**1.** **(c)** (degree of freedom)

$\Rightarrow f = 3 + 2 + 2 = 7$

$C_V = \frac{fR}{2} = \frac{7R}{2}$

**2.** **(d)** $e = B\vartheta l$

$e = 2 \times 8 \times 1$

$e = 16$ volt

**3.** **(a)**

$\underset{(nm)}{\lambda} = \frac{hc}{\Delta E} = \frac{1241}{\Delta E(ev)} = \frac{1241}{10} = 124.1$

**4.** **(c)** Both statement I and statement II are correct

**5.** **(a)** $v = \frac{dx}{dt} = 8t$

$v = 8 \times 5$

$v = 40$ m/s

**6.** **(d)**

$[Y] = \frac{F}{A} \cdot \frac{\Delta L}{L} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

$F = 6\pi\eta rv$

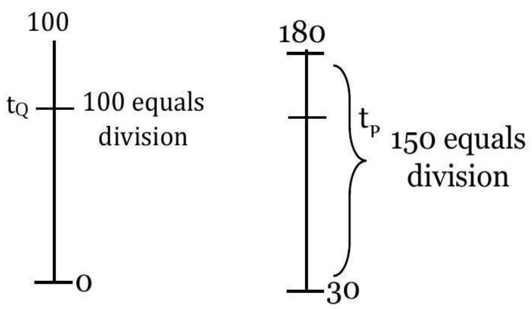
$[\eta] = \frac{F}{6\pi rv} = \frac{MLT^{-2}}{LLT^{-1}}$

$[\eta] = ML^{-1}T^{-1}$

$[h] = \frac{E}{f} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$

Work function $(\varphi) = ML^2T^{-2}$

**7.** **(a)**
**8.** **(b)**
**9.** **(d)**

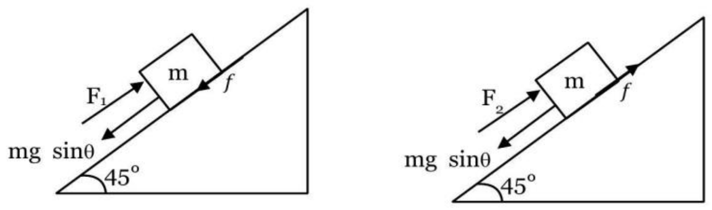



$$\frac{t_P - 30}{180 - 30} = \frac{t_Q - 0}{100 - 0}$$

$$\frac{t_P - 30}{150} = \frac{t_Q}{100}$$

$$\frac{t_Q}{100} = \frac{t_P - 30}{150}$$

**10. (d)**




$F_1 = mg\sin\theta + \mu mg\cos\theta$

$F_1 = mg\sin 45 + \mu mg\cos 45$

$F_2 = mg\sin 45 - \mu mg\cos 45$

$F_1 = 2\,F_2$

$$mg\left(\frac{1}{\sqrt{2}} + \frac{\mu}{\sqrt{2}}\right) = 2mg\left(\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right)$$

$1 + \mu = 2 - 2\mu$

$3\mu = 1$

$\mu = \frac{1}{3} = 0.33$

**11. (c)** By Newton's law $F = \frac{Gm_1\,m_2}{r^2}$

By kepler's law $T^2 \alpha a^3$

**12. (a)** $\theta = \frac{NBA}{C} I$

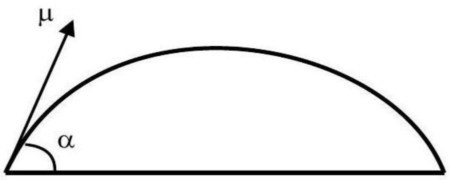
$$A = \frac{C\theta}{IBN}$$

$$= \frac{4 \times 10^{-5} \times .05}{10 \times 10^{-3} \times 0.01 \times 200}$$

$A = 10^{-4}\ m^2$

$= 1\ cm^2$

**13. (a)**
**14. (c)**



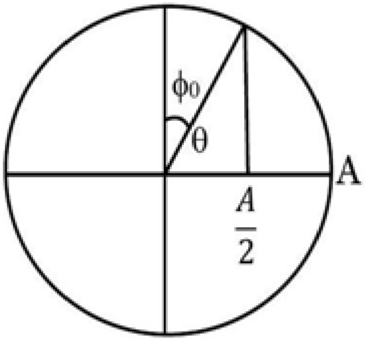

$$R_1 = \frac{u^2 \sin 2\alpha}{g}$$

$$R_2 = \frac{u^2 \sin 2\beta}{g} = \frac{u^2 \sin 2(90 - \alpha)}{g}$$

$$R_2 = \frac{u^2 \sin 2\alpha}{g} = R_1$$

$R_1 : R_2 = 1 : 1$

**15. (a)**




$$\cos\theta = \frac{A}{2 \times A} = \frac{1}{2} = \cos 60^\circ$$

$$\theta = 60 = \frac{\pi}{3}$$

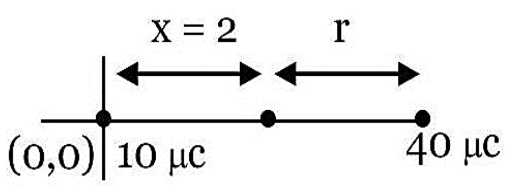
$$\phi_0 = 30 = \frac{\pi}{6}$$

$$o \to \frac{A}{2}, t = \frac{\frac{\pi}{6}}{\frac{2\pi}{T}} = \frac{T}{12} = 2$$

