

PHYSICS

SECTION-A

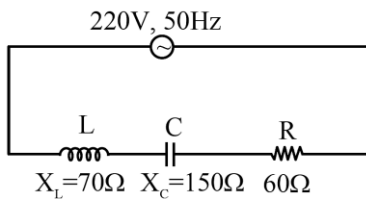
1. Match the List-I with List-II

List-I		List-II	
(A)	Magnetic induction	(I)	$MLT^{-2} A^{-2}$
(B)	Magnetic flux	(II)	$ML^2 T^{-2} A^{-2}$
(C)	Magnetic permeability	(III)	$ML^0 T^{-2} A^{-1}$
(D)	Self inductance	(IV)	$ML^2 T^{-2} A^{-1}$

Choose the correct answer from the options given below:

- (a) A-IV, B-III, C-I, D-II (b) A-III, B-IV, C-II, D-I
(c) A-I, B-III, C-IV, D-II (d) A-III, B-IV, C-I, D-II
2. Three charges $+2q$, $+3q$ and $-4q$ are situated at $(0, -3a)$, $(2a, 0)$ and $(-2a, 0)$ respectively in the xy plane. The resultant dipole moment about origin is ____
- (a) $2qa(3\hat{j} - \hat{i})$ (b) $2qa(3\hat{i} - 7\hat{j})$
(c) $2qa(7\hat{i} - 3\hat{j})$ (d) $2qa(3\hat{j} - 7\hat{i})$
3. A cylindrical block of mass M and area of cross section A is floating in a liquid of density ρ and with its axis vertical. When depressed a little and released the block starts oscillating. The period of oscillation is ____.
- (a) $2\pi\sqrt{\frac{M}{\rho Ag}}$ (b) $\pi\sqrt{\frac{2M}{\rho Ag}}$
(c) $\pi\sqrt{\frac{\rho A}{Mg}}$ (d) $2\pi\sqrt{\frac{\rho A}{Mg}}$
4. Density of water at $4^\circ C$ and $20^\circ C$ are 1000 kg/m^3 and respectively. The increase in internal energy of 4 kg water when it is heated from $4^\circ C$ to $20^\circ C$ is ____ J. (Specific heat capacity of water = 4.2 J/kg . and $1 \text{ atmospheric pressure} = 10^5 \text{ Pa}$)
- (a) 315826.2 (b) 234699.2
(c) 258700.8 (d) 268799.2
5. Two resistors of 100Ω each are connected in series with a 9 V battery. A voltmeter of 400Ω resistance is connected to measure the voltage drop across one of the resistors. The voltmeter reading is ____ V.
- (a) 3 (b) 4.5
(c) 4 (d) 2
6. The exit surface of a prism with refractive index n is coated with a material having refractive index $\frac{n}{2}$. When this prism is set for minimum angle of deviation it exactly meets the condition of critical angle. The prism angle is ____
- (a) 60° (b) 15°
(c) 30° (d) 45°
7. Two electrons are moving in orbits of two hydrogen like atoms with speeds $3 \times 10^5 \text{ m/s}$ and $2.5 \times 10^5 \text{ m/s}$ respectively. If the radii of these orbits are nearly same then the possible order of energy states are ____ respectively.
- (a) 6 and 5 (b) 9 and 8
(c) 8 and 10 (d) 10 and 12
8. In a microscope of tube length 10 cm two convex lenses are arranged with focal length of 2 cm and 5 cm . Total magnification obtained with this system for normal adjustment is $(5)^k$. The value of k is ____
- (a) 2 (b) 5
(c) 3.5 (d) 4

9. For the series LCR circuit connected with 220 V , 50 Hz a.c source as shown in the figure, the power factor is $\frac{\alpha}{10}$. The value of α is ____



- (a) 4 (b) 10
(c) 6 (d) 8

10. Match the List-I with List-II

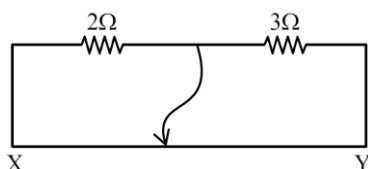
List-I		List-II	
(A)	Radio-wave	(I)	is produced by Magnetron valve
(B)	Micro-wave	(II)	Due to change in the vibrational modes of atoms
(C)	Infrared-wave	(III)	Due to inner shell electrons moving from higher energy level to lower energy level
(D)	X-ray	(IV)	Due to rapid acceleration of electrons

Choose the correct answer from the options given below:

- (a) A-II,B-IV,C-III,D-I (b) A-IV,B-III,C-I,D-II
(c) A-IV,B-I,C-II,D-III (d) A-IV,B-II,C-I,D-III

11. A spring of force constant 15 N/m is cut into two pieces. If the ratio of their length is 1: 3, then the force constant of smaller piece is ____ N/m
(a) 15 (b) 20
(c) 60 (d) 45

12. Two resistors 2Ω and 3Ω are connected in the gaps of bridge as shown in figure. The null point is obtained with the contact of jockey at some point on wire XY . When an unknown resistor is connected in parallel with 3Ω resistor, the null point is shifted by 22.5 cm toward Y . The resistance of unknown resistor is Ω .



