1. (c) $B = 3 \times 10^{10}$

$$-\frac{\Delta V}{V} = 0.02$$
$$B = \frac{\Delta P}{-\frac{\Delta V}{V}} \Rightarrow \Delta P = -B\left(\frac{\Delta V}{V}\right)$$
$$= (3 \times 10^{10})(0.02)$$
$$= 6 \times 10^{8}\ N/m^2$$

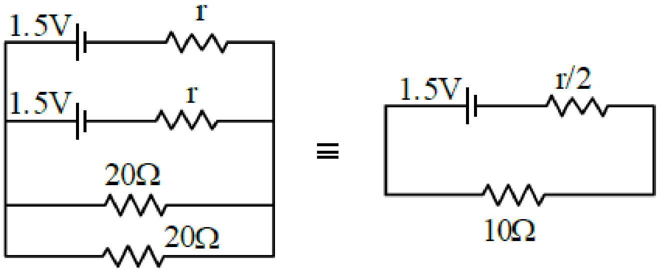
2. (a) $\vec{F} = q(\vec{v} \times \vec{B})$

$\vec{F} \perp \vec{v}$

Work done $= \vec{F} \cdot \vec{S}$

Work done $= 0$

3. (c)



$V = E - ir/2$

$1.2 = 1.5 - i\left(\frac{r}{2}\right)$

$i\frac{r}{2} = 0.3$

$i = \frac{1.5}{10 + \frac{r}{2}} \Rightarrow 10i + \frac{ir}{2} = 1.5$

$10i = 1.5 - 0.3$

$i = 0.12\ A$

$\Rightarrow r = \frac{0.6}{0.12} = 5\Omega$

4. (d)

$S = \frac{Q}{m\Delta T} = \frac{J}{Kg^{\circ}C}$

$L = \frac{Q}{m} = \frac{J}{Kg}$

5. (d)

$$R=\frac{V^2(2\sin\theta\cos\theta)}{g}$$
$$t=\frac{V\sin\theta}{g}\Rightarrow V=\frac{gt}{\sin\theta}$$
$$\Rightarrow R=\frac{g^2t^2}{\sin^2\theta}\cdot\frac{2\sin\theta\cos\theta}{g}$$
$$\tan\theta=\frac{2gt^2}{R}=\frac{20t^2}{R}$$
$$\cot\theta=\frac{R}{20t^2}$$

6. (b)

$$a=-\mu g=-0.5\times 9.8=-4.9\ \mathrm{m/s^2}$$

$$d=\frac{v^2}{2a}=\frac{9.8\times 9.8}{2(4.9)}$$

$$=9.8\ \mathrm{m}$$

7. (c) $T=M\omega^2R$

$T=80\ \mathrm{N}\ M=0.1\ \omega=?\ R=2\ \mathrm{m}$

$$80=0.1\omega^2(2)$$

$$\omega^2=400$$

$$\omega=20$$

$$2\pi f=20$$

$$f=\frac{10}{\pi}\frac{\mathrm{rev}}{\mathrm{s}}$$

$$=\frac{600}{\pi}\frac{\mathrm{rev}}{\mathrm{min}}$$

8. (b) $Mg=qE$

$$(0.1\times 10^{-3})(9.8)=4.9\times 10^5 q$$
$$\frac{2\times 10^{-4}}{10^5}=q$$
$$q=2\times 10^{-9}\mathrm{C}$$

9. (c) $F=4x\hat{i}+3y^2\hat{j}$

$$\mathrm{WD}=\Delta\mathrm{KE}$$

$$W=\int\vec{F}\cdot(dx\hat{i}+dy\hat{j})$$

$$=\int_1^2 4x\,dx+\int_2^3 3y^2 dx$$

$= (2x^2)_1^2 + (y^3)_2^3$

$= (8 - 2) + (27 - 8)$

$= 6 + 19 = 25\ \mathrm{J}$

10. (b) $Mg' = \frac{M}{3}\ g$

$g' = \frac{g}{3}$

$g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{3}$

$\frac{R}{R+h} = \frac{1}{\sqrt{3}}.$

$h = (\sqrt{3} - 1)R$

$= (1.732 - 1)6400$

$h = 4685\ \mathrm{km}$

11. (a) Considering sinusoidal AC.

Phase at maximum value $= \frac{\pi}{2}$

Phase at rms value $= \frac{3\pi}{4}$

Thus phase change $= \frac{3\pi}{4} - \frac{\pi}{2} = \frac{\pi}{4}$

Now $\omega = 2\pi f$

$= 2\pi \times 50$

$= 100\pi$

time taken $t = \frac{\theta}{\omega} = \frac{\pi/4}{100\pi} = \frac{1}{400}\ \mathrm{s}$

$t = 2.5 \times 10^{-3} = 2.5\ \mathrm{ms}$

12. (a) $A_1 = 5\ \ A_2 = 3$

$\Delta\theta = 2\pi(1.5) = 3\pi$

$A_{\text{net}} = \sqrt{A_1^2 + A_2^2 + 2\ A_1\ A_2 \cos(3\pi)}$

$= |A_1 - A_2|$

$= 2\ \mathrm{cm}$

13. (c) $\mu_r = 1.61\ \epsilon_r = 6.44$

$B = 4.5 \times 10^{-2}$

E = ?

$$C = \frac{1}{\sqrt{\mu_0 \in_0}} \quad V = \frac{1}{\sqrt{\mu \epsilon}}$$

$$\frac{C}{V} = \sqrt{\mu_r \in_r} = \sqrt{1.61 \times 6.44}$$

$$\frac{E}{B} = V = \frac{3 \times 10^8}{\sqrt{1.61 \times 6.44}} = 9.32 \times 10^7 \text{ m/s}$$

$$E = 4.5 \times 10^{-2} \times 9.32 \times 10^7$$

$$= 4.2 \times 10^6$$

14. **(c)** According to Rutherford, e^-revolves around nucleus in circular orbit. Thus $e^-$is always accelerating (centripetal acceleration). An accelerating change emits EM radiation and thus e^- should loose energy and finally should collapse in the nucleus.