$T = 24$

$$\frac{A}{2} \to A, t = \frac{\pi/3}{2\pi/T} = \frac{T}{6} = \frac{24}{6} = 4\text{sec}$$

**16. (b)**
**17. (d)**
**18. (b)**




$E_1 = E_2$

$$\frac{K \times 10}{(2)^2} = \frac{K \times 40}{4^2}$$

$r = 4$ cm

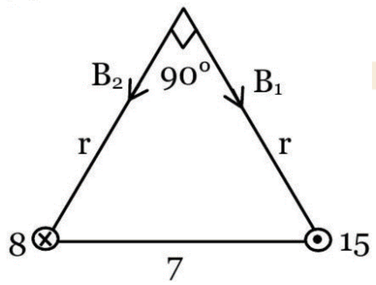
Distance from origin $= 2 + 4 = 6$ cm

**19. (b)**
$$\begin{aligned} R_{new} &= n^2 R \\ &= (5)^2 \times 5 \\ &= 125 \end{aligned}$$

**20. (c)**

$$\begin{aligned} h &= 2Re \\ \Delta U &= U_B - U_A^h \\ &= \frac{-GM_e m}{(R_e + h)} - \left(\frac{-GM_e m}{R_e}\right) \\ &= \frac{-GM_e m}{R_e + 2R_e} + \frac{GM_e m}{R_e} = \frac{2}{3}\frac{GM_e m}{R_e} \\ &= \frac{2}{3}\frac{GM_e m}{R_e^2} R_e \\ \Delta U &== \frac{2}{3} mgR_e \end{aligned}$$

**21. (60)**




$$r = \frac{7}{\sqrt{2}} \text{ cm}$$

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0 I_1}{2\pi r}\right)^2 + \left(\frac{\mu_0 I_2}{2\pi r}\right)^2}$$

$$= \frac{\mu_0}{2\pi r}\sqrt{8^2 + 15^2}$$

$$= \frac{4\pi \times 10^{-7} \times 17}{2\pi \times \frac{7}{\sqrt{2}} \times 10^{-2}} = 68 \times 10^{-6}$$

$= 68$

**22. (1)**

$U_I = T4\pi R^2 = T4\pi(1or)^2 = 100 \times T \times 4\pi r^2$

$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = 10r$

$\frac{u_f}{u_i} = \frac{1000 \times T \times 4\pi r^2}{100 \times T \times 4\pi r^2} = 10$

$\therefore x = 1$

**23. (2)**

$o = m_1 3v - m_2 2v$

$\frac{m_1}{m_2} = \frac{2}{3}$

$\frac{8v_1}{8v_2} = \frac{2}{3}$

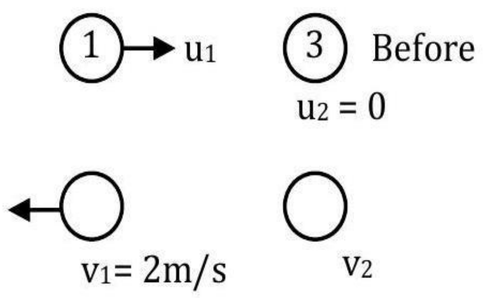
$\frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \frac{2}{3} = \frac{R_1}{R_2} = \left(\frac{2}{3}\right)^{\frac{1}{3}}$

$\therefore x = 2$

**24. (400)**

$f = \left(\frac{v \pm v_0}{v \pm v_s}\right) f_0 = \frac{330 \times 320}{330 - 66} = \frac{330 \times 320}{264} = 400$

**25. (4.00)**




$p_i = p_f$

$u_1 + 0 = -1 \times 2 + 3v_2$

$u_1 = 3v_2 - 2$

$e = 1 = \frac{v_2 - (-2)}{u_1 - 0}$

$v_2 = u_1 - 2$

$u_1 = 3(u_1 - 2) - 2$

$2u_1 = 8, u_1 = 4$

**26. (8)** $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2+(X_C-X_L)^2}}$

$= \frac{80}{\sqrt{(80)^2+(130-70)^2}} = \frac{80}{\sqrt{(80)^2+(60)^2}}$

$\cos\phi = \frac{80}{100} = \frac{8}{10}$

$x = 8$

**27. (5)**

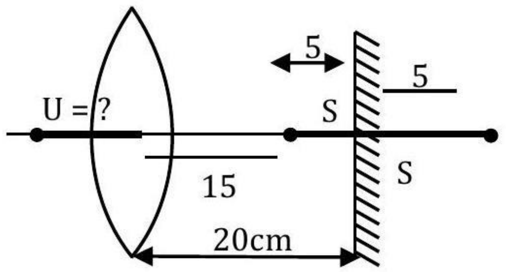
$I_{ss} = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2 = \frac{7}{5}\times 5\times R^2 = 7R^2$

$I_{Disc} = \frac{mR^2}{4} + mR^2 = \frac{5mR^2}{4} = \frac{5}{4}\times 4\times R^2 = 5R^2$

$\frac{I_{Disc}}{I_{SS}} = \frac{5R^2}{7R^2} = \frac{5}{7}$

$x = 5$

**28. (30)**




$\therefore$ for lens $v = 20 - 5 = 15$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{15} - \frac{1}{u} = \frac{1}{10}$

