Physics
Section - A

1. (a) $\beta = \left(\frac{\lambda_0}{\mu}\right) \times \frac{D}{d} = \frac{690\times10^{-9}\times0.72}{1.44\times1.5\times10^{-3}} = 0.23$ mm

2. (c)

γ-rays	X-rays	U.V rays	Visible rays	IR rays	Micro waves	Radio waves

$\xrightarrow{\quad\quad\quad}$
$\lambda\uparrow$

3. (b)

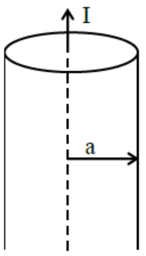
$\bar{F} = q(\bar{V} \times \bar{B})$
$\bar{F} = 0$
$\bar{V} \parallel \bar{B}$
$\theta = 0$ or 180

4. (b)

$$\frac{\text{Linear KE}}{\text{RotationalK.E}} = \frac{\frac{1}{2}mv_{cm}^2}{\frac{1}{2}I\omega^2}$$

$$\frac{mv_{cm}^2}{\frac{2}{5}mR^2\omega^2} = \frac{5}{2}\ (V = \omega R)$$

5. (a)



$$B_{in} = \frac{\mu_0(\bar{J} \times \bar{r})}{2}$$
$$B_{in} \propto r$$
$$B_{out} = \frac{\mu_0 I}{2\pi r}$$
$$B_{net} \propto \frac{1}{r}$$

6. (c)

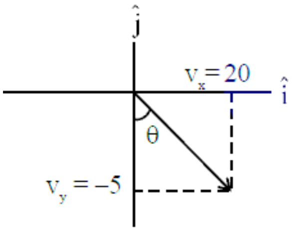
(A) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$
Temp increases
R) Free expansion is assumed fast, so Adiabatic

7. (d)

$$U_1 = \frac{4Kq_0^2}{a} + \frac{2Kq_0^2}{\sqrt{2}a} = \frac{Kq_0^2}{a}(4+\sqrt{2})$$
$$U_2 = \frac{Kq_0^2}{\left(\frac{a}{\sqrt{2}}\right)}(4+\sqrt{2}) = \frac{Kq_0^2}{a}(4\sqrt{2}+2)$$
$$U_2 - U_1 = \frac{Kq_0^2}{a}(3\sqrt{2}-2)$$

8. (c)

$\vec{r} = 5t^2\hat{i} - 5\hat{t}$
$\vec{v} = 10t\hat{i} - 5\hat{j}$
$\vec{v} = 20\hat{i} - 5\hat{j}$ at t = 2sec

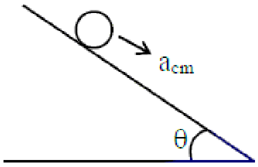


$$\tan\theta = \frac{20}{5} = 4$$

$$\theta = \tan^{-1} 4$$

From-veY-axis

9. (a)



$$t = \sqrt{\frac{2\ell}{a_{cm}}}$$

$$a_{cm} = \frac{g\sin\theta}{1 + \frac{I_{cm}}{MR^2}}$$

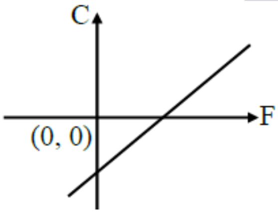
$$a_1 = a_{cm_1} = \frac{5\,g\sin\theta}{7} \text{ ..... Solid}$$

$$a_2 = a_{cm_2} = \frac{3\,g\sin\theta}{5} \text{ ..... Hollow}$$

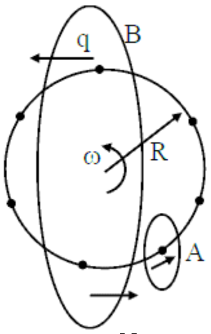
$$a_1 > a_2$$

$$t_1 < t_2$$

10. (b) $\frac{C}{5} = \frac{F-32}{9} \Rightarrow C = \frac{5F}{9} - \frac{160}{9}$



11. (c)



$$I_A = \frac{Nq}{\frac{2\pi}{\omega}}$$

$$I_A = \frac{Nq\omega}{2\pi}, I_B = 0$$

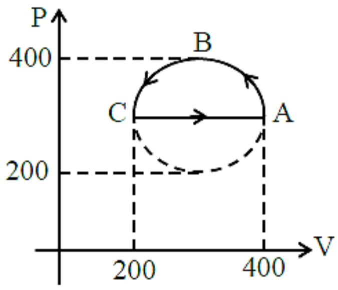
$$I_A - I_B = \frac{Nq\omega}{2\pi}$$

12. (c)

$hv = \phi + KE_{max}$
$KE_{max} = eV_0$
$V_0 = \frac{hv - \phi}{e}$

(A) $V_0 v/sV$ is linear correct
(B) Slope
$v_0 = \left(\frac{h}{e}\right)v - \frac{\phi}{e}$ Wrong
Slope $\frac{h}{e}$
(C) Correct (D) Incorrect (E) Correct

