1. **(b)** Surface tension (S) $= \frac{F}{l} \rightarrow kg\frac{M}{S^2}\cdot\frac{1}{M} \rightarrow Kgs^{-2}$

Impulse (J) $= \int Fdt \rightarrow N - S$

$\rightarrow Kgms^{-2}\cdot S$
$\rightarrow Kgms^{-1}$

Pressure (P) $= \frac{F}{A} \rightarrow Kgms^{-2},\ m^{-2}$

$\rightarrow Kgms^{-1}\ s^{-2}$

Viscocity$(\eta) = \frac{F}{6\pi rv}$

$\rightarrow \frac{kgms^{-2}}{m\cdot ms^{-1}}$
$\rightarrow kgm^{-1}\ s^{-1}$

2. **(c)** $\rho = \frac{M}{V}$ and $V = \frac{4}{3}\pi r^3$ when $r = R_0A^{1/3}$

$\therefore \rho = \frac{M}{\frac{4}{3}\pi R_0^3\ A}$

$\therefore \rho \propto \frac{M}{A}$

$\frac{\rho_0}{\rho_{He}} = \frac{M_o}{A_o}\times\frac{A_{He}}{M_{He}} = \frac{16}{8}\times\frac{2}{4} = 1$

3. **(d)**

$V_{rms} = \sqrt{\frac{3RT}{M_0}}$
$\therefore V_{rms} \propto \sqrt{T}$

4. **(a)**

(A) $B = \frac{\mu_0 I}{4\pi r}\times 2 - \frac{\mu_0 I}{2r}$

$= \frac{\mu I}{2r}\left(\frac{1}{\pi} - 1\right)$

$= \frac{\mu I}{2\pi r}(1-\pi)\odot$
$= \frac{\mu I}{2\pi r}(\pi-1)\otimes$

(B) $B = \frac{\mu_0 I}{4\pi r}\times\pi + \frac{\mu_0 I}{4\pi r}\times 2$

$= \frac{\mu_0 I}{4\pi r}(\pi+2)\odot$

(C) $B = \frac{\mu_0 I}{4\pi r} \cdot \pi + 0 + \frac{\mu_0 I}{4\pi r}$

$= \frac{\mu_0 I}{4\pi r}(\pi + 1) \otimes$

(D) $B = \frac{\mu_0 I}{4r} \odot$

5. (c)

Bandwidth $= 2 \times$ highest of base band frequency

$= 2 \times 5 = 10\text{kHZ}$

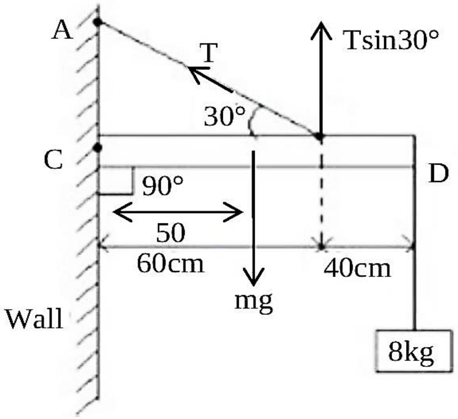
6. (c) The rod is in equilibrium. So, net torque about any point will be zero.

$\tau_c = 0$

$Mg \times 50 + 80 \times 100 = T\sin 30^\circ \times 30$

$20 \times 50 + 80 \times 100 = \frac{T}{2} \times 60$

$T = \frac{900}{3} = 300\text{ N}$



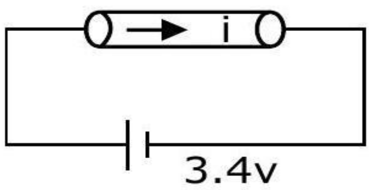
7. (b) Photo diodes are not used in forward bias.

8. (d) $\omega = \frac{1}{\sqrt{LC}}$

$\frac{\omega_1}{\omega_2} = \sqrt{\frac{L_2C_2}{L_1C_1}} = \sqrt{\frac{2\,L \cdot 8C}{L \cdot C}} = 4$

$\omega_2 = \frac{\omega_1}{4} = \frac{\omega_0}{4}$

9. (a)



Given, $i = 2$ A

$v = 3.4v$

$v = iR$

$R = \frac{v}{i} = \frac{3.4}{2} = 1.7\Omega$

$\text{volume} = \frac{\text{mass}}{\text{Density}} = \frac{8.92\times10^{-3}}{8.92\times10^{3}}\ m^3 = 10^{-6}\ m^3$

$\Rightarrow A\ell = 10^{-6}\ m^3$

$R = \frac{\rho\ell}{A}$

$\Rightarrow \frac{\rho}{R} = \frac{A}{\ell}$

$\frac{1.7\times10^{-8}}{1.7} = \frac{A}{\ell}$

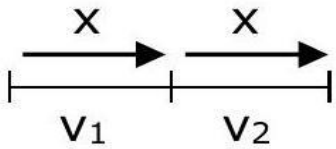
$\frac{A}{\ell} = 10^{-8}$

$\frac{\text{eq(i)}}{\text{eq(ii)}}$

$\ell^2 = 10^2$

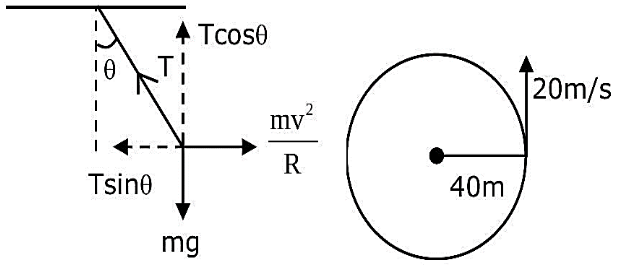
$\ell = 10$ m

10. (a)



$v_{avg} = \frac{\text{total Distance}}{\text{Total Time}}$
$= \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}}$
$= \frac{2v_1v_2}{v_1 + v_2}$

11. (c)



$T\cos\theta = mg$

$T\sin\theta = \frac{mv^2}{R}$

$\frac{eq(i)}{eq(ii)}; \frac{\cos\theta}{\sin\theta} = \frac{gR}{v^2}$

$\tan\theta = \frac{v^2}{Rg} = \frac{400}{40 \times 10} = 1$

$\theta = \frac{\pi}{4}$

12. (b) According to NLC,

$-\frac{d\theta}{dt} = k\theta$

$\frac{12}{2} = K\left(\frac{98+86}{2} - 22\right)$

$\Rightarrow 6 = K(92 - 22) = K \times 70$

$\Rightarrow K = \frac{6}{70}$

Now, $\frac{6}{t_2} = \frac{6}{70}\left(\frac{75+69}{2} - 22\right)$

$= \frac{6}{70} \times (72 - 22)$

$t_2 = \frac{6 \times 70}{6 \times 50}$

$\frac{7}{5} = 1.4$ min

13. (b) $B = \mu_0 nI$ and $n = \frac{1200}{2} = 600$

Magnetic field intensity $H = \frac{B}{\mu_0} = nI = 600 \times 2 = 1200 = 1.2 \times 10^3\ A\ m^{-1}$