15. **(d)** $Q = (B.E)_P - (B.E)_R$

$$= (105 + 115)(6.4) - (220)(5.6)$$

$$= 176\text{MeV}$$

16. **(c)** $f_c = 3.5\text{GHz}\ f_m = 3.5\text{MHz}$

Side band frequencies are $f_c - f_m \& f_c + f_m$. which are almost f_c

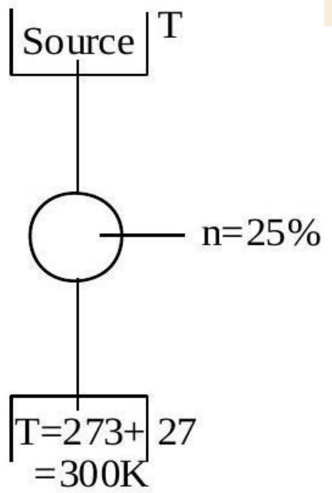
$$\lambda = \frac{c}{f_c}$$

Minimum length of antenna =

$$\frac{c}{f_c 4} = \frac{\lambda}{4} = \frac{3 \times 10^8}{3.5 \times 10^9 \times 4}$$

$$= 21.4 \text{ mm}$$

17. **(b)**



$$1-\frac{300}{T}=0.25$$

$$\frac{300}{T}=0.75$$

$$T=400\ K$$

If efficiency increased by 100% then new efficiency $\Rightarrow n'=50\%$

$$1-\frac{300}{T'}=0.5$$

$$T'=600\ K$$

Increase in temp $=600-400$

$=200$ K or $200^\circ C$

18. (d) $K=\frac{q}{A\epsilon_0 E}=\frac{7\times10^{-6}}{30\pi\times10^{-4}\times\frac{1}{4\pi\times9\times10^{9}}\times3.6\times10^{7}}$

$$K=\frac{36\times7}{30\times3.6}=2.33$$

19. (c) $B_C=\frac{\mu_0 I}{2r}, B_a=\frac{\mu_0 Ir^2}{2(x^2+r^2)^{3/2}}$

At $x=\frac{r}{2}$

$$B_a=\frac{\mu_0 Ir^2}{2\left(\frac{r^2}{4}+r^2\right)^{3/2}}$$

$$=\frac{\mu_0 Ir^2}{2\left(\frac{5}{4}r^2\right)^{3/2}}=\frac{\mu_0 I}{2r}\left(\frac{4}{5}\right)^{3/2}$$

$$=\frac{\mu_0 I}{2r}\left(\frac{2}{\sqrt{5}}\right)^3$$

20. (b) $\Delta T\propto R\,\alpha\frac{\ell}{k}$,

$$\frac{\Delta T_1}{\Delta T_2}=\frac{\ell_1}{k_1}\times\frac{k_2}{\ell_2}=\frac{16}{k_1}\times\frac{k}{8}$$

$$\frac{20}{80}=\frac{16}{k_1}\times\frac{k}{8}\to k_1=8k$$

21. (12) 0.056 kg $N_2=56$gm of $N_2=2$ mole of N_2

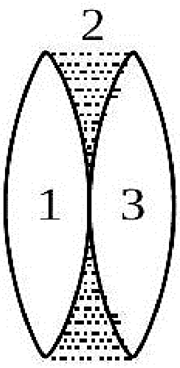
$T_1=400\ K, v\alpha\sqrt{T}$ so $T_2=4\ T_1=1600\ K$

$$Q=\frac{f}{2}nR\Delta T$$

$f = 5$

$Q = 12\text{kcal}$

22. (10)



$\frac{1}{f_1} = \frac{1}{15} = \left(\frac{3}{2} - 1\right)\left[\frac{2}{R}\right]$

$\frac{1}{R} = \frac{1}{15}$

$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$

$= \frac{1}{15} + \left(\frac{5}{4} - 1\right)\left[\frac{-2}{R}\right] + \frac{1}{15}$

$= \frac{1}{15} - \frac{1}{30} + \frac{1}{15}$

$= \frac{2 - 1 + 2}{30}$

$= \frac{3}{30} = \frac{1}{10}$

$= 10$

23. (750) $r_i = \frac{10\text{mV}}{10\mu\text{A}} = 10^3\Omega$

$\beta = \frac{1.5\text{ mA}}{10\mu\text{A}} = 150$

$A_V = \left(\frac{R_0}{r_i}\right)\beta = \left(\frac{5000}{1000}\right) \times 150 = 750$

24. (242) $f = 50\text{ Hz}$

$X_L = 2\pi fL$

$= 2\pi(50)(200 \times 10^{-3})$

$= 20\pi\Omega$

$$i_0 = \frac{V_0}{X_L} \Rightarrow \frac{V_{rms}\sqrt{2}}{X_L}$$

$$= \frac{(220)\sqrt{2}}{20\pi} = \frac{11\sqrt{2}}{\pi}$$

$$i_0 = \frac{\sqrt{242}}{\pi}$$

25. (3)

$$a\sin\theta = \frac{3\lambda}{2}$$

$$\frac{y}{L} = \theta = \frac{3\lambda}{2a}$$

$$L = 2\text{ m}$$

$$y_1 = \frac{3\lambda_1 L}{2a}$$

$$\lambda_2 = 655\text{ nm}$$

$$y_2 = \frac{3\lambda_2 L}{2a}$$

$$\lambda_1 = 650\text{ nm}$$

$$a = 0.5\text{ nm}$$

$$\Delta y = y_2 - y_1 = \frac{3(\lambda_2 - \lambda_1)}{2a}L$$

$$= \frac{3(655 - 650)}{2 \times 0.5 \times 10^{-3}} \times 2 \times 10^{-9}$$

$$= \frac{3 \times 5 \times 2}{1 \times 10^{-3}} \times 10^{-9}$$

$$= 3 \times 10^{-5}$$

26. (2)

