1. **(a)** Velocity gradient $= \frac{dV}{dx} = \frac{1}{S}$

$$\lambda = \frac{1}{S}$$

2. **(d)** $T' = T\left(\frac{3R}{R}\right)^{3/2} = 3\sqrt{3}T$

3. **(b)**

4. **(a)** $\frac{kq^2}{L^2} = \mu mg \Rightarrow L = \sqrt{\frac{k}{\mu mg}}q$

5. **(c)** $L = \frac{WD}{Q_H}$

$$\Rightarrow WD = Q_H\left(1 - \frac{T_L}{T_H}\right)$$
$$= 5 \times 10^3\left(1 - \frac{400}{1000}\right)$$
$$= 3000\text{kcal}$$

6. **(b)** $V_{max} = \omega A$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{9}{2} \times \frac{1}{2}} = \frac{3}{2}$$

7. **(b)** $\frac{4}{3} + 6 = \frac{22}{3}$

8. **(c)**

9. **(d)** $R = \frac{\sqrt{2\,km}}{qB} \propto \frac{\sqrt{m}}{q}$

$$\frac{\sqrt{m}}{e} : \frac{\sqrt{2\,m}}{e} : \frac{\sqrt{4\,m}}{2e}$$

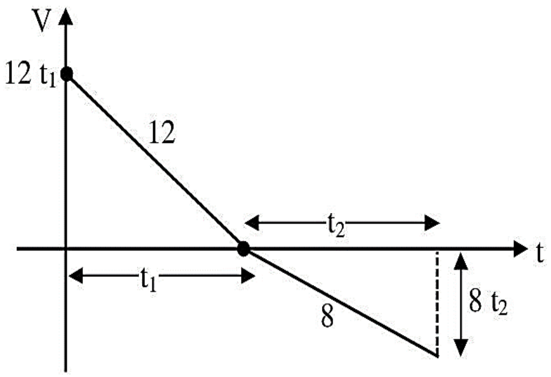
$1 : \sqrt{2} : 1$

10. **(c)** if $R = 0, P = 0$

11. **(c)** $\frac{-10\,A}{r^{11}} + \frac{5\,B}{r^6} = 0$

$$r^5 = \frac{10\,A}{5\,B} = \frac{2\,A}{B}$$

12. **(b)**



$6t_1^2 = 4t_2^2$

13. (b) $5 = \frac{1}{2}\alpha(1)^2$

$$\theta = \frac{1}{2}\alpha(2)^2$$

$\theta - 5 = 15$

14. (c) $\frac{1}{2} \times 1.5 \times 60^2 \times \frac{1}{4} = 0.1 \times 420 \times \Delta T$

15. (a)

$U \propto q^2$

$\Rightarrow q_f = 1.2q$

$q_f - q = 2$

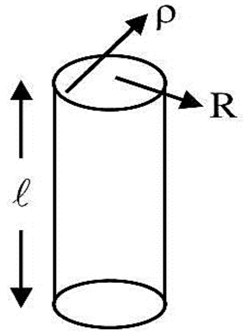
$\Rightarrow 1.2q - q = 2$

$q = 10$

16. (a) $E = 2\pi r\ell = \frac{\rho\pi r^2\ell}{\varepsilon_0}$

$$qE = \frac{q\rho R^2}{2\varepsilon_0 r} = \frac{mv^2}{r}$$

$$mv^2 = \frac{q\rho R^2}{2\varepsilon_0}$$



17. (b) $\frac{\eta P}{4\pi r^2} = \frac{cB_0^2}{2\mu_0}$

$$B_0 = \sqrt{\frac{\mu_0}{4\pi}\frac{\eta P}{c}\frac{1}{r}}$$

$$\Rightarrow B_0 = \frac{1}{4}\sqrt{\frac{10^{-7} \times 4 \times 3.5}{3 \times 10^8}} = 1.71 \times 10^{-8}\ \mathrm{T}$$

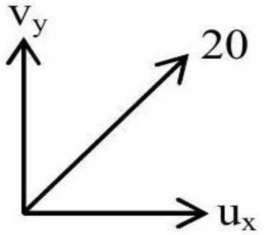
18. (b) $\sqrt{\frac{3.8-0.6}{1.4-0.6}} = \sqrt{\frac{3.2}{0.8}} = 2$

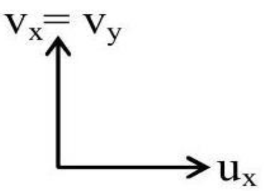
19. (d) $\sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

$$\left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = 5^2 = 25$$

20. (b)

21. (20)





$v_y = v_x - 20$

$\sqrt{(u_x - 20)^2 + u_x^2} = 20$

$\Rightarrow 2u_x^2 - 40u_x = 0$

$\therefore u_x = 20$

22. (6)

$$C' = \frac{C}{\sqrt{\mu_r \varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{6.25}} = \frac{3 \times 10^8}{2.5}$$

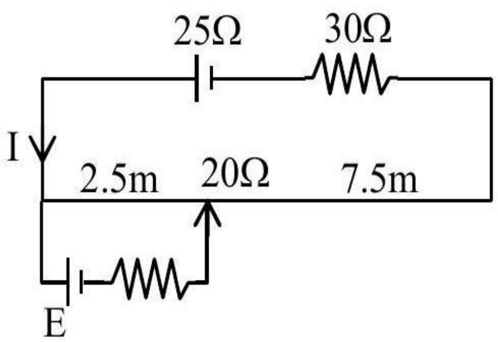
$f\lambda = 1.25 \times 10^8$ s

$\Rightarrow f(5 \times 10^{-3} \times 4) = 1.25 \times 10^8$

$f = 6.25\text{GHz}$

So $f \approx 6$

23. (25)



$I = \frac{25}{50} = \frac{1}{2}$ A

$\therefore \Delta V = 10$ V

10 m → 10 V

2.5 m → 2.5 V

24. (5) $10\cos\left(\frac{4\pi}{3}\right)$

25. (5) $I_L = \frac{5}{1000} = 5$ mA

26. (25)