14. (d) $5\beta = 5$ cm

$\Rightarrow \beta = 1$ cm

$$\frac{\lambda D}{d} = 1\text{ cm} = \frac{1}{100}\text{ m}$$

$\Rightarrow d = 600 \times 10^{-9} \times 100 \times 1$

$= 60 \times 10^{-6}$ m

$= 60\mu$m

15. (c) $\vec{B} \perp r\,\vec{E}$ and Direction of propagation is given by

$\vec{E} \times \vec{B}$

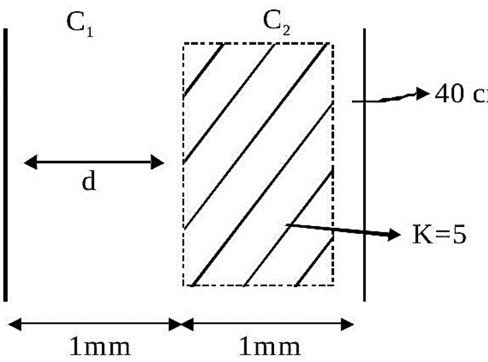
$\hat{j} \times \hat{i} = -k$

16. (b) $C_1 = \frac{\varepsilon_0 A}{d} = C_0$

$$C_2 = K\frac{\varepsilon_0 A}{d} = K\varepsilon_0$$

$$Ceq = \frac{C_1C_2}{C_1+C_2} = \frac{C_0 \times KC_0}{(K+1)\varepsilon_0} = \frac{KC_0}{K+1}$$

$$= \frac{5 \times \varepsilon_0 \times 40 \times 10^{-4}}{1 \times 10^{-3} \times 6}$$



$$= \frac{10}{3}\varepsilon_0\text{ F}$$

17. (d) Inside earth, force is given by $F = -\frac{GM_e mx}{R_e^3}$

And g_0 (on surface of earth) $= \frac{GM_e}{R_e^2}$

$$\therefore F = -\frac{g_0 m}{R_e}x$$

$$\Rightarrow a = -\frac{g_0}{R_e}x$$

$$\omega = \sqrt{\frac{g_0}{R_e}}$$

$\Rightarrow T = 2\pi\sqrt{\frac{R_e}{g_0}} = 2\pi\sqrt{\frac{6400 \times 10^3}{10}} = 2 \times 3.13 \times 8 \times 10^2 \text{sec} = 5024\text{sec} = 1.4\text{hr}$

$T = 1.4\text{hr} = 1\text{hr}24$ minutes

18. (c) $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$

$\Rightarrow \lambda \alpha \frac{1}{\sqrt{V}}$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}}$

$\Rightarrow \frac{\lambda_0}{\lambda_2} = \sqrt{\frac{40}{20}} = \sqrt{2}$

$\Rightarrow \lambda_2 = \frac{\lambda_0}{\sqrt{2}}$

19. (c)

$\eta = 1 - \frac{T_L}{T_H}$

$0.5 = 1 - \frac{T_L}{600} \Rightarrow T_L = (1 - 0.5) \times 600 \text{ K} = 300 \text{ K}$

Now $0.7 = 1 - \frac{300}{T_2}$

$\frac{300}{T_2} = 0.3 \Rightarrow T_2 = \frac{300}{0.3} = 1000 \text{ K}$

20. (b) $T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

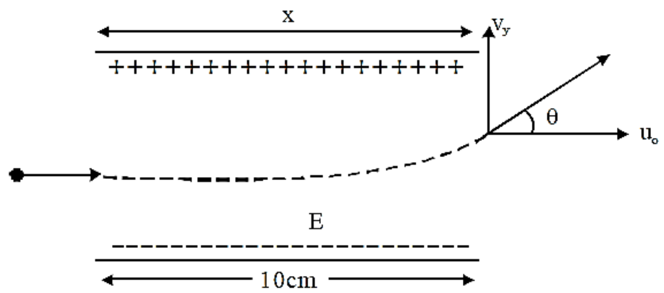
And $g_{\text{eff above earth's surface}} = \frac{GM}{(R+h)^2} = \frac{GM}{4R^2} = \frac{g_0}{4}$

Now $\frac{T_1}{T_2} = \sqrt{\frac{\frac{g_0}{4}}{g_0}} = \frac{1}{2}$

$T_2 = 2T_1$

$\therefore x = 2$

21. (45)



Force due to electric field is given by $\vec{F} = q\vec{E}$

$\therefore F = eE$

$\Rightarrow a = \frac{eE}{m}$

The electron will take a parabolic path i.e., projectile motion.

Here, $s_x = 10 \text{ cm} = 0.1 \text{ m}$

$\therefore t = \frac{0.1}{u_x} - - - - \text{(i)}$

Now $v_y = u_y + a_y t$

$\Rightarrow v_y = 0 + \frac{eE}{m} \times \frac{0.1}{u_x} - - - - \text{(ii)}$

Also $KE = \frac{1}{2}mv^2 = \frac{1}{2}mu_x^2$

$mu_x^2 = 2 \times KE = 2 \times 0.5e = e - - - \text{(iii)}$

From eq (i),(ii) and (iii), $\tan\theta = \frac{v_y}{u_x} = \frac{eE}{m} \times \frac{0.1}{u_x} \times \frac{1}{u_x} = \frac{0.1eE}{mu_x^2} = \frac{0.1eE}{e} = 0.1 \times 10 = 1$

$\Rightarrow \tan\theta = 1$

$\therefore \theta = 45^0$

22. (27) Bohr's energy is given by $E = -13.6 \times \frac{1}{n^2}$ for hydrogen atom.

And $E = \frac{hc}{\lambda}$

For 1^{st} condition, $\frac{hc}{\lambda_0} = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 13.6 \times \frac{5}{36} - - - - - \text{(i)}$

For 2^{nd} condition, $\frac{hc}{\lambda} = 13.6\left(\frac{1}{4} - \frac{1}{16}\right) = 13.6 \times \frac{3}{16} - - - - - \text{(ii)}$

Dividing equation (i) by (ii),

$\frac{\lambda}{\lambda_0} = \frac{5}{36} \times \frac{16}{3} = \frac{20}{27}$

$\Rightarrow \lambda = \frac{20}{27}\lambda_0$

$\Rightarrow n = 27$

23. (5) From graph, $\tan 45^\circ = \frac{\Delta l}{F}$

$\Rightarrow \frac{\Delta l}{F} = 1 - - - - -(i)$

Also, Young's modulus is given by $Y = \frac{Fl}{A\Delta l} = \frac{l}{A} \times \frac{F}{\Delta l} = \frac{l}{A} \times 1$

$\therefore Y = \frac{l}{A} = \frac{62.8 \times 10^{-2}}{\pi \times 4 \times 10^{-6}} = 5 \times 10^4 Nm^{-2}$

$\therefore x = 5$

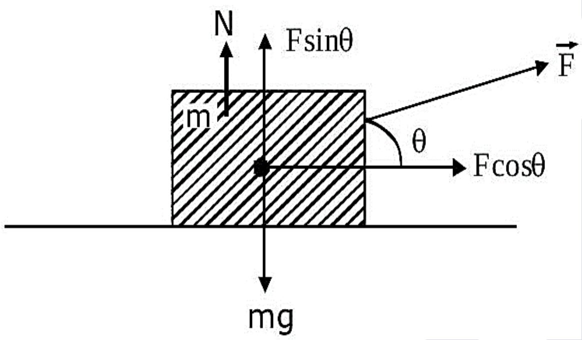
24. (17) $I_{CM} = \frac{MR^2}{2}$

$I_{AB} = \frac{MR^2}{2} + M\left(\frac{2}{3}R\right)^2 = \frac{MR^2}{2} + \frac{4MR^2}{9} = \frac{17MR^2}{18}$

As per question, $\frac{I_{AB}}{I_{CM}} = \frac{17}{9}$

$\therefore x = 17$

25. (2)



$F_x = 2\cos kx$

$F_y = 2\sin kx - mg$

According to Work Energy Theorem, $\Delta K = \Delta W$

Taking motion only along horizontal direction(X) i.e., linear motion as mentioned in question, $\Delta K = \int_0^x F_x dx$

$K_f - K_i = \int_0^x 2\cos kx dx = \frac{2\sin kx}{k}$

Hence $K_i = 0, \therefore K_f = \frac{2\sin kx}{k}$

$\therefore n = 2$

26. (100) At maximum current, there will be condition of resonance.