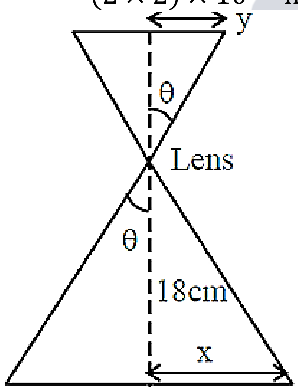
13. (b)



$W = \frac{1}{2}\pi R^2$
$= \frac{1}{2} \times \pi \times \left(\frac{200}{2} \times 10^3\right) \times \frac{200}{2} \times 10^{-6}$
$= \frac{10\pi}{2} = 5\pi\ J$

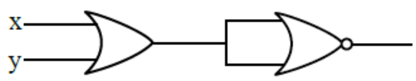
14. (a)

$(2 \times 2) \times 10^{-4}\ m^2$



$400 \times 10^6\ m^2$
$H = 18\ km$
Size of camera film $= 2\ cm \times 2\ cm$
$A_{fimage} = 400\ km^2$
$x = 20 \times 10^3\ m = 2 \times 10^4\ m$
$y = 2 \times 10^{-2}\ m$
$\frac{x}{y} = 10^6 = \frac{18Km}{f}$
$f = 18 \times 10^{-3}\ m = 18\ mm$
$f = 1.8\ cm$

15. (c)



x	y	

0	0	1
0	1	0
1	0	0
1	1	0

16. (c)

$$\frac{T_2 - T_1}{t} = K[T_{avg} - T_s]$$

$$T_1 = 24°C; T_2 = 40°C, t = 4,\ T_s = 16°C$$

$$\frac{40-24}{4} = K[32-16]$$

$$K == \frac{4}{16} = \frac{1}{4}$$

$$\text{Now } \frac{24-T}{4} = K\left[\frac{T+24}{2} - 16\right]$$

$$24 - T = \frac{T-16}{2} + 16$$

$$\frac{3\,T}{2} = 28$$

$$T = \frac{56}{3}C$$

17. (d)

$$[E] = M^1\ L^2\ T^{-2}$$

$$[Pc] = M^1\ L^1\ T^{-1} \cdot\ L^1\ T^{-1} = M^1\ L^2\ T^{-2}$$

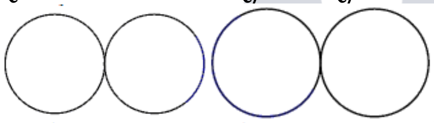
$$[mc^2] = M^1\ L^2 T^{-2}$$

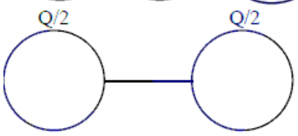
$$E^2 = M^1\ L^2\ T^{-2}$$

$$E^2 = P^2c^2 + m^2c^4$$

18. (a)

$Q = 4 \times 10^{-8}$ $Q/2$ $Q/2$





$$F = \frac{k\left(\frac{\theta}{2}\right)\left(\frac{\theta}{2}\right)}{r^2}$$

$$9 \times 10^{-3} = \frac{9 \times 10^9 \times (4 \times 10^{-8}) \times 4 \times 10^{-8}}{4 \times r^2}$$

$$r^2 = \frac{9 \times 10^9 \times 16 \times 10^{-16}}{4 \times 9 \times 10^{-3}} = 4 \times 10^{-4}$$

$$r = 2 \times 10^{-2}\ m \Rightarrow 2\ cm$$

19. (a)

$$x = x_0 \sin^2 \frac{t}{2} = x_0 \left(\frac{1 - \cos t}{2}\right)$$

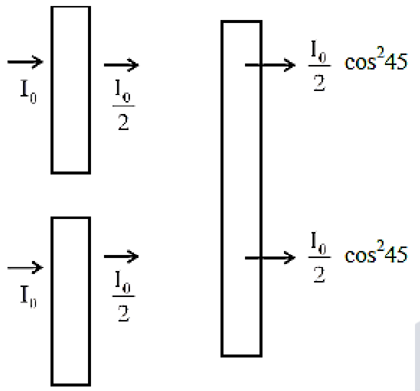
$$x - \frac{x_0}{2} = \frac{-\cos t}{2}$$

where $x_0 = 1$

$$x - \frac{1}{2} = \frac{-\cos t}{2}$$

Particle is oscillating between $x = 0$ to $x = 1$

20. (b)