$N_t = N_0(0.5)^{\frac{t}{t_{1/2}}}$

$= \frac{m}{M} \times N_A(0.5)^{\frac{t}{t/2}}$

$\frac{N_1}{N_2} = \frac{M_2}{M_1}(0.5)^t\left[\frac{1}{T_A} - \frac{1}{T_B}\right]$

$= 2(0.5)^{16\times\frac{1}{8}} = \frac{2}{4} = \frac{1}{2} = \frac{x}{100}$

27. (12)

$\ell = t\sin i\left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right]$

$\Rightarrow 4\sqrt{3} = t\sin 60^\circ\left[1 - \frac{\cos 60^\circ}{\sqrt{3 - \frac{3}{4}}}\right]$

28. (440)

$\epsilon_{max} = BAN\omega$

$= 0.07 \times 1 \times 10^3 \times 2\pi$

$= 140\pi \approx 440$

29. (2) $\Delta Q = \Delta E + WD \Rightarrow Q = \Delta E + \frac{Q}{4}$

$\Rightarrow n\frac{3R}{2}\Delta T = \Delta E = \frac{3Q}{4}$

$\therefore n\Delta T = \frac{Q}{2R}$

$\therefore C = 2R$

30. (16)

$T - T_0(T_i - T_0)e^{-\frac{Bt}{ms}}$

$6\lambda = \ln 1.5$

$40 = 60e^{-\lambda(6)} \Rightarrow 6\lambda = \ln 1.5$

$20 = 60e^{-\lambda t_2} \Rightarrow t_2\lambda = \ln 3$

$\frac{t_2}{6} = \frac{\ln 3}{\ln 1.5}$

$\therefore t_2 = 16.25$ min

So ≈ 16

31. (d) Given mass of organic compound $= 120$

mass of $CO_2(g) = 330$ g

mass of $H_2O(\ell) = 270$ g

mass of carbon $= n_{CO_2} \times 12$

$= \frac{330}{44} \times 12 = 90$ g

% of carbon $= \frac{90}{120} \times 100 = 75\%$

mass of hydrogen $= n_{H_2O} \times 2$

$= \frac{270}{18} \times 2 = 30$ g

% of hydrogen $= \frac{30}{120} \times 100 = 25\%$

32. (c) Energy of one mole of photons $= \frac{hc}{\lambda} \times N_A$

$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{300 \times 10^{-9}} \times 6.02 \times 10^{23}$

$= 399.13 \times 10^3$ Joule/mole

$= 399$ kJ/mole

33. (b)

Species	Bond order
C_2^{2-}	3
N_2^{2-}	2
O_2^{2-}	1

34. (c) $C_2H_6(g)+\frac{7}{2}O_2(g)\rightarrow 2CO_2(g)+3H_2O(\ell)$

$\Delta_C H(C_2H_6)=2\Delta_f HCO_2(g)+3\Delta_f H(H_2O,\ell)$

$-\Delta_f H(C_2H_6, g)$

$-1560=2(-394)+3(-286)-\Delta_f H(C_2H_6, g)$

$\Delta_f H(C_2H_6, g)=-86\ kJ/mole$

35. (c) Given $t_{0.90}=t_{0.90}=xt_{1/2}$

First order rate constant

$$K=\frac{\ln 2}{t_{1/2}}=\frac{1}{xt_{1/2}}\ln\frac{A_0}{A_0-A_0\times\frac{90}{100}}$$

$$\frac{\ln 2}{t_{1/2}}=\frac{\ln 10}{xt_{1/2}}$$

$$x=\frac{\ln 10}{\ln 2}=\frac{2.303}{2.303\times 0.3010}=3.32$$

36. (b) Hg, Ga, Cs are liquid near room temperature But Ag (silver) is solid.
37. (b) Hall Heroult process is the major industrial process for extraction of aluminium.
38. (a) Sodium hydroxide is generally prepared commercially by electrolysis of sodium chloride in castner Kellner cell.

at cathode: $Na+e^-\xrightarrow{Hg}Na-\text{amalgum}$

Anode: $Cl^-\longrightarrow\frac{1}{2}Cl_2+e^-$

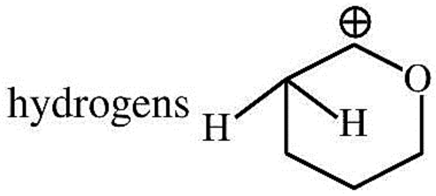
The Na-amalgam is treated with water to give sodium hydroxide and hydrogen gas:

$2Na\ (\text{amalgam})+H_2O\rightarrow 2NaOH+H_2+2Hg$

39. (a) Sodium hydrogencarbonate (Baking soda), $NaHCO_3$ is used in the fire extinguishers.
40. (b) PCl_5 forms five bonds by using the d-orbitals to "expand the octet". But NCl_5 does not exist because there are no d-orbitals in the valence shell $\left(2^{nd}\right.$ shell). Therefore, there is no way to expand the octet.
41. (d) CFSE of octahedral complexes with water is greater for 5 d series metal centre ion as compared to 3 d and 4 d series metal centre.
42. (d) CH_4, O_3 and H_2O causes global warming in Tropospheric level.

N_2 does not cause global warming.