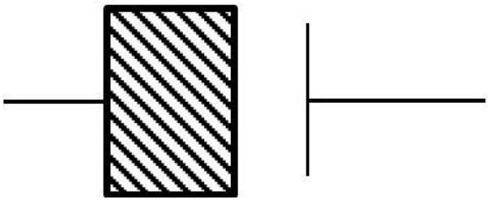
$\frac{1}{u} = \frac{1}{15} - \frac{1}{10} = \frac{2-3}{30} = \frac{1}{30}$

$u = -30$

$= 30$

**29. (6)** inserted between the plates. Capacitance of the capacitor in the presence of slab will be μF.

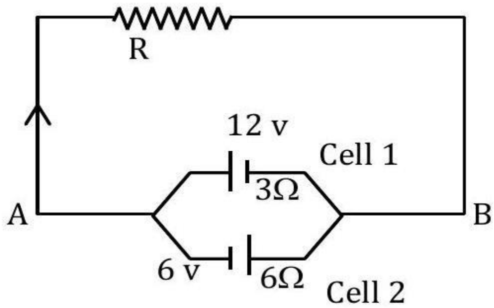
$$C_o = \frac{\epsilon_0 \ A}{d} = 5$$



$$C_1 = \frac{\frac{d}{2}}{\frac{\epsilon_0(1.5)A}{\frac{d}{2}}} = 3C_0, C_2 = \frac{\epsilon_0 \ A}{\frac{d}{2}} = 2Co$$

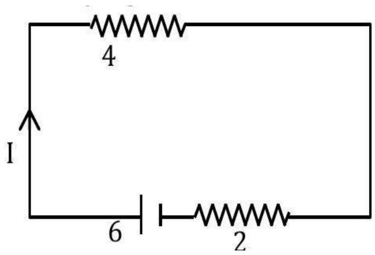
$$= \frac{3C_0 \times 2C_0}{5C_0} = \frac{6}{5} \times 5 = 6\mu f$$

**30. (1)**




$$v = \frac{12\times6-6\times3}{6+3} = \frac{54}{9} = 6 \text{ volt}$$

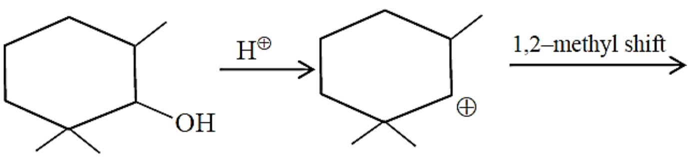
$$r_{eq} = \frac{6 \times 3}{6+3} = 2\Omega$$

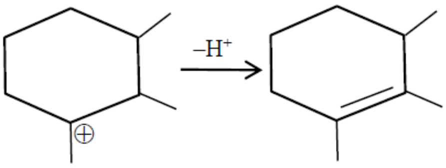



$$I = \frac{6}{4+2} = 1 \text{ A}$$

**31. (d)** If $[H^+] \rightarrow 10^3$ times then pH decreases by 3 units.

**32. (d)**





**33. (c)** (i) $CO_2$ is acidic as it from carbonic acid

$CO_2 + H_2O \rightarrow H_2CO_3$

(ii) CO is almost insoluble in water

**34. (a)** The alkali metal and their salts impart characteristic colour to an oxidizing flame. this is because the heat from the flame excites the outemost orbital electron to a higher energy level : when the excited electron comes back to the ground state, there is emission of radiation in the visible region. Alkali metal can therefore, be detected by the respective flame test and can be determined by flame photometry or atomic absorption spectroscopy.

**35. (d)**

$$Cr_2O_7^{-2} + 14H^+ + 6e^- \longrightarrow 2Cr^{+3} + 7H_2O$$
$$+6 \longrightarrow +3$$

**36. (d)**

(A) Glyptal → Paints and Lacquers (III)

(B) Neoprene → Gaskets (IV)

(C) Acrilan → Synthetic wool (II)

(D) LDP → Flexible pipes (I)

**37. (a)** Si < Be < Mg < K

Si is having Non-metallic character.

**38. (b)**

(A) Cobalt catalyst → methanol production.

(B) Syngas → coal gasification

(C) Nickel Catalyst → water gas production.

(D) Brine solution → $H_2 + Cl_2$ production.

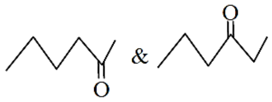
**39. (a)** Basicity order

$$Et-NH-Et > Et-\underset{\substack{|\\ Et}}{N}-Et > Et-NH_2 > \text{Aniline}$$

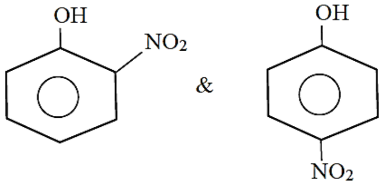
pKb: 3.00, pKb: 3.25 pKb: 3.29 pKb: 9.38

**40. (b)**

(A) C–C–C–$NH_2$ & C – NH – C – C : functional isomer (III)

(B)  : Metamer (I)

(C) $CH_3 - \underset{\underset{O}{\|}}{C} - NH_2$ & $CH_3 - \underset{\underset{OH}{|}}{C} = NH$ : Tautamer (IV)

(D)  : Position isomer (II)

