1. (c) $Z = \frac{A^2B^3}{C^4}$

In case of error

$$\frac{dZ}{Z} = \frac{2dA}{A} + \frac{3\,dB}{B} + \frac{4dC}{C}$$

$$\frac{\Delta Z}{Z} = \frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}$$

2. (c) $|\vec{A}| \neq 0$

$$\vec{A} \times \vec{A} = |\vec{A}||\vec{A}|\sin 0°\hat{n} = 0$$

3. (b) $|\hat{A} + \hat{B}| = \sqrt{|\hat{A}|^2 + |\hat{B}|^2 + 2|\hat{A}||\hat{B}|\cos\theta}$

$$= \sqrt{1 + 1 + 2\cos\theta}$$

$$= \sqrt{2(1 + \cos\theta)}$$

$$= \sqrt{2 \times 2\cos^2\frac{\theta}{2}}$$

$$= 2\cos\frac{\theta}{2}$$

$$|\hat{A} - \hat{B}| = \sqrt{|\hat{A}|^2 + |\hat{B}|^2 - 2|\hat{A}||\hat{B}|\cos\theta}$$

$$= \sqrt{2 - 2\cos\theta}$$

$$= 2\sin\frac{\theta}{2}$$

$$\frac{|\hat{A} + \hat{B}|}{|\hat{A} - \hat{B}|} = \cot\frac{\theta}{2}$$

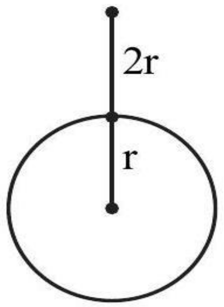
4. (b) $\vec{\tau} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 2 \\ 3 & 4 & -2 \end{vmatrix}$$
$$= \hat{\imath}(-2 - 8) - \hat{\jmath}(-4 - 6) + \hat{k}(8 - 3)$$
$$= -10\hat{\imath} + 10\hat{\jmath} + 5\hat{k}$$

5. (d) $g = \frac{Gm}{r^2}$

$$g' = \frac{Gm}{(3r)^2}$$

$$g' = \frac{Gm}{9r^2}$$



$g' = \frac{g}{9}$

6. **(c)** $v_t = \frac{2}{9}\frac{gr^2(\rho_p - \rho_1)}{\eta}$; $v_t \propto r^2$

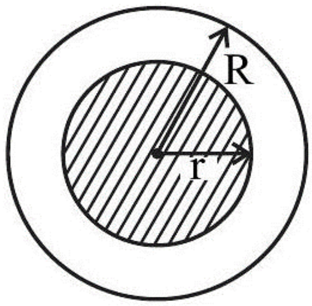
7. **(b)** $v_{rms} = \sqrt{\frac{3RT}{M}}$ and $v_{mp} = \sqrt{\frac{2RT}{M}}$

Thus $v_{rms} = \sqrt{\frac{3}{2}} v_{mp}$

8. **(c)** As the sheet is very large $\vec{E}$ is independent of distance from it. Thus $E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$

9. **(a)** AC generator converts mechanical energy into electrical energy. Galvanometer shows deflection when current passes through it so it is used to show presence of current in any wire. Transformer is used to step up or step down the voltage. Metals detectors contain inductor coils and use principle of induction and resonance in AC circuit.

10. **(B)** Use Ampere's law



$B.\ 2\pi r = \mu_0 \cdot \frac{I}{\pi R^2} \cdot \pi r^2$

Thus $B \propto r$

11. **(b)** Purely Inductive circuit

$\theta = \frac{\pi}{2}$

$\cos\frac{\pi}{2} = 0$

Average power $= 0$

12. **(b)** $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

$I = \frac{1}{2} \times (8.85 \times 10^{-12})(56.5)^2 \times (3 \times 10^8)$

$= 4.24 \mathrm{Wm}^{-2}$.