- (a) 3 (b) 2
(c) 4 (d) 1

13. Given below are two statements:

Statement I : For all elements, greater the mass of the nucleus, greater is the binding energy per nucleon.

Statement II : For all elements, nuclei with less binding energy per nucleon transforms to nuclei with greater binding energy per nucleon.

In the light of the above statements, choose the correct answer from the options given below:

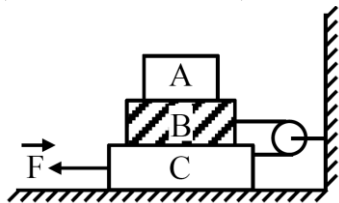
- (a) Both Statement I and Statement II are true
(b) Statement I is true but Statement II is false

- (c) Both Statement I and Statement II are false
(d) Statement I is false but Statement II is true
14. A brass wire of length 2 m and radius 1 mm at 27°C is held taut between two rigid supports. Initially it was cooled to a temperature of -43°C creating a tension T in the wire. The temperature to which the wire has to be cooled in order to increase the tension in it to $1.4 T$, is ____ $^{\circ}\text{C}$
(a) -86 (b) -71
(c) -65 (d) -80
15. The electrostatic potential in a charged spherical region of radius r varies as $V = ar^3 + b$, where a and b are constants. The total charge in the sphere of unit radius is $\alpha \times \pi a \epsilon_0$. The value of α is ____ .
(permittivity of vacuum is ϵ_0)
(a) -12 (b) -6
(c) -9 (d) -8
16. Two masses 400 g and 350 g are suspended from the ends of a light string passing over a heavy pulley of radius 2 cm . When released from rest the heavier mass is observed to fall 81 cm in 9 s . The rotational inertia of the pulley is ____ $\text{kg} \cdot \text{m}^2$.
($g = 9.8 \text{ m/s}^2$)
(a) 9.5×10^{-3} (b) 4.75×10^{-3}
(c) 1.86×10^{-2} (d) 8.3×10^{-3}
17. An unpolarised light is incident at an interface of two dielectric media having refractive indices of 2 (incident medium) and $2\sqrt{3}$ (medium) respectively. To satisfy the condition that reflected and refracted rays are perpendicular to each other, the angle of incidence is ____ .
(a) 60° (b) 10°
(c) 30° (d) 45°
18. A boy thrown a ball into air at 45° from the horizontal to land it on a roof of a building of height H . If the ball attains maximum height in 2 s and lands on the building in 3 s after launch, then value of H is ____ m.
($g = 10 \text{ m/s}^2$)
(a) 20 (b) 10
(c) 25 (d) 15
19. There are three co-centric conducting spherical shells A, B and C of radii a , b and c respectively. The potential of the spheres A, B and C respectively, are :
(a) $\frac{1}{4\pi\epsilon_0} \left(\frac{q_1+q_2+q_3}{a} \right), \frac{1}{4\pi\epsilon_0} \left(\frac{q_1+q_2+q_3}{b} \right), \frac{1}{4\pi\epsilon_0} \left(\frac{q_1+q_2+q_3}{c} \right)$
(b) $\frac{1}{4\pi\epsilon_0} \left(\frac{q_1+q_2+q_3}{a} \right), \frac{1}{4\pi\epsilon_0} \left(\frac{q_1+q_2}{b} + \frac{q_3}{c} \right), \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{a} + \frac{q_2}{b} + \frac{q_3}{c} \right)$
(c) $\frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{a} + \frac{q_2}{b} + \frac{q_3}{c} \right), \frac{1}{4\pi\epsilon_0} \left(\frac{q_1+q_2}{b} + \frac{q_3}{c} \right), \frac{1}{4\pi\epsilon_0} \left(\frac{q_1+q_2+q_3}{c} \right)$
(d) $\frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{a} + \frac{q_2}{b} + \frac{q_3}{c} \right), \frac{1}{4\pi\epsilon_0} \left(\frac{q_1+q_2+q_3}{b} \right), \frac{1}{4\pi\epsilon_0} \left(\frac{q_1+q_2+q_3}{c} \right)$
20. Three masses 200 kg, 300 kg and 400 kg are placed at the vertices of an equilateral triangle with sides 20 m . They are rearranged on the vertices of a bigger triangle of side 25 m and with the same centre. The work done in this process ____ J. (Gravitational constant $G = 6.7 \times 10^{-11} \text{ N m}^2/\text{kg}^2$)
(a) 9.86×10^{-6} (b) 2.85×10^{-7}
(c) 1.74×10^{-7} (d) 4.77×10^{-7}

