1. **(a)** Viscosity = pascal. second

$P^x A^y T^z = [M^1 L^{-1} T^{-1}]$

$[M^1 L^{+1} T^{-1}]^x [L^2]^y [T^1]^z = M^1 L^{-1} T^{-1}$

$M^x L^{+x+2y} T^{-x+z} = M^1 L^{-1} T^{-1}$

$x = 1 \; x + 2y = -1 \; -x + z = -1$

$y = -1$

$z = 0$

Viscosity $= P^1 A^{-1} T^0$

2. **(a)** $\vec{D} = \epsilon_0 \vec{E}$

$[D] = [\epsilon_0 E] = \left[\epsilon_0 \frac{\sigma}{\epsilon_0}\right]$

$[D] = [\sigma]$

→ Surface change density = σ.

3. **(b)** $d = R\theta$

$60 = R\left(\frac{3\pi}{4}\right)$

$R = \frac{60 \times 4}{3\pi} = \frac{80}{\pi} m$

Displacement $= \sqrt{R^2 + R^2 - 2R^2 \cos 135}$

$\Rightarrow \sqrt{2R^2 - 2R^2(-0.7)}$

$\Rightarrow \sqrt{3.4R^2} = \sqrt{3.4\left(\frac{80}{\pi}\right)^2}$

≈ 47 m

4. **(b)** $v_i = 3(0^2) + 4 = 4 \cong x = 0$

$v_F = 3(2)^2 + 4 \; \cong x = 2$

$= 16$

$W = \Delta K = \frac{1}{2} m(16^2 - 4^2)$

$= \frac{1}{2} \times \frac{1}{2}(256 - 16)$

$= \frac{240}{4} = 60\text{ J}$

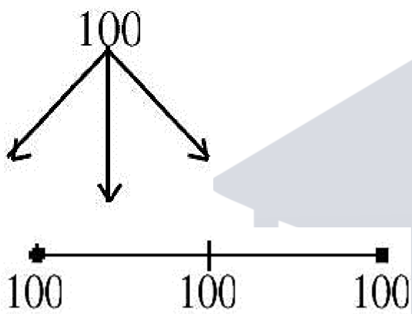
5. (d)

$$V = \sqrt{\frac{2gH}{1 + k^2/R^2}}$$

$$\frac{V_{cylinder}}{V_{sphere}} = \sqrt{\frac{(1 + k^2/R^2)_{sphere}}{(1 + k^2/R^2)_{cylinder}}}$$

$$= \sqrt{\frac{1 + 2/5}{1 + 1/2}} = \sqrt{\frac{7}{5} \times \frac{2}{3}} = \sqrt{\frac{14}{15}}$$

6. (b)



$$F = \frac{GMM}{r^2} + \sqrt{2}\frac{GMM}{(\sqrt{2}r)^2}$$

$$= \frac{GMM}{r^2}\left(1 + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{G \times 10^4}{13^2}\left(1 + \frac{1}{\sqrt{2}}\right)$$

$F \simeq 100G$

7. (b) $P_1 = 2 \times 10^7$ Pa

$P_1 V_1 = P_2 V_2$

Since $V_2 = 2V_1$ Hence $P_2 = P_1/2$ (isothermal expansion)

$P_2 = 1 \times 10^7$ Pa

$P_2(V_2)^\gamma = P_3(2V_2)^\gamma$

$$P_3 = \frac{1 \times 10^7}{2^{1.5}} = 3.536 \times 10^6$$

8. (a)

$KE_{avg} = \frac{3}{2}KT$

$P = \frac{1}{3}\rho V_{rms}^2$

9. **(a)** $T_1 = 2\pi\sqrt{\frac{3\,m}{2k}}$

$T_2 = 2\pi\sqrt{\frac{m}{3k}}$

$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{3\,m}{2k}}}{2\pi\sqrt{\frac{m}{3k}}} = \frac{3}{\sqrt{2}}$

10. **(a)** $Q = CV$

$V = \frac{1}{C}Q$

Straight line with slope $= \frac{1}{C}$

Slope $= \frac{1}{C} = \frac{1}{2\times10^{-6}} = 5\times10^5$

11. **(b)** Radius of circular path $R = \frac{\sqrt{2mk}}{qB}$

$q = \frac{\sqrt{2mk}}{RB}$

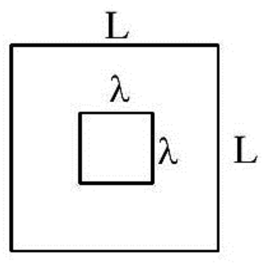
$\frac{q_1}{q_2} = \sqrt{\frac{m_1}{m_2}}\times\frac{R_2}{R_1} = \sqrt{\frac{9}{4}}\times\frac{5}{6} = \frac{5}{4}$

12. **(c)** $f = \frac{1}{2\pi\sqrt{LC}}$

To increase the resonating frequency product of L and C should decrease.