$$hv = hv_{th} + \frac{1}{2}mv^2$$

$$v = 2v_{th}$$

$$2hv_{th} = hv_{th} + \frac{1}{2}mv_1^2$$

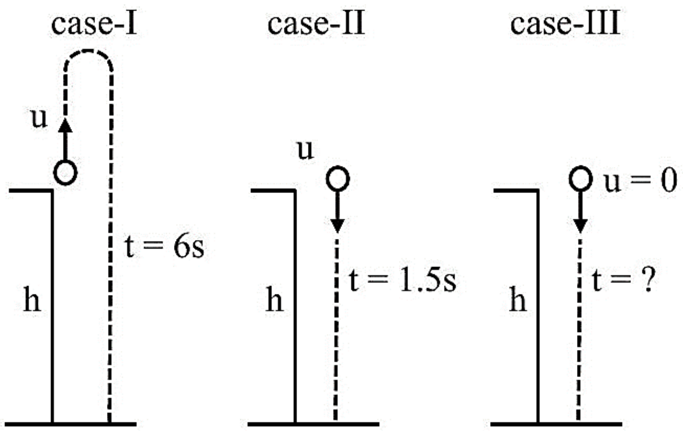
$$v = 5v_{th}$$

$$5hv_{th} = hv_{th} + \frac{1}{2}mv_2^2$$

$$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{hv_{th}}{4hv_{th}}$$

$$\left(\frac{v_1}{v_2}\right)^2 = \frac{1}{4} \Rightarrow v_2 = 2v_1$$

27. (3) Let height of tower be h and speed of projection in first two cases be u.



For case-I : 2nd equation $s = ut + \frac{1}{2}at^2$

$$h = -u(6) + \frac{1}{2}g(6)^2$$

$$H = -6u + 18g$$

For case-II : $h = u(1.5) + \frac{1}{2}g(1.5)^2$

$$h = 1.5u + \frac{2.25\ g}{2}$$

Multiplying equation (ii) by 4 we get

$$4\ h = 6u + 4.5\ g$$

equation (i) + equation (iii) we get $5\ h = 22.5\ g$

$h = 4.5\ g$ ... (iv)

For case-III :

$$h = 0 + \frac{1}{2}gt^2$$

Using equation (4) & equation (5)

$$4.5\ g = \frac{1}{2}gt^2$$

$$t^2 = 9 \Rightarrow t = 3\ s$$

28. (120) By energy conservation

$PE = KE$

$$mg\left(H+\frac{H}{2}\right)=\frac{1}{2}kx^2\left(x=\frac{H}{2}\right)$$

$$0.100\times10\times\frac{3}{2}(0.10)=\frac{1}{2}k(0.05\times0.05)$$

$$k=\frac{3\times0.10}{0.05\times0.05}$$

$$=\frac{3\times1000}{25}=120\text{ N/m}$$

29. (25)

$$\frac{\varepsilon_1}{\varepsilon_2}=\frac{\ell_1}{\ell_2}$$

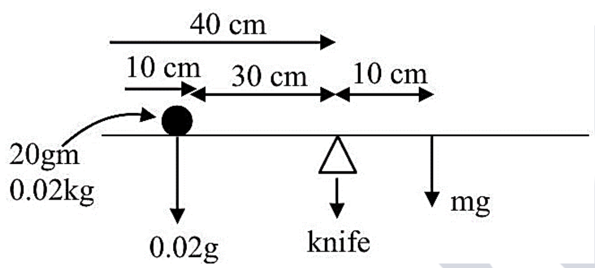
$$\frac{3}{2}=\frac{75\text{ cm}}{\ell_2}$$

$$\ell_2=50\text{ cm}$$

$$\ell_1-\ell_2=75-50$$

$$=25\text{ cm}$$

30. (6) Let mass of meter scale be m.



Balancing torque about knife edge

$$(0.02\text{ g})\times(30\times10^{-2})=mg\times(10\times10^{-2})$$

$$m=0.06\text{ kg}=6\times10^{-2}\text{ kg}$$

31. (c)

$$C_{15}H_{30}+\frac{45}{2}O_2\rightarrow15CO_2+15H_2O$$

Mass of fuel $=0.756\times1000$ g

No. of moles of fuel $=\frac{0.756\times1000}{210}$

Wt. of oxygen $=\frac{0.756\times1000}{210}\times\frac{45}{2}\times32=2592$ g

Wt of $CO_2=\frac{0.756\times1000}{210}\times15\times44=2376$ g

32. (b) Based on " $n+1$ " rule only (B) has pair of electron in degenerate orbitals

33. (b)

List - I		List - II	
(a)	$[PtCl_4]^{2-}$	(III)	dsp^2
(b)	BrF_5	(IV)	$sp^3\ d^2$
(c)	PCl_5	(I)	$sp^3\ d$
(d)	$[Co(NH_3)_6]^{3+}$	(II)	d^2sp^3

34. (b)

$$A(g) \rightleftharpoons B(g)+\frac{1}{2} C(g)$$

Initial: $\mathbf{P_i}$

0 0

At eq.: $P_i(1-\alpha)\ P_i \cdot \alpha\ P_i \frac{\alpha}{2}$

Now, equilibrium pressure (p),

$$P=P_i \times\left(1+\frac{\alpha}{2}\right)$$

$$\therefore P_A=\left(\frac{1-\alpha}{1+\frac{\alpha}{2}}\right) P$$

$$P_B=\left(\frac{\alpha}{1+\frac{\alpha}{2}}\right) P$$

$$P_C=\left(\frac{\frac{\alpha}{2}}{1+\frac{\alpha}{2}}\right) P \therefore K=\frac{P_c^{\frac{1}{2}} \times P_B}{P_A}$$

$$K=\frac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$$

35. (c) Statement I : Fact

Statement II: The principle emulsifying agents for O/W emulsions are proteins, gums natural and synthetic soaps etc...