SECTION-B

21. A short bar magnet placed with its axis at 30° with an external field of 800 Gauss, experiences a torque of 0.016 N.m. The work done in moving it from most stable to most unstable position is $\alpha \times 10^{-3} \text{ J}$. The value of α is ____ .
22. A gas of certain mass filled in a closed cylinder at a pressure of 3.23 kPa has temperature 50°C . The gas is now heated to double its temperature. The modified pressure is ____ Pa.
23. A voltage regulating circuit consisting of Zener diode, having break-down voltage of 10 V and maximum power dissipation of 0.4 W , is operated at 15 V . The approximate value of protective resistance in this circuit is ____ Ω .

24. In the given figure the blocks A, B and C weigh 4 kg, 6 kg and 8 kg respectively. The co-efficient of sliding friction between any two surfaces is 0.5. The force $\vec{F}$ required to slide the block C with constant speed is ____ N. (Used $g = 10 \text{ m/s}^2$)



25. Sixty four rain drops of radius 1 mm each falling down with a terminal velocity of 10 cm/s coalesce to form a bigger drop. The terminal velocity of bigger drop is ____ cm/s.

CHEMISTRY

SECTION-A

26. Given below are two statements :

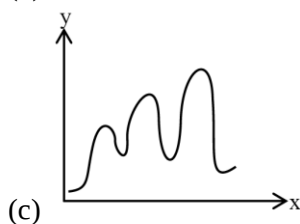
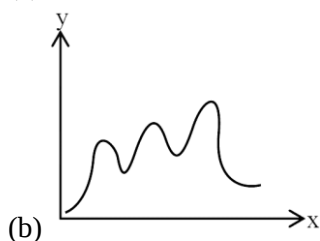
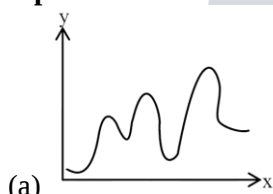
Statement-I Hybridisation, shape and spin only magnetic moment of $\text{K}_3[\text{Co}(\text{CO}_3)_3]$ is $\text{sp}^3 \text{d}^2$, octahedral and 4.9 BM respectively.

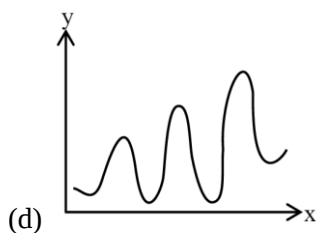
Statement-II Geometry, hybridisation and spin only magnetic moment values (BM) of the ions $[\text{Ni}(\text{CN})_4]^{2-}$, $[\text{MnBr}_4]^{2-}$ and $[\text{CoF}_6]^{3-}$ respectively are square planar, tetrahedral, octahedral : dsp^2 , sp^3 , $\text{sp}^3 \text{d}^2$ and 0, 5.9, 4.9.

In the light of the above statements, choose the correct answer from the options given below

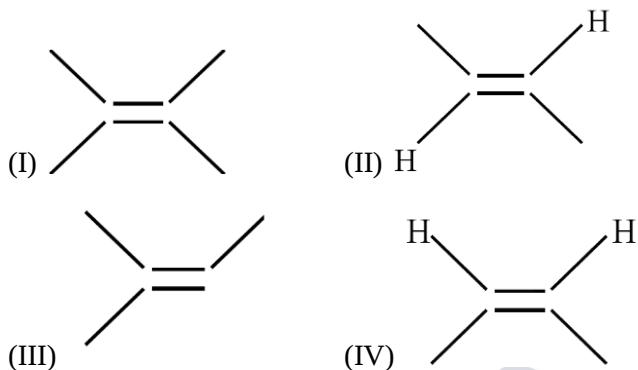
- (a) Both statement-I and statement-II are false
 (b) Statement I is false but statement-II is true
 (c) Both statement-I and statement-II are true
 (d) Statement-I is true but statement-II is false
27. $\text{A} \rightarrow \text{D}$ is an endothermic reaction occurring in three steps (elementary).
- (i) $\text{A} \rightarrow \text{B} \Delta H_i = +\text{ve}$
 (ii) $\text{B} \rightarrow \text{C} \Delta H_{ii} = -\text{ve}$
 (iii) $\text{C} \rightarrow \text{D} \Delta H_{iii} = -\text{ve}$

Which of the following graphs between potential energy (y-axis) vs reaction coordinate (x-axis) correctly represents the reaction profile of $\text{A} \rightarrow \text{D}$?