So, $\omega = \frac{1}{\sqrt{LC}}$

$\Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{4 \times \pi^2 \times 4 \times 10^6 \times 62.5 \times 10^{-9}} H = 0.1H = 100mH$

27. (18)

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

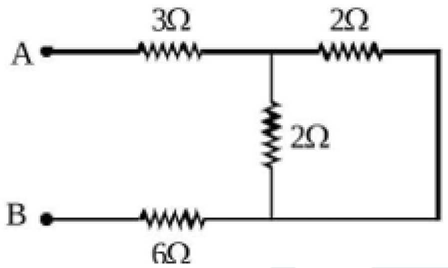
$\Rightarrow \frac{\pi}{3} = \frac{2\pi}{\lambda} \times 6$

$\Rightarrow \lambda = 36\text{ m}$

Now $v = f\lambda = 500 \times 36\text{ m/s} = 18000\text{ m/s} = 18\text{ km/s}$

28. (10) Due to short circuit, 3 resistances get vanished from the circuit.

The circuit is



$R_{eq} = 3 + 3 + 1 = 10\Omega$

29. (4)

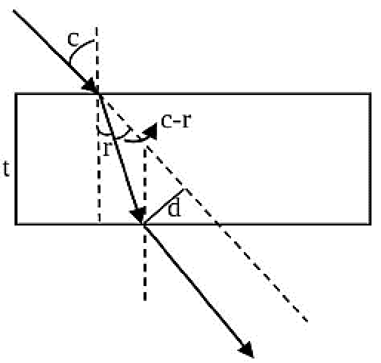
Let $\vec{C} = \vec{P} \times \vec{Q} = 3\sqrt{3}\hat{k} - 7.5\hat{j} - 4\sqrt{3}\hat{k} + 2.5\sqrt{3}\hat{i} + 8\hat{j} - 2\sqrt{3}\hat{i}$

$= \frac{1}{2}(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k})$

$|\vec{C}| = \frac{1}{2}\sqrt{3 + 1 + 12} = \frac{1}{2} \times 4 = 2$

$\therefore \hat{C} = \frac{\vec{C}}{|\vec{C}|} = \frac{1}{4}(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k})$

30. (52)



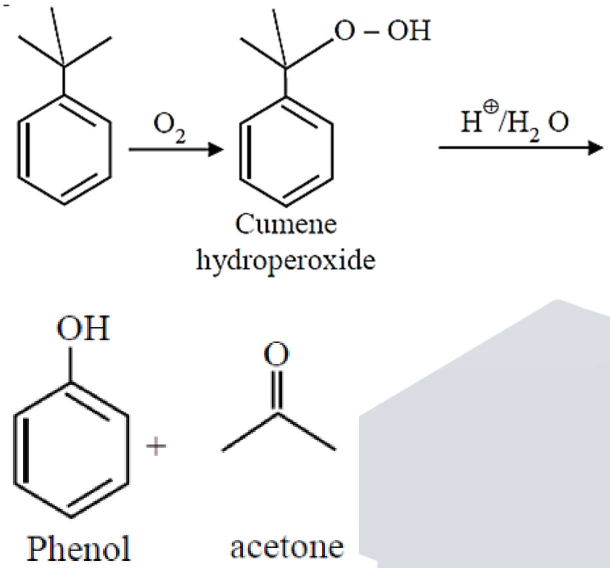
$\sin c = \frac{1}{\sqrt{2}} \Rightarrow c = 45^0$

Using Snell's law on 1st surface, $\sin c = \sqrt{2}\sin r$

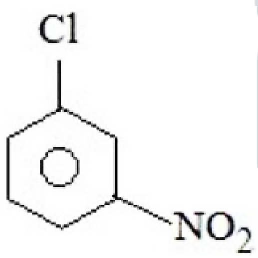
$\Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30^0$

$d = t\sec r \times \sin(c - r) = \sqrt{3} \times \frac{2}{\sqrt{3}} \times 0.26 = 0.52 \text{ cm} = 52 \times 10^{-2} \text{ cm}$

31. (c)



32. (c) Rate of nucleophilic aromatic substitution decrease by e^-withdrawing group

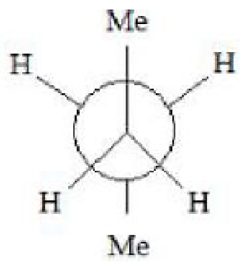


$-NO_2$ of meta shows $-I$ effect which is less dominating than $-M$

33. (b) Flame Test.

Metals	**Colour of flame test**
K	Violet
Ca	Brick Red
Sr	Crimson Red
Ba	Apple Green

34. (b)



Anti -position highly stable (bulky group maximum distance)

35. (d)

36. (a) Acetal and ketals are basically ether hence they must be stable in basic medium but should break down in acidic medium. Hence assertion is correct. Alkoxide ion (RO^-)is not considered a good leaving group hence reason must be false.

37. (d) Number of X-atom per unit cell $= 1 + 4 \times \frac{1}{8} = \frac{3}{2}$

Number of Y-atoms per unit cell $= 2 \times \frac{1}{2} = 1$

$\therefore$ Empirical formula of the solid is X_3Y_2.

38. (d)

Cations	Group No.	Group reagents
Pb^{2+}, Cu^{2+}	II	$H_2S + HCl$
Al^{3+}, Fe^{3+}	III	$NH_4Cl + NH_4OH$
Co^{2+}, Ni^{2+}	IV	$NH_4OH + H_2S$
Ba^{2+}, Ca^{2+}	V	NH_4OH, Na_2CO_3

39. (b) Antibiotic kill or inhibit the growth of microorganism

40. (b) In aqueous medium basic strength is dependent on electron density on nitrogen as well as solvation of cation formed after accepting H^+.After considering all these factors overall basic strength order is $Me_2NH > MeNH_2 > Me_3N > NH_3$

41. (b) $25VH_2O_2$ means: 1 lit of H_2O_2 on decomposition give 25 lit of $O_2(g)$ at STP.

$$2H_2O_2(\ell) \rightarrow 2H_2O(\ell) + O_2(g)$$
$$2\left[\frac{25}{22.4}\right] \text{ mole} \quad \left[\frac{25}{22.4}\right] \text{ mole}$$

Mass of $H_2O_2 = \frac{2\times25}{22.4} \times 34 = 75.89$ gram.