36. (b) $Na_2O=$ Basic

$As_2O_3=$ Amphoteric

N_2O = Neutral

NO = Neutral

37. (a)

	List - I		List - II
(a)	Sphalerite	(IV)	ZnS
(b)	Calamine	(III)	$ZnCO_3$
(c)	Galena	(II)	PbS
(d)	Siderite	(I)	$FeCO_3$

38. (b) Nitrogen. Around 55% of hydrogen around would goes to ammonia production

39. (a)

(a) Both LiCl and $MgCl_2$ are soluble in ethanol

(b) Li and Mg do not form superoxide

(c) LiF has high lattice energy

(d) Li_2O is least soluble in water than other alkali metal oxides

40. (c)

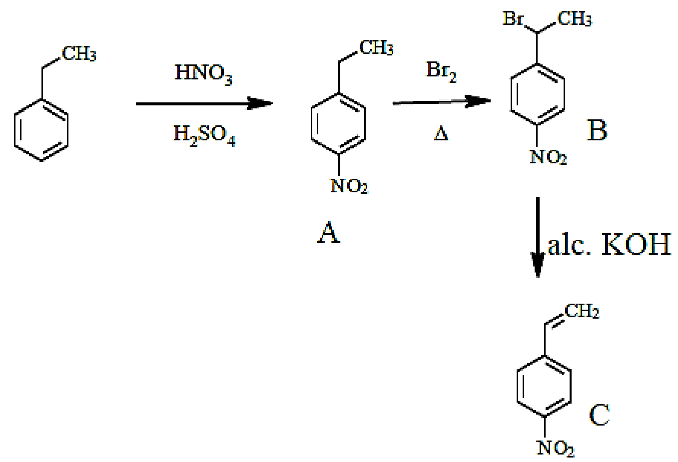
41. (a)

Order of stability: -

$NF_3 > NCl_3 > NBr_3 > NI_3$

42. (b) Calcium and phosphate are the major components of teeth enamel

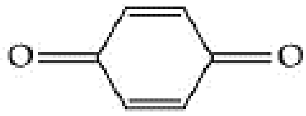
43. (b) C_8H_{10} $DU = 9 - 5 = 4$



44. (a) I. Better packing efficiency of monocarboxylic acids with even number of carbon atoms results in higher M.P

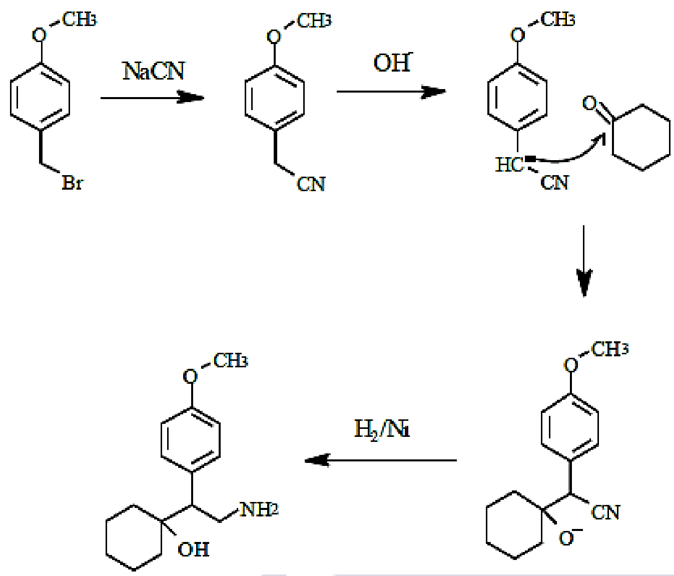
II. As molar mass increases hydrophobic part size increase hence solubility decreases.

45. (c)



is a conjugated diketone

46. (d)



47. (d)

48. (b) Cellulose contains β-glycosidic linkages only

49. (d) Penicillin G following is a narrow spectrum antibiotic

50. (a) $Ni^{2+} + DMG^{-} \rightarrow [Ni(DMG)_2] \downarrow$

(Bright red precipitate)

51. (43) $X \rightarrow 6\,Y \rightarrow \frac{2}{3} \times 2 \times 6 = 8$

$$\%X = \frac{6}{14} \times 100 = 42.8 \simeq 43\%$$

52. (747) $2O_3 \rightleftharpoons 3O_2(g)$

$\frac{2}{5}\ \frac{3}{5}$

$$k_p = \frac{P_{O_2}^3}{P_{O_3}^2}$$

$k_p = 1.35$

$\Delta G^\circ = -RT\ln k_p$

$= -8.3 \times 300 \times \ln 1.35$

$= -747\ J/mol$

53. (54) $\pi = C.R.T$

$7.47 = C \times 0.083 \times 300$

$C = 0.3M$

$= 0.3 \times 180 gL^{-1}$

$= 54 gL^{-1}$

54. (5)

Anode: $H_2 \rightarrow 2H^+ + 2e$

Cathode: $Cu^{2+} + 2e^- \rightarrow Cu$

$\overline{Cu^{2+} + H_2 \rightarrow 2H^+ + Cu}$

$$E_{cell} = E^0_{cell} - \frac{0.06}{2} \log \frac{[H^+]^2}{[Cu^{2+}]}$$

$$0.576 = 0.34 - \frac{0.06}{2} \log \left\{ \frac{[H^+]^2}{(0.01)} \right\}$$

$+3.93 - \log(H^+) + \log 0.1 \Rightarrow pH = 4.93 \simeq 5$

55. (154) $\ln k = \ln A - \frac{Ea}{10^3 RT} \times 10^3 = \ln A + \frac{10^3}{T}\left[-\frac{Ea}{10^3 RT}\right]$

From the graph

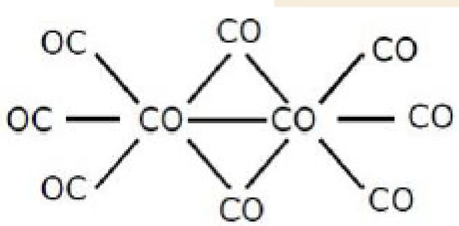
$$\frac{-Ea}{10^3 \times R} = -18.5$$

$Ea = 153.735$ kJ/mol

~ 154

56. (0) $\overset{+6}{Cr}O_4^{2-}, \overset{+6}{Cr}_2O_7^{2-}$ difference is zero