28. Arrange the following alkenes in decreasing order of stability.



Choose the correct answer from the options given below :

- (a) III > I > II > IV
 (b) III > II > I > IV
 (c) I > III > II > IV
 (d) I > III > IV > II

29. Given below are statements about some molecules/ions.

Identify the CORRECT statements.

- (A) The dipole moment value of NF_3 is higher than that of NH_3 .
 (B) The dipole moment value of BeH_2 is zero.
 (C) The bond order of O_2^{2-} and F_2 is same.
 (D) The formal charge on the central oxygen atom of ozone is -1.
 (E) In NO_2 , all the three atoms satisfy the octet rule, hence it is very stable.

Choose the correct answer from the options given below :

- (a) A, B, C, D & E (b) B & C only
 (c) B, C & D only (d) A, C & D only

30. A solution is prepared by dissolving 0.3 g of a nonvolatile non-electrolyte solute 'A' of molar mass 60 g mol^{-1} and 0.9 g of a non-volatile nonelectrolyte solute 'B' of molar mass 180 g mol^{-1} in $100 \text{ mL H}_2\text{O}$ at 27°C . Osmotic pressure of the solution will be

[Given : $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$]

- (a) 1.23 atm (b) 2.46 atm
 (c) 0.82 atm (d) 1.47 atm

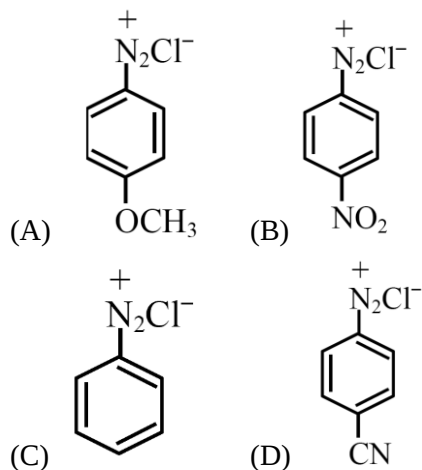
31. Among the following, the CORRECT combinations are :

- (A) $\text{IF}_3 \rightarrow \text{T-shaped} (sp^3 d)$
 (B) $\text{IF}_5 \rightarrow \text{Square pyramidal} (sp^3 d^2)$
 (C) $\text{IF}_7 \rightarrow \text{Pentagonal bipyramidal} (sp^3 d^3)$
 (D) $\text{ClO}_4^- \rightarrow \text{Square planar} (sp^2 d)$

Choose the correct answer from the options given below :

- (a) A, B and C only (b) A and B only
 (c) A, B, C and D (d) B, C and D Only

32. The correct stability order of the following diazonium salts is



- (a) $A > B > C > D$ (b) $C > D > B > A$
(c) $A > C > D > B$ (d) $C > A > D > B$

33. Consider a mixture 'X' which is made by dissolving 0.4 mol of $[\text{Co}(\text{NH}_3)_5\text{SO}_4]\text{Br}$ and 0.4 mol of $[\text{Co}(\text{NH}_3)_5\text{Br}]\text{SO}_4$ in water to make 4 L of solution. When 2 L of mixture 'X' is allowed to react with excess of AgNO_3 , it forms precipitate 'Y'. The rest 2 L of mixture 'X' reacts with excess BaCl_2 to form precipitate 'Z'. Which of the following statements is CORRECT.

- (a) 0.2 mol of 'Z' is formed
(b) 'Y' is BaSO_4 and 'Z' is AgBr
(c) 0.4 mol of 'Z' is formed
(d) 0.1 mol of 'Y' is formed

34. Given below are two statements

Statements-I : The number of paramagnetic species among $[\text{CoF}_6]^{3-}$, $[\text{TiF}_6]^{3-}$, V_2O_5 and $[\text{Fe}(\text{CN})_6]^{3-}$ is 3 .

Statement-II : $\text{K}_4[\text{Fe}(\text{CN})_6] < \text{K}_3[\text{Fe}(\text{CN})_6] < [\text{Fe}(\text{H}_2\text{O})_6]\text{SO}_4 \cdot \text{H}_2\text{O} < [\text{Fe}(\text{H}_2\text{O})_6]\text{Cl}_3$ is the correct order in terms of number of unpaired electron(s) in the complexes.

In the light of the above statements, choose the correct answer from the options given below.

