1. **(d)** Magnetic induction

$F = qvB$

$[B] = \left[\frac{F}{qV}\right]$

$[B] = [MT^{-2}\ A^{-1}]$

Magnetic Flux (ϕ)

$\phi = (B).(\text{Area})$

$[\phi] = [ML^2\ T^{-2}\ A^{-1}]$

Magnetic Permeability

$[\mu] = [MLT^{-2}\ A^{-2}]$

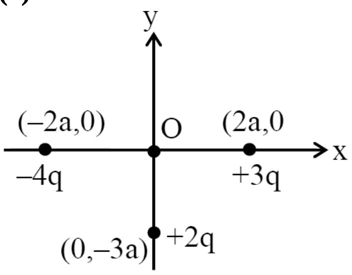
Self inductance

Using $U = \frac{1}{2}LI^2$

$[\text{Self inductance}] = [ML^2\ T^{-2}\ A^{-1}]$

A - III, B - IV, C - I, D - II

2. **(c)**

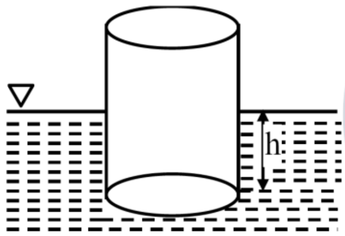


$\vec{p} = q_1\vec{r}_1 + q_2\vec{r}_2 + q_3\vec{r}_3$

$\vec{p} = (2q)(-3a)\hat{j} + (3q)(2a)\hat{i} + (-4q)(-2a)\hat{i}$

$\vec{p} = 2qa(7\hat{i} - 3\hat{j})$

3. **(a)**



At equilibrium

$\rho Ahg = Mg$

After displacing by x ,

$Ma = -\rho A(h + x)g + Mg$

$Ma = -\rho Ahg - \rho Axg + Mg$

$Ma = -\rho Axg$

$a = \left(\frac{-\rho Ag}{M}\right)x$

on comparing with,

$a = -\omega^2 x$

$\omega = \sqrt{\frac{\rho Ag}{M}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{\rho Ag}}$

4. **(d)** $Q = mS\Delta T = 4 \times 4200 \times 16\ J = 268800\ J$

$W = P\Delta V$

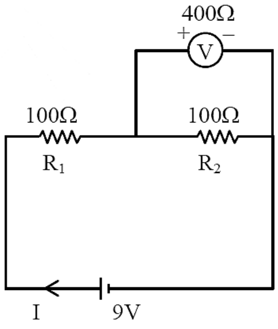
$\Delta V = \left(\frac{m}{\rho_f} - \frac{m}{\rho_i}\right) = 4\left[\frac{1}{998} - \frac{1}{1000}\right]$

$P = 10^5$ Pa.

$\therefore W = 10^5 \times 4 \times \left[\frac{1}{998} - \frac{1}{1000}\right] = \frac{8 \times 10^5}{10^3 \times 998} \approx 0.8$ J

$\Delta U = Q - W = 268799.2$ J

5. **(c)**



Current in circuit.

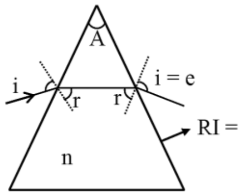
$I = \frac{E}{Req}$

$Req = 100 + \frac{400 \times 100}{400 + 100} = 180\Omega$

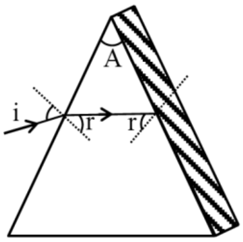
$\therefore I = \frac{9}{180} = \frac{1}{20}$ A

Reading of voltmeter $= V = I \times 80 = \frac{1}{20} \times 80 = 4$ V

6. **(a)** $i = e \& r = A/2$ for minimum deviation



For TIR ; $r = \theta_C$



$\text{Sin} r = \text{Sin}\theta_C$

$\text{Sin} r = \frac{n/2}{n}$

$\text{Sin} r = \frac{1}{2}$

$\text{Sin}\frac{A}{2} = \sin 30°$

$\frac{A}{2} = 30°$

$A = 60°$

7. **(a)** $V \propto \frac{Z}{n}$

$r \propto \frac{n^2}{Z}$

Thus ; $r \propto \frac{n}{V}$

Radii are same then

$$\frac{n_1}{V_1} = \frac{n_2}{V_2}$$

$$\frac{n_1}{n_2} = \frac{3 \times 10^5}{2.5 \times 10^5} = \frac{6}{5}$$

Possible order is 6 and 5

8. **(a)** $f_0 = 2\text{ cm}, fe = 5\text{ cm}$

$\ell = 10\text{ cm}, D = 25\text{ cm}$

$$M = \frac{\ell}{f_0} \cdot \frac{D}{f_e} = 25$$

9. **(c)** Power factor $= \frac{R}{Z}$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{60^2 + (150 - 70)^2} = 100\Omega$$

$$\therefore \text{ Power factor } = \frac{60}{100} = \frac{6}{10}$$

Then $a = 6$

10. **(c)** Radio wave ⇒ Produced by rapid acceleration of electrons

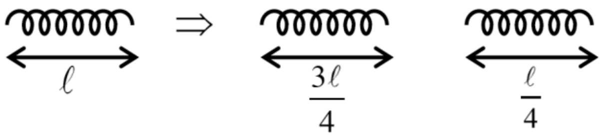
Micro wave ⇒ By magnetron valve

Infrared wave ⇒ Change in vibrational modes

X ray ⇒ Transition of inner shell electrons from high energy level to low energy level.