**41. (b)**

Case I

$$xgmC_2H_6O_2 \text{ present}$$
$$0.25 = \frac{x/62}{500 - x} \times 1000$$
$$125 = \left(\frac{1000}{62} + 0.25\right)x$$

Case II
y gm $C_2H_6O_2$ is present.
$$0.25 = \frac{y/62}{250 - y} \times 1000$$
$$62.5 - 0.25y = \frac{1000}{62}y$$
$$62.5 = \left(\frac{1000}{62} + 0.25\right)y$$
equation (1) ÷ equation (2)

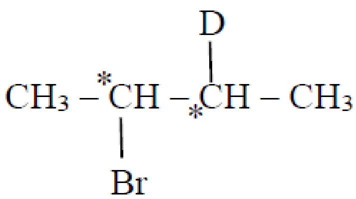
$$\frac{x}{y} = \frac{125}{62.5} = \frac{2}{1}$$

**42. (c)**

$$\Delta_o \uparrow \lambda \downarrow$$
$$\left(\text{splitting energy } = \frac{hc}{\lambda_{abs}}\right)$$

**43. (a)** The molecule BHA = Butylated hydroxyanisole commonly used as food preservatives which normally acts as antifungal and antiviral BHA reduces the rancidity of oil and fat which halps in retaining the nutrients (Butter contains saturated fats).

**44. (d)**



2 - Bromo-3-deuterobutane

**45. (b)**

MCl

Chloride Salt + $AgNO_3 \xrightarrow[HNO_3]{dil}$ $AgCl + MNO_3$

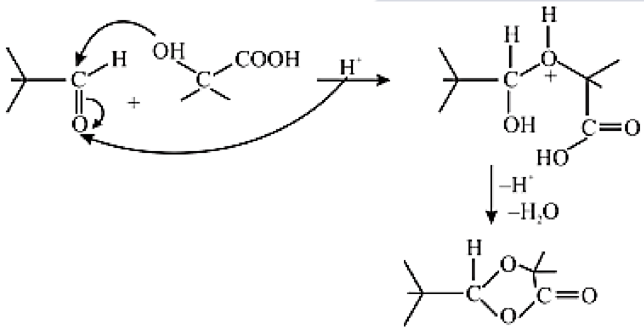
Curdy white PPt [A]

$\downarrow NH_4OH$

$[Ag(NH_3)_2]Cl$ [B] Clear Solution

**46. (b)** The crossed arrow of the dipole moment symbolizes the direction of the shift of electron density in the molecules.

**47. (b)**



**48. (c)**

(A) Ammonium salt are major component of both atmospheric nitrogen aerosols and wet deposited.

(C) PCB belongs to a broad family of man-made organic chemicals known. as chlorinated hydrocarbons.

**49. (a)** The rotating paddle in the froth flotation process violently agitates the suspension of powdered ore in water, as well the collectors and froth stablisers, generating frothing.

**50. (d)** Na metals is the weakest Reducing agent.

**51. (5)**

(i) $[Co(NH_3)_4Cl_2]Cl + AgNO_3 \rightarrow [Co(NH_3)_4Cl_2]^+ + AgCl$

(ii) $[Ni(H_2O)_6]Cl_2 + AgNO_3 \rightarrow [Ni(H_2O)_6]^{+2} + 2AgCl$

(iii) $[Pt(NH_3)_2Cl_2] + AgNO_3 \rightarrow$ no AgCl mole are ppt

(iv) $[Pd(NH_3)_4]Cl_2 + AgNO_3 \rightarrow [Pd(NH_3)_4]^{+2} + 2AgCl$

Total 5 mole AgCl are formed.

**52. (2)**

A & C are incorrect

$CaCl_2$ absorbs water vapour.

As adsorption proceeds,

$\Delta H$ becomes less negative.

**53. (12)**

$C \rightarrow 85.8\%$

$H \rightarrow 14.2\%$

mass of H in one molecule $= 84 \times \frac{14.2}{100} \approx 12$

No. of H − atoms $= \frac{12}{1}$

$= 12$

**54. (5)** Px, Py, Pz, $dz^2, dx^2 - y^2$ have Electron density along the axis.

**55. (847)**

Moles of mixture $= \frac{Pv}{RT} = \frac{1 \times 16.8}{0.0821 \times 298} = 0.6866$ moles

Moles of $CO_2 = \frac{1 \times 28}{0.0821 \times 298} = 1.144$ mole

$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

X X

$C_2H_4 + 3O_2 \longrightarrow 2CO_2 + 2H_2O$

$(0.6866 - x)\ 2(0.686 - x)$

Total $CO_2$ produced $= 1.144$

$x + 2(0.6866 - x) = 1.144$

$x = 1.3732 - 1.144$

$= 0.2292$

Moles of $CH_4 = 0.2292$

Moles of $C_2H_4 = 0.6866 - 0.2292$

$= 0.4574$

Total Heat produced

$= (900 \times 0.2292) + (0.4574 \times 1400)$

$= 206.28 + 640.36 = 846.64$

**56. (3)** Cell Reaction

$H_2 + M^{3+} \longrightarrow 2H^+ + M^+$

$$E_{cell} = E^o_{cell} - \frac{2.303RT}{2\,F}\log\frac{[M^+][H^+]^2}{[M^{3+}]}$$