42. (a)

$$R = 0.529 \times \frac{n^2}{Z}$$

$$r_{Li^{2+}\ n-2} = 0.529 \times \frac{(2)^2}{3} = x$$

$$r_{Be^{3+}\ n-3} = 0.529 \times \frac{(3)^2}{4}$$

$$\frac{r_{Li^{2+}\ n-2}}{r_{Be^{3+}\ n-3}} = \frac{\frac{r_0 \times (2)^2}{3}}{\frac{r_0 \times (3)^2}{4}}$$

$$\frac{x}{r_{Be^{3+}\ n-3}} = \frac{16}{27}$$

$$\therefore\ (r_{Be^{3+}})_{n=3} = \frac{27x}{16}$$

43. (d)

$$P_4 + 8SOCl_2 \longrightarrow 4PCl_3 + 4SO_2 + 2\ S_2Cl_2$$

(A)

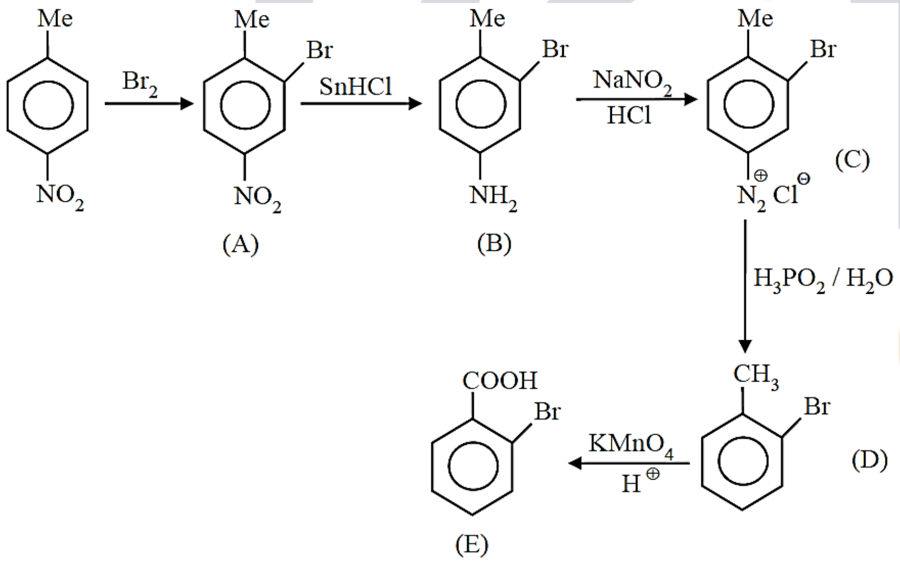
$$PCl_3 + 3H_2O \longrightarrow H_3PO_3 + 3HCl$$

(B)

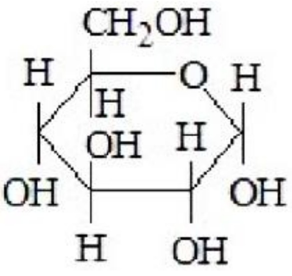
44. (b) Positive electron gain enthalpy. of inert gas is in order of

$Ne > Ar = Kr > Xe > He$

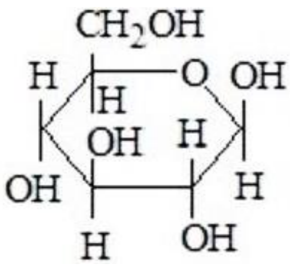
45. (c)



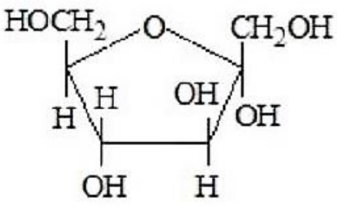
46. (d)



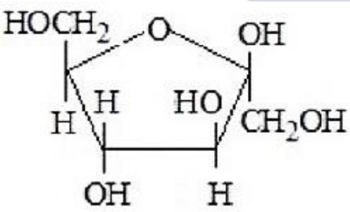
$\alpha - D - (-)$ Glucopyranose



$\beta - D - (-)$-Glucopyranose



$\alpha - \mathrm{D} - (-)$-Fructofuranose



$\beta - \mathrm{D} - (-) -$ Fructofuranose

47. (d)

$$\underset{\text{Impmily}}{CaO} + \underset{\text{Flux}}{SiO_2} \rightarrow \underset{\text{Slog}}{CaSiO_3}$$

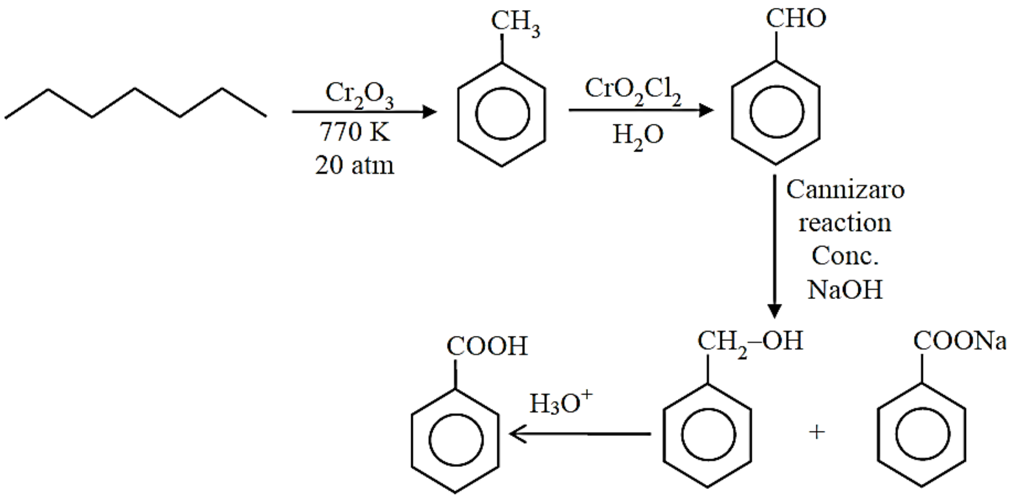
In metallurgy iron will occur not in metallurgy of Cu.

48. (b)

$$NO_2 \xrightarrow{hv} \overset{(x)}{O} + \overset{(y)}{NO}$$

$$O \xrightarrow{O_2\,(A)} O_3\,(B)$$

49. (c)



50. (d)

(A)

$Ca(OH)_2 + 2NH_4Cl \xrightarrow{\Delta} \underset{(B)}{2NH_3} + CaCl_2 + 2H_2O$

$2NH_3 \xrightarrow[\text{Excess}]{CO_2 + H_2O} \underset{(C)}{NH_4HCO_3} + NaCl \longrightarrow NH_4Cl + NaHCO_3$

51. (60)

$t_{75\%} = 2t_{1/2}$ [For 1st order reaction]

$t_{75\%} = 2 \times 30 = 60$ min.

