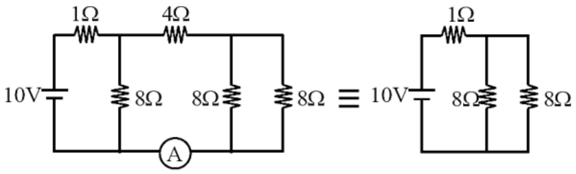
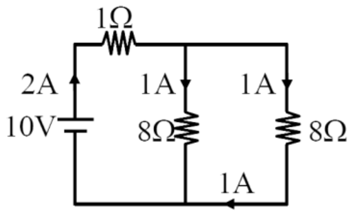
1. **(b)** In steady state



$I = 2\text{ A}$



Ammeter reading is 1A.

2. **(a)** $LC = 1MSD - 1MSD = 1MSD - \frac{48}{50}MSD$

$= \frac{2}{50}MSD = \frac{2}{50} \times .05\text{ mm} = 0.002\text{ mm}$

3. **(b)** $P = +5D$

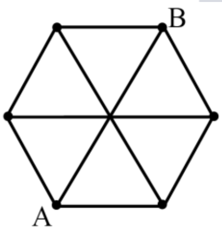
$\frac{1}{f} = 5 \Rightarrow f = 20\text{ cm}$

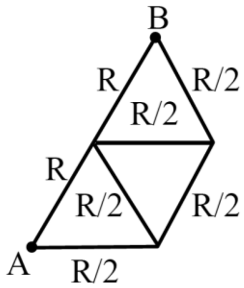
⇒ If object is between f & 2f image will be beyond 2f & magnified.

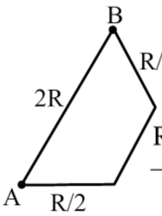
⇒ If object is beyond 2f, image will be between f & 2f & diminished.

Hence reading of P_3 are incorrect.

4. **(a)**



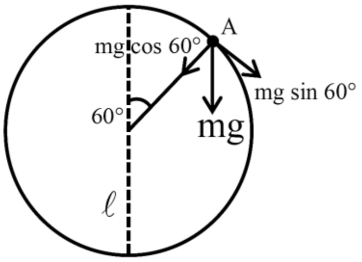




$$\frac{R \times \frac{R}{2}}{\frac{3R}{2}} = \frac{R}{3}$$

$$R_{eq} = \frac{2R \times \frac{4R}{3}}{2R + \frac{4R}{3}} = \frac{8R^2}{10R} = \frac{4}{5}R$$

5. **(b)**



$$T + mg\cos 60^\circ = \frac{mV^2}{\ell}$$

$T = 0$

$V^2 = \frac{g\ell}{2}$ here V is the speed at point A

M.E.C.

$$\frac{1}{2}mu^2 = mg(\ell + \ell\cos 60^\circ) + \frac{1}{2}mV^2$$

$$u^2 = 3\,g\ell + \frac{g\ell}{2}$$

$$u = \sqrt{\frac{7\,g\ell}{2}}$$

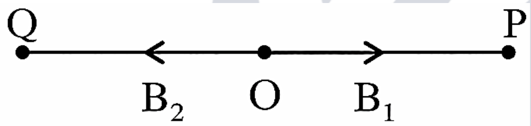
6. **(d)** $mg\frac{\ell}{6} = \frac{1}{2}I\omega^2$

Here $I = \frac{m\ell^2}{12} + \frac{m\ell^2}{36} = \frac{m\ell^2}{9}$

$$mg\frac{\ell}{6} = \frac{m\ell^2}{18}\omega^2 \Rightarrow \omega^2 = \frac{3\,g}{\ell}$$

$$\omega = \sqrt{\frac{3\,g}{\ell}}$$

7. **(d)** $B_{net} = B_1 - B_2$



$$= \frac{4\mu_0 iR^2}{2(R^2+R^2)^{3/2}} - \frac{\mu_0 iR^2}{2(R^2+R^2)^{3/2}}$$

$$= \frac{3\mu_0 i}{4\sqrt{2}R}$$

8. **(d)** $q.(3.2) = \frac{hc}{\lambda} - \phi$ .... (i)

$q(0.7) = \frac{hc}{2\lambda} - \phi$ .... (ii)

Eq. (i) - Eq. (ii)

$$q\cdot(2.5) = \frac{hc}{2\lambda}$$

$$2.5 = \left(\frac{hc}{e}\right)\left(\frac{1}{2\lambda}\right)$$

$$2.5 = \frac{12400}{2(\lambda)}$$

$$\lambda = \frac{12400}{5}\text{Å}$$

$\lambda = 2480\text{Å}$

$\lambda = 2.48 \times 10^{-7}$ m

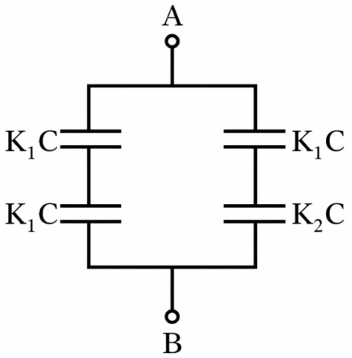
9. **(b)** $f_{5\text{ closed}} = f_{1\text{ open}}$

$$\frac{5v}{4\,L_{closed}} = \frac{v}{2\,L_{open}}$$

$\frac{L_{closed}}{L_{open}} = \frac{5}{2}$

$x = 2$

10. (d) For C_A :