57. (7)

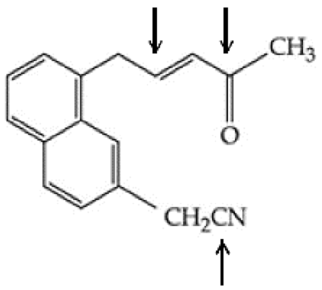


X=1

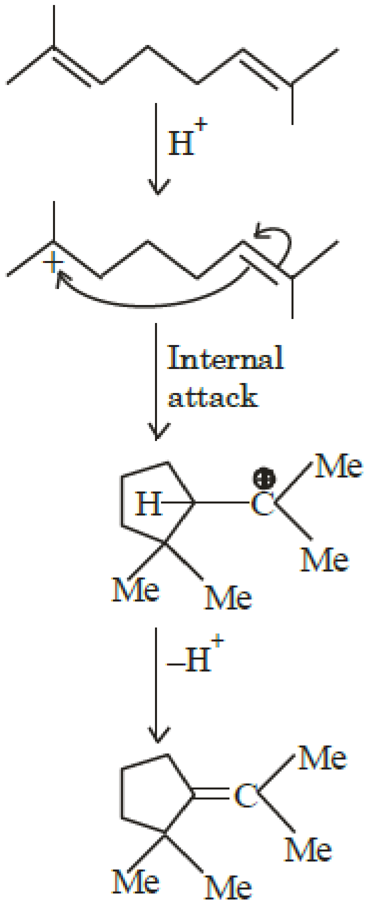
Y=6

58. (63)

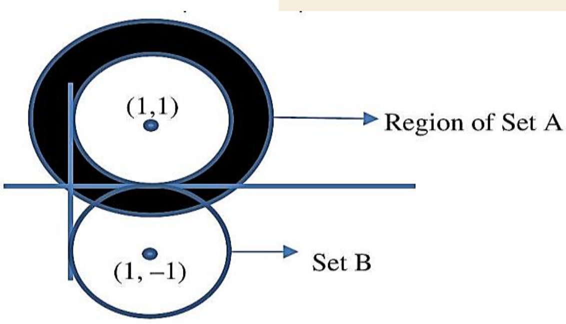
59. (3)



60. (2)



61. (d) $A = \{z \in C: 1 \leq |z - (1 + i)| \leq 2\}$



$B = \{z \in A: |z - (1 - i)| = 1\}$.

$A \cap B$ has infinite set.

62. (d) $3^{2022}=9^{1011}=(10-1)^{1011}=10m-1=10m-5+4=5(2\text{ m}-1)+4(\text{ m is integer })$ Remainder $=4$

63. (a) Let r be the radius of spherical balloon

$\mathrm{S}=$ Surface area

$S=4\pi r^2$

$\frac{\mathrm{dS}}{\mathrm{dt}}=8\pi\mathrm{r}\times\frac{\mathrm{dr}}{\mathrm{dt}}=\mathrm{k}(\text{ constant })$

$4\pi\mathrm{r}^2=\mathrm{kt}+\mathrm{C}(\mathrm{C}$ is constant of integration $)$

For $\mathrm{t}=0,\mathrm{r}=3\Rightarrow 36\pi=\mathrm{C}$

For $\mathrm{t}=5,\mathrm{r}=7\Rightarrow\mathrm{K}=32\pi$

$4\pi\mathrm{r}^2=32\pi\mathrm{t}+36\pi$

$\mathrm{r}^2=8\mathrm{t}+9$

for $\mathrm{t}=9$

$\mathrm{r}^2=81$

$\mathrm{r}=9$

64. (c) $\mathrm{E}_1=$ denotes selection for 1^{st} bag

$\mathrm{E}_2=$ denotes selection for 2^{nd} bag

$\mathrm{P}(\mathrm{E}_1)=\frac{1}{2},\mathrm{P}(\mathrm{E}_2)=\frac{1}{2}$

$\mathrm{A}=$ selected balls are 1 red &1 black

$$\mathrm{P}\left(\frac{\mathrm{A}}{\mathrm{E}_1}\right)=\frac{{}^3\mathrm{C}_1\times{}^1\mathrm{C}_1}{{}^6\mathrm{C}_2}=\frac{1}{5}$$

$$\mathrm{P}\left(\frac{\mathrm{A}}{\mathrm{E}_1}\right)=\frac{{}^3\mathrm{C}_1\times{}^2\mathrm{C}_1}{(\mathrm{n}+5)_{\mathrm{C}_2}}=\frac{12}{(\mathrm{n}+5)(\mathrm{n}+4)}$$

$$\mathrm{P}\left(\frac{\mathrm{E}_1}{\mathrm{A}}\right)=\frac{\mathrm{P}(\mathrm{E}_1)\times\mathrm{P}\left(\frac{\mathrm{A}}{\mathrm{E}_1}\right)}{\mathrm{P}(\mathrm{E}_1)\times\mathrm{P}\left(\frac{\mathrm{A}}{\mathrm{E}_1}\right)+\mathrm{P}(\mathrm{E}_2)\times\mathrm{P}\left(\frac{\mathrm{A}}{\mathrm{E}_2}\right)}$$

$$=\frac{\frac{1}{10}}{\frac{1}{10}+\frac{6}{(\mathrm{n}+5)(\mathrm{n}+4)}}=\frac{6}{11}$$

$\Rightarrow\mathrm{n}=4$

65. (a) $x^2+y^2+Ax+By+C=0$ is passing through $(0,6)$

$\Rightarrow 6\text{ B}+\mathrm{C}=-36$

The tangent of the parabola $y=x^2$ at $(2,4)$ is

$4x - y - 4 = 0$

The tangent of circle $x^2 + y^2 + Ax + By + C = 0$ at $(2,4)$ is

$(4 + A)x + (8 + B)y + 2A + 4B + 2C = 0$

From Equation (1) and (2)

$$\frac{4 + A}{4} = \frac{8 + B}{-1} = \frac{2A + 4B + 2C}{-4}$$

$A + 4B = -36$

$3A + 4B + 2C = -4$

From equation (3) and (4)