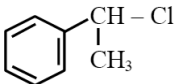
- (a) Both statement-I and statement-II are true
(b) Both statement-I and statement-II are false
(c) Statement-I is true but statement-II is false
(d) Statement-I is false but statement-II is true

35. Consider three metal chlorides x, y and z , where x is water soluble at room temperature, y is sparingly soluble in water at room temperature and z is soluble in hot water. x, y and z are respectively

- (a) MgCl_2 , AgCl and AlCl_3
(b) AgCl , Hg_2Cl_2 and PbCl_2
(c) AlCl_3 , PbCl_2 and BaCl_2
(d) CuCl_2 , AgCl and PbCl_2

36. Match the List-I with List-II

List-I (Chloro derivative)		List-II (Example)	
(A)	Vinyl Chloride	(I)	$\text{CH}_2 = \text{CH} - \text{CH}_2\text{Cl}$
(B)	Benzyl chloride	(II)	$\text{CH}_3 = \text{CH}(\text{Cl})\text{CH}_3$
(C)	Alkyl chloride	(III)	$\text{CH}_2 = \text{CHCl}$

(D)	Allyl chloride	(IV)	
-----	----------------	------	---

Choose the correct answer from the options given below:

- (a) A-IV, B-I, C-III, D-II
 (b) A-III, B-IV, C-I, D-II
 (c) A-III, B-IV, C-II, D-I
 (d) A-I, B-II, C-IV, D-III

37. 'W' g of a non-volatile electrolyte solid solute of molar mass 'M' g mol^{-1} when dissolved in 100 mL water, decreases vapour pressure of water from 640 mm Hg to 600 mm Hg. If aqueous solution of the electrolyte boils at 375 K and K_b for water is $0.52 \text{ K kg mol}^{-1}$, then the mole fraction of the electrolyte solute (x_2) in the solution can be expressed as

(Given : density of water = 1 g/mL and boiling point of water = 373 K)

- (a) $\frac{1.3}{8} \times \frac{W}{M}$ (b) $\frac{16}{2.6} \times \frac{W}{M}$
 (c) $\frac{2.6}{16} \times \frac{M}{W}$ (d) $\frac{1.3}{8} \times \frac{M}{W}$

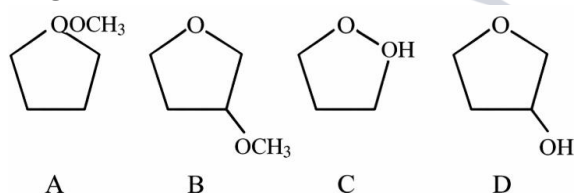
38. Match the List-I with List-II

List-I (Isothermal process for ideal gas system)		List-II Work done ($V_f > V_i$)	
(A)	Reversible expansion	(I)	$w = 0$
(B)	Free expansion	(II)	$w = -nRT \ln \frac{V_f}{V_i}$
(C)	Irreversible expansion	(III)	$w = -p_{\text{ex}}(V_f - V_i)$
(D)	Irreversible compression	(IV)	$w = -p_{\text{ex}}(V_i - V_f)$

Choose the correct answer from the options given below :

- (a) A-IV, B-I, C-III, D-II (b) A-IV, B-II, C-III, D-I
 (c) A-I, B-III, C-II, D-IV (d) A-II, B-I, C-III, D-IV

39. A student is given one compound among the following compounds that gives positive test with Tollen's reagent.



The compound is :

- (a) D (b) A
 (c) B (d) C

40. A hydroxy compound (X) with molar mass 122 g mol^{-1} is acetylated with acetic anhydride, using a large excess of the reagent ensuring complete acetylation of all hydroxyl groups. The product obtained has a molar mass of 290 g mol^{-1} . The number of hydroxyl groups present in compound (X) is :

- (a) 3 (b) 5
 (c) 2 (d) 4

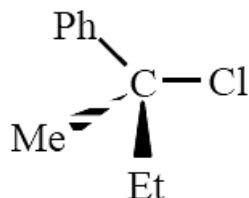
41. At 27°C in presence of a catalyst, activation energy of a reaction is lowered by 10 kJ mol^{-1} . The logarithm ratio of $\frac{k(\text{catalysed})}{k(\text{uncatalysed})}$ is (Consider that the frequency factor for both the reactions is same)

- (a) 17.41 (b) 1.741
(c) 3.482 (d) 0.1741

42. Given below are two statements :

Statement I: 'C – Cl' bond is stronger in $\text{CH}_2 = \text{CH} - \text{Cl}$ than $\text{CH}_3 - \text{CH}_2 - \text{Cl}$

Statement II : The given optically active molecule,

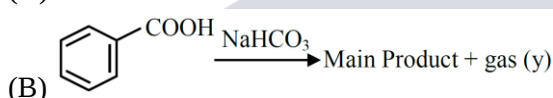
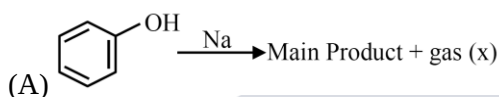


on hydrolysis gives a solution that can rotate the plane polarized light.