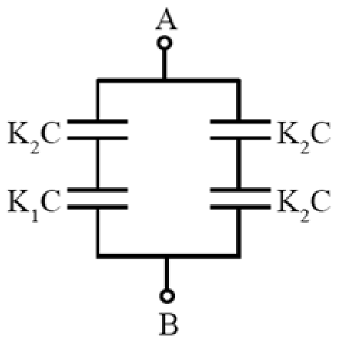


Let $\frac{\varepsilon_0 A}{d} = C$

$\therefore C_A = \frac{K_1 C}{2} + \frac{K_1 K_2 C}{K_1 + K_2}$

$= K_1 C\left[\frac{K_1 + 2K_2}{2(K_1 + K_2)}\right]$

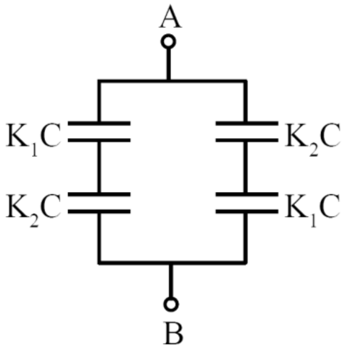
For C_B :



$C_B = \frac{K_2 C}{2} + \frac{K_1 K_2 C}{K_1 + K_2}$

$= K_2 C\left[\frac{K_1 + 2K_2}{2(K_1 + K_2)}\right]$

For C_C :



$C_C = \frac{2K_1 K_2 C}{(K_1 + K_2)}$

$C_A > C_C > C_B$

11. (a) $BE_{He^4} - BE_{He^3} = 20\,MeV$ .... (i)

$BE_{He^5} - BE_{He^4} = -0.9\,MeV$ .... (ii)

From eq (i) & (ii)

$BE_{He^4} > BE_{He^5} > BE_{He^3}$

$X_4 > X_5 > X_3$

12. (d) $Mg = F_b$

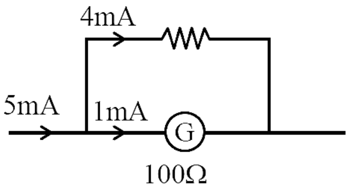
$dAHg = \rho Ahg$

$600 \times 8\text{ cm} = 900 \times h$

$h = \dfrac{16}{3}\text{ cm}$

$h = 5.3\text{ cm}$

13. (a)



$G = 100\Omega$

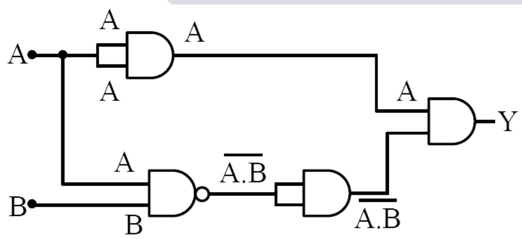
$i_g = 1\text{ mA}$

$i = 5\text{ mA}$

$$r_s = \frac{G}{\left(\frac{i}{i_g} - 1\right)}$$

$$= \frac{100}{\left(\frac{5}{1} - 1\right)} = 25\Omega$$

14. (d)

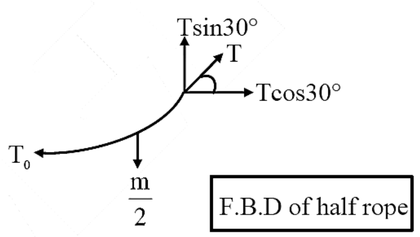


$y = A \cdot \overline{A \cdot B}$

$= A \cdot (\overline{A} + \overline{B}) = 0 + A\overline{B}$

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

15. (a)



$T\sin 30^\circ = \dfrac{m}{2}g$

$T\cos 30^\circ = T_0$

$\tan 30^\circ = \frac{mg}{2\,T_0}$

$T_0 = \frac{\sqrt{3}}{2} mg$

16. **(d)** $V \to 8\,V \Rightarrow R \to 2R$

$\Rightarrow A \to 4\,A$

$\Rightarrow I \to \frac{I_0}{4}$

17. **(b)** $d = 2$ mm

$D = 10$ m

$\lambda = 6000\text{Å}$

$y = \frac{d}{2}$(in front of one slit)

$I = 4I_0\cos^2\left(\frac{2\pi}{\lambda}\cdot\frac{y}{D}d\right)$

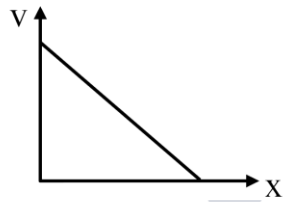
$\Rightarrow I = I_0$

18. **(c)** $\Delta Q = nC_v\Delta T$ (isochoric)

$= \frac{C_v}{R}\cdot nR\Delta T = \frac{C_v}{R}(P_2 - P_1)V$

$= \frac{21}{8.3} \times (30 - 21.7) \times 1 = 21$ J

19. **(b)**



Eq. of V vs x from graph

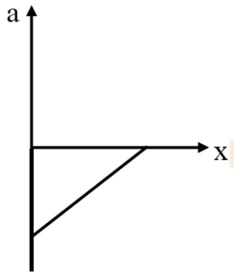
$V = C_1 - C_2x$

$a = V\frac{dV}{dx}$

$= (C_1 - C_2x) \times -C_2$

$a = C_2^2x - C_1C_2$

∴ graph is straight line + ve solve - ve intercept



20. **(d)** $m = -3 = \frac{v}{u}$

$v = -3u$

$|v| - |u| = 40$

$u = 20$ cm

$v = 60$ cm

$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{-60} + \frac{1}{-20} = \frac{1}{f}$

$f = -15$ cm

21. **(481)** $C_V = C_P - R = \frac{5}{2}R$

$\Delta Q = nC_V\Delta T$

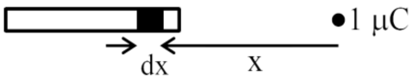
$300 = n \times \frac{5}{2} \times 8.314 \times 30$

$n = 0.48$

$\frac{m}{M} = 0.48$

We cannot find mass (m) because molar mass (M) not given.