A - IV, B - I, C - II, D - III

11. **(c)**



$K\ell = \text{constant}$

$$K\ell = K'\left(\frac{\ell}{4}\right)$$

$K' = 4\text{ K}$

$K' = 60\text{ N/m}$

12. **(b)** Initially, $\frac{2}{3} = \frac{x}{100-x}$

$\Rightarrow x = 40\text{ cm}$

Now when ' R ' connected in parallel

$$\frac{2}{\frac{3R}{3+R}} = \frac{40 + 22.5}{60 - 22.5} = \frac{62.5}{37.5}$$

$\therefore R = 2\Omega$

13. **(d)**

14. **(b)** $T = \alpha YA(27 - (-43))$ ………….. (i)

$1.4\ T = \alpha YA(27 - \theta)$ …………….. (ii)

using (ii)/(i)

$$1.4 = \frac{27 - \theta}{70}$$

$27 - \theta = 98 \ \therefore \theta = -71°\text{C}$

15. **(a)** $v = ar^3 + b$

$$E = -\frac{dv}{dr} = -3ar^2$$

$\phi_{closed} = \frac{q_{enc}}{\varepsilon_0}$

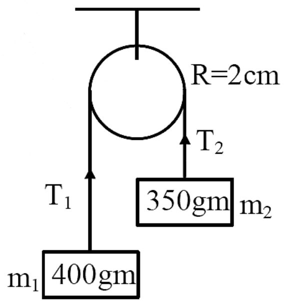
$q_{enc} = \varepsilon_0 \cdot E \cdot A$

$= \varepsilon_0(-3a \cdot (1)^2)4\pi(1)^2$

$= -12\pi a\varepsilon_0$

$\therefore x = -12$

16. (a)



$S = ut + \frac{1}{2}at^2$

$a = \frac{2\,s}{t^2} = \frac{2 \times 0.81}{81} = 0.02\ m/s^2$

$m_1\,g - T_1 = m_1 a$

$T_2 - m_2\,g = m_2 a$

$(T_1 - T_2)R = I \cdot \frac{a}{R}$

$\therefore a = \frac{(m_1 - m_2)g}{m_1 + m_2 + \frac{I}{R^2}}$

$0.02 = \frac{(400-350)(10^{-3})g}{(400+350)(10^{-3}) + \frac{I}{R^2}}$

$\frac{I}{R^2} = \frac{50 \times 10^{-3}\ g}{0.02} - 750 \times 10^{-3} = 23.75$

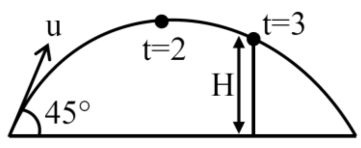
$I = 23.75 \times 4 \times 10^{-4} = 9.5 \times 10^{-3}\ kg - m^2$

17. (a) Brewester's law

$\tan\theta = \mu_{rel} = \sqrt{3}$

$\theta = 60^\circ$

18. (d) $T = \frac{2u_y}{g} = 4$



$\Rightarrow u_y = \frac{40}{2} = 20\ m/s$

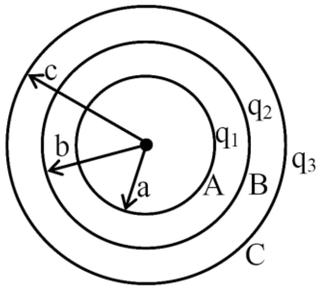
$y = u_y\Delta t - \frac{1}{2}\,g(\Delta t)^2$

$\Rightarrow H = 20 \times 3 - 5 \times 9$

$= 60 - 45$

$= 15\ m$

19. (c)

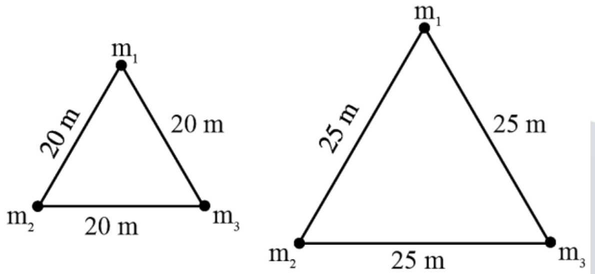


$V_A = \frac{Kq_1}{a} + \frac{Kq_2}{b} + \frac{Kq_3}{c} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{a} + \frac{q_2}{b} + \frac{q_3}{c}\right)$

$V_B = \frac{Kq_1}{b} + \frac{Kq_2}{b} + \frac{Kq_3}{c} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 + q_2}{b} + \frac{q_3}{c}\right)$

$V_C = \frac{Kq_1}{c} + \frac{Kq_2}{c} + \frac{Kq_3}{c} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 + q_2 + q_3}{c}\right)$

20. (c)



Work done by external agent :

$W_{ext} = \Delta U$

$U_i = -\frac{Gm_1\,m_2}{r_i} - \frac{Gm_2\,m_3}{r_i} - \frac{Gm_1\,m_3}{r_i} : r_i = 20\text{ m}$

$U_f = -\frac{Gm_1\,m_2}{r_f} - \frac{Gm_2\,m_3}{r_f} - \frac{Gm_1\,m_3}{r_f} : r_f = 25\text{ m}$

$U_i = \frac{-6.67 \times 10^{-11}}{20}[200 \times 300 + 300 \times 400 + 200 \times 400]$

$= \frac{-6.67 \times 10^{-11}}{20} \times 26 \times 10^4 = -86.71 \times 10^{-8}\text{ J}$

$U_f = \frac{-6.67 \times 10^{-11}}{0.25}[200 \times 300 + 300 \times 400 + 200 \times 400]$

$= \frac{-6.67 \times 10^{-11}}{0.25} \times 26 \times 10^4 = -693.68 \times 10^{-9}$

$= -69.36 \times 10^{-8}\text{ J}$

$\Delta U = U_f - U_i = 1.74 \times 10^{-7}\text{ J}$

21. (64) $\tau = \mu B \sin\theta \Rightarrow 0.016 = \mu \times B \times \frac{1}{2}$

$\Rightarrow \mu = \frac{0.032}{B}$

$W_{ext} = U_f - U_i = \mu B - (\mu B) = 2\mu B$

$= 2 \times \frac{0.032}{B} \times B$

$= 0.064\text{ J}$

22. (3730) As per NTA

V = constant

so $P \propto T$

$T_i = 50°C = 323\text{ K}$

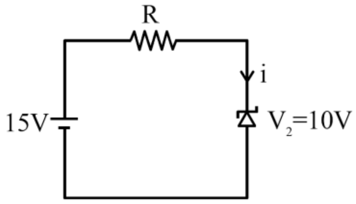
$T_f = 100°C = 373\text{ K}$

$\Rightarrow \frac{P_f}{P_i} = \frac{T_f}{T_i}$

$\Rightarrow \frac{P_f}{3.23kPa} = \frac{373}{323}$

$\Rightarrow P_f = 3730$ Pa

23. **(125)**

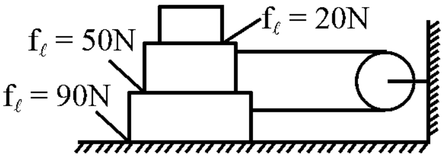


$P_D = 0.4W = 10i$

$i = 0.04$ A

$R = \frac{15-10}{0.04} = \frac{5}{0.04} = 125\Omega$

24. **(210)** For 8 kg to move with constant velocity $F_{net} = 0$.