$A + C = 16$

66. (b) $x + y + z = \alpha$

$\alpha x + 2\alpha y + 3z = -1$

$x + 3\alpha y + 5z = 4$

Has inconsistent solution

$$D = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix} = 0$$

$\Rightarrow (\alpha - 1)^2 = 0$

$\alpha = 1$

For $\alpha = 1$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 4 & 3 & 5 \end{vmatrix}$$

$= (10 - 9) - (-5 - 12) + (-3 - 8)$

$= 1 + 17 - 11 \neq 0$

For $\alpha = 1$ the system of equation has Inconsistent solution

67. (b) Here α, β roots of equation $3x^2 + \lambda x - 1 = 0$

$$\alpha + \beta = \frac{-\lambda}{3}, \alpha\beta = \frac{-1}{3}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = 15$$

$\lambda^2 = 9$

Now $6(\alpha^3+\beta^3)^2 = 6\left((\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)\right)^2$

$= 6\left(\frac{\lambda^2}{9}\right)\left\{\frac{\lambda^2}{9}+1\right\}^2 = 24$

68. (a) Let **S** $= (\tan^{-1}x)^3 + (\cot^{-1}x)^3$

$$= (\tan^{-1}x+\cot^{-1}x) - 3\tan^{-1}x\cdot\cot^{-1}x(\tan^{-1}x+\cot^{-1}x)$$
$$= \frac{\pi^3}{8} - \frac{3\pi}{2}\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right)$$
$$= \frac{3\pi}{2}\left(\tan^{-1}x-\frac{\pi}{4}\right)^2 + \frac{\pi^3}{32}$$
$$\Rightarrow \frac{\pi^3}{32} \le S < \frac{7}{8}\pi^3$$
$$= \frac{\pi^3}{32} \le K\pi^3 < \frac{7}{8}\pi^3$$
$$\frac{1}{32} \le K < \frac{7}{8}$$

69. (b) $S = \{\sqrt{n}: 1 \le n \le 50 \text{ and n is odd }\}$

$= \{\sqrt{1},\sqrt{3},\sqrt{5}\ldots\ldots\ldots\sqrt{49}\}$,25 terms

$$|A| = 1 + a^2$$
$$\sum_{a\in S}\det(\operatorname{adj}A) = \sum_{a\in S}|A|^2 = \sum (1+a^2)^2$$

$$= 22100 = 100\lambda$$
$$\lambda = 221$$

70. (c) $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5, x > 1$

$$f'(x) = \frac{4}{x-1} - 4(x-1)$$

For $1 < x < 2 \Rightarrow f'(x) > 0$

For $x > 2 \Rightarrow f'(x) < 0$ (option 1 is correct)

$f(x) = -1$ has two solution (option 2 is correct)

$f(e) > 0$

$f(e+1) < 0$

$f(e).f(e+1) < 0$ (option 4 is correct)

$$f'(e) - f''(2) = \frac{4}{e-1} - 4(e-1) + 8 > 0$$

71. (d) The tangent at (x_1, y_1) to the curve

$y = x^3 + 3x^2 + 5$

$y - y_1 = (3x_1^2 + 6x_1)(x - x_1)$ passing through origin

$-y_1 = (3x_1^3 + 6x_1)(-x_1)$

$y_1 = (3x_1^3 + 6x_1^2)$

And (x_1, y_1) lies on the curve

$y = x^3 + 3x^2 + 5$

$y_1 = x_1^3 + 3x_1^2 + 5$

From equation (1) and (2)

$$2y_1 = 3x_1^2 + \frac{15}{2}$$

Hence the equation of curve $y = \frac{3}{2}x^2 + \frac{15}{2}$

This curve does not intersect $\frac{x}{3} - y^2 = 2$

72. (b)

$f(x) = |2x^2 + 3x - 2| + \sin x \cos x$

$f(x) = |(2x - 1)(x + 2)| + \sin x \cos x$

$$f'(x) = \begin{cases} 4x + 3 + \dfrac{\cos 2x}{4}, & \dfrac{1}{2} < x < 1 \\ -(4x + 3) + \dfrac{\cos 2x}{4}, & 0 \le x < \dfrac{1}{2} \end{cases}$$

For $0 \le x < \frac{1}{2} \Rightarrow f'(x) < 0$

For $\frac{1}{2} < x \le 1 \Rightarrow f'(x) > 0$

f(x) local minima at $x = \frac{1}{2}$ and

local maxima at $x = 1$

$$f\left(\frac{1}{2}\right) + f(1) = 3 + \frac{1}{2}(1 + 2\cos 1)\sin 1$$

73. (b) $\sum_{i=1}^{n} a_i = \frac{n}{2}\{2a_1 + (n + 1)\} = 192$

$$\Rightarrow 2a_1 + (n - 1) = \frac{384}{n}$$

$$\sum_{i=1}^{n/2} a_{2i} = \frac{n}{4}\left[2a_1 + 2 + \left(\frac{n}{2} - 1\right)2\right] = 120$$

$$2a_1 + n = \frac{480}{n}$$

From equation (2) and (1)

$1=\frac{480}{n}-\frac{384}{n}$

$n=480-384=96$

74. (a) $y\frac{dx}{dy}=2x+y^3(y+1)e^y, x(1)=0$

$\frac{dx}{dy}-\frac{2}{y}x=y^2(y+1)e^y$

$I.f=e^{\int\frac{-2}{y}dy}=\frac{1}{y^2}$

$x\cdot\frac{1}{y^2}=\int(y+1)e^y dy$

$\frac{x}{y^2}=(y+1)e^y-e^y+c=y\cdot e^y+c$

$x=y^3e^y+cy^2$

For $x=0, y=1 \Rightarrow c=-e$

$x=y^3e^y-e\cdot y^2$

$x(e)=e^3(e^e-1)$

75. (d) $\lambda x-2y=\mu$ is a tangent to the curve

$a^2x^2-y^2=b^2$ then

$a^2x^2-\left(\frac{\lambda x-\mu}{2}\right)^2=b^2$

$(4a^2-\lambda^2)x^2+2\lambda\mu x-\mu^2-4b^2=0$

Disc. $=0$

$4\lambda^2\mu^2+4(4a^2-\lambda^2)(\mu^2+4b^2)=0$

$4\lambda^2b^2-4a^2\mu^2=16a^2b^2$

$\frac{\lambda^2}{a^2}-\frac{\mu^2}{b^2}=4$

76. (c) $|\hat{b}|^2=|\vec{c}+2(\vec{c}\times\hat{a})|^2$

$|\hat{b}|^2=|c|^2+4|\vec{c}\times\hat{a}|^2+4\vec{c}\cdot(\vec{c}\times\hat{a})$