22. **(90)**



$F = \int dF = \int_{2\text{ cm}}^{12\text{ cm}} \frac{kq\lambda dx}{x^2} = kq\lambda\left(\frac{1}{2 \times 10^{-2}} - \frac{1}{12 \times 10^{-2}}\right)$

$F = (9 \times 10^9)(10^{-6})\left(\frac{24 \times 10^{-6}}{10^{-1}}\right)\left(\frac{5}{12}\right) \times 10^2$

$= 9 \times 24 \times \frac{5}{12} = 90\text{ N}$

23. **(6)** In case I

$\frac{2}{3} = \frac{\ell}{(100-\ell)}$ .... (i)

$\ell = 40\text{ cm}$

In case II

$\frac{2}{R} = \frac{\ell + 10}{100 - (\ell + 10)}$

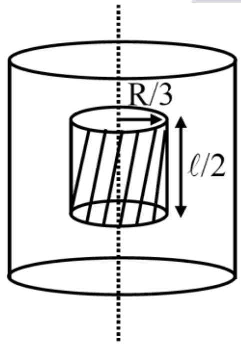
Put $\ell = 40$ cm& solve

$R = 2\Omega$

$\therefore \frac{3x}{3 + x} = 2$

$x = 6\Omega$

24. **(162)**



Original mass (M)

The removed mass (m)

$m = \rho \times \pi\left(\frac{R}{3}\right)^2 \times \frac{L}{2}$

$= \frac{\rho \cdot \pi R^2 L}{18} = \frac{M}{18}$

$I' = \frac{1}{2} \cdot \frac{M}{18} \cdot \frac{R^2}{9} = \frac{1}{324}MR^2$

$\frac{I}{I'} = \frac{\frac{1}{2}MR^2}{\frac{1}{324}MR^2} = 162$

25. **(11304)** $W = \Delta u$

$= S \times (8\pi r_2^2 - 8\pi r_1^2)$

$= 0.04 \times 2 \times \frac{22}{7}(147) \times 10^{-4}$

$W = 3696 \times 10^{-6} J$

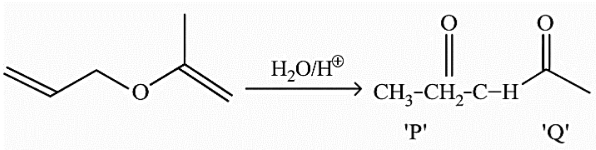
$3696 = 15000 - x$

$x = 11304 \mu J$

26. (a) C^{13} is not radioactive

C^{14} is radioactive

27. (d)



' P ' and ' Q ' can be differentiated by Fehling's test.

P gives positive Fehling test

Q gives negative Fehling test

28. (a) Gly ala val

Gly val ala

Val gly ala

Val ala gly

Ala val gly

Ala gly val

Total tri peptides = 6

29. (a) $\frac{Y_A}{Y_B} = \frac{P_A^0}{P_B^0} \cdot \frac{X_A}{X_B}$

$\frac{0.8}{0.2} = \frac{55}{15} \times \frac{X_A}{X_B}$

$\frac{X_A}{X_B} = \frac{60}{55} = \frac{12}{11}$

$X_A = \frac{12}{23} = 0.5217$

30. (a) $P_2(g) + Q_2(g) \rightleftharpoons 2PQ(g)$

$t = t_{eq}$ 2 mole 2 mole 2 mole

$K_{eq} = \frac{2^2}{2.2} = 1$

Now 1 mole of each P_2 and Q_2 is added

So reaction will move in forward direction

$P_2(g) + Q_2(g) \rightleftharpoons 2PQ(g)$

$t = t'_{eq.}$ $3 - x$ $3 - x$ $2 + 2x$

$\frac{2 + 2x}{3 - x} = 1$

$2 + 2x = 3 - x$

$x = \frac{1}{3}$

At new equilibrium :

Moles of $P_2 = \frac{8}{3} = 2.67$

Moles of $Q_2 = \frac{8}{3} = 2.67$

Moles of $PQ = \frac{8}{3} = 2.67$

31. (c)

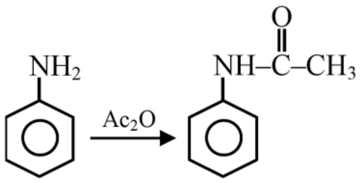
Species	Bond order	Magnetic Nature

$O_2^{\ +}$	2.5	Paramagnetic
$O_2^{\ -}$	1.5	Paramagnetic
$O_2^{\ +}$	2.5	Paramagnetic
$N_2^{\ -}$	2.5	Paramagnetic
$N_2^{\ 2-}$	2	Paramagnetic

32. (d) Wavelength of light absorbed increases as C.F.S.E of complex decreases.
$[Co(CN)_6]^{3-}$ has maximum CFSE
$[CoF_6]^{3-}$ has least CFSE
Ligand field strength ↑ ; C.F.S.E ↑
Correct wavelength order.
V > II > IV > I > III

33. (c) $Cl_2 + \quad 2KOH \rightarrow KCl + KClO + H_2O$
t = 0   1 mole   4 mole
t_f 0   2 mole   1 mole 1 mole
$[OH^-] = 1M$
$[Cl^-] = \frac{1}{2}M$
$[ClO^-] = \frac{1}{2}M$

34. (a)