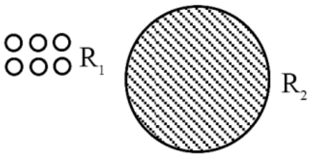
$\therefore F = 90 + T + 50$ (for 8kg block)

$T = 20 + 50$ (for 6 kg block)

$\therefore F = 210$ N.

25. **(160)** $V_T = \frac{2r^2 g}{9\eta}[\sigma - \rho]$

$V_T \propto r^2$



64 drop

$64\left(\frac{4}{3}\pi R_1^3\right) = \frac{4}{3}\pi R_2^3$

$R_2 = 4R_1$

$\frac{(V_T)_1}{(V_T)_2} = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{1}{4}\right)^2$

$\frac{10}{(V_T)_2} = \frac{1}{16}$

$(V_T)_2 = 160$ cm/sec

26. **(c)** In $K_3[Co(CO_3)_3] \Rightarrow sp^3\ d^2$ hybridized, octahedral

$\Rightarrow$ 4 unpaired electron

$\Rightarrow$ 4.9 B.M.

$[Ni(CN)_4]^{2-} \Rightarrow dsp^2$ hybridized , square plai

$\Rightarrow$ 0 unpaired electron

$\Rightarrow$ 0 B.M.

$[MnBr_4]^{2-} \Rightarrow sp^3$ hybridized , tetrahedral

$\Rightarrow$ 5 unpaired electron

$\Rightarrow$ 5.9 B.M.

$[CoF_6]^{3-} \Rightarrow sp^3\ d^2$ hybridized, octahedral

$\Rightarrow$ 4 unpaired electron

$\Rightarrow$ 4.9 B.M.

27. **(c)** Given :

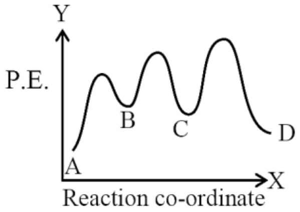
$A \rightarrow D; \Delta_r H = (+)ve = E_D - E_A \therefore E_D > E_A$

Mechanism

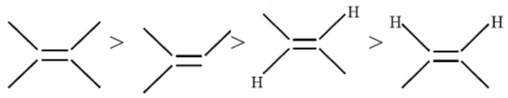
$A \rightarrow B; \Delta_r H = (+)ve \Rightarrow E_B > E_A$

$B \rightarrow C; \Delta_r H = (-)ve \Rightarrow E_C < E_B$

$C \rightarrow D; \Delta_r H = (-)ve \Rightarrow E_D < E_C$



28. **(c)** Stability order :



αH = 12 αH = 9 αH = 6 αH = 6

trans cis

Trans is more stable than cis

29. **(b)** (A) Dipole moment : $NF_3 < NH_3$

(B) BeH_2 is 'sp' hybridized, linear molecule with zero dipole moment.

(C) $O_2^{-2} \Rightarrow$ bond order $= 1$

$F_2 \Rightarrow$ bond order $= 1$

(D) Formal charge on central oxygen atom in O_3 is +1 .

(E) In NO_2; nitrogen does not follow octet rule.

30. **(b)** Mass of solute ' A ' = 0.3 g

Moles of solute $'A' = \frac{0.3\text{ g}}{60\text{ g/mol}} = \frac{1}{200}$ mol

Mass of solute $'B' = 0.9$ g

Moles of solute $'B' = \frac{0.9\text{gm}}{180\text{ g/mol}} = \frac{1}{200}$ mol

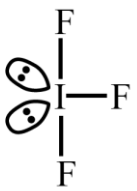
Total molarity of all solutes

$= \frac{2/200}{100} \times 1000 = \frac{1}{10}$ M

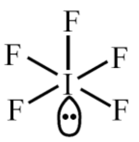
$\therefore \pi = \frac{1}{10} \times 0.082 \times 300$

$\pi = 2.46$ atm.

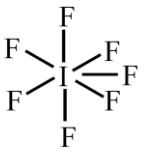
31. **(a)**



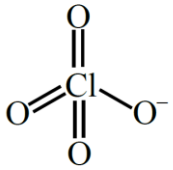
Hybridisation sp^3d
(T-shape)



Hybridisation sp^3d^2
(Squar pyramidal)



Hybridisation sp^3d^3
(Pentagonal bipyramidal)



Hybridisation sp^3
(Tetrahedral)

32. **(c)** Correct order of stability

$A > C > D > B$
$+M \quad -M \quad -M$
EDG increases stability
EWG decreases stability

33. (a) 0.4 mol$[Co(NH_3)_5SO_4]Br$ + 0.4 mol$[Co(NH_3)_5Br]SO_4$ are present in 4 lit. solution.
2 lit. of mixture will contain 0.2 mol of each complex.
2 lit. mixture on reaction with excess $AgNO_3$, 0.2 mole AgBr will be formed (Y).
2 lit. mixture on reaction with excess $BaCl_2$, 0.2 mole $BaSO_4$ will be formed (Z).