(Unpolerised light) (Polerised light)

after passing through third poleriser, Intensity of both the waves must be $\frac{I_0}{4}$

now, at a point where path diff is $\frac{\lambda}{3}$, phase difference

$$\Delta\phi = 2\,K\left(\frac{\Delta x}{\lambda}\right) = \frac{2\pi}{3}$$

$$\therefore I_{res} = \sqrt{\left(\frac{I_0}{4}\right)^2 + \left(\frac{I_0}{4}\right)^2 + 2\left(\frac{I_0}{4}\right)^2 \cos\frac{2\pi}{3}}$$

$$= \frac{I_0}{4}$$

Section - B

21. (250) Since time period of a revolving charge is $\frac{2\pi m}{qB}$

Where B = magnetic field

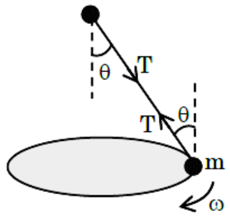
due to a solenoid $= \mu_0 nI$

$$\therefore T = \frac{2\pi\, m}{q(\mu_0 nI)}$$

$$75 \times 10^{-9} = \frac{(2\pi)(9 \times 10^{-31})}{1.6 \times 10^{-19} \times 4\pi \times 10^{-7} \times n \times 1.5}$$

$N = 250$

22. (36)



$T\cos\theta = mg$ ..... (1)

$T\sin\theta = M\omega^2 R$ ... (2)

Using equation (2)

$T\sin\theta = M\omega^2(L\sin\theta)$

$$T = M\omega^2\,L = M\left(\frac{3}{\pi} \times 2\pi\right)^2\,L$$

$T = 36ML$

23. (5) Since power emitting by a source is given as

$$= \frac{\text{Total energy emitted}}{\text{time}}$$

$$= \frac{(E_1 \text{ photon }) \times \text{ Number of photons (N)}}{t}$$

$$P_1 = (E_1)n$$

$$\frac{P_1}{P_2} = \frac{(E_1)n_1}{(E_2)n_2} = \frac{\left(\frac{hC}{\lambda_1}\right)n_1}{\left(\frac{hC}{\lambda_2}\right)n_2}$$

$$\frac{P_1}{P_2} = \left(\frac{\lambda_2}{\lambda_1}\right)\frac{n_1}{n_2}$$

Substituting the given values

$$2 = \left(\frac{300}{600}\right) \times \frac{2 \times 10^{15}}{n_2}$$

$$n_2 = \frac{1}{2} \times 10^{15} = 5 \times 10^{14} \text{ Photon /sec}$$

24. (43) Since bulk modulus is given as

$$B = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)}$$

$$2.15 \times 10^9 = \frac{-\Delta P}{-\left(\frac{0.2}{100}\right)}$$

$$\Delta P = 2.15 \times 10^9 \times 2 \times 10^{-3}$$

$$= 4.3 \times 10^6 = 43 \times 10^5 \text{ N/m}^2$$

25. (9) acceleration due to gravity on surface is given by

$$g = \frac{GM}{R_e^2}$$

Now since diameter is reduced to $1/3^{rd}$, radius also reduces to $1/3^{rd}$, keeping mass constant

New value of acceleration due to gravity on Earth's surface is

$$g' = \frac{GM}{\left(\frac{R_e}{3}\right)^2} = 9\frac{GMe}{R_e^2} = 9 \text{ g}$$

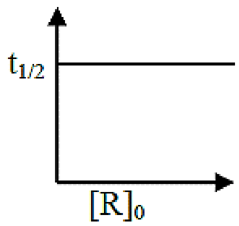
Chemistry
Section - A

26. (b) For strongest reducing agent

Reduction potential should be lowest

Hence Cr is the strongest reducing agent.

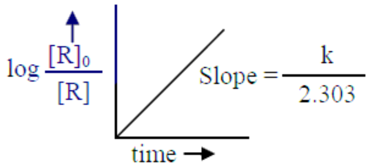
27. (d) For first order reaction $t_{1/2} = \frac{\ell n2}{k}$