$$0.1115 = 0.2 - \frac{0.059}{2}\log 10^a$$

$$\frac{0.059}{2}\log 10^a = 0.0885$$

$a = 3$

**57. (4)**

**58. (1)**

$k = 4.6 \times 10^{-3}\text{sec}^{-1}$

for Ist order :-

$t^1/2 = \frac{0.693}{k} = \frac{0.693}{4.6\times10^{-3}} = 150.65\text{sec.}$

$t_{completion} = \infty$

Degree of dissociation $(\propto) = \frac{x}{[A]_0} = \frac{[A]_0 - [A]_t}{[A]_0}$

$$= \frac{[A]_0 - [A]_0 e^{-kt}}{[A]_0} = 1 - e^{-kt}$$

rate and rate constant have different units

$$t_{10\%} = \frac{1}{K}\ln\frac{100}{90}$$

$$t_{90\%} = \frac{1}{K}\ln\frac{100}{10}$$

$$\frac{t_{10\%}}{t_{90\%}} = \frac{\log 10 - \log 9}{\log 10} = 0.045$$

$t_{10\%} = 0.045t_{90\%}$

**59. (2)** B & D option are correct.

**60. (3)**

**61. (a)**

**62. (a)**

$$\underbrace{3\hat{i} - 4\hat{j} + 2\hat{k}}_{P}, \underbrace{\hat{i} + 2\hat{j} - \hat{k}}_{Q}, \underbrace{-2\hat{i} - \hat{j} + 3\hat{k}}_{R}, \underbrace{5\hat{i} - 2\alpha\hat{j} + 4\hat{k}}_{S}$$

$\overrightarrow{PQ} = -2\hat{i} + 6\hat{\mathbf{j}} - 3\hat{k}$

$\overrightarrow{QR} = -3\hat{i} - 3\hat{j} + 4\hat{k}$

$\overrightarrow{RS} = 7\hat{i} + (1 - 2\alpha)\hat{j} + \hat{k}$

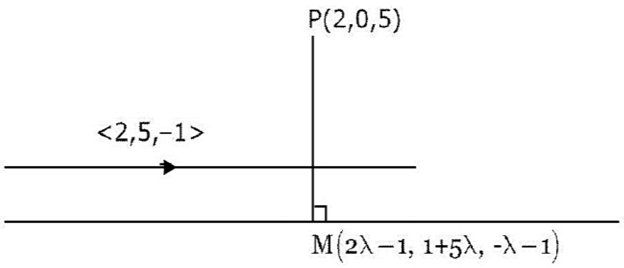
$[\overrightarrow{PQ}\overrightarrow{QR}\overrightarrow{RS}] = 0$

$$\begin{vmatrix} -2 & 6 & -3 \\ -3 & -3 & 4 \\ 7 & 1-2\alpha & 1 \end{vmatrix} = 0$$

$-2(-3+8\alpha-4)-6(-31)-3(6\alpha-3+21)=0$

$\alpha = \frac{73}{17}$

**63. (a)**



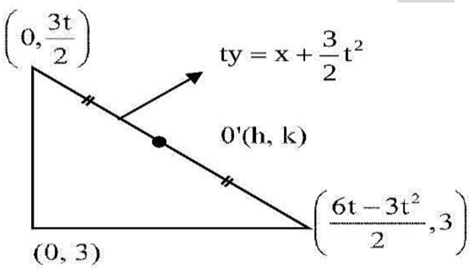

$\overrightarrow{PM}(2,5,-1)=0$
$(2\lambda-3, 5\lambda+1, -\lambda-6)\cdot(2,5,-1)=0$
$4\lambda-6+25\lambda+5+\lambda+6=0$
$\lambda=-\frac{1}{6}$

Now, $\alpha = 2\left(-\frac{1}{6}\right)-1=-\frac{4}{3}$

$\beta = \frac{1}{6}$

$\gamma = -\frac{5}{6}$

**64. (c)**




$2\text{ h} = \frac{6t-3t^2}{2}$

$4\text{ h} = 6t-3t^2$

$\&2k = \frac{3t+6}{2}$

$\frac{4k-6}{3} = t$

$4\text{ h} = 8k-12-\frac{1}{3}(16k^2-48k+36)$

$12\text{ h} = 24k - 36 - 16k^2 + 48k - 36$

$4y^2 - 18y + 3x + 18 = 0$

**65. (a)**

$133 = 2(4^n) + \lambda$
$255 = 2(5^n) + \lambda$
$122 = 2[5^n - 4^n]$
$\underline{5^n - 4^n = 61}$
$\Downarrow$
$\Downarrow$
$n = 3$
Now,
$f(c) = 2(3)^3 + \lambda$
$f(b) = 2(2)^3 + \lambda$
$f(c) - f(b) = 38 = 2 \times 19$
$(2^0 + 2^1)(19^0 + 19)$
$= 60$

**66. (c)**

${}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3$

add and subtract ${}^{45}C_4$

$({}^{45}C_4 + {}^{45}C_3) + {}^{46}C_3 + {}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 - {}^{45}C_4$ $({}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r)$

${}^{52}C_4 - {}^{45}C_4$

**67. (b)**

$f(x) = 2x^3 + (2p - 7)x^2 + 3(2p - 9)x - 6$

$f'(x) = 6x^2 + (4p - 14)x + 6p - 27 = 0 \searrow \alpha$

let $\alpha > 0 \& \beta < 0$

Products of roots $< 0 \Rightarrow$ (b)