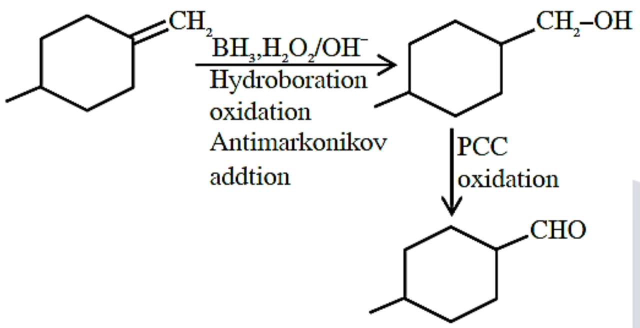
43. **(b)** Carbocation is stabilised by resonance with lone pairs on oxygen atom and +H effect of 2$\underline{\underline{a}}$



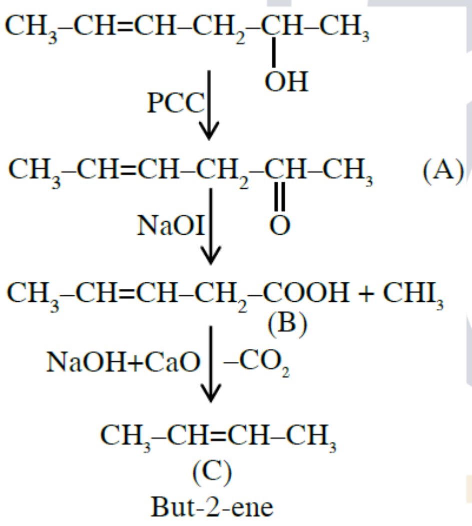
B > A > C

44. **(a)**

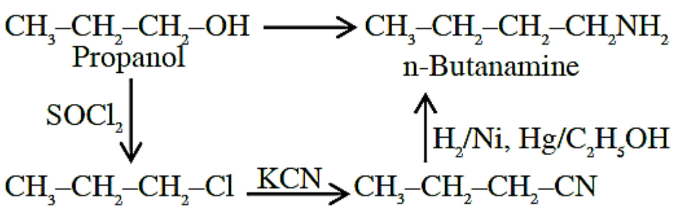
45. **(c)** $BH_3, H_2O_2/OH$ followed by PCC oxidation.



46. **(c)**



47. **(a)**



48. **(c)** Buna- N is an addition copolymer of 1,3-butadiene and acrylonitrile.

$$CH_2=CH-CH=CH_2 + \underset{\displaystyle CN}{\underset{|}{CH_2=CH}}$$

$$\underset{\text{Buna-N}}{[-CH_3-CH=CH-CH_2-CH_2-\underset{\displaystyle CN}{\underset{|}{CH}}-]_n}$$

49. (c)

50. (a)

Ion	Colour of the flame
(A) Cu^{+2}	green flame with blue center
(B) Sr^{2+}	Crimson Red
(C) Ba^{2+}	Apple green

51. (45)

Given: Ideal gas A and H_2 gas at same pressure and volume.

From ideal gas equation $pv = nRT$

$n_1\ T_1 = n_2\ T_2$

$$\frac{3}{\text{GMM of A}} \times 300 = \frac{0.2}{2} \times 200$$

$\text{GMM of A} = 45\ \text{g/mole}$

52. (1221 or1222)

From Henry law

$P = K_H X_{CO_2}$

$$0.835 = 1.67 \times 10^3 \times 1.67 \times 10^3 \times \frac{w_{CO_2}/44}{\frac{w_{CO_2}}{44} + \frac{1000}{18}}$$

$w_{CO_2} = 1.2228\ g = 1222.8 \times 10^{-3}\ g$

Or

$$P = K_H X_{CO_2}$$
$$0.835 = 1.67 \times 10^3 \times \frac{n_{CO_2}}{n_{CO_2} + n_{H_2O}}$$
$$0.835 = 1.67 \times 10^3 \times \frac{W_{CO_2}/44}{\frac{1000}{18}}$$

$w_{CO_2} = 1.2222\ g = 1222.2 \times 10^{-3}\ g$

53. (1107) Given: 2 mole of N_2 gas was present as inert gas.

Equilibrium pressure $= 2.46$ atm

$PCl_5(g) \rightleftharpoons PCl_3(g) + C\ell_2(g)$

$t = 0 \quad 5 \quad 0 \quad 0$

$t = Eq^m\ 5 - x\ x\ x$

from ideal gas equation

$PV = nRT$

$2.46 \times 200 = (5 - x + x + x + 2) \times 0.082 \times 600$

$x = 3$

$$K_P = \frac{n_{PCl_3} \times n_{Cl_2}}{n_{PCl_5}} \times \left[\frac{P_{total}}{n_{total}}\right]$$

$$\frac{3 \times 3}{2} \times \frac{2.46}{10} = 1.107 = 1107 \times 10^{-3}$$

54. (266) $K = \frac{1}{R} \times$ cell constan t

$0.152 \times 10^{-3} = \frac{1}{1750}$ cell constant

cell constant $= 266 \times 10^{-3}$

55. (2) weight of wood charcoal $= 0.6$ g

Mass of acetic acid adsorbed $= \frac{M_1 V_1 - M_2 V_2}{1000} \times 60$

$$= \frac{0.2 \times 200 - 0.1 \times 200}{1000} \times 60$$

$= 1.2$ g

Mass of acetic acid adsorbed per gram of carbon $= \frac{1.2}{0.6} = 2$

56. (3)

(a) Baryte : $BaSO_4$

$$\left.\begin{array}{l}\text{(b) Galena : PbS} \\ \text{(c) Zinc blende : ZnS} \\ \text{(d) Copper pyrite : } CuFeS_2\end{array}\right] \begin{array}{c}\text{sulphide } (S^{2-}) \\ \text{ores}\end{array}$$

57. (3) MnO_4^{2-} disproportionates in a neutral or acidic solution to give $MnO_4^-$and Mn^{+4} $3MnO_4^{2-} + 3H^+ \longrightarrow 2MnO_4^- + MnO_2 + 2H_2O$

O.S. of Mn in $MnO_4^- = +7$

O.S. of Mn in $MnO_2 = +4$

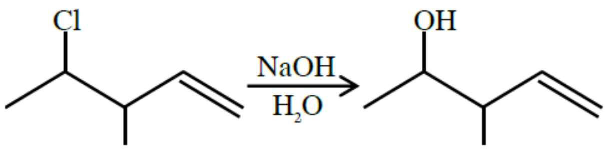
difference $= 3$

58. (14) weight of organic compound $= 0.2$ g mass of $N_2(g)$ evolved $= \frac{22.4\times10^{-3}}{22.4} \times 28$

$= 28 \times 10^{-3}$ g

% of N $= \frac{28\times10^{-3}}{0.2} \times 100 = 14$

59. (2)



60. (4) There are Five amino acids and four peptide linkages.