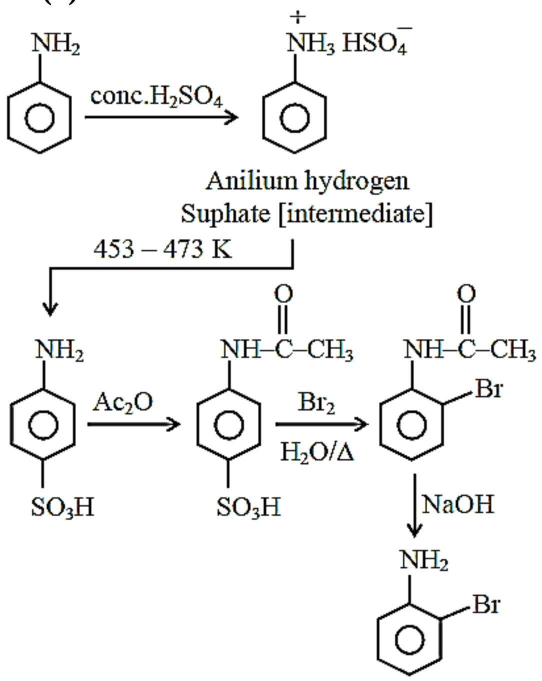
For first order reaction

$$\log\frac{[R]_0}{[R]} = \frac{1}{2.303}kt$$

$$\log\frac{[R]_0}{[R]} = \left(\frac{k}{2.303}\right) \times t$$



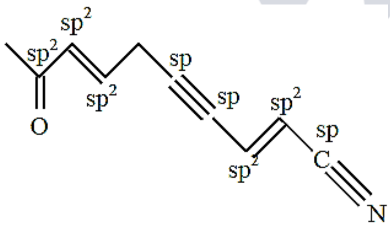
28. (b)



29. (d) In hydrogen atom the orbitals in a shell are degenerate means energy depends only on ' n '

$\therefore E_{3p_s} = E_{3\,d_{x^2-y^2}} = E_{3\,d_{z^2}}$

30. (b)



Number of sp and sp^2 hybridised carbon atom are 3 and 5 .

31. (a) Higher the number of milli moles of acid or base reacted higher will be temperature rise.

Option (4) n_{acid} or n_{base} reacted $= 30$ m mol

Option (2) n_{acid} or n_{base} reacted $= 30$ m mol

but less energy will be released by neutralisation reaction of weak acid hence option (2) can not be correct.

Option (3) $\Rightarrow 20$ m mol

Option (4) $\Rightarrow 25$ m mol

32. (a) Due to resonance bond length is identical in ozone. Therefore statement I is true and statement II is false

33. (b)

34. (b) $H_2 + I_2 \rightleftharpoons 2HI$

Concentration of H_2 and I_2 decreases untill equilibrium condition and concentration of HI increases till equilibrium condition and after equilibrium concentration of all the reactant and products remain constant.

35. (b)

$Sc^{+3} = 3\,d^{\circ} \quad \therefore \mu_{spin} = 0$

$V^{+2} = 3\,d^3 \quad \therefore \mu_{spin} = 3.87$ B.M.

$Ni^{+2} = 3\,d^8 \quad \therefore \mu_{spin} = 2.84$ B.M.

$Ti^{+3} = 3\,d^1 \quad \therefore \mu_{spin} = 1.73$ B.M.

36. (c)

C	:	H	:	O
$\frac{54.2}{12}$	:	9.2	:	$\frac{36.6}{16}$
4.516	:	9.2	:	2.287
$\frac{4.516}{2.287}$	:	$\frac{9.2}{2.287}$	:	$\frac{2.287}{2.287}$
1.97	:	4.02	:	1

$C_2H_4O \Rightarrow$ Empirical formula
E.F. mass $= 24 + 4 + 16 = 44$
and molar mass $= 132$
Hence molecular formula $= (C_2H_4O)_3$
$= (C_6H_{12}O)_3$

37. (c)
$[Ni(H_2O)_6]^{+2}_{(aq)} + en_{(aq)} \rightarrow [Ni(H_2O)_4(en)]^{+2}{}_{(aq)} + 2H_2O$
Green
$[Ni(H_2O)_4(en)]^{+2}_{(\,(aq)} + en_{(aq)} \rightarrow [Ni(H_2O)_2(en)_2]^{+2}_{(aqq)} + 2H_2O$
Pale Blue
$[Ni(H_2O)_2(en)]^{+2}_{(aq)} + en_{(aq)} \rightarrow [Ni(en)_3]^{+2}_{(aq)} + 2H_2O$
Blue / purple

38. (a) For $3d^4$
If ligand is SFL : $t_{2\,g}{}^{4}e_g{}^{0}$ (Low spin)
If ligand is WFL : $t_{2\,g}{}^{3}e_g{}^{1}$ (High spin)

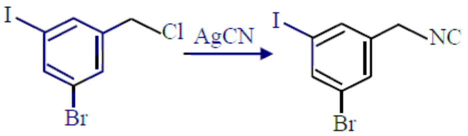
39. (a) (B) -CN is meta directing But -OH is ortho / pera directing.
(E) Halides are deactivating groups.