34. (a) Paramagnetic species :
$[CoF_6]^{3-}, [TiF_6]^{3-}, [Fe(CN)_6]^{3-}$
Diamagnetic species : V_2O_5
In $K_4[Fe(CN)_6] \Rightarrow$ No. of unpaired electron $= 0$
$K_3[Fe(CN)_6] \Rightarrow$ No. of unpaired electron $= 1$
$[Fe(H_2O)_6]SO_4 \cdot H_2O \Rightarrow$ No. of unpaired electron $= 4$
$[Fe(H_2O)_6]Cl_3 \Rightarrow$ No. of unpaired electron $= 5$

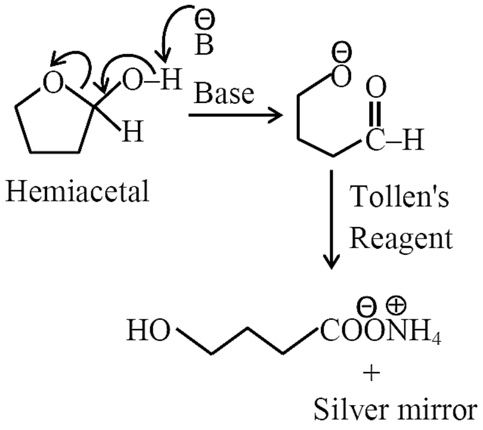
35. (d) $MgCl_2, AlCl_3, CuCl_2$ are water soluble at room temperature.
$AgCl, Hg_2Cl_2$ are sparingly soluble in water $PbCl_2$ is soluble in hot water.

36. (c)

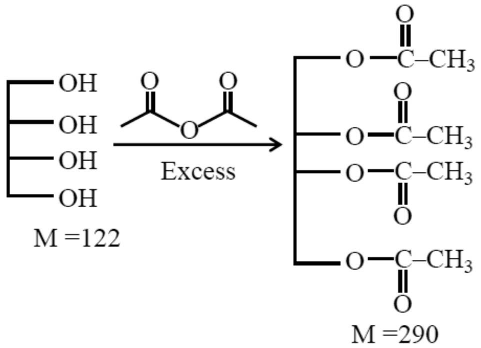
37. (a) $P^\circ = 640$ mmHg
$P_s = 600$ mmHg
$\Delta P = 40$ mmHg
moles of solute $= \frac{W}{M}$
$\frac{\Delta P}{P^o} = i.X_{solute}$
Again :
$\Delta T_b = i \times k_b \times m$
$2 = i \times 0.52 \times \frac{W/M}{100} \times 1000$
$i = \frac{2}{5.2} \times \frac{M}{W}$
$X_{solute} = \frac{40}{640} \times \frac{1}{i} = \frac{1}{16} \times \frac{5.2}{2} \times \frac{W}{M}$
$X_{solute} = \frac{1.3}{8} \times \frac{W}{M}$

38. (d) (A) $W_{Rev.} = -\int P_{gas}\, dV$
$W_{Rev.\ Isot.\ Exp.} = -nRT\ln\left[\frac{V_f}{V_i}\right]$
(A) → (II)
(B) Free expansion
$W_{irrev.} = -P_{ext}\,\Delta V$
$P_{ext} = 0$
$W = 0$
(B) → (I)
(C) Irreversible expansion
$W_{irrev.} = -P_{ext}\,\Delta V$
$W_{irrev.} = -P_{ext}\,(V_f - V_i)$
(C) → (III)
(D) Irreversible compression
$W_{irrev.} = -P_{ext}\,\Delta V$
$W_{irrev.} = -P_{ext}\,(V_i - V_f)$

39. (d)



40. (d)



No. of OH groups $= \frac{290-122}{42} = 4$

41. (b) $\frac{K_{catalyst}}{K_{uncatalyst}} = e^{\frac{\Delta E_a}{RT}}$

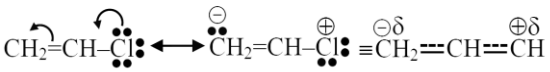
$\ln \frac{K_{catalyst}}{K_{uncatalyst}} = \frac{\Delta E_a}{RT}$

$\log \frac{K_{catalyst}}{K_{uncatalyst}} = \frac{\Delta E_a}{2.303RT}$

$= \frac{10 \times 1000}{2.303 \times 8.314 \times 300}$

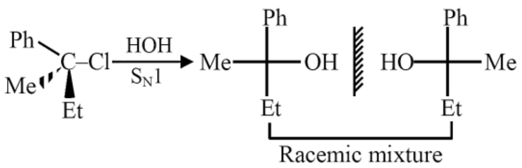
$\log \frac{K_{catalyst}}{K_{uncatalyst}} = 1.741$

42.(d) Statement-I :



C – Cl bond is strong in vinyl chloride because of double bond character.

Statement-II :



Racemic mixture is optically inactive, which can not rotate PPL.

43. (c) $x = H_2$ (gas), $y = CO_2$ (gas)

Sum of molar mass $= 2 + 44 = 46$

44. (c) Electron withdrawing groups increases stability of carbanion and electron donating groups decreases stability of carbanion.

45. **(d)** Metallic character of s-block elements is greater than p -block elements.
Anionic radius is greater than atomic radius but cationic radius is always less than atomic radius for any element.

46. **(111)** Eq of Cu = Eq of O_2 $\frac{300\times10^{-3}\times2}{63.54} = n_{O_2}\times4$

$2.36\times10^{-3} = n_{O_2}$

When current is further passed

$n_{O_2}\times4 = \frac{600\times28\times60}{96500\times1000}$

$n_{O_2} = 2.611\times10^{-3}$

Total O_2 released

$= [10^{-3}\times(2.36+2.611)]\times22400\text{ml}$

$= 111.35\text{ml}$

47. **(4)** $XS(s) \rightleftharpoons X^{+2}$ (aq.) $+S^{2-}$ (aq.)

For precipitation of XS(s)

$[X^{+2}][S^{2-}] \geq K_{SP}(XS)$

$[S^{2-}] \geq \frac{1\times10^{-22}}{0.01} = 10^{-20}$

$YS(s) \rightleftharpoons Y^{+2}(aq) + S^{2-}(aq)$

For precipitation of YS(s)

$[Y^{+2}][S^{2-}] \geq K_{SP}(YS)$

$[S^{2-}] \geq \frac{4\times10^{-16}}{10^{-2}} = 4\times10^{-14}$

Now, $H_2\,S(aq) \rightleftharpoons 2H^+(aq) + S^{2-}(aq)$

$\frac{[S^{2-}][H^+]^2}{H_2\,S} = K_{a_1}\times K_{a_2} = 1\times10^{-21}$

$[S^{2-}] = \frac{1\times10^{-21}\times[H_2\,S]}{[H^+]^2} \geq 4\times10^{-14}$

$[H^+]^2 \leq \frac{1}{4}\times10^{-7}\times10^{-1}$

$[H^+] \leq \frac{1}{2}\times10^{-4} \Rightarrow pH \geq 4.3$

48. **(54)** $\Delta E(L_1) = 13.6\times Z^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = 13.6Z^2\times\frac{3}{4}$