9.3 gm   11 gm
MW = 93   MW = 135
$n = \frac{9.3}{93} = 0.1 \; n = \frac{11}{135} = 0.08148$
$\%\ \text{yield} = \frac{0.08148}{0.1} \times 100 = 81.5\%$

35. (d) Statement B & D are not true

36. (b) To compare second ionization potential configuration of mono-cation is observed

C^+	N^+	O^+	F^+
[He] $2\ s^2sp^1$	[He] $2\ s^22p^2$	[He]$2\ s^22p^3$ Half-filled stable.	[He]$2\ s^22p^4$

2^{nd} IE order
$O > F > N > C$

37. (c) To identify Ba^{2+}&Ca^{2+}
Reagent $(NH_4)_2CO_3 + NH_4Cl$ is used $BaCO_3$ & $CaCO_3$ are obtained as precipitates

38. (d) $n_1 \rightarrow$ lower energy level
$n_2 \rightarrow$ higher energy level
$n_1 + n_2 = 4, n_2 = 3$
$n_2 - n_1 = 2, n_1 = 1$
Rydderg's formula :

$\frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\frac{1}{\lambda} = R_H (3)^2 \left[\frac{1}{1^2} - \frac{1}{3^2}\right]$

$\frac{1}{\lambda} = 8R_H$

$\lambda = \frac{1}{8R_H}$

$\lambda = \frac{1}{8 \times 1.1 \times 10^5}$

$\lambda = \frac{1000}{8.8} \times 10^{-8}$ cm

$\lambda = 113.63 \times 10^{-8}$ cm

$\lambda \simeq 1.14 \times 10^{-6}$ cm

39. **(a)** Statement I : False

Cross aldol can give 2 or 4 products

Statement II : False

Benzaldehyde & Acetone both react with semi carbazide.

40. **(d)** **Statement I** : False

$R - NC \xrightarrow{H_3O^+} R - NH_2 + HCOOH$

Statement II : True

$R - CN \xrightarrow{H_3O^+} RCOOH \xrightarrow{SOCl_2} RCOCl$

$\xrightarrow{NH_3} RCONH_2 \xrightarrow{NaOCl/OH^-} RNH_2$

$\xrightarrow{CHCl_3 + KOH} R - NC$

41. **(b)** II, IV & V are secondary alcohol.

42. **(b)** $CH_4(g) \rightarrow C(g) + 4H(g);\ \Delta_r H = xkJ/$ mole $C_2H_6(g) \rightarrow 2C(g) + 6H(g); \Delta_r H = ykJ/mole 1000x = 4 \times \varepsilon_{C-H}$

$1000y = 1 \times \varepsilon_{C-C} + 6 \times \varepsilon_{C-H}$

$\varepsilon_{C-C} = \left[y - \frac{3x}{2}\right] \times 1000 = \frac{hc}{\lambda}.N_A$

(' λ ') wavelength of photon $= \frac{hcN_A}{[4y-6x]\times 250}$

43. **(d)** $P_{N_2} = K_H \cdot X_{N_2}$

$P_{N_2} = 0.8 \times 10 = 8$ atm

$8 \times 760 = 6.5 \times 10^7 \times X_{N_2}$

$X_{N_2} = \frac{8 \times 760}{6.5 \times 10^7}$

$X_{N_2} = 9.35 \times 10^{-5}$.

44. **(a)** Lightest element of Group 7 $\Rightarrow$ Mn

$K_2MnO_4 \Rightarrow$ Green

45. **(d)** Both Statements are correct.

46. **(73)** $C_xH_yO_z + O_2 \rightarrow CO_2 + H_2O$

$t = 0$ 0.25gm

t_{eq} – 0.18gm 0.15gm

Mass of ' C ' $= \frac{0.18}{44} \times 12 = 0.049 \simeq 0.05$gm

Mass of ' H ' $= \frac{0.15}{18} \times 2 = 0.016 \simeq 0.017$gm.

Mass of ' O ' $= 0.25 - 0.05 - 0.017 = 0.1833$gm

Mass % of ' O ' $= \frac{0.1833}{0.25} \times 100 = 73.32\%$.

47. **(102)** $t_{1/2}=\frac{\ln 2}{K}$

$K=\frac{\ln 2}{245}$

$t=\frac{1}{K}\ell n\frac{a_0}{a_t}$

$t_{25\%}=\frac{1}{K}\ell n\frac{4}{3}$

$t_{25\%}=\frac{1}{\frac{\ln 2}{245}}\ln\frac{4}{3}$

$t_{25\%}=245\frac{\ln\frac{4}{3}}{\ln 2}=245\left[\frac{2\log 2-\log 3}{\log 2}\right]$

$=245\left[\frac{2\times 0.3010-0.4771}{0.3010}\right]=101.66$ day.

48. **(3)**$[Cr(H_2O)_5Cl]Cl_2\cdot H_2O+AgNO_3\rightarrow 2AgCl$ $2.75/266.5$

$=0.0103$ moles

0.02063 mole

Mass of AgCl $=0.02063\times 143.5=2.96$gm

49. **(b)** $K_a(HQ)=C_1\alpha_1^2$

$\alpha_1=\frac{\lambda_m(HQ)}{\lambda_m^\infty(HQ)}$

$K_a(HZ)=C_2{\alpha_2}^2$

$\alpha_2=\frac{\lambda_m(HZ)}{\lambda_m^\infty(HZ)}$

$\frac{K_a(HQ)}{K_a(HZ)}=\frac{C_1}{C_2}\cdot\left(\frac{\alpha_1}{\alpha_2}\right)^2=\frac{0.18}{0.02}\cdot\left[\frac{\lambda_m(HQ)}{\lambda_m(HZ)}\right]^2$