40. (a) Order of I.E. (in KJ/mol) :
$C > Si > Ge > Sn < Pb$
1086 786 761 708 715

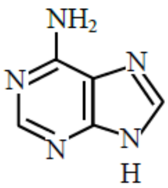
41. (b)
$$\underset{(g)}{SO_2} + \underset{(g)}{\frac{1}{2}O_2} \longrightarrow \underset{(g)}{SO_3} \quad \Delta H = -y$$
$\Delta H_r = (\Delta H_f)_{SO_3} - (\Delta H_f)_{SO_2}$
$-y = -2x - (\Delta H_f)_{SO_2}$
$(\Delta H_f)_{SO_2} = y - 2x$

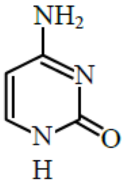
42. (a)

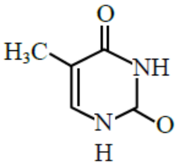
43. (b)

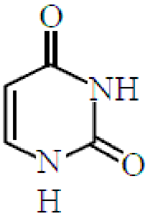


44. (a)

(A) Adenine (III) 

(B) Cytosine (IV) 

(C) Thymine (II) 

(D) Uracil (I) 

45. (b) The IE_4 is suddenly very high therefore element must have 3 valence $e^-(s)$ and it belong to group 13

Section - B

46. (75)

$MX_2 \rightarrow M^{+2} + 2X^-$

$i = \dfrac{\text{normal molar mass}}{\text{observed molar mass}}$

$i = \dfrac{164}{65.6}$

$1 + (3-1)\alpha = \dfrac{164}{65.6}$

$2\alpha = \dfrac{98.4}{65.6}$

$\alpha = 0.75$

percent dissociation $= 75\%$

47. (4)

n(stereogenic unit) $= 2, 2^2 = 4$ stereoisomers are possible.

48. (20)

$$K = \sqrt{\frac{K_1\,K_3}{K_2}}$$

$$A \cdot e^{-Ea/RT} = \sqrt{\frac{A_1 e^{-Ea/RT} \times A_3 e^{-Ea/RT}}{A_2 e^{-Ea_2/RT}}}$$

By comparinig exponential term

$$\frac{E_a}{RT}=\frac{1}{2}\times\left(\frac{E_{a_1}}{RT}+\frac{E_{a_3}}{RT}-\frac{E_{a_2}}{RT}\right)$$
$$E_a=\left(E_{a_1}+E_{a_3}-E_{a_2}\right)/2$$
$$E_a=(60+10-30)/2=20\text{ kJ mol}^{-1}$$

49. (3) Mass of carbon $=\frac{80\times 90}{100}=72\text{gm}$

Number of C-atoms $=\frac{72}{12}=6$

Mass of hydrogen $=\frac{80\times 10}{800}=8\text{gm}$

Number of H -atoms $=\frac{8}{1}=8$

So molecular formula C_6H_8

D.U. $=6+1-8/2=7-4=3$

50. (255)

$$\%\text{ Bromine }=\frac{\text{Molar Mass of Bromine}}{\text{Molar Mass of Silver bromide}}\times\frac{\text{Weight of AgBr}}{\text{Weight of sample}}\times 100$$
$$=\frac{80}{188}\times\frac{0.165}{0.25}\times 100$$
$$=\frac{4800}{188}=25.53=255\times 10^{-1}$$

Mathematics

Section - A

51. (a) Equation of chord with given middle point

$$T=S_1$$
$$\Rightarrow\frac{3x}{25}+\frac{y}{16}-1=\frac{9}{25}+\frac{1}{16}-1$$
$$48x+25y=144+25$$
$$48x+25y=169\text{ Ans}$$

52. (a)

$$f(x)=\frac{2^{2x}-1}{2^{2x}+1}$$
$$=1-\frac{2}{2^{2x}+1}$$
$$f'(x)=\frac{2}{(2^{2x}+1)^2}\cdot 2\cdot 2^{2x}\cdot\ln 2\text{ i.e always }+\text{ve}$$

so $f(x)$ is ↑ function

$$\therefore f(-\infty)=-1$$
$$f(\infty)=1$$

$\therefore f(x)\in(-1,1)\neq$ co-domain

so function is one-one but not onto

53. (d)

$$\Rightarrow\cot^{-1}\left(\frac{\alpha\beta+1}{\alpha-\beta}\right)+\cot^{-1}\left(\frac{\beta\gamma+1}{\beta-\gamma}\right)+\cot^{-1}\left(\frac{\alpha\gamma+1}{\gamma-\alpha}\right)$$
$$\Rightarrow\tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right)+\pi+\tan^{-1}\left(\frac{\gamma-\alpha}{1+\gamma\alpha}\right)$$
$$\Rightarrow(\tan^{-1}\alpha-\tan^{-1}\beta)+(\tan^{-1}\beta-\tan^{-1}\gamma)+(\pi+\tan^{-1}\gamma-\tan^{-1}\alpha)$$
$$\Rightarrow\pi$$

54. (c)

$$x^2 f'(x) - 2x f(x) = 3$$

$$\left(\frac{x^2 f'(x) - 2x f(x)}{(x^2)^2}\right) = \frac{3}{(x^2)^2}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{f(x)}{x^2}\right) = \frac{3}{x^4}$$