In the light of the above statements, choose the correct answer from the options given below

- (a) Statement I is false but Statement II is true
(b) Both Statement I and Statement II are true
(c) Both Statement I and Statement II are false
(d) Statement I is true but Statement II is false

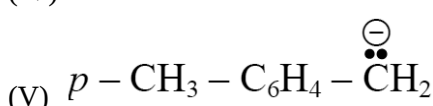
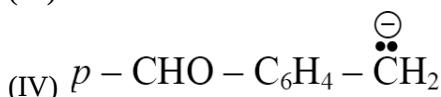
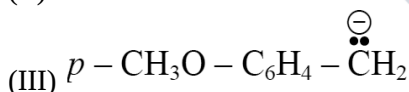
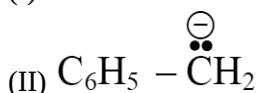
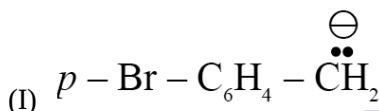
43. Consider the following two reactions A and B .



Numerical value of [molar mass of x + molar mass of y] is ____.

- (a) 4 (b) 88
(c) 46 (d) 160

44. Arrange the following carbanions in the decreasing order of stability



Choose the correct answer from the options given below :

- (a) I > II > IV > V > III (b) I > IV > II > V > III
(c) IV > I > II > V > III (d) IV > II > I > III > V

45. Given below are two statements :

Statement I: $\text{K} > \text{Mg} > \text{Al} > \text{B}$ is the correct order in terms of metallic character.

Statement II : Atomic radius is always greater than the ionic radius for any element.

In the light of the above statements, choose the correct answer from the options given below

- (a) Both Statement I and Statement II are true
(b) Both Statement I and Statement II are false

- (c) Statement I is false but Statement II is true
(d) Statement I is true but Statement II is false

SECTION-B

46. Electricity is passed through an acidic solution of Cu^{2+} till all the Cu^{2+} was exhausted, leading to the deposition of 300 mg of Cu metal. However, a current of 600 mA was continued to pass through the same solution for another 28 minutes by keeping the total volume of the solution fixed at 200 mL. The total volume of oxygen evolved at STP during the entire process is ____ mL. (Nearest integer)
[Given : $\text{Cu}^{2+}(\text{aq}) + 2\text{e}^- \rightarrow \text{Cu}(\text{s})$ $E_{\text{red}}^0 = +0.34 \text{ V}$
 $\text{O}_2(\text{g}) + 4\text{H}^+ + 4\text{e}^- \rightarrow 2\text{H}_2\text{O}$ $E_{\text{red}}^0 = +1.23 \text{ V}$
Molar mass of Cu = 63.54 g mol^{-1}
Molar mass of O_2 = 32 g mol^{-1}
Faraday Constant = 96500 C mol^{-1}
Molar volume at STP = 22.4 L]
47. Consider two Group IV metal ions X^{2+} and Y^{2+} .
A solution containing 0.01MX^{2+} and 0.01MY^{2+} is saturated with H_2S . The pH at which the metal sulphide YS will form as a precipitate is ____ (Nearest integer)
(Given : $K_{\text{sp}}(\text{XS}) = 1 \times 10^{-22}$ at 25°C , $K_{\text{sp}}(\text{YS}) = 4 \times 10^{-16}$ at 25°C , $[\text{H}_2\text{S}] = 0.1\text{M}$ in solution, $K_{\text{a1}} \times K_{\text{a2}}(\text{H}_2\text{S}) = 1.0 \times 10^{-21}$, $\log 2 = 0.30$, $\log 3 = 0.48$, $\log 5 = 0.70$)
48. The hydrogen spectrum consists of several spectral lines in Lyman series ($L_1, L_2, L_3, \dots$; L_1 has lowest energy among Lyman series). Similarly it consists of several spectral lines in Balmer series ($B_1, B_2, B_3, \dots$; B_1 has lowest energy among Balmer lines). The energy of L_1 is x times the energy of B_1 . The value of x is ____ $\times 10^{-1}$ (Nearest integer)
49. In Dumas method for estimation of nitrogen, 0.50 g of an organic compound gave 70 mL of nitrogen collected at 300 K and 715 mm pressure. The percentage of nitrogen in the organic compound is ____ % (Aqueous tension at 300 K is 15 mm).
50. X and Y are the number of electrons involved, respectively during the oxidation of I^- to I_2 and S^{2-} to S by acidified $\text{K}_2\text{Cr}_2\text{O}_7$. The value of $X + Y$ is ____ .