By joining capacitor in series, capacitor will decrease

13. **(c)** Assuming current I in outer loop magnetic field at centre $= 4\times\frac{\mu_0 i}{4\pi\times\frac{L}{2}}\times(2\sin 45^\circ) = \frac{2\sqrt{2}\mu_0 i}{\pi L}$



$$M = \frac{\text{Flux through inner loop}}{i}$$

$$M = \frac{2\sqrt{2}\mu_0 \ell^2}{\pi L}$$

14. **(b)** Current in capacitor $I = \frac{V}{X_C}$

$$I = (V) \times (\omega C)$$

$$C = \frac{I}{V\omega} = \frac{6.9 \times 10^{-6}}{230 \times 600} = 50\text{pF}$$

15. **(a)** In primary rainbow, red colour is at top and violet is at bottom. Intensity of secondary rainbow is less in comparison to primary rainbow.

16. **(a)** $\frac{\mu_A}{\mu_B} = \frac{c/V_A}{c/V_B} = \frac{V_B}{V_A} = \frac{1}{2}$

Let the thickness is d

$$\frac{d}{v_B} - \frac{d}{v_A} = 5 \times 10^{-10}$$

$$d = \frac{5 \times 10^{-10} \times v_A v_B}{v_A - v_B}$$

As $v_A = 2v_B \Rightarrow d = 5 \times 10^{-10} \times 2v_B$

Or $d = 5 \times 10^{-10} \times v_A$

17. **(c)** $k_1 = \frac{1230}{800} - \phi$

$$k_2 = 2k_1 = \frac{1230}{500} - \phi$$

Eliminating k_1 from

(1) and (2) we get

$$0 = \frac{1230}{500} - \frac{1230}{400} + \phi$$

$$\phi = 0.615\text{eV}$$

18. **(a)** Angular momentum is integral multiple of $\frac{h}{2\pi}$ $mvr = \frac{nh}{2\pi}$ So momentum $mv = \frac{nh}{2\pi r}$

19. **(b)** Ratio of magnetic moment and angular momentum $\frac{\vec{\mu}}{\vec{L}} = \frac{q}{2m}$

For e^-

$$\vec{\mu} = -\frac{e}{2m}\vec{L}$$

20. **(a)** When both A and B have logical value ' 1 ' both diode are reverse bias and current will flow in resistor hence output will be 5 volt i.e. logical value ' 1 '. In all other case conduction will take place, hence output will be zero volt i.e. logical value ' 0 '.

So, truth table is

A	B	Y	
0	0	0	
0	1	0	(AND gate)
1	0	0	
1	1	1	

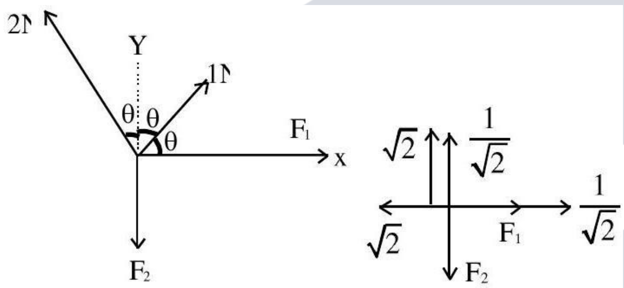
21. (3) Stopping distance $= \frac{v^2}{2a} = d$

If speed is made $\frac{1}{3}$rd

$d' = \frac{1}{9}$ d. $d' = \frac{27}{9} = 3.$

Braking acceleration remains same

22. (3)



$\theta = 45°$

Taking components along x&y

$$F_1 = \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{2-1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$F_2 = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$F_1 : F_2 = 1:3$

$x = 3$

23. (5)

$$\Delta\ell_1 = \frac{F\ell}{AY} = \frac{F\ell}{\pi r^2 Y} = 5 \text{ cm}$$

$$\Delta\ell_2 = \frac{4\,F4\ell}{\pi 16 r^2 Y} = \frac{F\ell}{\pi r^2 Y} = 5 \text{ cm}$$

24. (60) $\ell_B(1 + \alpha_B \Delta T) - \ell_i(1 + \alpha_i \Delta T) = \ell_B - \ell_i$

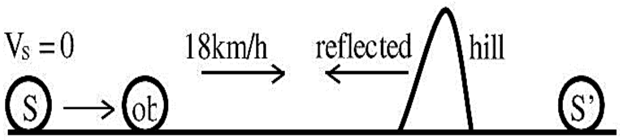
$\alpha_B \ell_B = \ell_i \alpha_i$

$1.8 \times 10^{-5} \times 40 = \ell_i \times 1.2 \times 10^{-5}$

$\ell_i = \dfrac{1.8 \times 10^{-5} \times 40}{1.2 \times 10^{-5}} = \dfrac{3 \times 40}{2} = 60$

$\ell_i = 60 \text{ cm}$

25. (20)