61. (b) $(x * 1) * 1 = x * (1 * 1)$

$(x^2 + 1) * 1 = x * (2)$

$(x^2 + 1)^2 + 1 = x^2 + 8$

$x^4 + x^2 - 6 = 0 \Rightarrow (x^2 + 3)(x^2 - 2) = 0$

$x^2 = 2$

$$\Rightarrow 2\sin^{-1}\left(\frac{x^4 + x^2 - 2}{x^4 + x^2 + 2}\right) = 2\sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3}$$

62. (b) $(e^{2x} - 4)(6e^{2x} - 3e^x - 2e^x + 1) = 0$

$(e^{2x} - 4)(3e^x - 1)(2e^x - 1) = 0$

$e^{2x} = 4$ or $e^x = \frac{1}{3}$ or $e^x = \frac{1}{2}$

$\Rightarrow$ sum of real roots $= \frac{1}{2}\ln 4 + \ln\frac{1}{3} + \ell n\frac{1}{2}$

$= -\ln 3$

63. (c)

$$\Delta = \begin{vmatrix} 1 & 1 & \alpha \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(\alpha + 3)$$

$$\Delta_1 = \begin{vmatrix} 2 & 1 & \alpha \\ 4 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(3 + \alpha)$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & \alpha \\ 3 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -(\alpha + 3)$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 4 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$\alpha \neq -3, x = 1, y = 1, z = 0,$

Now points $(\alpha, 1), (1, \alpha) \& (1, -1)$ are collinear

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \alpha(\alpha + 1) - 1(1 - 1) + 1(-1 - \alpha) = 0$

$\alpha^2 + \alpha - 1 - \alpha = 0$

$\alpha = \pm 1$

64. (d)

Using AM $\geq$ GM

$$\frac{x + x + x + y + y}{5} \geq (x^3 \cdot y^2)^{\frac{1}{5}}$$

$$\frac{3x + 2y}{5} \geq (2^{15})^{\frac{1}{5}}$$

$(3x + 2y)_{\min} = 40$

65. (c) $f(x) = \begin{cases} \frac{\sin(x+2)}{x+2} & , \quad x \in (-2, -1) \\ \max\{2x, 0\} & , \quad x \in (-1, 1) \\ 1, & \quad \text{otherwise} \end{cases}$

$f(-2^+) = \lim_{h \to 0} f(-2 + h) = \lim_{h \to 0} \frac{\sin h}{h} = 1$

f is continuous at $x = -2$

$f(-1^-) = \lim_{h \to 0} \frac{\sin(-1 - h + 2)}{(-1 - h + 2)} = \sin 1$

$f(-1) = f(-1^+) = 0$

$f(1^+) = 1 \& f(1^-) = 0 \Rightarrow$ f is not continuous at $x = 1$

f is continuous but not diff. at $x = 0$

$\left.\begin{array}{l} \Rightarrow \text{f is discontinuous at } x = -1 \& 1 \\ \quad \& \text{f is not diff. at } x = -1, 0 \& 1 \end{array}\right\} \Rightarrow \begin{array}{l} m = 2 \\ n = 3 \end{array}$

66. (c)

$$I = \int_{-\pi/2}^{0} \frac{dx}{(1 + e^x)(\sin^6 x + \cos^6 x)} + \int_{0}^{\pi/2} \frac{dx}{(1 + e^x)(\sin^6 x + \cos^6 x)}$$

Put $x = -t$

$$= \int_{\pi/2}^{0} \frac{-dt}{(1+e^{-t})(\sin^6 t+\cos^6 t)} + \int_{0}^{\pi/2} \frac{dx}{(1+e^{x})(\sin^6 x+\cos^6 x)}$$

$$= \int_{0}^{\pi/2} \frac{(e^x+1)dx}{(1+e^{x})(\sin^6 x+\cos^6 x)}$$

$$= \int_{0}^{\pi/2} \frac{dx}{(\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x)}$$

$$= \int_{0}^{\pi/2} \frac{(1+\tan^2 x)\sec^2 x dx}{(\tan^4 x-\tan^2 x+1)}$$

Put $\tan x = t$

$$= \int_{0}^{\infty} \frac{(1+t^2)dt}{(t^4-t^2+1)}$$

$$= \int_{0}^{\infty} \frac{\left(1+\frac{1}{t^2}\right)dt}{t^2-1+\frac{1}{t^2}} = \int_{0}^{\infty} \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+1}$$

Put $t-\frac{1}{t} = z$

$$\left(1+\frac{1}{t^2}\right)dt = dz$$

$$= \int_{-\infty}^{\infty} \frac{dz}{1+z^2} = (\tan^{-1} z)_{-\infty}^{\infty}$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$$