Integrating both sides

$$\frac{f(x)}{x^2} = -\frac{1}{x^3} + C$$

$$f(x) = -\frac{1}{x} + Cx^2$$

put $x = 1$

$$4 = -1 + C \Rightarrow C = 5$$

$$f(x) = -\frac{1}{x} + 5x^2$$

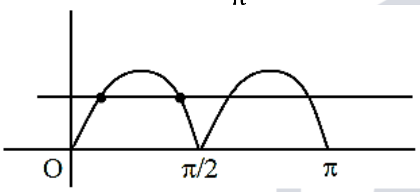
Now $2 \times f(2) = 2 \times \left[-\frac{1}{2} + 5 \times 2^2\right] = 39$

55. (c)

A: $\log_{2\pi}|\sin x| + \log_{2\pi}|\cos x| = 2$

$\Rightarrow \log_{2\pi}(|\sin x \cdot \cos x|) = 2$

$\Rightarrow |\sin 2x| = \frac{8}{\pi^2}$



Number of solution 4

B : let $\sqrt{x} = t < 2$

Then $\sqrt{x}(\sqrt{x} - 4) + 3(\sqrt{x} - 2) + 6 = 0$

$\Rightarrow t^2 - 4t + 3t - 6 + 6 = 0$

$\Rightarrow t^2 - t = 0, t = 0, t = 1$

$x = 0, x = 1$

again let $\sqrt{x} = t > 2$

then $t^2 - 4t - 3t + 6 + 6 = 0$

$\Rightarrow t^2 - 7t + 12 = 0$

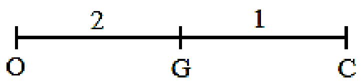
$\Rightarrow t = 3,4$

$x = 9,16$

Total number of solutions

$n(A \cup B) = 4 + 4 = 8$

56. (a) We know that



O (orthocentre) $\frac{\vec{p} + \vec{q} + \vec{r}}{4}$

C (circum centre) $\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}$

C(centroid) $= \frac{\vec{p} + \vec{q} + \vec{r}}{3}$

by relation

$$\Rightarrow 2(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r})+\frac{\vec{p}+\vec{q}+\vec{r}}{4}=3\left(\frac{\vec{p}+\vec{q}+\vec{r}}{3}\right)$$

$$\Rightarrow 8(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r})=3(\vec{p}+\vec{q}+\vec{r})$$

$$\Rightarrow 8\alpha=3, 8\beta=3, 8\gamma=3$$

$$\alpha=\frac{3}{8},\beta=\frac{3}{8},\gamma=\frac{3}{8}$$

$$\therefore \alpha+2\beta+3\gamma$$

$$\frac{3}{8}+\frac{6}{8}+\frac{15}{8}=\frac{24}{8}=3$$

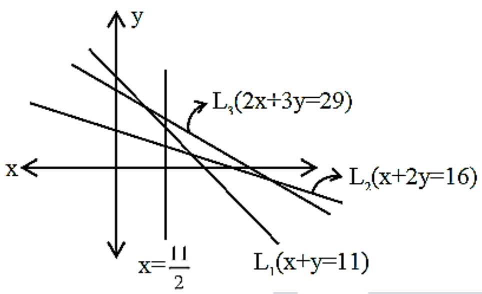
57. (c) $f(x)=[x]+|x-2| \quad -2<x<3$

$$f(x)=\begin{cases}-x, & -2<x<-1\\ -x+1, & -1\le x<0\\ -x+2, & 0\le x<1\\ -x+3, & 1\le x<2\\ x, & 2\le x<3\end{cases}$$

So $f(x)$ is not continuous at 4 points and not differentiable at 4 point

So $m+n=4+4=8$

58. (c)



Point of intersection of $x=\frac{11}{2}$ with L_1&L_3 gives, $\alpha_{min}=\frac{11}{2}$

and $\alpha_{max}=6$

$$\therefore \alpha_{min}\cdot\alpha_{max}=\frac{11}{2}\times 6=33$$

59. (b) Let a& d are first term and common diff of an AP.

$$S_{40}=\frac{40}{2}[2a+39d]=1030 \quad \ldots\ldots(1)$$

$$S_{12}=\frac{12}{2}[2a+11d]=57 \quad \ldots\ldots(2)$$

by (1) & (2)

$$a=-\frac{7}{2}\ d=\frac{3}{2}$$

$$\therefore S_{30}-S_{10}=\frac{30}{2}[2a+29d]-\frac{10}{2}[2a+9d]$$

$$=20a-390d$$

$$=515$$

60. (c)

$$\text{Let } S=5+\frac{1}{7}(5+\alpha)+\frac{1}{7^2}(5+2\alpha)+\cdots$$

$$\frac{1}{7}S=\frac{1}{7}(5)+\frac{1}{7^2}(5+\alpha)+\cdots\infty$$

$$\frac{6}{7}(S)=5+\frac{1}{7}\alpha\left(\frac{1}{1-\frac{1}{7}}\right)$$

$$6=5+\frac{\alpha}{6}\Rightarrow\alpha=6$$

61. (d)