$1=|c|^2+4|c|^2\sin^2\frac{\pi}{12}+0$

$1=|c|^2+4|c|^2\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$ $|c|^2=\frac{1}{3-\sqrt{3}}=\frac{3+\sqrt{3}}{6}$

So $6^2|c|^2=6(3+\sqrt{3})$

77. (a) n = 33, let probability of success is p and q = 1 − p

$3p(x=0) = p(x=1)$
$3.\ {}^{33}C_0(q)^{33} = {}^{33}C_1 pq^{32}$

$p = \frac{1}{12}, q = \frac{11}{12}, \frac{q}{p} = 11$

$\frac{p(x=15)}{p(x=18)} - \frac{p(x=16)}{p(x=17)}$

$\frac{{}^{33}C_{15}p^{15}q^{18}}{{}^{33}C_{18}p^{18}q^{15}} - \frac{{}^{33}C_{16}p^{16}q^{17}}{{}^{33}C_{17}p^{17}q^{16}} = \left(\frac{q}{p}\right)^3 - \left(\frac{q}{p}\right)$

$= (11)^3 - 11$

$= 1320$

78. (d)

79. (b) $\sin\theta\tan\theta + \tan\theta = \sin 2\theta$

$\tan\theta(\sin\theta + 1) = \frac{2\tan\theta}{1+\tan^2\theta}$

$\tan\theta = 0 \Rightarrow \theta = -\pi, 0, \pi$

$(\sin\theta + 1) = 2\cdot\cos^2\theta = 2(1+\sin\theta)(1-\sin\theta)$

$\sin\theta = -1$ which is not possible

$\sin\theta = \frac{1}{2}\ \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$n(s) = 5\ T = \cos 0 + \cos 2\pi + \cos 2\pi + \cos\frac{\pi}{3} + \cos\frac{5\pi}{3}$

$T = 4$

$T + n(s) = 9$

80. (b) For tautology $((p\Delta \sim q) \vee ((\sim p)\Delta q))$ must be true.

This is possible only when $\Delta = \vee$ & $\Rightarrow$

81. (31)

$2f(a) + 3f(c) = f(d) - f(b)$

Using fundamental principle of counting

Number of one-one function is 31

82. (40)

$x_1 + x_2 + x_3 + x_4 + x_5 = 5$

Only one possibilities $3,3,3,-2,-2$

Number of ways is $=\frac{5!}{3!2!}\times 2\times 2=40$

83. (8) $A=\left(\frac{3}{\sqrt{a}},\sqrt{a}\right)$

$B=\left(\frac{-3}{\sqrt{a}},\sqrt{a}\right)$

$C=\left(-\frac{3}{\sqrt{a}},-\sqrt{a}\right)$

Area of ACD

$$\frac{1}{2}\begin{vmatrix} \frac{3}{\sqrt{a}} & \sqrt{a} \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} \\ 3\cos\theta & a\sin\theta \\ \frac{3}{\sqrt{a}} & \sqrt{a} \end{vmatrix}$$

$$\frac{1}{2}6\sqrt{a}(\cos\theta-\sin\theta)$$

$3\sqrt{a}(\cos\theta-\sin\theta)$

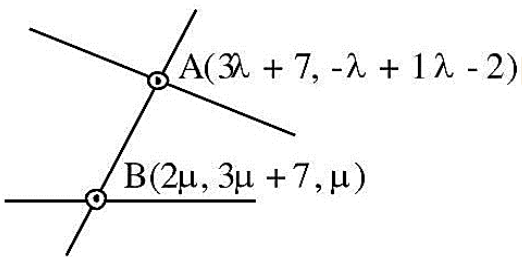
max values of function is $3\sqrt{a}\sqrt{2}$

$3\sqrt{a}\sqrt{2}=12$

$2a=16$

$a=8$

84. (84)



DR's of AB

$$(3\lambda-2\mu+7,-\lambda-3\mu-6,\lambda-\mu-2)$$
$$\frac{3\lambda-2\mu+7}{1}=\frac{-\lambda-3\mu-6}{-4}=\frac{\lambda-\mu-2}{2}$$

Taking first (2) $-12\lambda+8\mu-28=-\lambda-3\mu-6$

$\lambda-\mu+2=0$

Taking second & third

$-2\lambda - 6\mu - 12 = -4\lambda + 4\mu + 8$

$\lambda - 5\mu - 10 = 0$

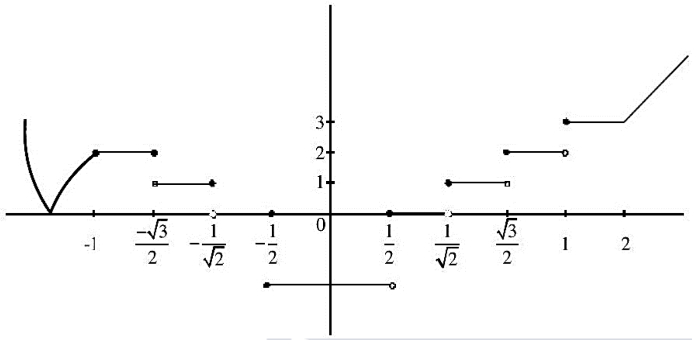
After solving above two equation $\lambda = -5, \mu = -3$