67. (a) $\lim_{n\to\infty}\left(\sum_{r=1}^{n} \frac{n^2}{(n^2+r^2)(n+r)}\right)$

$$= \lim_{n\to\infty}\left(\sum_{r=1}^{n} \frac{1}{n\left(1+\left(\frac{r}{n}\right)^2\right)\left(1+\left(\frac{r}{n}\right)\right)}\right)$$

$$= \int_{0}^{1} \frac{dx}{(1+x^2)(1+x)} = \frac{1}{2}\int_{0}^{1} \frac{1-x}{1+x^2}dx + \frac{1}{2}\int_{0}^{1} \frac{1}{1+x}dx$$

$$= \frac{1}{2}\int \left(\frac{1}{1+x^2} - \frac{x}{1+x^2}\right)dx + \frac{1}{2}(\ln(1+x))_0^1$$

$$= \frac{1}{2}\left[\tan^{-1} x - \frac{1}{2}\ln(1+x^2)\right]_0^1 + \frac{1}{2}\ln 2$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2}\ln 2\right] + \frac{1}{2}\ln 2$$

$$= \frac{\pi}{8} + \frac{1}{4}\ln 2$$

68. (a) Let Point $P(x, y)$

$Y - y = y'(X - x)$

$Y = 0 \Rightarrow X = x - \frac{y}{y'}$

$Q\left(x - \frac{y}{y'}, 0\right)$

Mid Point of PQ lies on y axis

$x - \frac{y}{y'} + x = 0$

$y' = \frac{y}{2 \cdot x} \Rightarrow 2\frac{dy}{y} = \frac{dx}{x}$

$2\ell\, ny = \ell nx + \ell nk$

$y^2 = kx$

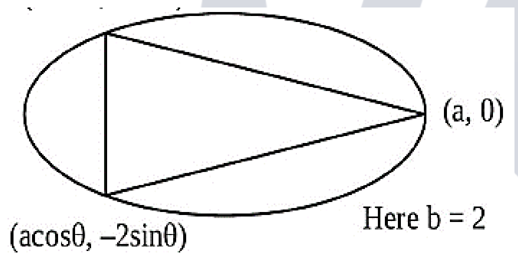
It passes through $(3,3) \Rightarrow k = 3$

curve $c \Rightarrow y^2 = 3x$

Length of L.R. $= 3$

Focus $= \left(\frac{3}{4}, 0\right)$

69. (a) $(a\cos\theta, 2\sin\theta)$



$A = \frac{1}{2}a(1 - \cos\theta)(4\sin\theta)$

$A = 2a(1 - \cos\theta)\sin\theta$

$\frac{dA}{d\theta} = 2a(\sin^2\theta + \cos\theta - \cos^2\theta)$

$\frac{dA}{d\theta} = 0 \Rightarrow 1 + \cos\theta - 2\cos^2\theta = 0$

$\cos\theta = 1$ (Reject)

OR

$\cos\theta = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

$$\frac{d^2 A}{d\theta^2} = 2a(2\sin^2\theta - \sin\theta)$$

$\frac{d^2 A}{d\theta^2} < 0$ for $\theta = \frac{2\pi}{3}$

Now, $A_{max} = \frac{3\sqrt{3}}{2} a = 6\sqrt{3}$

$a = 4$

Now, $e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{\sqrt{3}}{2}$

70. (c) $\frac{1}{2}\begin{vmatrix} \alpha & 0 & 1 \\ 1 & \alpha & 1 \\ 0 & \alpha & 1 \end{vmatrix} = \pm 4$

$\alpha = \pm 8$

Now given points $(8, -8), (-8, 8), (64, \beta)$

OR $(-8, 8), (8, -8), (64, \beta)$

are collinear $\Rightarrow$ Slope $= -1$.

$\beta = -64$

71. (d) $x^7 - 7x - 2 = 0$

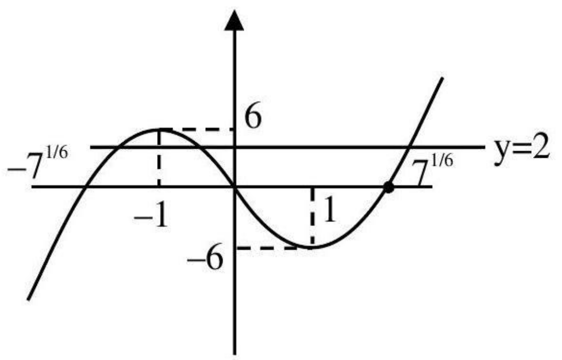
$x^7 - 7x = 2$

$f(x) = x^7 - 7x$ (odd) &$y = 2$

$f(x) = x(x^2 - 7^{1/3})(x^4 + x^2 \cdot 7^{1/3} + 7^{2/3})$

$f'(x) = 7(x^6 - 1) = 7(x^2 - 1)(x^4 + x^2 + 1)$

$f'(x) = 0 \Rightarrow x = \pm 1$



$f(x) = 2$ has 3 real distinct solution.

72. (a)

$$P\left(\frac{1 < x < 4}{x \le 2}\right) = \frac{P(1 < x < 4 \cap x \le 2)}{P(x \le 2)}$$

$= \frac{P(1 < x \le 2)}{P(x \le 2)} = \frac{P(x = 2)}{P(x \le 2)}$

$= \frac{4k}{k + 2k + 4k} = \frac{4}{7}$

73. (d) $\cos\left(\frac{\pi}{3} + x\right)\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4}\cos^2 2x$

$x \in [-3\pi, 3\pi]$

$4\left(\cos^2\left(\frac{\pi}{3}\right) - \sin^2 x\right) = \cos^2 2x$

$4\left(\frac{1}{4} - \sin^2 x\right) = \cos^2 2x$

$1 - 4\sin^2 x = \cos^2 2x$

$1 - 2(1 - \cos 2x) = \cos^2 2x$

let $\cos 2x = t$

$-1 + 2\cos 2x = \cos^2 2x$

$t^2 - 2t + 1 = 0$

$(t - 1)^2 = 0$

$t = 1 \cos 2x = 1$

$2x = 2n\pi$

$x = n\pi$

$n = -3, -2, -1, 0, 1, 2, 3$

74. (a) SHORTEST distance $\frac{|(a_2 - a_1)\cdot(b_1 \times b_2)|}{|b_1 \times b_2|}$ $a_1 = (1,2,3)$

$a_2 = (2,4,5)$

$\vec{b}_2 = 2\hat{\imath} + 3\hat{\jmath} + \lambda\hat{k}$

$\vec{b}_2 = \hat{\imath} + 4\hat{\jmath} + 5\hat{k}$

S.D. $= \frac{|((2-1)\hat{\imath} + (4-2)\hat{\jmath} + (5-3)\hat{k})\cdot(\vec{b}_1 \times \vec{b}_2)|}{|b_1 \times b_2|}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & \lambda \\ 1 & 4 & 5 \end{vmatrix}$