52. (2) $(Cr^{+3} \& Ni^{+3})$

$V^{3+} =$ 3d: [↑][↑][ ][ ][ ] 4s: [ ]

$Cr^{3+} =$ 3d: [↑][↑][↑][ ][ ] 4s: [ ]

$Fe^{2+} =$ 3d: [↑↓][↑][↑][↑][↑] 4s: [ ]

$Ni^{3+} =$ 3d: [↑↓][↑↓][↑][↑][↑] 4s: [ ]

53. (42) Organic compound $\rightarrow BaSO_4$

Weight $= 0.417$ g Weight $= 1.44$ g

Moles $BaSO_4 = \frac{1.44}{233} =$ moles of Sulphur

Weight Sulphur $= \frac{1.44}{233} \times 32$ gram

$$\% \text{ S} = \frac{\text{weight of sulphur}}{\text{weight of organic}} \times 100$$

$$\Rightarrow \frac{1.44 \times 32}{233 \times 0.471} \times 100$$

$$\Rightarrow \frac{46.08}{109.743} \times 100$$

$$\Rightarrow 41.98 \simeq 42$$

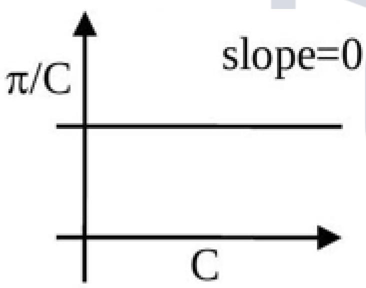
54. (41500)

$$\pi = \text{M}'\text{RT} = \left(\frac{\text{W/M}}{\text{V}}\right)\text{RT}$$

$$\Rightarrow \pi = \left(\frac{\text{W}}{\text{V}}\right)\left(\frac{1}{\text{M}}\right)\text{RT} = \text{C}\left(\frac{\text{RT}}{\text{M}}\right)$$

$$\Rightarrow \frac{\pi}{\text{C}} = \frac{\text{RT}}{\text{M}} \neq \text{f(c)}$$

If we assume graph between $\frac{\pi}{\text{C}}$ and C



Assuming π vs C graph

$$\text{Slope} = \frac{\text{RT}}{\text{M}} = \frac{0.083 \times 300}{\text{M}} = 6 \times 10^{-4}$$

$$\therefore \text{M} = \frac{0.083 \times 300}{6 \times 10^{-4}} = \frac{830 \times 300}{6}$$

$$= 41{,}500$$

55. (12) Molarity of acid $= \frac{1.2 \times 10^3}{24.2} = \frac{1000}{20} = 50\text{M}$

Neutralization reaction:

$$\text{HA} + \text{NaOH} \rightarrow \text{NaA} + \text{H}_2\text{O}$$

$$\text{M}_1\ \text{V}_1 = \text{M}_2\ \text{V}_2$$

$$[50] \times \text{V} = [0.24 \times 25]$$

$$\text{V} = 00.12\text{ml}$$

56. (360)

$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O(\ell)$

$n = \dfrac{100}{180}$

Energy needed to perspire water $= 1800 \times \frac{1}{2}$

Moles of water evaporated $= \frac{900}{45} = 20$ moles

Weight of water evaporated $\Rightarrow 20 \times 18$

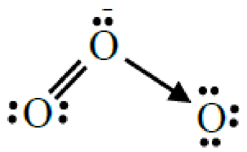
$\Rightarrow 360$ gram

57. (4)
58. (10)

$$\text{Anode } \Rightarrow \frac{1}{2}H_2(g) \rightarrow H^+(aq) + e^-$$
$$\text{Cathode } \Rightarrow Fe^{3+} + e^- \rightarrow Fe^{2+}$$
$$\text{Overall } \frac{1}{2}H_2 + Fe^{3+} \rightarrow H^+ + Fe^{2+}$$
$$E_{cell} = E^0_{cell} - \frac{0.059}{1}\log\frac{[Fe^{2+}]}{[Fe^{3+}]} \times \frac{[H^+]}{[P_{H_2}]^{\frac{1}{2}}}$$
$$0.712 = 0.771 - 0.059\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$
$$\log\frac{[Fe^{2+}]}{[Fe^{3+}]} = 1$$
$$\text{So } \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$

59. (6)

Not 1.p. e^-in O_3 is $= 6$



60. (9)
61. (d) Only possibilities is $\alpha = 0, \beta = 1$

Equation of circle

$(x-0)(x-1) + (y-0)(y-1) = 0$

$x^2 + y^2 - x - y = 0$

Image of circle in live $x + y + 2 = 0$

$x^2 + y^2 + 5x + 5y + 12 = 0$

62. (d)

$$\left.\begin{aligned} y^2 &= \frac{x}{2} \\ y^2 &= x - 1 \end{aligned}\right]$$

Tangent to $y^2 = \frac{x}{2}$ is $y = mx + \frac{1}{8m}$

$$y^2 = x - 1 \text{ is } y = m(x-1) + \frac{1}{4m}$$
$$y = mx - m + \frac{1}{4m}$$

(a) & (b)

$$\frac{1}{8m} = -m + \frac{1}{4m}$$
$$m = \frac{1}{4m} - \frac{1}{8m}$$
$$m = \frac{1}{8m} \Rightarrow m^2 = \frac{1}{8} \Rightarrow m = \frac{1}{2\sqrt{2}} (m > 0)$$

From (a)

$$y = \frac{1}{2\sqrt{2}}x + \frac{1}{2\sqrt{2}}$$

distance from $(6, -2\sqrt{2})$

$$\frac{\left|\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{1}{2\sqrt{2}}\right|}{\sqrt{1 + \frac{1}{8}}} = \frac{6 + 8 + 1}{3} = \frac{15}{3} = 5$$

63. (b)

$$(\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c} = \frac{\bar{b}}{2} - \frac{\bar{c}}{2}$$

$$\bar{a} \cdot \bar{c} = \frac{1}{2}, \bar{a} \cdot \bar{b} = \frac{1}{2}$$

$$\bar{b}.\bar{d} = \frac{1}{2}$$

$$\begin{aligned}(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d}) &= \bar{a} \cdot [\bar{b} \times (\bar{c} \times \bar{d})] \\ &= \bar{a} \cdot [(\bar{b} \cdot \bar{d})\bar{c} - (\bar{b} \cdot \bar{c})\bar{d}] \\ &= \bar{a} \cdot [\bar{c}/2] \\ &= \frac{1}{2}(\bar{a} \cdot \bar{c}) \\ &= \frac{1}{4}\end{aligned}$$

64. (c) $\bar{b} = \lambda\bar{a} + \mu\hat{j}$

$= \lambda(-\hat{i} + 2\hat{j} + \hat{k}) + \mu\hat{j}$

$\bar{b} = -\lambda\hat{i} + (2\lambda + \mu)\hat{j} + \lambda\hat{k}$

$|\bar{a}| = |\bar{b}|$

$|\bar{a}|^2 = |\bar{b}|^2 \;\Rightarrow 6 = \lambda^2 + (2\lambda + \mu)^2 + \lambda^2$

$\because \bar{a}\cdot\bar{b} = 0 \;\Rightarrow \lambda + 2(2\lambda + \mu) + (a)(\lambda) = 0$

$\Rightarrow 6\lambda + 2\mu = 0$

$\Rightarrow \mu = -3\lambda$

from (a) & (b)

$3\lambda^2 = 6$
$\lambda^2 = 2 \quad \Rightarrow \lambda = \pm\sqrt{2}$
$\Rightarrow \mu = \pm 3\sqrt{2}$

Projection of $3\bar{a} + 2\bar{b}$ on $\bar{c}$ is $= \frac{(3\bar{a}+\sqrt{2}\bar{b})\cdot\bar{c}}{|\bar{c}|}$

$$= \frac{3\bar{a}\cdot\bar{c} + \sqrt{2}\bar{b}\cdot\bar{c}}{|\bar{c}|}$$

$$= \frac{18 + \sqrt{2}(-6\sqrt{2})}{\sqrt{50}}$$

$$= \frac{6}{\sqrt{50}} = \frac{6}{5\sqrt{2}} = \frac{3\sqrt{2}}{5}$$

Case I :