$\frac{K_a(HQ)}{K_a(HQ)}=9\times\left(\frac{1}{30}\right)^2=\frac{1}{100}$

$pK_a(HQ)-pK_a(HZ)=2$

50. **(6)**$1.4dm^3$ (or 1.4 mL) occupied by 1 gm

Molecular weight of $Q=\frac{22.4}{1.4}=16$

Q is CH_4 gas and Grignard reagent is CH_3MgBr

$CH_3MgBr\xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CO_2\text{ (dryice)}}CH_3COOH$

(Molecular weight 60)

$\therefore$ Weight of 0.1 mole of $CH_3COOH=6$

51. **(d)** $f(x)=\frac{\int\left(\frac{7}{x^8}+\frac{9}{x^{10}}\right)}{\left(\frac{1}{x^9}+\frac{1}{x^7}+2\right)^2}dx$

Put $t=\frac{1}{x^9}+\frac{1}{x^7}+2\Rightarrow\frac{dt}{dx}=\frac{-9}{x^{10}}-\frac{7}{x^8}$

$f(x)=\int\frac{-dt}{t^2}=\frac{1}{t}+C$

$f(x)=\frac{1}{\frac{1}{x^9}+\frac{1}{x^7}+2}+C$

$=\frac{x^9}{1+x^2+2x^9}+C$

Given $f(1)=\frac{1}{4}=\frac{1}{4}+C\Rightarrow C=0$

$f(x)=\frac{x^9}{1+x^2+2x^9}$

$$f'(x) = \frac{(1+x^2+2x^9) - 9x^8 - x^9(2x+18x^8)}{(1+x^2+2x^9)^2}$$

$$f'(x) = \frac{36-20}{16} = 1$$

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 4 & 1 & 1 \\ \alpha^2 & \frac{1}{4} & 1 \end{pmatrix}$$

$|A| = |1-\alpha^2| = 3$

$1-\alpha^2 = 3, -3 \Rightarrow \alpha^2 = -2, 4$

Value of $\alpha^2 = 4$

$B = \operatorname{adj}(\operatorname{adj}A)$

$|B| = 81 = |A|^4 \Rightarrow |A| = 3$

52. (c) $f(t) = \frac{-3}{4} + 2t - t^2$

$f(t)|_{\text{maximum}} = \frac{1}{4} = e \Rightarrow e^2 = \frac{1}{16} \Rightarrow \frac{a^2-b^2}{a^2} = \frac{1}{16}$..

$\because \frac{2b^2}{a} = 30 \Rightarrow b^2 = 15a$

By (i) & (ii)

$16(a^2 - 15a) = a^2 \Rightarrow 15a^2 - 16 \times 15a = 0$

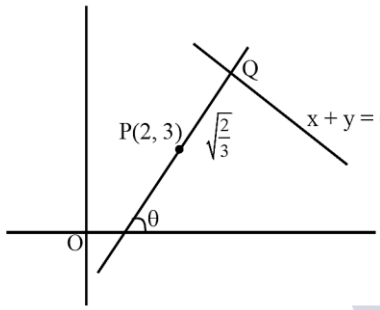
$a = 16$

$b^2 = 240$

$a^2 + b^2 = 256 + 240$

$= 496$

53. (c)



Let Q is $\left(\sqrt{\frac{2}{3}}\cos\theta + 2, \sqrt{\frac{2}{3}}\sin\theta + 3\right)$

so, $x + y = 6$

$\sqrt{\frac{2}{3}}(\cos\theta + \sin\theta) + 5 = 6$

$\sin\theta + \cos\theta = \sqrt{\frac{3}{2}}$

$1 + \sin 2\theta = \frac{3}{2}$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{12} \& \frac{5\pi}{6}$

So $\theta_1 + \theta_2 = \frac{\pi}{2}$

54. (c) $\vec{v} = x\vec{a} + y\vec{b} = x(2\hat{i} - \hat{j} - \hat{k}) + y(\hat{i} + 3\hat{j} - \hat{k})$

$\vec{v} = (2x + y)\hat{i} + (3y - x)\hat{j} + (-x - y)\hat{k}$

$\left|\frac{\vec{v} \cdot \vec{C}}{|\vec{C}|}\right| = \frac{1}{\sqrt{14}}$

$\vec{v} \cdot \vec{c} = 2(2x + y) + 3y - x - 3x - 3y$

$= 2y$

$\left|\frac{2y}{\sqrt{14}}\right| = \frac{1}{\sqrt{14}} \Rightarrow |2y| = 1$

$|\vec{v}| = \sqrt{(2x + y)^2 + (3y - x)^2 + (x + y)^2}$

$= \sqrt{6x^2 + 11y^2 + 4xy - 6xy + 2xy}$

$= \sqrt{6x^2 + \frac{11}{4}} = \frac{\sqrt{24x^2 + 11}}{2}$

Now if we take $x^2 = 1$ then option $\frac{\sqrt{35}}{2}$ matches most probably NTA thought could been this.