$= \hat{\imath}(15 - 4\lambda) + \hat{\jmath}(\lambda - 10) + \hat{k}(5)$

$= (15 - 4\lambda)\hat{\imath} + (\lambda - 10)\hat{\jmath} + 5\hat{k}$

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(15-4\lambda)^2 + (\lambda-10)^2 + 25}$

Now

S.D. $= \frac{|(\hat{i}+2\hat{j}+2\hat{k})\cdot[(15-4\lambda)\hat{i}+(\lambda-10)\hat{j}+5\hat{k}]|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}$

$$\frac{|15-4\lambda+2\lambda-20+10|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}} = \frac{1}{\sqrt{3}}$$

square both side

$3(5-2\lambda)^2 = 225 + 16\lambda^2 - 120\lambda + \lambda^2 + 100 - 20\lambda + 25$

$12\lambda^2 + 75 - 60\lambda = 17\lambda^2 - 140\lambda + 350$

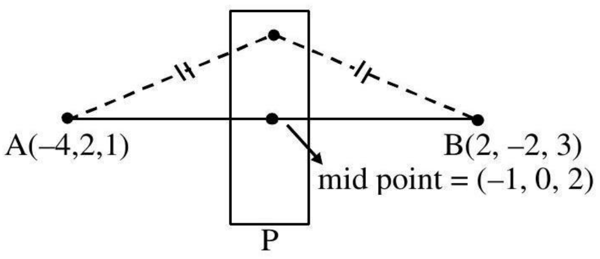
$5\lambda^2 - 80\lambda + 275 = 0$

$\lambda^2 - 16\lambda + 55 = 0$

$(\lambda-5)(\lambda-11) = 0$

$\Rightarrow \lambda = 5,11$

75. (c)



Normal vector $= \overrightarrow{AB} = (\overrightarrow{OB} - \overrightarrow{OA})$

$= (6\hat{i} - 4\hat{j} + 2\hat{k})$

or $2(3\hat{i} - 2\hat{j} + \hat{k})$

$P \equiv 3(x+1) - 2(y) + 1(z-2) = 0$

$P \equiv 3x - 2y + z + 1 = 0$

$P' \equiv 2x + y + 3z - 1 = 0$

angle between $P \& P' = \left|\frac{\hat{n}_1\cdot\hat{n}_2}{|n_1||n_2|}\right| = \cos\theta$

$$\theta = \cos^{-1}\left(\frac{6-2+3}{\sqrt{14}\times\sqrt{14}}\right)$$

$$\theta = \cos^{-1}\left(\frac{7}{14}\right) == \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

76. (c) $|(\hat{a}+\hat{b}) + 2(\hat{a}\times\hat{b})| = 2, \theta \in (0,\pi)$

$((\hat{a}+\hat{b})+2(\hat{a}\times\hat{b}))\cdot((\hat{a}+\hat{b})+2(\hat{a}\times\hat{b}))=4$

$|\hat{a}+\hat{b}|^2+4|(\hat{a}\times\hat{b})|^2+0=4$

Let the angle be θ between $\hat{a}$ and $\hat{b}$

$2+2\cos\theta+4\sin^2\theta=4$

$2+2\cos\theta-4\cos^2\theta=0$

Let $\cos\theta=t$ then

$2t^2-t-1=0$

$2t^2-2t+t-1=0$

$2t(t-1)+(t-1)=0$

$(2t+1)(t-1)=0$

$t=-\frac{1}{2}$ or $t=1$

$$\cos\theta=-\frac{1}{2} \quad \Bigg| \quad \text{not possible as } \theta\in(0,\pi)$$

$$\theta=\frac{2\pi}{3}$$

Now,

S_1 $2|\vec{a}\times\vec{b}|=2\sin\left(\frac{2\pi}{3}\right)$

$|\hat{a}-\hat{b}|=\sqrt{1+1-2\cos\left(\frac{2\pi}{3}\right)}$

$=\sqrt{2-2\times\left(-\frac{1}{2}\right)}$

$=\sqrt{3}$

S_1 is correct.

S_2 projection of $\hat{a}$ on $(\hat{a}+\hat{b})$.

$$\frac{\hat{a}.(\hat{a}+\hat{b})}{|\hat{a}+\hat{b}|}=\frac{1+\cos\left(\frac{2\pi}{3}\right)}{\sqrt{2+2\cos\frac{2\pi}{3}}}$$

$$=\frac{1-\frac{1}{2}}{\sqrt{1}}$$

$= \frac{1}{2}$

77. (b) $y = \tan^{-1}(\sec x^3 - \tan x^3)$

$= \tan^{-1}\left(\frac{1-\sin x^3}{\cos x^3}\right)$

$= \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-x^3\right)}{\sin\left(\frac{\pi}{2}-x^3\right)}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x^3}{2}\right)\right)$

Since $\frac{\pi}{4}-\frac{x^3}{2} \in \left(-\frac{\pi}{2}, 0\right)$

$y = \left(\frac{\pi}{4}-\frac{x^3}{2}\right)$

$y' = \frac{-3x^2}{2}, y'' = -3x$

$4y = \pi - 2x^3$

$4y = \pi - 2x^2\left(\frac{-y''}{3}\right)$

$12y = 3\pi + 2x^2y''$

$x^2y'' - 6y + \frac{3\pi}{2} = 0$

78. (b) $\sim((A \wedge C) \rightarrow B)$

$\sim(\sim(A \wedge C) \vee B)$

Using De-Morgan's law

$(A \wedge C) \wedge (\sim B)$

79. (d) Slope of normal $= \frac{-dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1}$