MATHEMATICS

SECTION-A

51. If the function $f(x) = \frac{e^{x(e^{\tan x} - 1)} + \log_e(\sec x + \tan x) - x}{\tan x - x}$ is Continuous at $x = 0$, then the value of $f(0)$ is equal to
(a) 2 (b) $\frac{2}{3}$
(c) $\frac{1}{2}$ (d) $\frac{3}{2}$
52. Let a circle of radius 4 pass through the origin O, the points $A(-\sqrt{3}a, 0)$ and $B(0, -\sqrt{2}b)$, where a and b are real parameters and $ab \neq 0$. Then the locus of the centroid of $\triangle OAB$ is a circle of radius
(a) $\frac{5}{3}$ (b) $\frac{7}{3}$
(c) $\frac{8}{3}$ (d) $\frac{11}{3}$
53. Let the lines $L_1: \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$, $\lambda \in \mathbb{R}$ and $L_2: \vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$, $\mu \in \mathbb{R}$, intersect at the point R. Let P and Q be the points lying on lines L_1 and L_2 , respectively, such that $|\overrightarrow{PR}| = \sqrt{29}$ and $|\overrightarrow{PQ}| = \sqrt{\frac{47}{3}}$. If the point P lies in the first octant, then $27(\text{QR})^2$ is equal to
(a) 340 (b) 360
(c) 320 (d) 348
54. Let 729, 81, 9, 1, be a sequence and P_n denote the product of the first n terms of this sequence. If $2 \sum_{n=1}^{40} (P_n)^{\frac{1}{n}} = \frac{3^\alpha - 1}{3^\beta}$ and $\text{gcd}(\alpha, \beta) = 1$, then $\alpha + \beta$ is equal to

- (a) 73 (b) 74
(c) 75 (d) 76
55. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \vec{a} \times \vec{b}$. Let $\vec{d}$ be a vector such that $|\vec{d} - \vec{a}| = \sqrt{11}$, $|\vec{c} \times \vec{d}| = 3$ and the angle between $\vec{c}$ and $\vec{d}$ is $\frac{\pi}{4}$. Then $\vec{a} \cdot \vec{d}$ is equal to
(a) 11 (b) 3
(c) 0 (d) 1
56. If the domain of the function $f(x) = \log_{(10x^2 - 17x + 7)}(18x^2 - 11x + 1)$ is $(-\infty, a) \cup (b, c) \cup (d, \infty) - \{e\}$, then $90(a + b + c + d + e)$ equals:
(a) 170 (b) 177
(c) 307 (d) 316
57. Let each of the two ellipses $E_1: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, ($a > b$) and $E_2: \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, ($A < B$) have eccentricity $\frac{4}{5}$. Let the lengths of the latus recta of E_1 and E_2 be ℓ_1 and ℓ_2 , respectively, such that $2\ell_1^2 = 9\ell_2$. If the distance between the foci of E_1 is 8, then the distance between the foci of E_2 is
(a) $\frac{96}{5}$ (b) $\frac{32}{5}$
(c) $\frac{16}{5}$ (d) $\frac{8}{5}$
58. The value of $\frac{\sqrt{3}\operatorname{cosec}20^\circ - \sec 20^\circ}{\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}$ is equal to
(a) 32 (b) 16
(c) 64 (d) 12
59. Let $S = \{z \in \mathbb{C} : \left| \frac{z-6i}{z-2i} \right| = 1 \text{ and } \left| \frac{z-8+2i}{z+2i} \right| = \frac{3}{5}\}$. Then $\sum_{z \in S} |z|^2$ is equal to
(a) 398 (b) 413
(c) 423 (d) 385
60. If $\cot x = \frac{5}{12}$ for some $x \in \left(\pi, \frac{3\pi}{2}\right)$, then $\sin 7x \left(\cos \frac{13x}{2} + \sin \frac{13x}{2}\right) + \cos 7x \left(\cos \frac{13x}{2} - \sin \frac{13x}{2}\right)$ is equal to
(a) $\frac{4}{\sqrt{26}}$ (b) $\frac{6}{\sqrt{26}}$
(c) $\frac{1}{\sqrt{13}}$ (d) $\frac{5}{\sqrt{13}}$
61. Let $f(t) = \int \frac{(1 - \sin(\log_e t))}{(1 - \cos(\log_e t))} dt$, $t > 1$. If $f(e^{\pi/2}) = -e^{\pi/2}$ and $f(e^{\pi/4}) = \alpha e^{\pi/4}$, then α equals
(a) $-1 - \sqrt{2}$ (b) $-1 - 2\sqrt{2}$
(c) $1 + \sqrt{2}$ (d) $-1 + \sqrt{2}$
62. Let R be a relation defined on the set $\{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$ by $R = \{((a, b), (c, d)) : 2a + 3b = 3c + 4d\}$. Then the number of elements in R is
(a) 6 (b) 18
(c) 12 (d) 15
63. Let $A(1, 0)$, $B(2, -1)$ and $C\left(\frac{7}{3}, \frac{4}{3}\right)$ be three points. If the equation of the bisector of the angle ABC is $\alpha x + \beta y = 5$, then the value of $\alpha^2 + \beta^2$ is
(a) 8 (b) 5
(c) 13 (d) 10
64. Let $S = \frac{1}{25!} + \frac{1}{3!23!} + \frac{1}{5!21!} + \dots$ up to 13 terms. if $13S = \frac{2^k}{n!}$, $k \in \mathbb{N}$, then $n + k$ is equal to
(a) 51 (b) 52
(c) 49 (d) 50