$V_S = 0,\ V_{ob} = 5 \text{ m/s}$

$f_{direct} = \left(\dfrac{320 - 5}{320}\right) 640 = 630 \text{ Hz}$

$f_{reflected} = \left(\dfrac{320 + 5}{320}\right) 640 = 650 \text{ Hz}$

$f_{beat} = 650 - 630 = 20 \text{ Hz}$

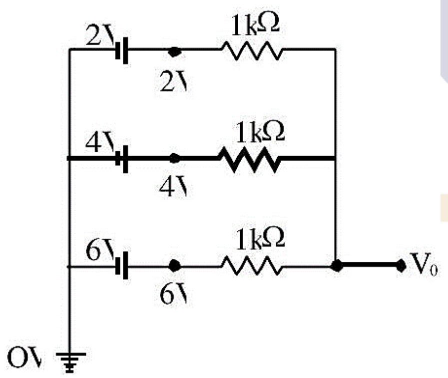
26. (45) No. of electric field lines per unit area = electric field.

$E = \frac{\rho r}{3\epsilon_0}$, for $r = R$

$E = \dfrac{\rho R}{3\ \epsilon_0} = \dfrac{2 \times 6}{3 \times 8.85 \times 10^{-12}} = 0.45 \times 10^{12} NC^{-1}$

$= 45 \times 10^{10}$ N/C

27. (d)



By nodal analysis $\frac{V_0 - 2}{1k\Omega} + \frac{V_0 - 4}{1k\Omega} + \frac{V_0 - 6}{1k\Omega} = 0$

$3\ V_0 - 12 = 0$

$V_0 = 4$

28. (4)

Each wire has resistance $= \rho \frac{4\ell}{\pi d^2} = r$

Eight wire in parallel, then equivalent resistance is $\frac{r}{8}=\frac{\rho \ell}{2\pi d^2}$

Single copper wire of length 2l has resistance $R=\rho\frac{2\ell\times 4}{\pi d_1^2}=\frac{\rho \ell}{2\pi d^2}$

$\Rightarrow d_1 = 4\ d$

29. (3)

$$E_g=\frac{hc}{\lambda}=\frac{1242}{\lambda(nm)}=\frac{1242}{400}=3.105$$

Answer rounded to 3eV

30. (150)

$d=\sqrt{2Rh}$

$d=\sqrt{2\times 6400\times h\times 10^{-3}}$(h in m)

$\text{Area}=\pi d^2$

$=(\pi\times 2\times 6400\times h\times 10^{-3})\text{km}^2$

$6.03\times 100000=100\times\pi\times 2\times 6400\times 10^{-3}\ h$

$$h=\frac{6.03\times 10^5}{10\times\pi\times 128}$$

$h=150\ m$

31. (c)

Let $n(SO_2Cl_2)=x$ moles

$\therefore n(H_2SO_4)=x, n(HCl)=2x$

$\Rightarrow n(H^+)=4x$

For Neutralisation

$\Rightarrow n(H^+)=n(OH^-)$

$\Rightarrow 4x=16$

$\Rightarrow \mathbf{x}=4$

32. (c) $1=0,1,2\ldots\ldots(n-1)$

$\therefore$ for $n=3$

$1=0,1,2$

$\Rightarrow 1=3,$

not possible for $n=3$

33. (c) $[HCOOH] = 0.5 ml l^{-1}$

$\Rightarrow (0.5 ml \times 1.05\ g ml^{-1}) HCOOH$ in 1 L

$\Rightarrow 0.525\ g HCOOH$ in 1 L

$m = \frac{(0.525/46)}{1\ kg}$ mol [Assuming dilute solution]

$\therefore \Delta T_f = iK_f m \Rightarrow i = \frac{\Delta T_f}{k_f m} = \frac{0.0405 \times 46}{1.86 \times 0.525} = 1.9$

34. (c) $NH_4OH + HCl \rightarrow NH_4Cl + H_2O$

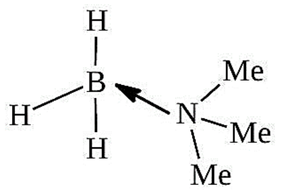
mmole 2 2

$[NH_4^+] = \frac{2 mmole}{60 ml} = \frac{1}{30} M$

$pH = \frac{pK_w - pK_b - \log C}{2} = \frac{14 - 5 + 1.48}{2} = 5.24$

35. (c)
36. (d) Atomic Number 103
37. (d) $FeO + SiO_2 \rightarrow FeSiO_3$
38. (a) $H_2O_2 + MnO_4^- \rightarrow Mn^{2+} + O_2$ (unbalanced)
39. (a)
40. (b) $BF_3 + NaH \xrightarrow{450\ K} \underset{\text{(diborane)}}{B_2H_6} + NaF$