$D=\begin{vmatrix}1 & 2 & -3\\ 2 & \lambda & 5\\ 14 & 3 & \mu\end{vmatrix}=0, \lambda\mu+42\lambda-4\mu+107=0$

$D_1=2\lambda\mu+99\lambda-10\mu+255$

$D_2=13-\mu$

$D_3=5\lambda+5$

$D_2=0 \Rightarrow \mu=13 \& D_3=0 \Rightarrow \lambda=-1$

check & verify for these values $D \& D_2=0$

62. (b)

$$f'(x)=\frac{2}{x-2}-2x+a\geq 0$$

$$f''(x)=\frac{-2}{(x-2)^2}-2<0$$

$f'(x)\downarrow$

$f'(3)\geq 0$

$2-6+a\geq 0$

$a\geq 4$

$a_{min}=4$

$g(x)=(x-1)^3(x+2-a)^2$

$g(x)=(x-1)^3(x-2)^2$

$g'(x)=(x-1)^3 2(x-2)+(x-2)^2 3(x-1)^2$

$=(x-1)^2(x-2)(2x-2+3x-6)$

$=(x-1)^2(x-2)(5x-8)<0$

$$x\in\left(\frac{8}{5},2\right)$$

$$100(a+b-c)=100\left(4+\frac{8}{5}-2\right)=360$$

63. (b)

$A={}^{2n-1}C_{29}\quad B={}^{2n-1}C_{11}$

$2\,{}^{2n-1}C_{29}=5\,{}^{2n-1}C_{11}$

$$2\frac{(2n-1)!}{29!\,(2n-30)!}=5\frac{(2n-1)!}{(2n-12)!\,11!}$$

$$\frac{1}{29\ldots12\cdot5}=\frac{1}{(2n-12)(2n-13)\ldots(2n-29)2}$$

$$\frac{1}{30\cdot29\ldots12}=\frac{1}{(2n-12)(2n-13)\ldots(2n-29)12}$$

$2n-12=30$

$n=21$

64. (c)

$\vec{b}=\vec{a}\times(\hat{\imath}-3\hat{k})$

$$=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 3 & -1 & 2\\ 1 & 0 & -2\end{vmatrix}=2\hat{\imath}+8\hat{\jmath}+\hat{k}$$

$\vec{c}=\vec{b}\times\hat{k}=8\hat{\imath}-2\hat{\jmath}$

$\vec{c}-2\hat{\jmath}=8\hat{\imath}-4\hat{\jmath}$

Projection of $(\hat{\imath}-2\hat{\jmath})$ on $\vec{a}$

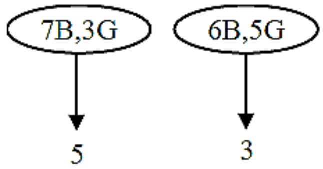
$$(\vec{c}-2\hat{\jmath})\cdot\hat{a}=\frac{\langle 8,-4,0\rangle\cdot\langle 3,-1,2\rangle}{\sqrt{14}}$$

$$=\frac{28}{\sqrt{14}}=2\sqrt{14}$$

65. (d)

$$\lim_{x\to 0} f(x) = \begin{vmatrix} a+1 & 1 & b \\ a & 1+1 & b \\ a & 1 & b+1 \end{vmatrix}$$
$= (a+1)(2(b+1)-b) + 1(ab - a(b+1)) + ba$
$= (a+1)(b+2) - a + ab$
$= b + a + 2 = \lambda + \mu a + vb$
$\lambda = 2, \mu = 1, v = 1 \Rightarrow (\lambda + \mu + v)^2 = 16$

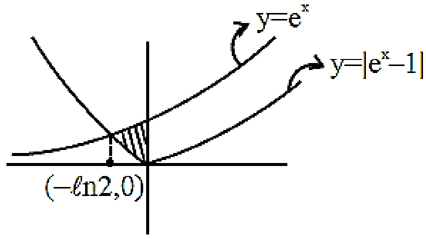
66. (c)



C-I (3G&2 B)&(1G&2 B)
C-II (2G&3 B)&(2G&1 B)
C-III (1G&4 B)&(3G&0 B)
Total $= C - I + C - II + C - III$

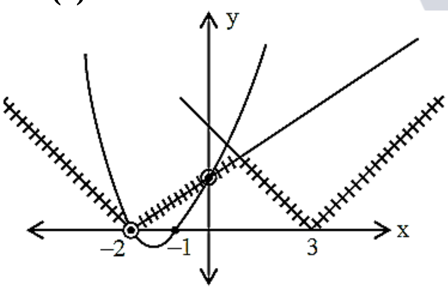
$= {}^7C_2 \cdot {}^3C_3 \cdot {}^6C_2 \cdot {}^5C_1 + {}^7C_3 \cdot {}^3C_2 \cdot {}^6C_1\, {}^5C_2 + {}^7C_4 \cdot {}^3C_1 \cdot {}^6C_0 \cdot {}^5C_3$
$= 8925$