$\Delta E(B_1) = 13.6\times Z^2\times\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = 13.6\times Z^2\times\frac{5}{4\times9}$

$\frac{\Delta E(L_1)}{\Delta E(B_1)} = \frac{3}{5}\times9 = \frac{27}{5} = x$

$x = \left(\frac{27}{5}\times10\right)\times10^{-1} = 54\times10^{-1}$

49. **(15)** $P_{N_2} = (715-15)\text{mm} = \frac{700}{760}$ atm

$V_{N_2} = 70\text{ml} = \frac{70}{1000}\text{l}$

$n_{N_2} = \frac{PV}{RT} = \frac{\left(\frac{700}{760}\right)\times\left(\frac{70}{1000}\right)}{0.0821\times300}$

$W_{N_2} = \frac{700}{760}\times\frac{\frac{70}{1000}}{0.0821\times300}\times28$

$\%N = \frac{W_{N_2}}{0.5}\times100 = \frac{700}{760}\times\frac{\frac{70/1000}{0.0821\times300}\times28}{0.5}\times100$

$= 14.65\% \approx 15$

50. **(12)** $Cr_2O_7^{\ 2-} + 14H^+ + 6I^- \rightarrow 2Cr^{3+} + 3I_2 + 7H_2O$

no. of moles e^-involved $= x = 6$

$Cr_2O_7^{\ 2-} + 3\ S^{2-} + 14H^+ \rightarrow +S + 2Cr^{3+} + 7H_2O$

No. of moles e^-involved $= y = 6$

Sum of $x + y = 6 + 6 = 12$

51. **(d)** $f(0) = \text{Lim}_{x\rightarrow 0}\frac{e^{\tan x} - e^x + \ell n(\sec x + \tan x) - x}{\tan x - x}$

Applying L'hospital rule

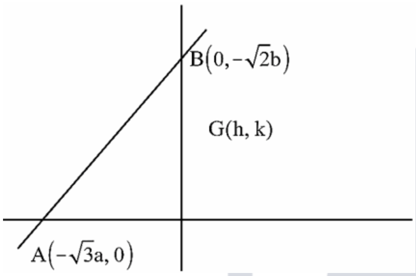
$$\Rightarrow f(0) = \text{Lim}_{x\rightarrow 0}\frac{e^{\tan x}\cdot \sec^2 x - e^x + \sec x - 1}{\sec^2 x - 1}$$

$$\Rightarrow f(0) = \text{Lim}_{x\rightarrow 0}\frac{e^{\tan x}(\sec^2 x - 1) + (e^{\tan x} - e^x) + \sec x - 1}{\tan^2 x}$$

$$\Rightarrow f(0) = \text{Lim}_{x\rightarrow 0}\left(e^{\tan x} + \frac{e^x(e^{\tan x - x} - 1)}{\tan^2 x} + \frac{1}{\sec x + 1}\right)$$

$$\Rightarrow f(0) = 1 + 0 + \frac{1}{2} = \frac{3}{2}$$

52. **(c)**



$AB = 8$

$3a^2 + 2b^2 = 64$

Centroid $G(h, k)$

$$h = -\frac{\sqrt{3}a}{3}, k = -\frac{\sqrt{2}b}{3}$$

$$a = -\sqrt{3}\ h, b = \frac{-3}{\sqrt{2}}k$$

$9h^2 + 9k^2 = 64$

$$x^2 + y^2 = \frac{64}{9}$$

$$r = \frac{8}{3}$$

53. **(b)** For POI

$2\lambda + 1 = 5\mu + 4; 3\lambda + 2 = 2\mu + 1; 4\lambda + 3 = \mu$

$\Rightarrow \lambda = \mu = -1$

$R(-1, -1, -1)\ P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

$PR^2 = 29 \Rightarrow (2\lambda + 2)^2 + (3\lambda + 3)^2 + (4\lambda + 4)^2 = 29$

$\Rightarrow \lambda = 0$ or $\lambda = -2$ (Reject)

$\Rightarrow P(1,2,3)$

$Q(5\mu + 4, 2\mu + 1, \mu)$

$$|PQ| = \sqrt{\frac{47}{3}} \Rightarrow PQ^2 = \frac{47}{3}$$

$$\Rightarrow (5\mu + 3)^2 + (2\mu - 1)^2 + (\mu - 3)^2 = \frac{47}{3}$$

$$\Rightarrow \mu = -\frac{1}{3}$$

$Q = \left(\frac{7}{3}, \frac{1}{3}, -\frac{1}{3}\right)$

$(QR)^2 = \left(\frac{7}{3}+1\right)^2 + \left(\frac{1}{3}+1\right)^2 + \left(-\frac{1}{3}+1\right)^2$

$= \frac{100+16+4}{9} = \frac{120}{9}$

$\Rightarrow 27 \times (QR)^2 = 27 \times \frac{120}{9} = 360$

54. (a) $P_n = 729.81.9 \ldots \ldots .$ (n terms)

$= 3^6.3^4.3^2 \ldots \ldots . . 3^{-2n+8}$

$P_n = 3^{6+4+2+\cdots\ldots+(-2n+8)} = 3^{n(7-n)}$

$P_n{}^{1/n} = 3^{7-n}$

$\Rightarrow \sum_{n=1}^{40} (P_n)^{\frac{1}{n}} = 3^6 + 3^5 + \cdots . . +(40 \text{ terms })$

$= 3^6 \left[\frac{1-\left(\frac{1}{3}\right)^{40}}{1-\frac{1}{3}}\right]$

$= \frac{3^6[3^{40}-1] \times 3^1}{3^{40} \times 2}$

$\sum (P_n)^{\frac{1}{n}} = \frac{(3^{40}-1)}{2 \times 3^{33}}, \quad \alpha = 40 \quad \beta = 33$