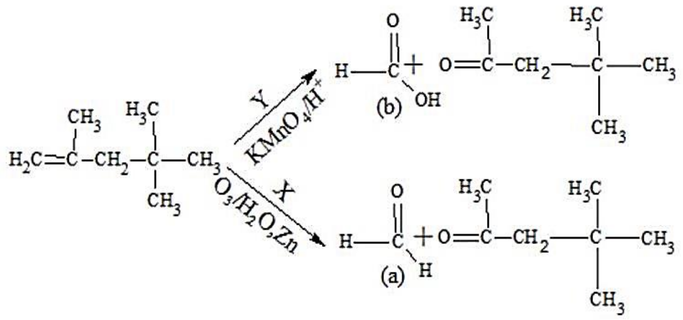
$$B_2H_6 + NMe_3 \longrightarrow 2\underset{\text{symmetrical cleavage}}{\left[BH_3 \leftarrow NMe_3\right]}$$



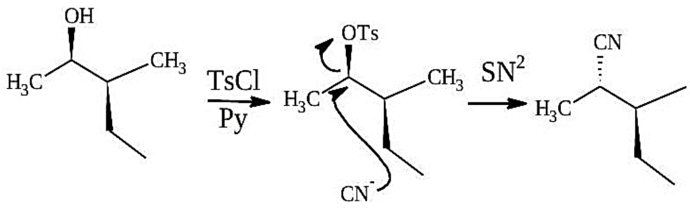
41. (b)

$Br_2 + 5 \underset{\text{(excess)}}{F_2} \longrightarrow BrF_5^{+5} \rightarrow HBrO_3$ (Forms bromate)

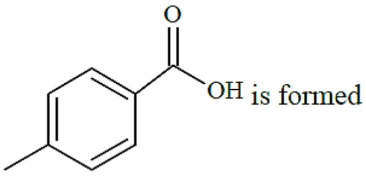
42. (c)
43. (d)



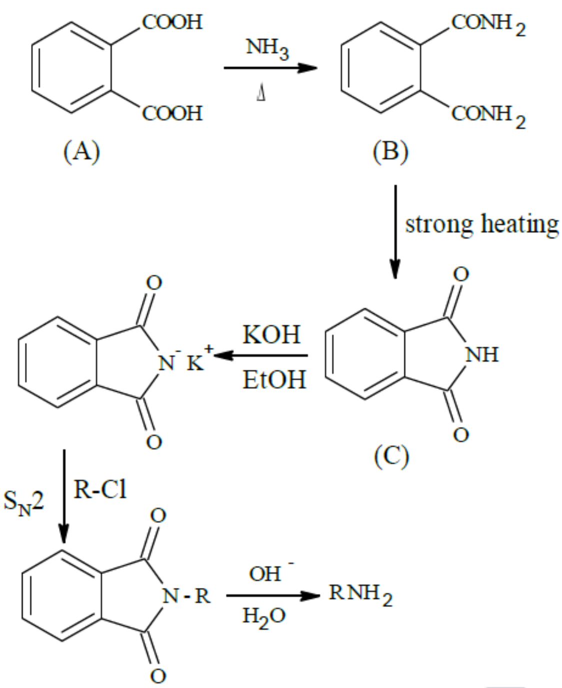
44. (b)



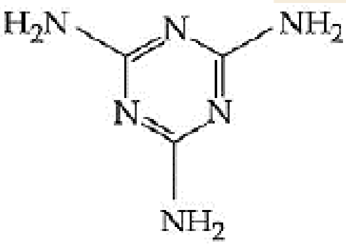
45. (d)



46. (c) Gabriel Pthalimide reaction



47. (a) Melamine:



Formaldehyde HCHO
Melamine formaldehyde Resin is melamine polymer

48. (a) Primary structure remains intact during denaturation of proteins

49. (b)

50. (b) Acrolein has a pungent, suffocating odour. Acrolein is used to detect presence of glycerol

51. (2) Diamagnetic species are: $N_2, O_2{}^{2-}$

52. (104)

$3C_{(gr)} + 4H_{2(g)} \rightarrow C_3H_{8(g)}$

$= -103.7 \text{ kJ mol}^{-1}$

53. (22) $[P_{gas}]_0 + \text{V.P.} = 4$

$[P_{gas}]_0 = 4 - 0.4 = 3.6$

As volume is doubled, $[P_{gas}]_{new} = 1.8$ atm

New Total Pressure $= 1.8 + 0.4 = 2.2$ atm

54. (4) $E = E^0 - \frac{2.303RT}{nF} \log Q$

Here, $E = +0.801 \text{ V}, E^0 = 0.008 - (-0.763)$

$= +0.771 \text{ V}$

$\therefore 0.801 = +0.771 - \frac{0.06}{n} \log 10^{-2}$

$\Rightarrow n = 4$

55. (1)

$(t_{1/2})_{500 \text{ torr}} = 240\text{sec} = 4 \text{ min.}$

$(t_{1/2})_{250 \text{ torr}} = 4 \text{ min.}$

$t_{1/2} \propto a^{1-n}$

As $t_{1/2}$ is independent of initial pressure. Hence, order is 1 st order.