67. (d)



For Area $\int_{-\ln 2}^{0} [e^x - (1 - e^x)]dx$
$\int_{-\ln 2}^{0} (2e^x - 1)dx = [2e^x - x]_{-\ln 2}^{0}$
$= (2 - (1 + \ell n2))$
$= 1 - \ell \ln 2$

68. (a)



Only two solutions

69. (d) C-I $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \to$ 4 ways

C-II $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ & $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \to$ 2 ways

$$P = \frac{\text{favourable}}{\text{total}} = \frac{6}{16} = \frac{3}{8}$$

70. (d) Equation of axis $y - 3 = 2\left(x - \frac{3}{2}\right)$
$y - 2x = 0$

foot of directrix
$y - 2x = 0$
& $\Rightarrow (0,0)$
$2y + x = 0$

Focus $= (3,6)$

$PS^2 = PM^2$

$$(x-3)^2 + (y-6)^2 = \left(\frac{x+2y}{\sqrt{5}}\right)^2$$

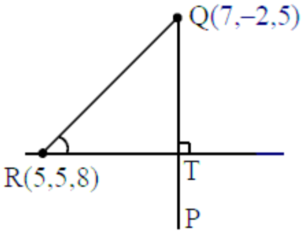
$4x^2 + y^2 - 4xy - 30x - 60y + 225 = 0$

$\Rightarrow \alpha = 4, \beta = 1, \gamma = 4 \Rightarrow \alpha + \beta + \gamma = 9$

Section - B

71. (392)

72. (957)



Let $R(2\lambda+1, 3\lambda-1, 4\lambda)$

$2\lambda + 1 = 5$

$\lambda = 2$

$R(5,5,8)$

let $T(2\lambda+1, 3\lambda-1, 4\lambda)$

$\overrightarrow{QT} = (2\lambda-6)\hat{i} + (3\lambda+1)\hat{j} + (4\lambda-5)\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{QT} \cdot \vec{b} = 0$

$4\lambda - 12 + 9\lambda + 3 + 16\lambda - 20 = 0$

$\lambda = 1$

$T(3,2,4)$

$QT = \sqrt{33}\ RT = \sqrt{29}$

$$(\text{ area of } \Delta PQR)^2 = \left(\frac{1}{2}\sqrt{29} \cdot 2\sqrt{33}\right)^2$$

$= 957$

73. (1)

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

$\text{I.F.} = e^{2\int \tan x dx} = \sec^2 x$

$$y\sec^2 x = \int \frac{\sin x}{\cos^2 x} dx$$

$= \int \tan x \sec x dx$

$= \sec x + C$

$C = -2$

$y = \cos x - 2\cos^2 x$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} - 1$$

$y' = -\sin x + 4\cos^x x \sin x$

$$y'\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} + 2$$

$$y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right) = 1$$

74. (55)

$$\frac{2\,b^2}{a}=15\sqrt{2}$$

$$1+\frac{b^2}{a^2}=\frac{5}{2}$$

$$a=5\sqrt{2}$$

$$b=5\sqrt{3}$$

$$\frac{2\,A^2}{B}=12\sqrt{5}$$

$$2a\cdot 2\,B=100\sqrt{10}$$

$$2.5\sqrt{2}\cdot 2\,B=100\sqrt{10}$$

$$B=5\sqrt{5}$$

$$A=5\sqrt{6}$$

$$e_2^2=1+\frac{A^2}{B^2}$$

$$=1+\frac{150}{125}$$

$$e_2^2=1+\frac{30}{25}$$

$$25e_2^2=55$$

75. (16)

$$2x^2+5x+9=A(x^2+x+1)+B(2x+1)+C$$

$$A=2\;B=\frac{3}{2}\;C=\frac{11}{2}$$

$$2\int\sqrt{x^2+x+1}dx+\frac{3}{2}\int\frac{2x+1}{\sqrt{x^2+x+1}}dx+\frac{11}{2}\int\frac{dx}{\sqrt{x^2+x+1}}$$

$$2\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx+3\sqrt{x^2+x+1}+\frac{11}{2}\int\frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$2\left(\frac{x+\frac{1}{2}}{2}\sqrt{x^2+x+1}+\frac{3}{8}\ln\left(x+\frac{1}{2}+\sqrt{x^2+x+1}\right)\right)+3\sqrt{x^2+x+1}$$

$$+\frac{11}{2}\ell n\left(x+\frac{1}{2}+\sqrt{x^2+x+1}\right)+C$$

$$\alpha=\frac{7}{2}\quad\beta=\frac{25}{4}$$

$$\alpha+2\beta=16$$