**68. (d)** Now, $M^2 = (A^TBA)(A^TBA) = A^TB^2A$

$AA^T = 1$

$\Rightarrow M^{2023} = A^TB^{2023}A$

Let $D = A^{2023}A^T = AA^TB^{2023}AA^T$ $AA^T = 1$

$D = B^{2023}$

Now, $B^2 = \begin{bmatrix} 1 - & i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 - & i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$

Now, $B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

$D^{-1} = \begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

**69. (c)** $\vec{a} \times (\vec{a} \times \vec{b}) = \begin{vmatrix} i & j & k \\ -1 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \hat{i} + \hat{j} + 2\hat{k}$

$\vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$-3\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$

$-6\vec{b} = 4\hat{i} + 4\hat{j} + 2\hat{k}$

Now, $\vec{a} - 6\vec{b} = 3(\hat{i} + \hat{j} + \hat{k})$

**70. (b)** $16\int_1^2 \frac{dx}{x^3 x^4\left(1+\frac{2}{x^2}\right)^2}$

Let, $1 + \frac{2}{x^2} = t \Rightarrow -\frac{4}{x^3}dx = dt$

$\frac{-4}{4}\int_3^{3/2} \frac{(t-1)^2}{t^2}dt = \int_{3/2}^{3} \frac{t^2 - 2t + 1}{t^2}dt$

$\Rightarrow \int_{3/2}^{3}\left(1 - \frac{2}{t} + \frac{1}{t^2}\right)dt$

$\Rightarrow 3 - \frac{3}{2} - 2(\ln 3 - \ln 3/2) - \frac{1}{3} + \frac{2}{3}$

$\Rightarrow \frac{11}{6} - 2\ln 2 \Rightarrow \frac{11}{6} - \ln 4$

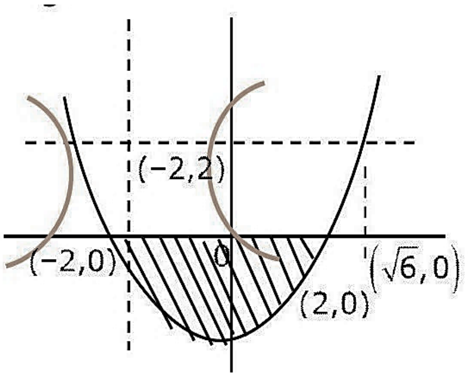
**71. (a)**

$$16(x^2 + 4x) - (y^2 - 4y) + 44 = 0$$
$$16\{(x+2)^2 - 4\} - (y-2)^2 + 4 + 44 = 0$$
$$16(x+2)^2 - (y-2)^2 = 16$$
$$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16}$$

Area $= \int_{-2}^{\sqrt{6}}(y_2 - y_1)dx$

$= \int_{-2}^{\sqrt{6}}\left(2 - (x^2 - 4)\right)dx$

$= 4\sqrt{6} + \frac{28}{3}$




**72. (d)**

$$3N = N^2 - 4$$
$$N^2 - 3N - 4 = 0$$
$$N = 4$$

Sum should be equal to 4 so possible outcomes are $\{(1,3),(2,2),(3,1)\}$

$\Rightarrow \text{Prob} = \frac{3}{36} = \frac{1}{12} = \frac{k}{48}$

$K = 4$

**73. (d)**
**74. (b)**
**75. (d)**

$$\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$$

L.D.E (Linear differential equation)

$\text{I.F.} = e^{\int \alpha \cdot dt} = e^{\alpha t}$

$$y(e^{\alpha t}) = \int \gamma e^{-\beta t} \cdot e^{\alpha t} \cdot dt$$
$$\Rightarrow y e^{\alpha t} = \gamma \frac{e^{(\alpha-\beta)t}}{\alpha - \beta} + C$$

$$\Rightarrow y(t) = \frac{\gamma}{\alpha - \beta} e^{-\beta t} + C \cdot e^{-\alpha t}$$

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} \left\{\frac{\gamma}{\alpha - \beta} e^{-\beta t} + c \cdot e^{-\alpha t}\right\}$$

$= 0 + 0$

$\Rightarrow \lim_{t\to\infty} y(t) = 0$

**76. (c)**

$$\left|\frac{x + i(y-2)}{x + i(y+1)}\right| = 2$$

$$x^2 + (y-2)^2 = 4(x^2 + (y+1)^2)$$

$3x^2 + 4y^2 + 4 + 8y - y^2 - 4 + 4y = 0$

$3(x^2 + y^2) + 12y = 0$

$x^2 + y^2 + 4y = 0$

$C(0, -2)$

**77. (a)**

$A^T = A,\ B^T = -B,\ C^T = -C$

$(S_1)$: $(A^{13}\ B^{26} - B^{26}\ A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$