56. (0) $\Delta_0 \propto \frac{1}{\lambda}$ Here, CN^- being SFL will have maximum CFSE So, $[Co(CN)_6]^{3-}$ will be $d^2sp^3, \mu = 0$

57. (5)

Co^{3+} can't liberate H_2.

It has d^6 configuration,

Number of unpaired electrons $= 4$

$\mu = \sqrt{4 \times 6} = 4.92$ B.M.

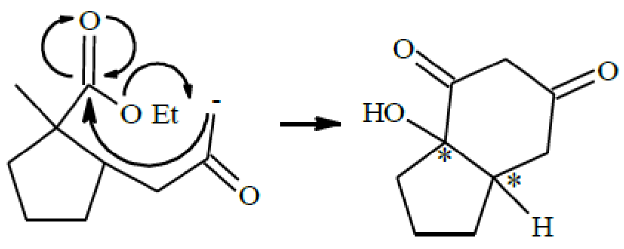
58. (56)

. $\% N = \frac{1.4(N_1 V_1)}{\text{mass of organic compound}}$

$$\% \text{ N} = \frac{1.4(2.5 \times 2 \times 2)}{0.25} = 56$$

59. (1)

60. (2)



2 chiral carbons

61. (b) $A = \{1,2,3,4\}$

$B = \{1,2,3,4,5,6\}$

Here $f(3)$ can be 2, 3, 4, 5, 6

$f(3) = 2, (f(1), f(2)) \rightarrow (1,1) \rightarrow 6$ cases

$f(3) = 3, (f(1), f(2)) \rightarrow (1,2), (2,1)$

$\rightarrow 2 \times 6 = 12$ cases

$f(3) = 4, (f(1), f(2)) \rightarrow (1,3), (3,1), (2,2)$

$\rightarrow 3 \times 6 = 18$ cases

$f(3) = 5, (f(1), f(2)) \rightarrow (1,4), (4,1), (2,3), (3,2)$

$\rightarrow 4 \times 6 = 24$ cases

$f(3) = 6, (f(1), f(2)) \rightarrow (1,5), (5,1), (2,4), (4,2), (3,3)$

$\rightarrow 5 \times 6 = 30$ cases

Total number of cases $= 6 + 12 + 18 + 24 + 30 = 90$

62. (b)

$\alpha, \beta, \gamma, \delta$ root of the equation

$x^4 + x^3 + x^2 + x + 1 = 0$

Which are 5^{th} roots of unity except 1.

then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} =$

$\alpha + \beta + \gamma + \delta = -1$

63. (d)

$S_n: |z - (3 - 2i)| = \frac{n}{4}$ is a circle center $C_1(3, -2)$

and radius $n/4$

$T_n: |z-(2-3i)| = \frac{1}{n}$ is a circle center $C_2(2,-3)$

and radius $1/n$

Here $S_n \cap T_n = \phi$

Both circles do not intersect each other

Case-1 : $C_1C_2 > n/4 + 1/n$

$$\sqrt{2} > \frac{n}{4} + \frac{1}{n}$$

then $n = 1,2,3,4$

Case-2 : $C_1C_2 < \left|\frac{n}{4} - \frac{1}{n}\right|$

$$\Rightarrow \sqrt{2} < \left|\frac{n^2-4}{4n}\right|$$

$\Rightarrow$ n has infinite solutions for $n \in \mathbf{N}$

64. (b) The system of equation has no solution.

$$D = \begin{vmatrix} 3\sin 3\theta & -1 & 1 \\ 3\cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix} = 0$$

$$21\sin 3\theta + 42\cos 2\theta - 42 = 0$$

$$\sin 3\theta + 2\cos 2\theta - 2 = 0$$

Number of solution is 7 in $(0,4\pi)$

65. (c) $\lim_{n\to\infty} n\left(1-\frac{n+1}{n^2}\right)^{\frac{1}{2}} + \alpha n + \beta = 0$

$$\lim_{n\to\infty}\left\{1 - \frac{1}{2}\left(\frac{n+1}{n^2}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{n+1}{n^2}\right)^2 + \cdots.\right\} + \alpha n + \beta = 0$$

$$\lim_{n\to\infty} n - \frac{1}{2} + \frac{1}{n} + \cdots. + n\alpha + \beta = 0$$

$$\alpha = -1, \beta = \frac{1}{2}$$

$$8(\alpha+\beta) = -4$$

66. (b) $f'(x) = e^{(4x^3-12x^2-180x+31)}(12(x^2-2x+7)(x+3)(x-5)+2(x-1))$

for $x \in [-3,0]$

$\Rightarrow f'(x) < 0$

$f(x)$ is decreasing function on $[-3,0]$

The absolute maximum value of the function $f(x)$ is at $x = -3$

$\Rightarrow \alpha = -3$

67. (**a**) $y(x) = ax^3 + bx^2 + cx + 5$ is passing through $(-2,0)$ then $8a - 4b + 2c = 5$