55. (a) a_1, a_2, a_3, a_4 as $a - 3\,d, a - d, a + d, a + 3\,d$

where $d = \frac{\ell}{2}$

$\because a_1 + a_2 + a_3 + a_4 = 48 \Rightarrow 4a = 48 \Rightarrow a = 12$

$\& a_1a_2a_3a_4 + \ell^4 = 361 \Rightarrow (a^2 - 9d^2)(a^2 - d^2) + 16d^4$

$= 361$

$\Rightarrow (144 - 9\,d^2)(144 - d^2) + 16\,d^4 = 361$

$\Rightarrow 25\,d^4 - 1440\,d^2 + (144)^2 = 361$

$(5\,d^2 - 144)^2 = 19^2$

$\therefore 5\,d^2 - 144 = 19$ or -19

$d^2 = \frac{163}{5}$ or $d^2 = \frac{125}{5} = 25$

$d = \sqrt{\frac{163}{5}}$ or $d = 5$

$\therefore \ell = 2\sqrt{\frac{163}{5}}$ or $\ell = 10$

(rejected)

$\because$ common difference is an integer

$\therefore$ largest term $= 12 + 15 = 27$

56. (c) Parametric point P on $x^2 = 4y$ is $P(2t, t^2)$

$\therefore$ mirror image of P in $x - y = 1$ is

$Q \equiv \left(2t - \frac{2 \cdot 1 \cdot (2t - t^2 - 1)}{2}, t^2 + \frac{2 \cdot 2(1) \cdot (2t - t^2 - 1)}{2}\right)$

$Q \equiv (t^2 + 1, 2t - 1) \equiv (h, k)$

$\therefore$ locus of Q is $x = \frac{(y+1)^2}{4} + 1$ which is the required parabola.

$\therefore (y + 1)^2 = 4(x - 1)$

$\therefore a = 1, b = 4, c = 1$

$\therefore a + b + c = 6$

57. (a) Let $a = \frac{4}{7}, b = \frac{1}{3}$

Multiply N^r and D^r by $(a - b) = \frac{4}{7} - \frac{1}{3} = \frac{5}{21}$

$\frac{1}{a - b}[(a^2 - b^2) + (a^3 - b^3) + (a^4 - b^4) + \cdots .\infty]$

$$\frac{1}{a-b}\left[\frac{a^2}{1-a}-\frac{b^2}{1-b}\right]=\frac{21}{5}\left[\frac{\frac{16}{49}}{1-\frac{4}{7}}-\frac{\frac{1}{9}}{1-\frac{1}{3}}\right]$$

$$=\frac{21}{5}\left[\frac{16}{21}-\frac{1}{6}\right]=\frac{21}{5}\left[\frac{96-21}{21.6}\right]$$

$$=\frac{75}{5.6}=\frac{15}{6}=\frac{5}{2}$$

58. (b) $\begin{vmatrix} 2p_{11} & 2^2p_{12} & 2^3p_{13} \\ 2^2p_{21} & 2^3p_{22} & 2^4p_{23} \\ 2^3p_{31} & 2^4p_{32} & 2^5p_{33} \end{vmatrix}=2^{10}$

$$2^2\cdot 2\cdot 2^3\begin{vmatrix} p_{11} & p_{12} & p_{13} \\ 2p_{21} & 2p_{22} & 2p_{23} \\ 2^2p_{31} & 2^2p_{32} & 2^2p_{33} \end{vmatrix}=2^{10}$$

$$2^9\begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix}=2^{10}\Rightarrow |P|=2$$

$$|\mathrm{adj}(\mathrm{adj}(P))|=|P|^{(n-1)^2}=|P|^4=2^4=16$$

59. (a) ADIPRUU

$A\rightarrow\frac{6!}{2!}=360$

$D\rightarrow\frac{6!}{2!}=360$

$P\rightarrow\frac{6!}{2!}=360$

$R\rightarrow\frac{6!}{2!}=360$

$UA\rightarrow 5!=120$

$UDAP\rightarrow 3!=6$

$UDAR\rightarrow 3!=6$

$UDAU\rightarrow 3!=6$

UDAYPRU → 1

UDAYPUR → 1

$\text{Total}=1580$

60. (d) $40^n=2^{3n}\times 5^n$

$$E_2(60!)=\left[\frac{60}{2}\right]+\left[\frac{60}{2^2}\right]+\left[\frac{60}{2^3}\right]+\left[\frac{60}{2^4}\right]+\left[\frac{60}{2^5}\right]$$

$$=30+15+7+3+1=56$$

$$E_5(60!)=\left[\frac{60}{5}\right]+\left[\frac{60}{5^2}\right]$$

$$=12+2=14$$

$$40^n=(2^3)^n\times 5^n=(2^3\times 5)^n$$

$$60!=2^{56}\times 5^{14}\ldots=2^{14}\cdot(2^3\cdot 5)^{14}$$

∴ Maximum value of n is 14 .

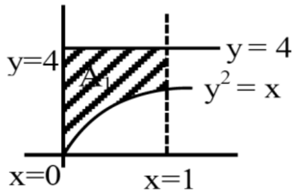
61. (b) $\sqrt{2}=\dfrac{\begin{vmatrix} -1 & 1 & 3 \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}}$

$$\sqrt{2}=\frac{-1(-2\alpha+2\alpha)-1(2\alpha^2+\alpha^2)+3(2\alpha+\alpha)}{\left|\hat{i}(-2\alpha+2\alpha)-\hat{j}(2\alpha^2+\alpha^2)+\hat{k}(2\alpha+\alpha)\right|}$$

$$\sqrt{2}=\frac{-3\alpha^2+9\alpha}{\sqrt{9\alpha^4+9\alpha^2}}$$

$\sqrt{2} = \frac{-\alpha + 3}{\sqrt{\alpha^2 + 1}}$
$\Rightarrow 2\alpha^2 + 2 = \alpha^2 + 9 - 6\alpha$
$\alpha^2 + 6\alpha - 7 = 0$
$(\alpha + 7)(\alpha - 1) = 0$
$\alpha = -7, 1$
sum $= -7 + 1 = -6$

62. (c) at $\alpha = 0 \Rightarrow f(0)$
$x = 0, x = 1, y^2 = x$
$y = |0 \cdot x - 5| - |1 - 0 \cdot x| + 0 \cdot x^2$
$y = 4$