$A = (-8,6,7)$

$B = (-6,-2,-3)$

$(AB)^2 = 4 + 64 + 16 = 84$

85.(7)



86. (1) $f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t)dt$

$f(\theta) = \sin\theta + \sin\theta \int_{-\pi/2}^{\pi/2} f(t)dt + \cos\theta \int_{-\pi/2}^{\pi/2} tf(t)dt$

Let $A = \int_{-\pi/2}^{\pi/2} f(t)dt,\ B = \int_{-\pi/2}^{\pi/2} tf(t)dt$

$f(\theta) = \sin\theta + A\sin\theta + B\cos\theta$

$f(\theta) = (A+1)\sin\theta + B\cos\theta$

$A = \int_{-\pi/2}^{\pi/2} (A+1)\sin t + B\cos t dt$

$A = 2B$

$B = \int_{-\pi/2}^{\pi/2} t((A+1)\sin t + B\cos t)$

$B = \int_{-\pi/2}^{\pi/2} t(A+1)\sin t$

$B = (A+1)2\int_{0}^{\pi/2} t\sin t dt$

$B = (A+1)2.1$

$2A + 2 - B = 0$

After solving

$\mathrm{B} = -\frac{2}{3}, \mathrm{A} = -\frac{4}{3}$

$\left|\int_0^{\pi/2} \mathrm{f}(\theta) d\theta\right| = \left|\int_0^{\pi/2} -\frac{1}{3}\sin\theta - \frac{2}{3}\cos\theta\right|$

$= 1$

87. (34)

$y = \frac{9 - x^2}{5 - x} = 5 + x + \frac{16}{x - 5}$

$\frac{\mathrm{dy}}{\mathrm{dx}} = 1 - \frac{16}{(\mathrm{x} - 5)^2}$

So critical point is $\mathrm{x} = 1$ in $[0,2]$

$\mathrm{y}(0) = \frac{9}{5}, \mathrm{y}(1) = 2, \mathrm{y}(2) = \frac{5}{3}$

So $\alpha = 2$ and $\beta = \frac{5}{3}$

$I = \int_{-1}^{3} \max\left(\frac{9 - x^2}{5 - x}, x\right)$

$\mathrm{I} = \int_{-1}^{9/5} \frac{9 - \mathrm{x}^2}{5 - \mathrm{x}} \mathrm{dx} + \int_{9/5}^{3} \mathrm{xdx}$

$I = \int_{-1}^{9/5} 5 + x + \frac{16}{x - 5} dx + \int_{9/5}^{3} x dx$

After solving

$\mathrm{I} = 14 + \frac{28}{25} + 16\ln\left(\frac{8}{15}\right) + \frac{72}{25}$

$\alpha_1 = 18$ and $\alpha_2 = 16$

88. (2929)

$\alpha = \frac{1}{5}\cos\theta, \beta = \frac{1}{2}\sin\theta$

Equation of tangent to $y^2 = 4x$

$\mathrm{y} = \mathrm{mx} + \frac{1}{\mathrm{m}}$

It passes through (α, β)

$\frac{1}{2}\sin\theta = \mathrm{m}\frac{1}{5}\cos\theta + \frac{1}{\mathrm{m}}$

$m^2\left(\frac{\cos\theta}{5}\right) - m\left(\frac{1}{2}\sin\theta\right) + 1 = 0$

It has two roots m_1 and m_2 where $m_1 = 4\,m_2$

$$m_1 + m_2 = \frac{\frac{1}{2}\sin\theta}{\frac{\cos\theta}{5}}$$

$$m_1\,m_2 = \frac{5}{\cos\theta}$$

After eliminating m_1 and m_2

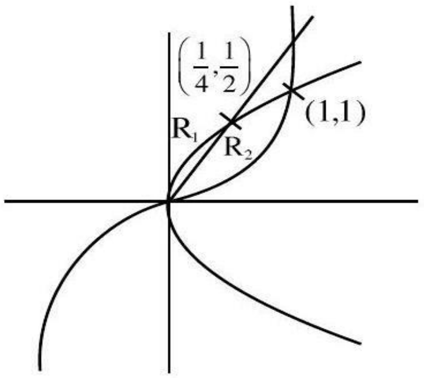
$$\cos\theta = \frac{-5 \pm \sqrt{29}}{2}$$

$$\alpha = \frac{-5 \pm \sqrt{29}}{10} \Rightarrow 10\alpha + 5 = \pm\sqrt{29}$$

$$\beta^2 = \frac{1}{4}\sin^2\theta \Rightarrow 16\beta^2 = -50 \pm 10\sqrt{29}$$

$$(10\alpha + 5)^2 + (16\beta^2 + 50)^2 = 2929$$

89. (19)



$$S = \int_0^1 \sqrt{x} - x^3$$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^4}{4}\right]_1^0$$

$$= \frac{5}{12}$$

$$R_1 = \int_0^{1/4} (\sqrt{x} - 2x)dx$$

$$= \left[\frac{2x^{3/2}}{3} - x^2\right]_0^{1/4} = \frac{1}{48}$$

$$\therefore R_2 = \frac{19}{48}$$

So, $\frac{R_2}{R_1} = 19$

90. (2)

$a_1 = (-1,0,3)$

$a_2 = (0,-1,2)$

$b_1 = (1,-a,0)$dr 's of line (1)

$b_2 = (1,-1,1)$dr 's of line (2)

$\bar{a}_2 - \bar{a}_1 = (1,-1,-1)$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix}$$

$\bar{b}_1 \times \bar{b}_2 = \hat{i}(-a) - \hat{j} + k(a-1)$

$|\bar{b}_1 \times \bar{b}_2| = \sqrt{a^2 + 1 + (a-1)^2}$

$a_2 - a_1 \cdot \bar{b}_1 \times \bar{b}_2 = 2 - 2a$

$$\frac{2(1-a)}{\sqrt{a^2+1+(a-1)^2}} = \sqrt{\frac{2}{3}}$$

Squaring an both the side

After solving $a = 2, \frac{1}{2}$