$y'(x) = 3axx^2 + 2bx + c$ touches x-axis at $(-2,0)$

$12a - 4b + c = 0$

again, for $x = 0, y'(x) = 3$

$c = 3$

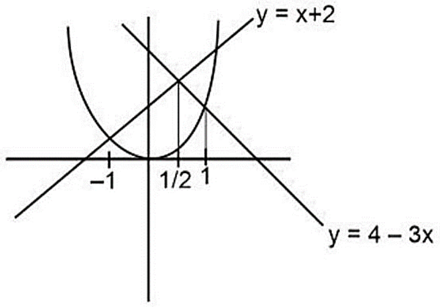
Solving eq. (1), (2) & (3) $a = -\frac{1}{2}, b = -\frac{3}{4}$

$y'(x) = -\frac{3}{2}x^2 - \frac{3}{2}x + 3$

$y(x)$ has local maxima at $x = 1$

$y(1) = \frac{27}{4}$

68. (b)



$A = \int_{-1}^{\frac{1}{2}} (x + 2 - x^2)dx + \int_{\frac{1}{2}}^{1} (4 - 3x - x^2)dx = \frac{17}{6}$

69. (a) $f(x)$ is periodic function whose period is 2

$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x dx = \frac{\pi^2}{10} \times 10\int_{0}^{2} f(x)\cos \pi x dx$

$= \pi^2 \left(\int_0^1 (1 - x)\cos \pi x dx + \int_1^2 (x - 1)\cos \pi x dx\right)$

Using by parts

$= \pi^2 \times \frac{4}{\pi^2} = 4$

70. (b)

$\frac{dy}{dx}=\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$

$$\frac{dy}{dx}=e^{2x}-\frac{6e^{x}}{2e^{2x}+9}$$

$$y=\frac{e^{2x}}{2}-\tan^{-1}\left(\frac{\sqrt{2}e^{x}}{3}\right)+c$$

If C passes through the point $\left(0,\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$

$$c=-\frac{\pi}{4}-\tan^{-1}\frac{\sqrt{2}}{3}$$

Again C passes through the point $\left(\alpha,\frac{1}{2}e^{2\alpha}\right)$

then $e^{\alpha}=\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

71. (a) $(x-y^2)dx+y(5x+y^2)dy=0$

$\frac{dy}{dx}=\frac{y^2-x}{y(5x+y^2)}$. Let $y^2=v$

$$\frac{2ydy}{dx}=2\left(\frac{y^2-x}{5x+y^2}\right)$$

$$\frac{dv}{dx}=2\left(\frac{v-x}{5x+v}\right)\quad v=kx$$

$$k+x\frac{dk}{dx}=2\left(\frac{kx-x}{5x+kx}\right)$$

$$x\frac{dk}{dx}=-\frac{(k^2+3k+2)}{k+5}$$

$$\int\frac{(5+k)}{(k+1)(k+2)}dk=\int-\frac{dx}{x}$$

$$\int\left(\frac{4}{k+1}-\frac{3}{k+2}\right)dk=-\int\frac{dx}{x}$$

$$4\ln(k+1)-3\ln(k+2)=-\ln x+\ln c$$

$$\frac{(k+1)^4}{(k+2)^3}=-\ln x+\ln c$$

$$c(y^2+2x)^3=(y^2+x)^4$$

72. (c) Let $B(x_1,x_1-2)$

$$\sqrt{(x_1-4)^2+(x_1-2-3)^2}=\frac{\sqrt{29}}{3}$$

Squaring on both side

$18x_1^2 - 162x_1 + 340 = 0$

$x_1 = \frac{51}{9}$ or $x_1 = \frac{10}{3}$

$y_1 = \frac{33}{9}$ or $y_1 = \frac{4}{3}$

Option (c) will satisfy $\left(\frac{10}{3}, \frac{4}{3}\right)$

73. (c) $(\alpha - 0)^2 + (\beta - 1)^2 = (\beta + 1)^2$

$\alpha^2 = 4\beta$

$x^2 = 4y$

$A = 2\int_0^4 \left(4 - \frac{x^2}{4}\right) dx = \frac{64}{3}$

74. (c) $a(x-3) + b(y+4) + c(z-7) = 0$

P: $\begin{array}{r} 9a - b - 5c = 0 \\ -11a - b + 5c = 0 \end{array}$

After solving DR's $\propto (1, -1, 2)$

Equation of plane

$x - y + 2z = 21$

$d = \frac{8}{\sqrt{6}}$

$d^2 = \frac{32}{3}$

75. (d) $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -\vec{a}$

$|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} = |\vec{a}|^2$

$|\vec{c}|^2 = 36$

$|\vec{c}| = 6$

S1: $|\vec{a} \times \vec{b} + \vec{c} \times \vec{b}| - |\vec{c}|$

$|(\vec{a} + \vec{c}) \times \vec{b}| - |\vec{c}|$

$|-\vec{b} \times \vec{b}| - |\vec{c}|$

$0 - 6 = -6$

S2: $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -\vec{a}$

$|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(\angle ACB) = |\vec{c}|^2$