$x^2y^2dx + dx - xydx = x^2dy$

$x^2y^2dx + dx = x^2dy + xydx$
$x^2y^2dx + dx = x(xdy + ydx)$

$x^2y^2dx + dx = xd(xy)$

$\frac{dx}{x} = \frac{d(xy)}{1 + x^2y^2}$

$\ln kx = \tan^{-1}(xy)$

passes though (1,1)

$\ln k=\frac{\pi}{4} \Rightarrow k=e^{\frac{\pi}{4}}$

equation (i) be becomes

$\frac{\pi}{4}+\ln x=\tan ^{-1}(x y)$

$x y=\tan \left(\frac{\pi}{4}+\ell n x\right)$

$x y=\left(\frac{1+\tan (\ell n x)}{1-\tan (\ell n x)}\right)$

put $x=e$ in (ii)

$\therefore$ ey (e) $=\frac{1+\tan 1}{1-\tan 1}$

80. (d) $f_{\lambda}(x)=4 \lambda x^{3}-36 \lambda x^{2}+36 x+48$

$f_{\lambda}{ }^{\prime}(x)=12 \lambda x^{2}-72 \lambda x+36$

$f_{\lambda}{ }^{\prime}(x)=12\left(\lambda x^{2}-6 \lambda x+3\right) \geq 0$

$\therefore \lambda>0 \& D \leq 0$

$36 \lambda^{2}-4 \times \lambda \times 3 \leq 0$

$9 \lambda^{2}-3 \lambda \leq 0$

$3 \lambda(3 \lambda-1) \leq 0 \ \lambda \in\left[0, \frac{1}{3}\right]$

$\therefore \lambda_{\text{largest}}=\frac{1}{3}$

$f(x)=\frac{4}{3} x^{3}-12 x^{2}+36 x+48$

$\therefore f(1)+f(1)=72$

81. (80) $|z-3| \leq 1$

represent pt. i/s circle of radius 1& centred at (3,0)

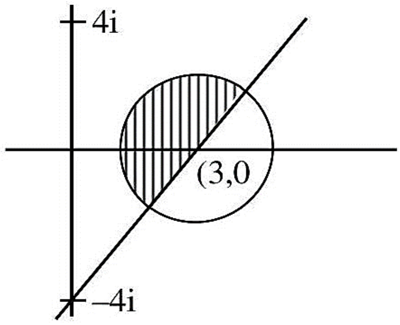
$z(4+3 i)+\bar{Z}(4-3 i) \leq 24$

$(x+i y)(4+3 i)+(x-i y)(4-3 i) \leq 24$

$4 x+3 x i+4 i y-3 y+4 x-3 i x-4 i y-3 y \leq 24$

$8 x-6 y \leq 24$

$4 x-3 y \leq 12$



minimum of (0,4) from circle $= \sqrt{3^2+4^2} - 1 = 4$ will lie along line joining (0,4)&(3,0)

∴ equation line

$$\frac{x}{3}+\frac{y}{4}=1 \Rightarrow 4x+3y=12$$

equation circle $(x-3)^2+y^2=1$

$$\left(\frac{12-3y}{4}-3\right)^2+y^2=1$$
$$\left(\frac{-3y}{4}\right)^2+y^2=1$$

$$\frac{25\mathrm{y}^2}{16}=1 \Rightarrow \mathrm{y}=\pm\frac{4}{5}$$

for minimum distance $y=\frac{4}{5}$

$$\therefore \mathrm{x}=\frac{12}{5}$$

$$\therefore 25(\alpha+\beta)=25\left(\frac{4}{5}+\frac{12}{5}\right)$$

$$=16\times 5=80$$

82. (100) $A=\begin{bmatrix}-1 & \mathrm{a}\\ 0 & \mathrm{b}\end{bmatrix}$

$$A^2=\begin{bmatrix}-1 & \mathrm{a}\\ 0 & \mathrm{b}\end{bmatrix}\begin{bmatrix}-1 & \mathrm{a}\\ 0 & \mathrm{b}\end{bmatrix}$$

$$=\begin{bmatrix}1 & -\mathrm{a}+\mathrm{ab}\\ 0 & \mathrm{b}^2\end{bmatrix}$$

$$\therefore \mathrm{T_n}=\left\{\mathrm{A}\in \mathrm{S}; \mathrm{A}^{(\mathrm{n}(\mathrm{n}+1))}=\mathrm{I}\right\}$$

∴ b must be equal to 1

∴ In this case A^2 will become identity matrix and a can take any value from 1 to 100

∴ Total number of common element will be 100

83.(576) Digits are 1, 2, 3, 4, 5, 7, 9

Multiple of 11 → Difference of sum at even & odd place is divisible by 11.

Let number of the form abcdefg

$\therefore (a+c+e+g)-(b+d+f)=11x$

$a+b+c+d+e+f=31$

$\therefore$ either $a+c+e+g=21$ or 10

$\therefore b+d+f=10$ or 21

Case- 1

$a+c+e+g=21$

$b+d+f=10$

$(b,d,f)\in\{(1,2,7)(2,3,5)(1,4,5)\}$

$(a,c,e,g)\in\{(1,4,7,9),(3,4,5,9),(2,3,7,9)\}$

$\therefore$ Total number in case- $1=(3!\times 3)(4!)=432$

Case- 2

$a+c+e+g=10$

$b+d+f=21$

$(a,b,e,g)\in\{1,2,3,4)\}$

$(b,d,f)\&\{(5,7,9)\}$

$\therefore$ Total number in case $2=3!\times 4!=144$

$\therefore$ Total numbers $=144+432=576$

84. (1633) $\text{HCF}(\alpha,24)=1$

Now, $24=2^2\cdot 3$

$\rightarrow \alpha$ is not the multiple of 2 or 3

Sum of values of α

$=S(U)-\{S$ (multiple of 2) $+S$ (multiple of 3)