$= (-B)^{26}(\ A)^{13} - (A)^{13}(-B)^{26}$

$= B^{26}A^{13} - A^{13}B^{26}$

$= -(A^{13}B^{26} - B^{26}A^{13})$

$(S_1 \rightarrow \text{false})$

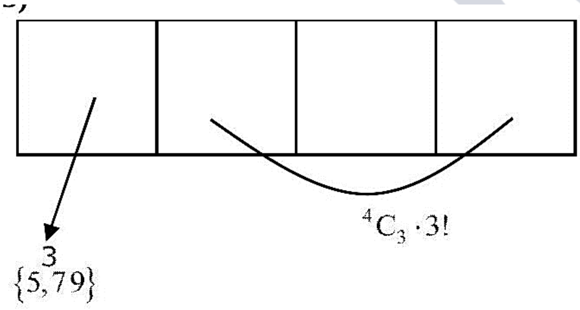
$(S_2)$: $(A^{26}C^{13} - C^{13}\ A^{26})^T$

$= (A^{26}C^{13})^T - (C^{13}A^{26})^T$

$= (C^T)^{13}(A^T)^{26} - (A^T)^{26}(C^T)^{13}$

$= -C^{13}\ A^{26} - A^{26}(-C)^{13}$
$= A^{26}C^{13} - C^{13}\ A^{26}$ ( $S_2 \rightarrow$ True )

**78. (c)**




No. of ways $= 3.4 \times 3! = 3.4! = 72$

**79. (a)**

$\because -\sqrt{2} \le \sin x - \cos x \le \sqrt{2}$

$\Rightarrow -2 \le \sqrt{2}(\sin x - \cos x) \le 2$

$\Rightarrow m - 4 \le \sqrt{2}(\sin x - \cos x) + m - 2 \le m$

$\Rightarrow \log_{\sqrt{m}}^{(m-4)} \leq \log_{\sqrt{m}}\left\{\sqrt{2(\sin x - \cos x) + m - 2}\right\} \leq \log_{\sqrt{m}}^{m}$

0

$\Rightarrow \log_{\sqrt{m}}^{(m-4)} = 0$

$\Rightarrow m = 5$

**80. (b)** $\frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}}, \; \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$

$$d = \left|\frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$$

$\vec{a} = (-1,0,0), \vec{b} = (0,-2,1)$

$$\vec{p} = \left(1, \frac{1}{2}, \frac{-1}{12}\right), \vec{q} = \left(1,1,\frac{1}{6}\right)$$

$$\vec{b} - \vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & \frac{1}{2} & -\frac{1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix}$$

$$= \hat{\imath}\left(\frac{1}{12} + \frac{1}{12}\right) - \hat{\jmath}\left(\frac{1}{6} + \frac{1}{12}\right) + \hat{k}\left(1 - \frac{1}{2}\right)$$

$$= \frac{\hat{\imath}}{6} - \frac{\hat{\jmath}}{4} + \frac{\hat{k}}{2}$$

$$|\vec{p} \times \vec{q}| = \sqrt{\frac{1}{36} + \frac{1}{16} + \frac{1}{4}} = \frac{7}{12}$$

$$d = \left|\frac{(\hat{\imath} - 2\hat{\jmath} + \hat{k}) \cdot \left(\frac{\hat{\imath}}{6} - \frac{\hat{\jmath}}{4} + \frac{\hat{k}}{2}\right)}{\frac{7}{12}}\right|$$

$$d = \left|\frac{\frac{1}{6} + \frac{1}{2} + \frac{1}{2}}{\frac{7}{12}}\right| = \frac{\frac{7}{6}}{\frac{7}{12}} = 2$$

**81. (9)** P( smoker ) $= \frac{1}{4}$

P( non smoker ) $= \frac{3}{4}$

Probability that a smoker has lung cancer

$$P\left(\frac{C}{S}\right) = 27P\left(\frac{C}{NS}\right)$$

Probability that a person is smoker when he has lung cancer

$$= \frac{P(S) \cdot P\left(\frac{C}{S}\right)}{P(S) \cdot P\left(\frac{C}{S}\right) + P(NS) \cdot P\left(\frac{C}{NS}\right)}$$

$$= \frac{\frac{1}{4} \times P\left(\frac{C}{S}\right)}{\frac{1}{4} \times P\left(\frac{C}{S}\right) + \frac{3}{4} P\left(\frac{C}{NS}\right)}$$

$$= \frac{\frac{1}{4} \times 27P\left(\frac{C}{NS}\right)}{\frac{1}{4} \times 27P\left(\frac{C}{NS}\right) + \frac{3}{4} P\left(\frac{C}{NS}\right)}$$

$$\frac{27}{30} = \frac{k}{10}$$

$k = 9$

**82. (7)**

$2023 = 289 \times 7$

2023 is a multiple of 7

$n = (2023)^{2023}$ is multiple of 7

and $(2023)^{2023} = (-2)^{2023} = -2(2^2)^{1011}$

$$= -2(5-1)^{1011}$$
$$= -2[\,{}^5C_0 5^{1011} - \,{}^5C_1 5^{1010} + \cdots .. - {}^{1011}C_{1011}]$$

$(2023)^{2023}$ when divided by 5

gives remainder 2

If $n = (2023)^{2023}$ divided by $35 = 7 \times 5$

$n = 7k$

$n - 7 = 7(k-1) \rightarrow n7$ is multiple of 7

and $n = 5\,m + 2$

so $n - 7 = 5\,m - 5 =$ multiple of 5

so $n - 7$ is multiple of 35 so when n is divided by 35 , reminder $= 7$

**83. (45)**

$\alpha + \beta = -60^{\frac{1}{4}}$ and $\alpha\beta = a$

$\alpha^2 + \beta^2 = 60^{\frac{1}{2}} - 2a$

$\alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 604a^2 - 4a \cdot 60^{\frac{1}{2}}$