$\alpha + \beta = 73$

55. (c) $\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$

$\vec{c} = 2\hat{i} + 2\hat{k} + \hat{k}, |c| = 3$

$|\vec{c} \times \vec{d}| = 3$

$|\vec{c}||\vec{d}|\sin\frac{\pi}{4} = 3 \Rightarrow |\vec{d}| = \sqrt{2}$

$|\vec{d} - \vec{a}| = \sqrt{11}$

$\Rightarrow |\vec{a}|^2 + |\vec{d}|^2 - 2\vec{a}\cdot\vec{d} = 11$

$9 + 2 - 2\vec{a}\cdot\vec{d} = 11$

$\vec{a}\cdot\vec{d} = 0$

56. (d) $18x^2 - 11x + 1 > 0$

$(2x-1)(9x-1) > 0$

$x < \frac{1}{9}$ or $\frac{1}{2} < x$

Also $10x^2 - 17x + 7 > 0$

$(x-1)(10x-7) > 0$

$x < \frac{7}{10}$ or $1 < x$

$\&10x^2 - 17x + 7 \neq 1$

$x \in \left(-\infty, \frac{1}{9}\right) \cup \left(\frac{1}{2}, \frac{7}{10}\right) \cup (1, \infty) - \left\{\frac{6}{5}\right\}$

$90(a+b+c+d+e) = 90\left(\frac{1}{9} + \frac{1}{2} + \frac{7}{10} + 1 + \frac{6}{5}\right)$

$= 10 + 45 + 63 + 90 + 108 = 316$

57. (b) $2ae = 8 \Rightarrow a = 5$

$b^2 = a^2(1-e^2)$

$b^2 = a^2 \times \frac{9}{25} \quad b^2 = 9$

$E_1: \frac{x^2}{25} + \frac{y^2}{9} = 1$

$\ell_1: \frac{2\,b^2}{a} = \frac{2 \times 9}{5} = \frac{18}{5}$

$A^2 = B^2(1 - e^2) \Rightarrow A^2 = \frac{9}{25}\ B^2 \Rightarrow\ A = \frac{3}{5}\ B$

$2\ell^2 = 9\ell_2 \Rightarrow 2\left(\frac{18}{5}\right)^2 = 9\ell_2 \Rightarrow \ell_2 = \frac{4 \times 18}{25}$

$\frac{2\,A^2}{B} = \frac{72}{25} \Rightarrow\ A^2 \frac{36}{25}\ B$

$\frac{9}{25}\ B^2 = \frac{36\,B}{25} \Rightarrow\ B = 4,$

Distance between focii $2Be = 2 \times \frac{4}{5} \times 4 = \frac{32}{5}$

58. (c) $E = \frac{\frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}}{\frac{1}{2}\cdot\frac{1}{4}\cdot\cos 60^\circ}$

$= \frac{\left(\sqrt{3}\cos 20^\circ - \sin 20^\circ\right)}{\cos 20^\circ \cdot \sin 20^\circ} 16$

$= \frac{\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right) 32 \times 2}{2\cos 20^\circ \cdot \sin 20^\circ}$

$= \frac{\sin 40^\circ}{\sin 40^\circ} \times 64 = 64$

59. (d) Solving $\left|\frac{z-6i}{z-2i}\right| = 1 \Rightarrow y = 4$ ………………. (i)

(where $z = x + iy$)

Now solving $\left|\frac{z-8+2i}{z+2i}\right| = \frac{3}{5}$

$\Rightarrow x^2 + y^2 - 25x + 4y + 104 = 0$ ………………. (ii)

Solving (i) & (ii) $\Rightarrow z = 17 + 4i \& 8 + 4i$

$\Rightarrow \Sigma|z|^2 = (17)^2 + (4)^2 + (8)^2 + (4)^2 = 385$

60. (c) $\cot x = \frac{5}{12} \Rightarrow \cos x = \frac{-5}{13} = 2\cos^2\frac{x}{2} - 1$

$\cos\left(\frac{x}{2}\right) = -\frac{2}{\sqrt{13}}$ or $\frac{2}{\sqrt{13}}$ (rejected)

$\left\{\because \frac{x}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)\right\}$

$\left(\sin 7x \frac{\sin 13x}{2} + \cos 7x \frac{\cos 13x}{2}\right) + \left(\sin 7x \frac{\cos 13x}{2} - \cos 7x \frac{\sin 13x}{2}\right)$

$\cos\left(7x - \frac{13x}{2}\right) + \sin\left(7x - \frac{13x}{2}\right)$

$\cos\frac{x}{2} + \sin\left(\frac{x}{2}\right)$

$\frac{3}{\sqrt{13}} - \frac{2}{\sqrt{13}} = \frac{1}{\sqrt{13}}$

61. (a) $f(t) = \int \frac{1 - \sin(\ln t)}{1 - \cos(\ln t)} dt$

Let $\ln t = x \Rightarrow t = e^x \Rightarrow dt = e^x dx$

$= \frac{1}{2}\int \left(\operatorname{cosec}^2\frac{x}{2} - 2\cot\frac{x}{2}\right) e^x dx - t\cot\left(\frac{\ln t}{2}\right) + C$

$(\therefore \int (f(x) + f'(x))e^x dx = f(x) \cdot e^x + c)$

Now $f(e^{\pi/2}) = -e^{\pi/2}\cot\left(\frac{\pi}{4}\right) + C = -e^{\frac{\pi}{2}}$ (given)

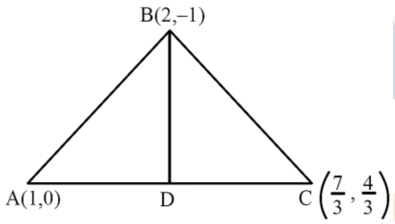
$CC = 0$

Now $f\left(e^{\pi/4}\right) = -e^{\pi/4}\cot\left(\frac{\pi}{8}\right) + C = -e^{\frac{\pi}{4}}(\sqrt{2}+1)$

62. (c)

(a, b)	(c, d)
(1,1)	x
(1,2)	x
(1,3)	(1,2)
(1,4)	(2,2)
(2,1)	(1,1)
(2,2)	(2,1)
(2,3)	(3,1)
(2,4)	(4,1)
(3,1)	x
(3,2)	x
(3,3)	(1,3)
(3,4)	(2,3)
(4,1)	(1,2)
(4,2)	(2,2)
(4,3)	(3,2)
(4,4)	(4,2)