$\cos(\angle ACB) = \sqrt{\frac{2}{3}}$

76. (c) $np + npq = 24$

$np \cdot npq = 128$

Solving (1) and (2) :

We get $p = \frac{1}{2}, q = \frac{1}{2}, n = 32.$

Now,

$P(X = 1) + P(X = 2)$

$= {}^{32}C_1 pq^{31} + {}^{32}C_2 p^2 q^{30}$

$= \frac{33}{2^{28}}$

77. (a)

$x^2 + \alpha x + \beta > 0, \forall x \in R$

$D = \alpha^2 - 4\beta < 0$

$\alpha^2 < 4\beta$

Total cases $= 6 \times 6 = 36$

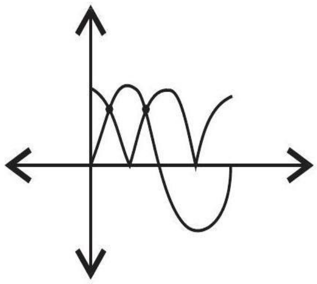
Fav. cases $= \beta = 1, \alpha = 1$

$\beta = 2, \alpha = 1,2$
$\beta = 3, \alpha = 1,2,3$
$\beta = 4, \alpha = 1,2,3$
$\beta = 5, \alpha = 1,2,3,4$
$\beta = 6, \alpha = 1,2,3,4$

Total favourable cases $= 17$

$P(x) = \frac{17}{36}$

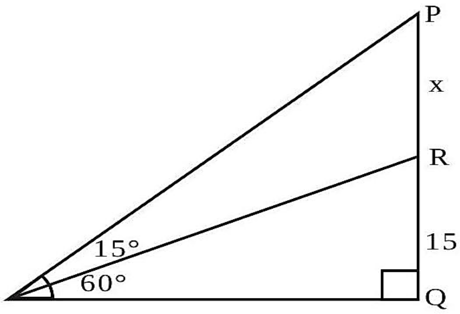
78. (c)



2 solutions in $(0,2\pi)$

So, 8 solutions in $[-4\pi, 4\pi]$

79. (a)



$$\frac{15}{AQ} = \tan 60^\circ$$

$$\frac{15 + x}{AQ} = \tan 75^\circ$$

$$\frac{(1)}{(2)} \Rightarrow x = 10\sqrt{3}$$

So, $PQ = 5(2\sqrt{3} + 3)$m

80. (d)

81. (17)

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow A^2 = A \Rightarrow A^n = A.$$

$\forall n \in \{1,2, \ldots ,100\}$

$$\text{Now, } B = A - I = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

$B^2 = -B$

$\Rightarrow B^3 = -B^2 = B$

$\Rightarrow B^5 = B$

$\Rightarrow B^{99} = B$

Also, $\omega^{3k} = 1$

So, $n =$ common of $\{1,3,5, \ldots ,99\}$ and

$\{3,6,9, \ldots ,99\} = 17$

82. (1492)

M	A	N	K	I	N	D

$$\left(\frac{4 \times 6!}{2!}\right) + (5! \times 0) + \left(\frac{4! \times 3}{2!}\right) + (3! \times 2) + (2! \times 1) + (1! \times 1) + (0! \times 0) + 1 = 1492$$

83. (6006)

$$\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$$

$$T_{r+1} = {}^{15}C_r \left(t^2 x^{\frac{1}{5}}\right)^{15-r} \cdot \frac{(1-x)^{\frac{r}{10}}}{t^r}$$

For independent of t,

$30 - 2r - r = 0$

$\Rightarrow r = 10$

So, Maximum value of ${}^{15}C_{10} x(1-x)$ will be at $x = \frac{1}{2}$

i.e. 6006

84. (38)

$x^2 - 8ax + 2a = 0$ $\quad$ $x^2 + 12bx + 6b = 0$

$p + r = 8a$ $\quad$ $q + s = -12b$

$pr = 2a$ $\quad$ $qs = 6b$

$\frac{1}{p} + \frac{1}{r} = 4$ $\quad$ $\frac{1}{q} + \frac{1}{s} = -2$

$\frac{2}{q} = 4$ $\quad$ $\frac{2}{r} = -2$

$q = \frac{1}{2}$ $\quad$ $r = -1$

$p = \frac{1}{5}$ $\quad$ $s = \frac{-1}{4}$

Now, $\frac{1}{a} - \frac{1}{b} = \frac{2}{pr} - \frac{6}{qs} = 38$

85. (27560)