$-30 + 2a^2 = 60 + 4a^2 - 4a\sqrt{60}$

$a^2 - 2a\sqrt{60} + 45 = 0$

Product = 45

**84. (3)** $b^2 = \frac{a}{18}$

$$20 = \frac{1}{a} + \frac{1}{b}$$

$$a = \frac{b}{20b - 1}$$

$$b^2 = \frac{1}{18} \times \frac{b}{20b - 1}$$

$$360b^2 - 18b - 1 = 0$$

$$360b^2 - 30b + 12b - 1 = 0$$

$$(12b - 1)(30b + 1) = 0$$

$b = \frac{1}{12}, \frac{-1}{30}$ (rejected)

$$a = \frac{1}{8}$$

$$16a + 12b = 2 + 1 = 3$$

**85. (25)**

$$2\cos 2\theta \cos\frac{\theta}{2} = 2\cos 3\theta \cos\frac{9\theta}{2}$$

$$\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}$$

$$\cos\frac{5\theta}{2} - \cos\frac{15\theta}{2} = 0$$

$\sin 5\theta = 0$ or $\sin\frac{5\theta}{2} = 0$

$\theta = \frac{n\pi}{5}$ or $\frac{2n\pi}{5}$

$$\theta = 0, \pm\frac{\pi}{5}, \pm\frac{2\pi}{5}, \pm\frac{3\pi}{5}, \pm\frac{4\pi}{5}, \pm\pi$$

$$m = n = 5$$

$$mn = 25$$

**86. (18)**

A (1,2,3)
B (2,3,4)

Equation of line AB

$$\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}$$

Given line

$$\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-2}{0}$$

shortest distance $=\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_2\times\vec{b}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$

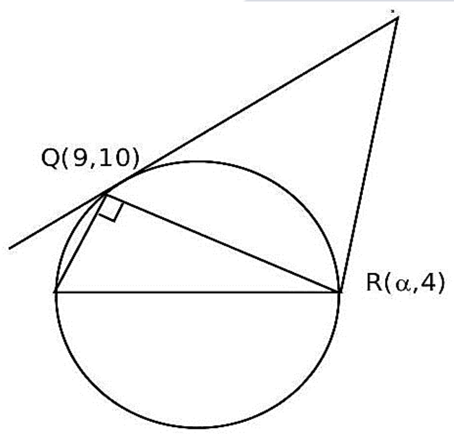
$$=\left|\frac{(3\hat{j}-\hat{k})\cdot(\hat{i}+2\hat{j}-3\hat{k})}{\sqrt{1+4+9}}\right|$$

$$\alpha=\frac{3}{\sqrt{14}}$$

$$28\alpha^2=28\times\frac{9}{14}=18$$

**87. (3)** Equation of circle is

$(x+3)(x-\alpha)+(y-2)(y-4)=0$




Q lies on it

$12(9-\alpha)+8\times 6=0$

$\alpha=13$

$x^2+y^2-10x-6y-31=0$

Equation of Tangent at Q

$x.9+y.10-5(x+9)-3(y+10)-31=0$

$4x+7y=106$

Equation of Tangent at R

$x.13+y.4-5(x+13)-3(y+4)-31=0$

$8x+y=108$

Solution (a) and (b)

$s = \left(\frac{25}{2}, 8\right)$

which lies on $2x - ky = 1$

$k = 3$

**88.** ( **6860)**

Three cases are possible

$1R1\ W3O + 2R1\ W2O + 1R2\ W2O$

${}^{7}C_1 \cdot {}^{5}C_1 \cdot {}^{8}C_3 + {}^{7}C_2 \cdot {}^{5}C_1 \cdot {}^{8}C_2 + {}^{7}C_1 \cdot {}^{5}C_2 \cdot {}^{8}C_2$

$= 6860$

**89. (20)**

$$\int_{\frac{1}{3}}^{3} |\log_e x| dx$$
$$= \int_{\frac{1}{3}}^{1} (-\ln x) dx + \int_{1}^{3} (\ln x) dx$$
$$-[x\ln x - x]_{\frac{1}{3}}^{1} + [x\ln x - x]_{1}^{3}$$
$$= \frac{4}{3}\ln\left(\frac{9}{e}\right) = \frac{m}{n}\ln\left(\frac{n^2}{e}\right)$$

$m = 4$ and $n = 3$

so $m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

**90. (31)**

$3x + 4y = 60$

$x = 1, 4y = 57, y = 14.2$

$x = 1, y = 1,2,3, \dots \dots .14 \rightarrow 14$ points

$x = 2, 4y = 54, y = 13.5$

$x = 2, y = 2,4,6,8,10,12 \rightarrow 6$ points

$x = 3, y = 3,6,9,12 \rightarrow 4$ points

$x = 4, y = 4,8 \rightarrow 2$ points

$x = 5, y = 5,10 \rightarrow 2$ points

$x = 6, y = 6 \rightarrow 1$ points

$x = 7, y = 7 \rightarrow 1$ points

$x = 8, y = 8 \rightarrow 1$ points

$x = 9, 4y = 23, y = 5.7$ $\times$ no point

Total points $= 14 + 6 + 4 + 2 + 2 + 1 + 1 + 1 = 31$