63. (d)



$\frac{BD}{DC} = \frac{AB}{AC} = \frac{\sqrt{2}\times 3}{5\sqrt{2}} = \frac{3}{5}$

$D = \left(\frac{12}{8}, \frac{4}{8}\right) = \left(\frac{3}{2}, \frac{1}{2}\right)$

Slope of AD $= \frac{-3/2}{\frac{1}{2}} = -3$

$3x + y = 5$

$\alpha = 3, \beta = 1; \alpha^2 + \beta^2 = 10$

64. (c) $\frac{1}{26!}\left(\frac{26!}{25!1!} + \frac{26!}{3!23!} + \frac{26!}{5!21!} + \cdots..+13 \text{ terms}\right)$

$\frac{1}{26!}\left({}^{26}C_1 + {}^{26}C_3 + {}^{26}C_5 + \cdots.+13 \text{ terms}\right)$

$\frac{1}{26!}\left({}^{26}C_1 + {}^{26}C_5 + \cdots.+{}^{26}C_{25}\right)$

$\Rightarrow S = \frac{1}{26!} \times 2^{25}$

$\Rightarrow 13\ S = \frac{2^{24}}{25!}$

so $n + k = 25 + 24 = 49$

65. (c) $S_n = \frac{n}{2}[a_1 + a_n] = \frac{525}{2}, d = \frac{-3}{4}$

$\frac{n}{2}\left[a_1 + \frac{a_1}{4}\right] = \frac{525}{2}$

$\frac{5a_1 n}{4} = 525$

$a_1 n = 420$

$a_n = a_1 + (n-1)\left(\frac{-3}{4}\right)$

$\Rightarrow \frac{-3}{4} a_1 = \left(\frac{-3}{4}\right)(n-1) \Rightarrow a_1 = n - 1$

$n(n-1) = 420$

$n^2 - n - 420 = 0$

$(n-21)(n+20) = 0$

$n = 21, a_1 = 20$

$\sum_{i=1}^{17} a_i = \frac{17}{2}[2a_1 + 16d]$

$= \frac{17}{2}\left[40 + 16\left(\frac{-3}{4}\right)\right]$

$= \frac{17}{2}[40 - 12]$

$= 17 \times 14 = 238$

66. (b) $f(x) = \begin{cases} 2\alpha x^2 + 2\beta x - 4\alpha; & x < 1 \\ (\alpha + 3)x + \alpha - \beta; & x \geq 1 \end{cases}$

$f(1^+) = 2\alpha - \beta + 3, f(1^-) = -2\alpha + 2\beta$

$2\alpha - \beta + 3 = 2\beta - 2\alpha \Rightarrow 4\alpha - 3\beta + 3 = 0$ ……………. (i)

$f'(1^+) = 4\alpha + 2\beta, f'(1^-) = \alpha + 3$

$4\alpha + 2\beta = \alpha + 3 \Rightarrow 3\alpha + 2\beta - 3 = 0$ ………………….. (ii)

Solving (i) & (ii)

We get $\alpha = \frac{3}{17}, \beta = \frac{21}{17}$

$\Rightarrow 34(\alpha + \beta) = 34 \times \frac{27}{17} = 48$

67. (d) 10 defective & 90 non-defective

Req. probability = (7 def 1 fair) or (8 defective)

Req. probability $= \frac{(10^7 \times 90) \times 8 + 10^8}{100^8}$

$= \frac{72 \times 10^8 + 10^8}{100^8} = \frac{73}{10^8}$

68. (d) Let first 10 numbers are $x_1, x_2, \ldots \ldots x_9, \alpha$

$\Rightarrow \alpha + \sum_{i=1}^{9} x_i = 100 \Rightarrow \sum_{i=1}^{9} x_i = 100 - \alpha$

Variance $= \left(\frac{\sum x_i^2}{n}\right) - \left(\frac{\sum x_i}{n}\right)^2$

$\Rightarrow \frac{\sum x_i^2}{n} = 98$

$\Rightarrow x_1^2 + x_2^2 + \cdots \ldots x_9^2 + \alpha^2 = 1020 \Rightarrow \sum x_i^2 = 1020 - \alpha^2$

In second case, let number are

$x_1, x_2, \ldots \ldots x_9, \beta$

$100 - \alpha + \beta = 101\alpha - \beta + 1 = 0$

$\dfrac{\sum x_i^2 + \beta^2}{10} - (10.1)^2 = 1.99$

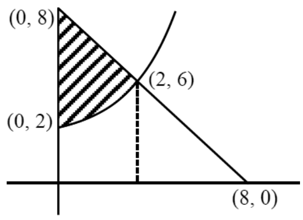
$\beta^2 - \alpha^2 = 20$

$\alpha = \dfrac{19}{2}$

$\beta = \dfrac{21}{2}$

$\alpha + \beta = \dfrac{19+21}{2} = 20$

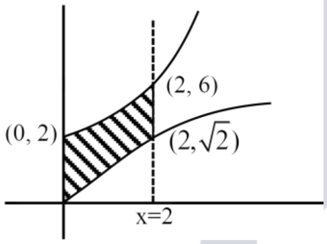
69. (a)



$A_1 = \int_0^2 \left((8-x) - (x^2+2)\right)dx$

$= A_1 = \int_0^2 (6 - x - x^2)dx$

$A_1 \left(6x - \dfrac{x^2}{2} - \dfrac{x^3}{3}\right)_0^2 = 12 - 2 - \dfrac{8}{3} = 10 - \dfrac{8}{3} = \dfrac{22}{3}$



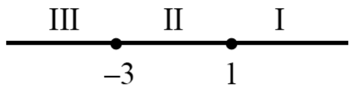
$A_2 = \int_0^2 (x^2+2)dx - \dfrac{2}{3}(2\sqrt{2})$

$A_2 = \left(\dfrac{x^3}{3} + 2x\right)_0^2 - \dfrac{4\sqrt{2}}{3}$

$A_2 = \dfrac{8}{3} + 4 - \dfrac{4\sqrt{2}}{3} = \dfrac{20}{3} - \dfrac{4\sqrt{2}}{3}$

$A_1 - A_2 = \dfrac{2}{3} + \dfrac{4\sqrt{2}}{3}$

70. (a)



