (19) Differentiate both sides

$4(x+2)f(x) + 2(x+2)^2 f'(x) - 6(x+2) = 10(x+2)f(x)$

$2(x+2)^2 f'(x) - 6(x+2)f(x) = 6(x+2)$

$(x+2)\frac{dy}{dx} - 3y = 3$

$\int \frac{dy}{dx} = 3\int \frac{dx}{x+2}$

$\ln(y+1) = 3\ln(x+2) + C$

$(y+1) = C(x+2)^3$

$f(0) = \frac{3}{2}$

$f(2) = 19$

72. (14)

If $\left(\tan(\tan^{-1}(\alpha)) + 1\left(\cot(\cot^{-1}\beta)\right)^2 = 36\right.$

$\alpha^2 + \beta^2 = 34$

$\alpha\beta = 15$

$\alpha = 3, \beta = 5$

$\therefore \alpha^2 + \beta = 9 + 5 = 14$

73. (125) No, of 3 digits $= 999 - 99 = 900$

No. of 3- digit numbers divisible by 2&3 i.e. by 6

$\frac{900}{6} = 150$

No. of 3 -digit numbers divisible by 4&9 i.e. by 36

$\frac{900}{36} = 25$

No of 3 -digit numbers divisible by 2&3 but not by 4&9

$150 - 25 = 125$

74. (44)

75. (5120)

Let $\frac{y}{x} = \lambda$

$y = \lambda x$

$= 10 \times ({}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + \cdots . + {}^9C_9)$

$= 10 \times (2^9)$

10×512

5120