$a_1 = b_1 = 1$

$a_2 = a_1 + 2 = 3$

$a_3 = a_2 + 2 = 5$

$a_4 = a_2 + 2 = 7$

$\Rightarrow a_n = 2n - 1$

$b_2 = a_1 + b_1 = 4$

$b_3 = a_3 + b_2 = 9$

$b_4 = a_4 + b_3 = 16$

$b_n = n^2$

$\sum_{n=1}^{15} a_n b_n$

$\sum_{n=1}^{15} (2n-1)n^2$

$\sum_{n=1}^{15} (2n^3 - n^2)$

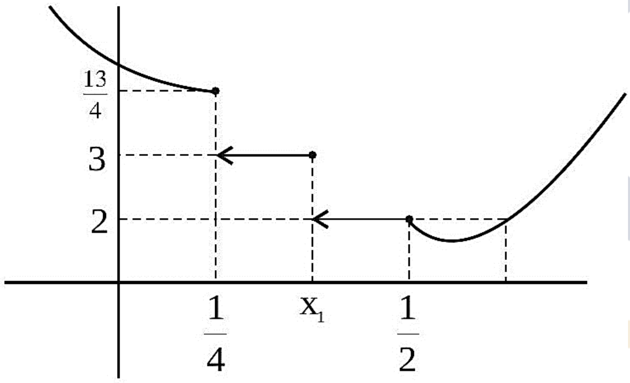
$= 2\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$

Put $n = 15$

$= \frac{2 \times 225 \times 16 \times 16}{4} - \frac{15 \times 16 \times 31}{6}$

$= 27560$

86. (3)



ND at $\frac{1}{4}, x_1, \frac{1}{2}$

87. (5) LHS

$\lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}} [nk \cdot n + 1 + 2 + \cdots + n]$

$= \lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}} \cdot \left[n^2 k + \frac{n(n+1)}{2}\right]$

$= \lim_{n\to\infty} \frac{(n+1)^{k-1}\cdot n^2\left(k+\frac{\left(1+\frac{1}{n}\right)}{2}\right)}{n^{k+1}}$

$\Rightarrow \lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(k+\frac{\left(1+\frac{1}{n}\right)}{2}\right)$

$\Rightarrow \left(k+\frac{1}{2}\right)$

RHS

$\Rightarrow \lim_{n\to\infty}\frac{1}{n^{k+1}}\left(1^k+2^k+\cdots+n^k\right)=\frac{1}{k+1}$

LHS = RHS

$\Rightarrow k+\frac{1}{2}=33\cdot\frac{1}{k+1}$

$\Rightarrow (2k+1)(k+1)=66$

$\Rightarrow (k-5)(2k+13)=0$

$\Rightarrow k=5 \text{ or } -\frac{13}{2}$

88. (2) $2p+f-1=0$

$2-pf-4p=0$

$2=p(f+4)$

$p=\frac{2}{f+4}$

$2p=1-f$

$\frac{4}{f+4}=1-f$

$f^2+3f=0$

f = 0 or -3

Hyperbola $3x^2-y^2=3, x^2-\frac{y^2}{3}=1$

$y=mx\pm\sqrt{m^2-3}$

It passes (1,0)

$o=m\pm\sqrt{m^2-3}$

m tends ∞

It passes (1,3)

$3 = m \pm \sqrt{m^2 - 3}$

$(3 - m)^2 = m^2 - 3$

$m = 2$

89. (10) $x^2 = \frac{64.5}{75}\left(y - \frac{3}{5}\right)$

equation of tangent at $\left(\frac{8}{5}, \frac{6}{5}\right)$

$x \cdot \frac{8}{5} = \frac{64}{15}\left(\frac{y+\frac{6}{5}}{2} - \frac{3}{5}\right)$ $3x - 4y = 0$

equation of family of circle is

$$\left(x - \frac{8}{5}\right)^2 + \left(y - \frac{6}{5}\right)^2 + \lambda(3x - 4y) = 0$$

It touches y axis so $f^2 = c$

$$x^2 + y^2 + x\left(3\lambda - \frac{16}{5}\right) + y\left(-4\lambda - \frac{12}{5}\right) + 4 = 0$$

$$\frac{\left(4\lambda + \frac{12}{5}\right)^2}{4} = 4$$

$\lambda = \frac{2}{5}$ or $\lambda = -\frac{8}{5}$

$\lambda = \frac{2}{5}$, $r = 1$

$\lambda = -\frac{8}{5}$, $r = 4$

$d_1 + d_2 = 10$

90. (3) DR's of line of shortest distance

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 2 & 2 & 1 \end{vmatrix} = -\hat{i} + 2\hat{j} - 2\hat{k}$$

angle between line and plane is $\cos^{-1}\sqrt{\frac{2}{27}} = \alpha$

$\cos\alpha = \sqrt{\frac{2}{27}}, \sin\alpha = \frac{5}{3\sqrt{3}}$

DR's normal to plane $(1, -1, -1)$

JEE MAINS PYQs

$$\sin\alpha = \left|\frac{-a-2+2}{\sqrt{4+4+1}\sqrt{a^2+1+1}}\right| = \frac{5}{3\sqrt{3}}$$

$$\sqrt{3}|a| = 5\sqrt{a^2+2}$$

$$3a^2 = 25a^2 + 50$$

No value of (a)