(I) $x^2 + 3x + x - 1 - 2 = 0$

$x^2 + 4x - 3 = 0$

$x = -2 + \sqrt{7}$ (rejected) , $-2 - \sqrt{7}$ (rejected)

(II) $x^2 + 3x + 1 - x - 2 = 0$

$x^2 + 2x - 1 = 0$

$x = -1 + \sqrt{2}, -1 - \sqrt{2}$

$-x^2 - 3x + 1 - x - 2 = 0$

$x^2 + 4x + 1 = 0$

$x = -2 - \sqrt{3}, -2 + \sqrt{3}$ (rejected)

71. (64) $\int_0^{36} f\left(\frac{tx}{36}\right)dt = 4\alpha f(x)$, Put $\frac{tx}{36} = y$

$\frac{dy}{dt} = \frac{x}{36}$

$\int_0^x \frac{f(y)36dy}{x} = 4\alpha f(x)$

$\int_0^x f(y)dy = \frac{\alpha f(x)x}{9}$

$f(x) = \frac{\alpha}{9}(f(x) + xf'(x))$

$\left(1 - \frac{\alpha}{9}\right) f(x) = \frac{\alpha x}{9} f'(x) \Rightarrow (9 - \alpha)f(x) = \alpha x f'(x)$

$\frac{f'(x)}{f(x)} = \left(\frac{9}{\alpha} - 1\right)\frac{1}{x}$

$\log_e f(x) = \left(\frac{9}{\alpha} - 1\right)\log_e x + \log_e c$

$f(x) = cx\left(\frac{9}{\alpha} - 1\right)$ for standard parabola

$\frac{9}{\alpha} - 1 = 2$

$\alpha = 3$

$f(x) = cx^{2}$

passing through $(2,1)$

$1 = 4c \Rightarrow c = 1/4$

$y = \frac{x^2}{4}$ passing through $(-4, \beta)$

$\beta = 4$

$\beta^x = 4^3 = 64$

72. (6) $d_1 =< 4,1,1 >$ and $d_2 =< 1,1,0 >$

$d_L = d_1 \times d_2 = \begin{vmatrix} i & j & k \\ 4 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \langle -1,1,3 \rangle$

Line L passes through $P < 1,1,1 >$ with $d_2 =< -1,1,3 >$

$r(t) =< 1,1,1 > +t < -1,1,3 >$

$=< 1 - t, 1 + t, 1 + 3t >$

$=< 1 - t, 1 + t, 1 + 3t >$

For point Q; $x = 0$

$\Rightarrow t = 1$

$Q =< 0,2,4 >$

Another

Line parallel to L passes through $S < 1,0,-1 >$

with $d_L = \langle -1,1,3 \rangle$

$r'(u) =< 1,0,-1 > +\mu < -1,1,3 >$

$=< 1 - \mu, \mu, -1 + 3\mu >$

for point R, $x = 0$

$\Rightarrow \mu = 1$

$R < 0,1,2 >$

Area of parallelogram with adjacent vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

$\overrightarrow{PQ} = \langle -1,1,3 \rangle$

$\overrightarrow{PS} =< 0,-1,-2 >$

Area of parallelogram

$\overrightarrow{PQ} \times \overrightarrow{PS} = \begin{vmatrix} i & j & k \\ -1 & 1 & 3 \\ 0 & -1 & -2 \end{vmatrix} = \langle 1,-2,1 \rangle$

Area $= \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$

73. **(42)**(1) all different

$5,0,1,9 \Rightarrow \lfloor 3 = 6$ ways

(2) 2 alike, 2 different

$5,0,0,1 \Rightarrow 3$ ways

$5,1,1,2 \Rightarrow 3$ ways

$5,2,2,0 \Rightarrow 3$ ways

$5,2,2,9 \Rightarrow 3$ ways

$5,5,0,2 \Rightarrow 6$ ways

$5,5,2,9 \Rightarrow 6$ ways

$5,1,9,9 \Rightarrow 3$ ways

(3) 3 alike, 1 different

$5,5,5,0 \Rightarrow 3$ ways

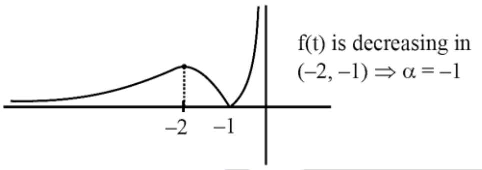
$5,5,5,9 \Rightarrow 3$ ways

(4) 2 alike, 2 other alike

$5,5,1,1 \Rightarrow 3$ ways

Total ways $= 42$

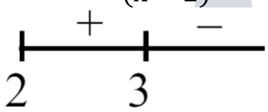
74. **(4)** Drawing graph of $f(t)$ for $t < 0$



$g(x) = \log_e(x-2) - x^2 + 4x + 1; x > 2$

$g'(x) = \dfrac{2}{x-2} - (2(x-2); x > 2$

$g'(x) = \dfrac{1-(x-2)^2}{(x-2)} = \dfrac{-(x-3)(x-1)}{(x-2)}$



maxima occur at $x = 3$

$g(3) = 2\log_e 1 - 9 + 12 + 1 = 4$

75. **(312)** $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}_{3\times 2}$

$A^TA = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}_{2\times 3} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}_{3\times 2}$

$= \begin{pmatrix} a_1^2 + a_2^2 + a_3^2 & \_\_\_\_ \\ \_\_\_\_ & b_1^2 + b_2^2 + b_3^2 \end{pmatrix}$

$\mathrm{Tr}(A^TA) = a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 = 5$

$\{2,1,0,0,0,0\}$

$\{2,-1,0,0,0,0\}$

$\{-2,1,0,0,0,0\}$

$\{-2,-1,0,0,0,0\}$

$\{1,1,1,1,1,0\}$

No. of ways $= \frac{6!}{4!} \times 4 + 2 \times \frac{6!}{5!} + 2 \times \frac{6!}{4!} + 2 \times \frac{6!}{3!2!}$

$= \dfrac{6!}{3!} + 2 \times 6 + 2 \times 15 \times 2 \times \dfrac{6!}{3!}$

$= 120 + 120 + 12 + 60 = 312$