$(\bar{a}\cdot\bar{c} = -5 + 8 + 3 = 6)$

$$\frac{18 + \sqrt{2}(-6\sqrt{2})}{\sqrt{50}}$$

$\lambda = \sqrt{2}\;\bar{b} = -\sqrt{2}\hat{i} + 12\sqrt{2} - 3\sqrt{2}\hat{j} + \sqrt{2}\hat{k}$

$\bar{b} = -\sqrt{2}\hat{i} - \sqrt{2}\hat{j} + \sqrt{2}\hat{k}$

$\bar{b}\cdot\bar{c} = -5\sqrt{2} - 4\sqrt{2} + 3\sqrt{2}$

$= -6\sqrt{2}$

Case II : $\left.\begin{array}{l}\lambda = -\sqrt{2}\\ \mu = 3\sqrt{2}\end{array}\right]\; \bar{b} = \sqrt{2}\hat{i} + (\sqrt{2})\hat{j} + (-\sqrt{2})\hat{k}$

$$= \frac{18 + \sqrt{2}(6\sqrt{2})}{\sqrt{50}}$$

$$= \frac{30}{\sqrt{50}} = \frac{30}{5\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2} \text{ Ans.}$$

65. (d) Let

$z = x + iy$
$z - z_1 = (x-2) + i(y-3)$
$|z - z_1|^2 = (x-2)^2 + (y-3)^2$
$z - z_2 = (x-3) + i(y-4)$
$|z - z_2|^2 = (x-3)^2 + (y-4)^2$
$((x-2)^2 + (y-3)^2) - ((x-3)^2 + (y-4)^2) = 2$
$\Rightarrow 2x + 2y = 14$
$= x + y = 7$
straight line with sum of intercept on C.A $= 14$

66. (a)

Let Number of observations is $= n$

$$\frac{\sum x_i}{n} = 10 \quad \frac{\sum x_i - 8 + 12}{n} = 10.2$$

$$\sum x_i = (10.2)n - 4$$

$\sum x_i = 10n$

$10n = (10.2)n - 4$

$\Rightarrow (.2)n = 4 \Rightarrow n = 20$

Given $\frac{\sum x_i^2}{20} - (10)^2 = 4 \Rightarrow \sum x_i^2 = 2080$

After Change

$$\sum x_i^2 = 2080 - 8^2 + (12)^2$$
$$= 2160$$

$$\text{New vanance} = \frac{\sum x_i{}^2}{20} - (\bar{x})^2$$
$$= \frac{2160}{20} - (10.2)^2$$
$$= 108 - (10.2)^2$$
$$= 3.96$$

67. (d)

$$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$
$\Delta = a(15a^2 + 31a + 36) = 0$
$a = 0$

$\Delta \neq 0$ for all $a \in \mathbb{R} - \{0\}$

$\therefore S_1 = \mathbb{R} - \{0\}, S_2 = \Phi$

68. (a)

$$\lim_{n\to\infty} \frac{(1 + 2 + 4 + 5 + \cdots + (3n-2) + (3n-1) - 3 + 6 + \cdots + 3n)}{\sqrt{2n^4 + 4n + 3} - \sqrt{n^4 + 5n + 4}}$$

Let $N^r = \sum_{n=1}^{\infty} (3n-2) + (3n-1) - 3n$

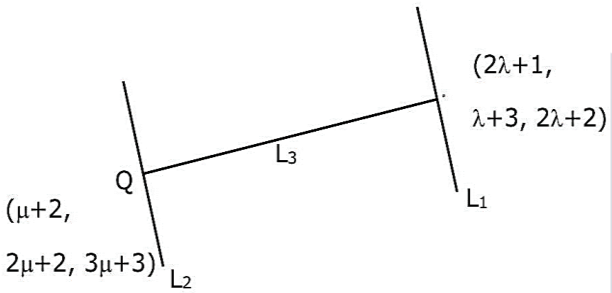
$$= \sum_{n=1}^{\infty} (3n-3)$$

$$= \frac{3n(n+1)}{2} - 3n = \frac{3}{2}(n^2 - n)$$

$$\frac{3}{2} \lim_{n\to\infty} \frac{n^2\left(1-\frac{1}{n}\right)}{n^2\left(\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}} - \sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}\right)}$$

$$= \frac{3}{2(\sqrt{2}-1)} \text{ or } \frac{3}{2}(\sqrt{2}+1) \text{ Ans.}$$

69. (a)
70. (d)



D.R's of PQ are $= (2\lambda - \mu - 1, \lambda - 2\mu + 1, 2\lambda - 3\mu - 1)$

given D.R's are $= (1, -1, -2)$

$$\frac{2\lambda - \mu - 1}{1} = \frac{\lambda - 2\mu + 1}{-1} = \frac{2\lambda - 3\mu - 1}{-2}$$

$\lambda = \mu = 3$

$P = (7,6,8)$

$Q = (5,8,12)$

$PQ = 2\sqrt{6}$

71. (d) Let $x^2 = t$

$2xdx = dt$

$$f(x) = \int \frac{dt}{(t+1)(t+3)}$$

$$= \frac{1}{2}\int \left(\frac{1}{t+1} - \frac{1}{t+3}\right) dt$$

$$= \frac{1}{2}\ln\left|\frac{t+1}{t+3}\right| + C$$

$$f(x)=\frac{1}{2}\ln\left|\frac{x^2+1}{x^2+3}\right|+C$$

$x=3$

$$\frac{1}{2}\ln\left(\frac{5}{6}\right)=\frac{1}{2}\ln\left(\frac{5}{6}\right)+C\Rightarrow C=0$$

$$f(x)=\frac{1}{2}\ln\left(\frac{x^2+1}{x^2+3}\right)$$

$$f(x)=\frac{1}{2}\ln\left(\frac{17}{19}\right)$$

$$=\frac{1}{2}[\ln 17-\ln 19]$$

72. (b)

Case I

$$f(x)-\int_0^2 e^{-(x-4)}dt$$
$$-e^{-x}\int_0^2 e^t dt-e^{-x}(e^2-1)$$

Case II

$$(0<x<2)\quad f(x)-\int_0^x e^{x-t}dt+\int_x^2 e^{-(x-t)}dt$$
$$-e^x(-e^{-t})_0^x+e^{-x}[e^t]_x^2$$
$$-e^x[-e^{-x}+1]+e^{-x}[e^2-e^x]$$
$$=-1+e^x+e^{2-x}-1$$

Case III

$$x\geq 2\quad f(x)-\int_0^2 e^{(x-t)}dt$$
$$-e^x[-e^{-4}]_0^2$$
$$-e^x[-e^{-2}+1]$$
$$-e^x(1-e^{-2})$$

$$f(x)-\begin{bmatrix} e^{-x}(e^2-1), & x\leq 0\rightarrow & (e^2-1)\\ e^x+e^{2-x}-2, & 0\leq x\leq 2\rightarrow & 2(e-1)\\ ex(1-e^{-2}), & x\geq 2\rightarrow & (e^2-1)\end{bmatrix}$$

Minimum value $=2(e-1)$

73. (d) Let $a,b\rightarrow 2$ positive number

$$\frac{a+b}{2}\geq\sqrt{ab}$$
$$\sqrt{ab}\leq 33$$
$$ab\leq(33)^2\ M=(33)^2$$

$x(66-x) \geq \frac{5}{9}(33)^2$

$66x - x^2 \geq 605$

$0 \geq x^2 - 66x + 605$

$(x-11)(x-55) \leq 0$

$x \in [11,55]$

$A = \{12,15,18 \ldots 54\}$

Total number in $A = 15$

$P(A) = \frac{15}{45} = \frac{1}{3}$ Ans.