65. Consider an A.P.: $a_1, a_2, \dots, a_n$; $a_1 > 0$. If $a_2 - a_1 = \frac{-3}{4}$, $a_n = \frac{1}{4}a_1$, and $\sum_{i=1}^n a_i = \frac{525}{2}$, then $\sum_{i=1}^{17} a_i$ is equal to
 (a) 476 (b) 952
 (c) 238 (d) 136
66. Let $\alpha, \beta \in \mathbb{R}$ be such that the function $f(x) = \begin{cases} 2\alpha(x^2 - 2) + 2\beta x & , x < 1 \\ (\alpha + 3)x + (\alpha - \beta) & , x \geq 1 \end{cases}$ be differentiable at all $x \in \mathbb{R}$.
 Then $34(\alpha + \beta)$ is equal to
 (a) 84 (b) 48
 (c) 36 (d) 24
67. From a lot containing 10 defective and 90 nondefective bulbs, 8 bulbs are selected one by one with replacement. Then the probability of getting at least 7 defective bulbs is :-
 (a) $\frac{7}{10^7}$ (b) $\frac{81}{10^8}$
 (c) $\frac{67}{10^8}$ (d) $\frac{73}{10^8}$
68. The mean and variance of a data of 10 observations are 10 and 2, respectively. If an observations α in this data is replaced by β , then the mean and variance become 10.1 and 1.99, respectively. Then $\alpha + \beta$ equals.
 (a) 10 (b) 15
 (c) 5 (d) 20
69. Let A_1 be the bounded area enclosed by the curves $y = x^2 + 2$, $x + y = 8$ and y-axis that lies in the first quadrant. Let A_2 be the bounded area enclosed by the curves $y = x^2 + 2$, $y^2 = x$, $x = 2$, and y-axis that lies in the first quadrant. Then $A_1 - A_2$ is equal to
 (a) $\frac{2}{3}(2\sqrt{2} + 1)$ (b) $\frac{2}{3}(4\sqrt{2} + 1)$
 (c) $\frac{2}{3}(\sqrt{2} + 1)$ (d) $\frac{2}{3}(3\sqrt{2} + 1)$
70. The number of the real solutions of the equation : $x|x + 3| + |x - 1| - 2 = 0$ is
 (a) 3 (b) 2
 (c) 5 (d) 4

SECTION-B

71. Let a differentiable function f satisfy the equation $\int_0^{36} f\left(\frac{tx}{36}\right) dt = 4\alpha f(x)$.
 If $y = f(x)$ is a standard parabola passing through the points (2,1) and (-4, β),
 Then β^α is equal to ____.
72. Let a line L passing through the point $P(1,1,1)$ be perpendicular to the lines $\frac{x-4}{4} = \frac{y-1}{1} = \frac{z-1}{1}$ and $\frac{x-17}{1} = \frac{y-71}{1} = \frac{z}{0}$. Let the line L intersect the yz -plane at the point Q . Another line parallel to L and passing through the point $S(1,0,-1)$ intersects the yz -plane at the point R . Then the square of the area of the parallelogram $PQRS$ is equal to ____.
73. The number of numbers greater than 5000, less than 9000 and divisible by 3, that can be formed using the digits 0,1,2,5,9, if the repetition of the digits is allowed, is ____
74. Let $(2\alpha, \alpha)$ be the largest interval in which the function $f(t) = \frac{|t+1|}{t^2}$, $t < 0$, is strictly decreasing. Then the local maximum value of the function $g(x) = 2\log_e(x-2) + \alpha x^2 + 4x - \alpha$, $x > 2$, is ____
75. The number of 3×2 matrices A , which can be formed using the elements of the set $\{-2, -1, 0, 1, 2\}$ such that the sum of all the diagonal elements of $A^T A$ is 5, is ____