$-S$ (multiple of 6)$\}$

$=(1+2+3+\cdots\ldots.100)-(2+4+6\ldots\ldots+100)-(3$

$+6+\cdots..99)+(6+12+\cdots..+96)$

$$=\frac{100\times 101}{2}-50\times 51-\frac{33}{2}\times(3+99)+\frac{16}{2}(6+96)$$

$= 5050 - 2550 - 1683 + 816 = 1633$

85. (4) $\frac{1.(3^{2022}-1)}{2} = \frac{9^{1011}-1}{2}$

$= \frac{(10-1)^{1011}-1}{2}$

$= \frac{100\lambda + 10110 - 1 - 1}{2}$

$= 50\lambda + \frac{10108}{2}$

$= 50\lambda + 5054$

$= 50\lambda + 50 \times 101 + 4$

$\text{Rem}(50) = 4.$

86. (18)

$\mathrm{x} = 4 - \mathrm{y}$

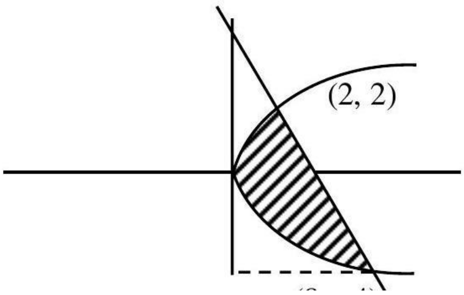
$y^2 = 2(4-y)$

$\mathrm{y}^2 = 8 - 2\mathrm{y}$

$\mathrm{y}^2 + 2\mathrm{y} - 8 = 0$

$y = -4, y = 2$

$\mathrm{x} = 8, \mathrm{x} = 2$



$(8, -4)$

$\int_{-4}^{2}\left[(4-y) - \frac{y^2}{2}\right]dy$

$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6}\right]_{-4}^{2}$

$= 8 - 2 - \frac{8}{6} + 16 + \frac{16}{2} - \frac{64}{6}$

$= 22 + 8 - \frac{72}{6}$

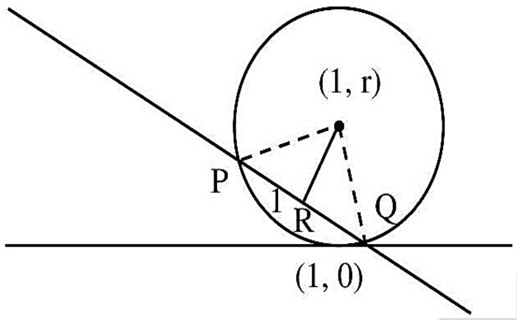
$= 30 - 12 = 18$

87. (7) $k = r$

$h = 1$

$OP = r, PR = 1$

$OR = \left|\frac{r+1}{\sqrt{2}}\right|$



$r^2 = 1 + \frac{(r+1)^2}{2}$

$2r^2 = 2 + r^2 + 1 + 2r$

$r^2 - 2r - 3 = 0$

$(r-3)(r+1) = 0$

$r = 3, -1$

$h + k + r = 1 + 3 + 3$

$= 7$

88. (479)

$A = \{1,2,3,4\}: P(a) = \frac{3}{4} \rightarrow$ Correct

$B = \{5,6,7,8,9,10\}; P(b) = \frac{1}{4}$ Correct

8 Correct Ans.:

$(4,4)$: ${}^4C_4\left(\frac{3}{4}\right)^4 \cdot {}^6C_4 \cdot \left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)^2$

$(3,5)$: ${}^4C_3\left(\frac{3}{4}\right)^3 \cdot \left(\frac{1}{4}\right)^1 \cdot {}^6C_5\left(\frac{1}{4}\right)^5 \cdot \left(\frac{3}{4}\right)$

$(2,6)$: ${}^4C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2 \cdot {}^6C_6\left(\frac{1}{4}\right)^6$

Total $= \frac{1}{4^{10}}[3^4 \times 15 \times 3^2 + 4 \times 3^3 \times 6 \times 3 + 6 \times 3^2]$

$= \frac{27}{4^{10}}[2.7 \times 15 + 72 + 2]$

$\Rightarrow K = 479$

89. (42) $\frac{x^2}{a^2} - \frac{y^2}{1} = 1 \; \frac{x^2}{4} + \frac{y^2}{3} = 1$

$e_H = \sqrt{1 + \frac{1}{a^2}} \; e_E = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$\ell.R. = \frac{2}{a} \; \ell R = \frac{2 \times 3}{2} = 3$

$\frac{2}{a} = 3$

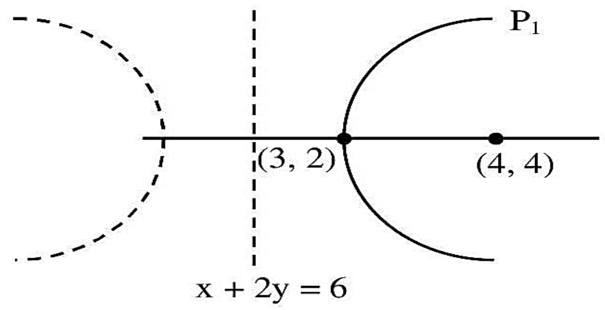
$a = \frac{2}{3}$

$e_H = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$

$12(e_H^2 + e_E^2) = 12\left(\frac{13}{4} + \frac{1}{4}\right)$

$= \frac{12 \times 14}{4} = 42$

90. (10)



P_1 : Directorix :

$x + 2y = k$

$x + 2y - k = 0$

$\left|\frac{3 + 4 - K}{\sqrt{5}}\right| = \sqrt{5}$

$|7 - k| = 5$

$7 - K = 5 \; 7 - K = -5$

$k = 2 \; k = 12$

Accepted Rejected

Passes through

focus

$$\left.\begin{array}{l} D_1 = x + 2y = 2 \\ \ell = x + 2y = 6 \\ D_2 = x + 2y = C \end{array}\right] \Rightarrow d \Rightarrow c = 10$$