74. (c)

$$\begin{aligned} f'(x) &= 8x^3 - 36x + 8 \\ &= 4[2x^3 - 9x + 2] \\ &= 4[(x-2)(2x^2+4x-1)] \\ &= 4\left[(x-2)\left(x-\left(-\frac{2-\sqrt{6}}{2}\right)\left(x-\left(\frac{-2+\sqrt{6}}{2}\right)\right)\right)\right] \end{aligned}$$

$$\frac{-\ +}{\frac{-2-\sqrt{6}}{2}} \quad \frac{+2+\sqrt{6}}{2} \quad \overset{2}{\underset{\text{maximum}}{\bigcup}}$$

$$M = 2\left(\frac{-2+\sqrt{6}}{2}\right)^4 - 18\left(\frac{-2+\sqrt{6}}{2}\right)^2 + 8\left(\frac{-2+\sqrt{6}}{2}\right) + 12$$

$$= 12\sqrt{6} - \frac{33}{2}$$

75. (1)

$$f(x) = \frac{1}{1-e^{-x}}$$

$$\begin{aligned} g(x) &= (f(-x) - f(x)) \\ &= \frac{1}{1-e^{x}} - \frac{1}{1-e^{-x}} \\ &= \frac{1}{1-e^{x}} - \frac{e^x}{e^x - 1} \end{aligned}$$

$$g(x) = \frac{1+e^x}{1-e^x}$$

$$g'(x) = \frac{(1-e^x)(e^x) - (1+e^x)(-e^x)}{(1-e^x)^2}$$

$$= \frac{e^x - e^{2x} + e^x + e^{2x}}{(1-e^x)^2}$$

$$g'(x) = \frac{2e^x}{(1-e^x)^2}$$

$g'(x) > 0 \Rightarrow g(x) \uparrow$

$g(x)$ is one-one

76. (d) $f(x) = y = \frac{(1-x)(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})}{(1-x)}$

$$f(x) = y = \frac{(1-x^{32})}{1-x} \Rightarrow f(-1) = 0$$

$(1-x)y = 1 - x^{32}$

differentiate both side

$(1-x)y' + y(-1) = -32x^{31}\ x = -1 \Rightarrow y' = 16$

differentiate both side

$(1-x)y' + y'(-1) - y' = -(32)(31)x30$

Put $x = -1$

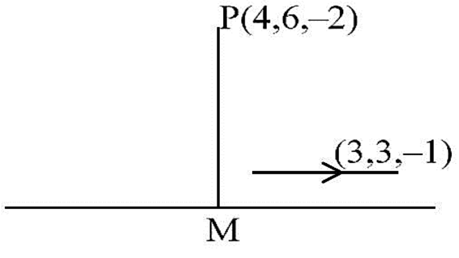
$2y'' - 2y' = -(32)(31)$

$y'' - y' = -(16)(31)$

$y' - y'' = 496$

77. (a) equation of line

$\bar{r} = (-3,2,3) + \lambda(3,3,-1)$
$\overline{PM} \cdot (3,3,-1) = 0$
$(3\lambda - 7, 3\lambda - 4, 5 - \lambda) \cdot (3,3,-1) = 0$
$\Rightarrow 3(3\lambda - 7) + 3(3\lambda - 4) - 1(15 - \lambda) = 0$
$\Rightarrow 19\lambda = 38 \Rightarrow \lambda = 2$
$M = (3,8,1)$
$PM = \sqrt{1+4+9} = \sqrt{14}$



$(-3+3\lambda, 2+3\lambda, 3-\lambda)$

78. (a) $|\operatorname{adj}(\operatorname{adj} A^2| = |A^2|^{(3-1)^2} = |A|^8$

$$|A| = \begin{vmatrix} 1 & \frac{\ln y}{\ln} & \frac{\ln z}{\ln x} \\ \frac{\ln x}{\ln y} & 2 & \frac{\ln z}{\ln y} \\ \frac{\ln x}{\ln z} & \frac{\ln y}{\ln z} & 3 \end{vmatrix}$$

$$= \frac{1}{\ln x \ln y \ln z} \begin{vmatrix} \ln x & \ln y & \ln z \\ \ln x & 2\ln y & \ln z \\ \ln x & \ln y & 3\ln z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$|A| = 2$

$\therefore |\text{adj}(\text{adj}\, A)| = 2^8$ Ans.

79. (d)

$$\sum_{r=1}^{10} r^3 \left[\frac{a_r}{a_{r-1}}\right]^2$$

$$\because \frac{a_r}{a_{r-1}} = \frac{10-r+1}{r} = \frac{11-r}{r}$$

$\because (1+x)^{10} \Rightarrow {}^{10}C_r n^r$

$\Rightarrow {}^{10}C_{10-r} n^{10-r}$

$= {}^{10}C_{10-r}$ or $= {}^{10}C_r x^{10-r}$

$a_r = {}^{10}C_r$

$$\sum_{r=1}^{10} r^3 \left[\frac{11-r}{r}\right]^2$$

$$\sum_{r=1}^{10} r(11-r)^2$$

$$\sum_{r=1}^{10} [r^3 - 22r^2 + 121r]$$

$$= \left(\frac{(10)(11)}{2}\right)^2 - 22\left(\frac{(10)(11)(21)}{6}\right) + \left(\frac{(10)(11)}{2}\right)(121)$$

$= 1210$ Ans.

80. (d)

$$\frac{dy}{dx} - \frac{y}{x}[1 + xy^2(1 + \ln x)]$$

$$\frac{dy}{dx} - \frac{y}{x} - y^3(1 + \ln x)$$

$$\frac{1}{y^3}\frac{dy}{dx} - \frac{1}{x} - \frac{1}{y^2} - 1 + \ln x$$

$$-\frac{1}{y^2} - t \Rightarrow \frac{2}{y^3}\frac{dy}{dx} - \frac{dt}{dx}$$

From (1)

$$\frac{1}{2}\frac{dy}{dx} + t\left(\frac{1}{x}\right) - 1 + \ln x$$
$$\frac{dy}{dx} + t\left(\frac{2}{x}\right) - 2(1 + \ln x)$$
$$IF = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$$
$$t(x^2) = \int \ [2(1 + \ln x) - x^2]dx$$
$$\Rightarrow t \cdot x^2 - \frac{2x^3}{3} + 2\int \ x^2 \ln x dx$$
$$\Rightarrow \frac{-x^2}{y^2} - \frac{2x^3}{3} + 2\left[\ln x \cdot \frac{x^3}{3} - \frac{x^2}{9}\right] + C$$
$$x = 1, y = 3 \ \Rightarrow c - \frac{-5}{9}$$
$$\frac{-x^2}{y^2} - \frac{2x^3}{3} + 2\left[\ln x \cdot \frac{x^3}{3} - \frac{x^3}{9}\right]\frac{-5}{9}$$

81. (1080)

General term $= \frac{5!}{r_1!r_2!r_3!}(2x)^{r_1}(1/x)^{r_2}(3x^2)^{r_3}$

$$= \frac{5!}{r_1!\,r_2!\,r_3!}2^{r_1} \cdot 3^{r_3}\left[x^{r_1 - 7^{r_2} + 2^{r_3}}\right]$$
$$\left.\begin{array}{r} r_1 - 7r_2 + 2r_3 = 0 \\ r_1 + r_2 + r_3 = 5 \end{array}\right]$$