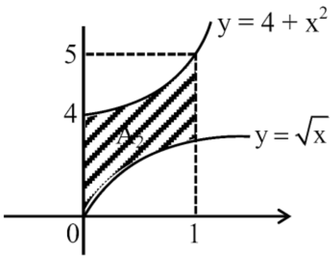


$A_1 = \int_0^1 (4 - \sqrt{x})dx$

$= 4x - \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^1$

$= 4 - \frac{2}{3}(1) = \frac{10}{3}$

at $\alpha = 1 \Rightarrow f(1)$
$x = 0, x = 1, y^2 = x,$
$y = |x - 5| - |1 - x| + x^2$ in $x \in (0,1)$
$y = 5 - x - (1 - x) + x^2$
$y = 4 + x^2$



$A_2 = \int_0^1 \left((4 + x^2) - (\sqrt{x})\right) dx$

$= 4x + \frac{x^3}{3} - \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^1$

$= 4 + \frac{1}{3} - \frac{2}{3} = \frac{11}{3}$

$|f(0) + f(1)| = |A_1 + A_2| = \left|\frac{10}{3} + \frac{11}{3}\right| = \left|\frac{21}{3}\right| = 7$

63. (b) $-1 \leq \frac{2}{x^2 - 2x - 2} \leq 1$

$\frac{1 + x^2 - 2x - 2}{x^2 - 2x - 2} \geq 0 \Rightarrow \frac{(x-1)^2 - 2}{(x-1)^2 - 3} \geq 0$

$\Rightarrow \frac{(x - 1 - \sqrt{2})(x - 1 + \sqrt{2})}{(x - 1 - \sqrt{3})(x - 1 + \sqrt{3})} \geq 0$

$x \in (-\infty, 1 - \sqrt{3}) \cup [1 - \sqrt{2}, 1 + \sqrt{2}] \cup (1 + \sqrt{3}, 0)$ ………….. (i)

$$1-\frac{1}{x^2-2x-2}\geq 0 \Rightarrow \frac{x^2-2x-3}{x^2-2x-2}\geq 0$$
$$\Rightarrow \frac{(x+1)(x-3)}{(x-1+\sqrt{3})(x-1-\sqrt{3})}\geq 0$$
$x \in (-\infty,-1] \cup (1-\sqrt{3},\sqrt{3}+1) \cup [3,\infty)$ …….. (ii)

(i) ∩ (ii)

$\Rightarrow x \in (-\infty,-1] \cup [1-\sqrt{2},1+\sqrt{2}] \cup [3,\infty)$

$\therefore \alpha+\beta+\gamma+\delta = 4$

64. (d) $\Sigma y_i = a\Sigma x_i + \Sigma b$

$= a \times (1+2+\cdots..+19) + 19\,b$
$$\frac{\Sigma y_i}{19} = \frac{a\times 19\times 20}{2\times 19} + b$$
$30 = 10a + b$ ………….. (i)

Variance of X $= \frac{\Sigma x_i^2}{19} - \left(\frac{\Sigma x_i}{19}\right)^2$
$$= \frac{19\times 20\times 39}{19\times 6} - (10)^2 = 30$$
Variance of Y $= a^2$ (variance of X)

$750 = a^2 \times 30$

$a^2 = 25 \Rightarrow a = \pm 5$

if $a = +5 \Rightarrow\ b = 30 - 50 = -20$ ... from (i)

if $a = -5 \Rightarrow\ b = 30 + 50 = 80$ ... from (i)

sum of values of b $= 80 - 20 = 60$

65. (d) $f(x) = |\ell nx| - |x-1|$
$$= \begin{cases} \ell nx - (x-1) & x \geq 1 \\ -\ell nx + (x-1) & 0 < x < 1 \end{cases}$$
$$= \begin{cases} \ln x - x + 1 & x \geq 1 \\ -\ln x + x - 1 & 0 < x < 1 \end{cases}$$
$$f'(x) = \begin{pmatrix} \frac{1}{x} - 1 & x \geq 1 \\ -\frac{1}{x} + 1 & 0 < x < 1 \end{pmatrix}$$
$f'(1^+) = f'(1^-) = 0 \Rightarrow f(x)$ is differentiable $\forall x > 0$

$f'(x) < 0\ \forall x > 1$

$f'(x) < 0\ \forall 0 < x < 1$

$\Rightarrow f(x)$ is decreasing $\forall x \in (0,\infty)$

66. (b) $\lim_{t\to x} \frac{2tf(x) - x^2 f'(t)}{-1} = 3$

$x^2 f'(x) - 2xf(x) = 3$
$$\frac{dy}{dx} - \frac{2y}{x} = \frac{3}{x^2}$$
I.F. $= e^{-\int \frac{2}{x}dx} = e^{-2\log_e x} = 1/x^2$
$$y\cdot\frac{1}{x^2} = \int \frac{3}{x^4}dx$$
$$\frac{y}{x^2} = -\frac{1}{x^3} + c \Rightarrow y = cx^2 - \frac{1}{x} = f(x)$$
$f(1) = 2 = c - 1 \Rightarrow c = 3$
$$f(x) = 3x^2 - \frac{1}{x}$$
$$f(2) = 12 - \frac{1}{2} \Rightarrow 2f(2) = 23$$

67. (c) $m = \frac{9\times10\times19}{6} = 15 \times 19$

$3f(x) + 2f\left(\frac{15}{x}\right) = 5x$

Replace x by $\frac{15}{x}$

$3f\left(\frac{15}{x}\right) + 2f(x) = \frac{75}{x}$

$9f(x) - 4f(x) = 15x - \frac{150}{x}$

$5f(x) = 15x - \frac{150}{x}$

$f(x) = 3x - \frac{30}{x}$

$f(5) = 15 - \frac{30}{5} = 9$

$f(2) = 6 - 15 = -9$

$f(5) - f(2) = 18$

68. (d) $x^4 - ax^2 + 9 = 0$ ……….. (i)

let $x^2 = t$

$t^2 - at + 9 =$ ……….. (ii)

for roots of equation (i) to be real & distinct roots of equation (ii) must be positive & distinct.

(i) $D > 0 \Rightarrow a^2 - 36 > 0 \Rightarrow a \in (-\infty, -6) \cup (6, \infty)$

(ii) $\frac{-b}{2a} > 0 \Rightarrow \frac{a}{2} > 0 \Rightarrow a > 0$

(iii) $f(0) > 0 \Rightarrow 9 > 0 \Rightarrow a \in R$

By (i) ∩ (ii) ∩ (iii)

$\therefore a \in (6, \infty)$

∴ least integral value of a is 7

69. (c) $2(\vec{a} \times \vec{c}) + 3(\vec{b} \times \vec{c}) = 0$

$\Rightarrow (2\vec{a} + 3\vec{d}) \times \vec{c} = 0 \Rightarrow \vec{c} = \lambda(2\vec{a} + 3\vec{d})$

$\Rightarrow \vec{c} = \lambda(7i - 13j + 19k)$

Now $(\vec{a} - \vec{b}) \cdot \vec{c} -= \lambda(7 + 52 + 38) + 97\lambda = -97$

$\Rightarrow \lambda = -1$

Now $\vec{c} = -7i + 13j - 19k$

$\Rightarrow \vec{c} \times \vec{k} -= 7j + 13i \Rightarrow |\vec{c} \times \hat{k}|^2 = 7^2 + 13^2 = 218$

70. (c) $f(0) = a$

$RHL = \lim_{x\to0^+} \frac{\sin x(1 - \cos x)}{x^3} = \frac{1}{2}$

$LHL = \lim_{x\to0^-} \left(b^2 \sin\frac{\pi}{2}\left[\frac{\pi}{2}(\sin x + \cos x)\cos x\right]\right) = b^2$

$a = \frac{1}{2} \& b^2 = \frac{1}{2}$

so $(a^2 + b^2) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

71. (2) $f(x) = e^x + \int_0^1 yf(y)dy + xe^x \int_0^1 f(y)dy$

$f(x) = e^x + A + Bxe^x$

$A = \int_0^1 yf(y)dy = \int_0^1 y(A + e^y + Bye^y)dy$

$A = \frac{A}{2} + (0 - (-1)) + B(e - 1)$

$\frac{A}{2} + B(1 - e) = 1$

$B=\int_0^1 f(y)dy$

$B=\int_0^1 (e^y+A+Bye^y)dy$

$B=(e-1)+A+B(0-(-1))$

$B=e-1+A+B \Rightarrow A=1-e$

$f(x)=e^x+A+Bxe^x$

$f(0)=1+A=1-e+1=2-e$

$e+f(0)=2$

72. **(9)** Let $P \equiv (2\cos\theta, 2\sin\theta)$

$\therefore$ coordinates of $Q=(4\cos\theta+1, 6\sin\theta+3)$

$\therefore$ locus of Q is $\left(\frac{x-1}{4}\right)^2+\left(\frac{y-3}{6}\right)^2=1$

$\therefore e^2=1-\frac{16}{36}=\frac{5}{9}$

$\therefore \frac{5}{e^2}=9$

73. **(5)** $|z|^2=2^3.5^2.13.17$

$\prod_{r=1}^{n}(1+r^2)=2^3\cdot 5^2\cdot 13\cdot 17=(2)\cdot(5)\cdot(2\cdot 5)\cdot(17)\cdot(2\cdot 13)=2\cdot 5\cdot 10\cdot 17\cdot 26$

so $n=5$

74. **(15)** $S=\{a,b,c,d,e\}$

P(S) contains 32 elements

both set A and set B are subsets of P(S)

Every element has 4 choices

A	B
✓	✓
✓	x
x	✓
x	x

Favourable cases $=3^5$

Total cases $=4^5$

$P=\frac{3^5}{4^5}=\frac{3^5}{2^{10}}$

$m=5, n=10$

$m+n=15$

75. **(4)** $\frac{\tan(x+100^\circ)}{\tan x}=\tan(x+50^\circ)\tan(x-50^\circ)$

$\frac{\sin(x+100^\circ)\cos x}{\cos(x+100^\circ)\sin x}=\frac{\sin(x+50^\circ)\sin(x-50^\circ)}{\cos(x+50^\circ)\cos(x-50^\circ)}$

Apply C & D

$\frac{\sin(2x+100^\circ)}{\sin 100^\circ}=\frac{\cos 100^\circ}{-\cos 2x}$

$2\sin(2x+100^\circ)\cos 2x+\sin 200^\circ=0$

$\sin(4x+100^\circ)+\sin 100^\circ+\sin 200^\circ=0$

$\sin(4x+100^\circ)=-2\sin 150^\circ\cos 50^\circ$

$\sin(4x+100^\circ)=-\cos 50^\circ=\sin(-40^\circ)$

$\therefore 4x+100^\circ=n\pi+(-1)^n\cdot(-40^\circ)$

$x=\frac{n\pi+(-1)^{n+1}(40^\circ)-100^\circ}{4}$

$\therefore x = 30°, 55°, 120°, 145°$ in $(0, \pi)$
$\therefore$ no. of solutions $= 4$
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