$r_1 = 1, r_2 = 1, r_3 = 3$

Constant term $= 1080$

82. (2039)

Let $f(g(x)) = h(x)$

$f(g(x)) = 2x^3 + 7$

$a(x^b + c) - 3 = 2x^3 + 7$

$a = 2, b = 3, ac = 10$

$c = 5$

$g(f(x))(c) = 32$

$f(g(10)) = 2007$

Sum $= 2039$

83. (43) No. of element $1 = \{3\}$

No. of element $2 = \{(3\text{ K} + 1), (3k + 2)\}$

$(c)\ (c) = 9$

No. of element 3 $= \{3k, 3k+1, 3K+2\} = (a)(c)(c) = 9$

$= \{(3k+1), (3k+1), (3k+1)\} = 1$

$= \{(3K+2), (3k+2), (3k+2)\} = \frac{1}{11}$

No. of element 4 $= \{3k, 3k+1, 3k+1, 3k+1\} \rightarrow 1$

$= \{3k, 3k+2, 3k+2, 3k+2\} \rightarrow 1$
$= (3k+1, 3k+2, 3k+2, 3k+1\} \rightarrow {}^3C_2 \times {}^3C_2 = 9$

No. of element 5 $= 9$, no. of element 6 $= 1$, no. of element 7 $= 1$

Total $= 43$.

84. (9) Equation of plane is

$(x-2y-z-5) + b(x+y+3z-5) = 0$
$$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow b = 12$

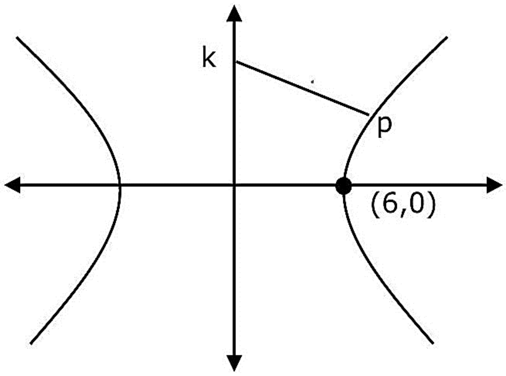
Plane is $13x + 10y + 35z = 65$

Distance From given point is $= 9$

85. (2)

$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1} x$
$x \in (0,1)\ \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1} x$
$x \in (-1,0)\ \cot^{-1}\left(\frac{1-x^2}{2x}\right) = IT + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = IT + 2\tan^{-1} x$
$x \in (0,1)\ 2\tan^{-1} x + 2\tan^{-1} x = \pi/3$
$\tan^{-1} x = \frac{\pi}{12}$
$x = 2 - \sqrt{3}$
$x \in (-1,0)\ 2\tan^{-1} x + IT + 2\tan^{-1} x = \pi/3$
$4\tan^{-1} x = \frac{-2\pi}{3}$
$\tan^{-1} x = -\pi/6$
$x = -1/\sqrt{3}$
$(2-\sqrt{3}) + \left(-\frac{1}{\sqrt{3}}\right) = \alpha - \frac{4}{\sqrt{3}}$
$2 - 4/\sqrt{3} = \alpha - 4/\sqrt{3}$
$\alpha = 2$

86. (216)



$H: \frac{x^2}{36} - \frac{y^2}{9} = 1$

Equation of normal is

$6x\cos\theta + 3y\cot\theta = 45$

$M = -2\sin\theta = -\sqrt{2}$
$\theta = \pi/4$

Equation of normal is

$\sqrt{2}x + y = 15$

$P(a\sec\theta, b\tan\theta)$
$P(6\sqrt{2}, 3), k(0,15)$
$d^2 = 216$

87. (120)

$x + y = 5\lambda$

X	y	No. of ways
5λ	5λ	20
$5\lambda+1$	$5\lambda+4$	25
$5\lambda+2$	$5\lambda+3$	25
$5\lambda+3$	$5\lambda+2$	25
$5\lambda+4$	$5\lambda+1$	25
		1200

Total Ways = 120

88. (25)

$$\log_2\left[\frac{9^{2\alpha-4}+13}{3^{2\alpha-4}\cdot\frac{5}{2}+1}\right]=2$$

$$=\frac{9^{2\alpha-4}+13}{3^{2\alpha-4}\cdot\frac{5}{2}+1}=4$$

$= 9^{2\alpha-4}+13=10\cdot 3^{2\alpha-4}+4$

$t^2-10t+9=0$

$t=1,9$

$3^{2\alpha-4}=3^0,3^2$

$2\alpha-4=0,2$

$=\alpha=2,3$

$x^2-2(25)x+25\beta=0$

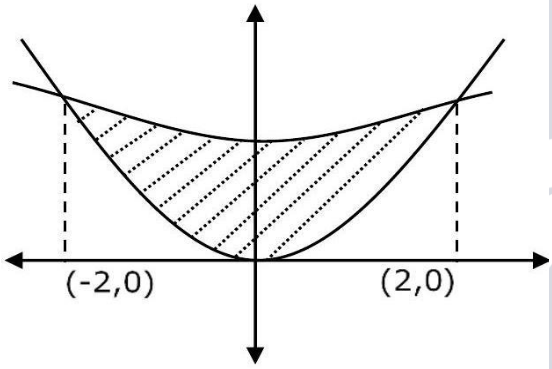
$D\geq 0$

$(2)^2(25)^2-4(25)(\beta)\geq 0$

$\beta\leq 25$

$\beta_{max}=25$

89. (600)



$y=\frac{5x^2}{2}$, $y=x^2+6$

$\frac{5x^2}{2}=x^2+6$

$3x^2=12\Rightarrow x^2=4$

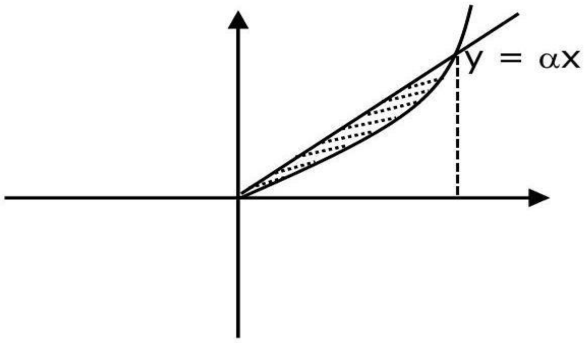
$x=\pm 2$

$$A_1=2\int_0^2\left(x^2+6-\frac{5}{2}x^2\right)dx$$

$$=2\int_0^2\left(6-\frac{3x^2}{2}\right)dx$$

$=2[6x-x^3/2]_0^2=2[12-4]$

$=16$



$y=\frac{5}{2}x^2, y=\alpha x(\alpha>0)$

area $=\frac{8}{3}[a^2\ m^3]$

$=\frac{8}{3}[1/10]^2\cdot\alpha^3$

$=\frac{8}{300}-\alpha^3=\frac{2}{75}\alpha^3$

$\because \frac{2}{75}-\alpha^3=16 \Rightarrow \alpha^3=8\times 75$

$\alpha^3=600$

90. (495)

$$\begin{vmatrix} A+6d & 7 & 1 \\ 21(A+1+8d) & 17 & 1 \\ A+2+16d & 17 & 1 \end{vmatrix}+70=0$$

$A=-7, d=6$

$\therefore c-a-b=20$

$\therefore 5_{20}=495$