13. (b) $I_P = I + 9I + 2\sqrt{I \times 9I} \cos\frac{\pi}{2}$

$\mathrm{I_P} = 10\mathrm{I}$

$\mathrm{I_Q} = \mathrm{I} + 9\mathrm{I} + 2\sqrt{\mathrm{I} \times 9\mathrm{I}} \cos\pi$

$= 10\mathrm{I} - 6\mathrm{I} = 4\mathrm{I}$

$\therefore \mathrm{I_P} - \mathrm{I_Q} = 10\mathrm{I} - 4\mathrm{I} = 6\mathrm{I}$

14. (d) For total internal reflection, $\mathrm{i} > \theta_\mathrm{C}$

$\Rightarrow \sin\mathrm{i} > \sin\theta_\mathrm{C}$

$\Rightarrow \sin\mathrm{i} > \frac{\mu_R}{\mu_D}$

Also $\mu = \sqrt{\mu_\mathrm{r} \in_\mathrm{r}}$

$$\frac{\mu_\mathrm{R}}{\mu_\mathrm{D}} = \frac{\sqrt{1 \times 1}}{\sqrt{4 \times 1}} = \frac{1}{2}$$

From (1), $\sin\mathrm{i} > \frac{1}{2} \Rightarrow \mathrm{i} > 30^\circ, \mathrm{i} = 60^\circ$

15. (a) In Davisson-Germer experiment the electrons exhibit diffraction there by proving that electrons have wave nature.

16. (d) $\mathrm{v} \propto \frac{\mathrm{Z}}{\mathrm{n}} \propto \mathrm{Z}$ (n = constant)

$$\Rightarrow \frac{\mathrm{v_{He^+}}}{\mathrm{v_H}} = \frac{\mathrm{Z_{He^+}}}{\mathrm{Z_H}} = \frac{2}{1}$$

17. (d) Very small change in minority charge carriers produces high value of reverse bias current.

18. (b) Optical fibre frequency range is 1THz to 1000 THz.

19. (c) $v = \frac{c}{\mu}$

$$\Rightarrow \mathrm{v_B} = \frac{3 \times 10^8}{1.47} = 2.04 \times 10^8 = 20.4 \times 10^7 \mathrm{~m/s}$$

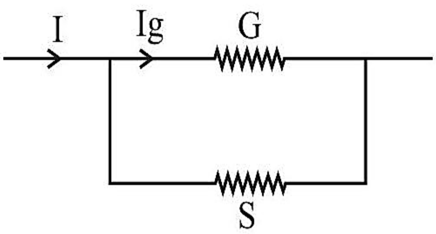
$\because \mathrm{v_A} - \mathrm{v_B} = 2.6 \times 10^7 \mathrm{~m/s}$

$\therefore \mathrm{v_A} = (20.4 + 2.6) \times 10^7 = 23 \times 10^7 \mathrm{~m/s}$

$$\therefore \frac{\mu_\mathrm{B}}{\mu_\mathrm{A}} = \frac{\mathrm{v_A}}{\mathrm{v_B}} = \frac{23 \times 10^7}{20.4 \times 10^7} = 1.13$$

20. (b) In galvanometer

$\Rightarrow (\mathrm{I} - \mathrm{I_g})\mathrm{S} = \mathrm{I_g}\mathrm{G}$



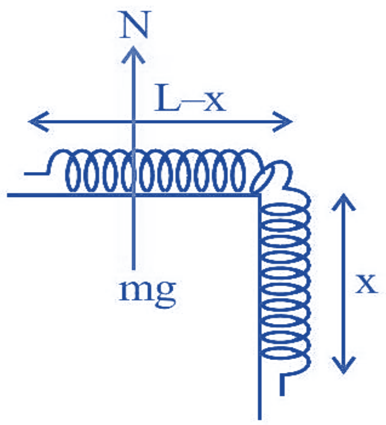
$$\frac{I_g}{I} = \frac{S}{S+G}$$

$$\Rightarrow \frac{1}{3} = \frac{\mathrm{S}}{\mathrm{S+G}} \Rightarrow \mathrm{S+G = 3\,S \Rightarrow G = 2\,S}$$

21. (2) Mass per unit length $= \lambda$

$\mathrm{N = mg} = \lambda(\mathrm{L-x})\mathrm{g}$

$\mathrm{fs_{max}} = \mu_s \mathrm{N}$



$\mathrm{fs_{max}} = (0.5)(\lambda)(\mathrm{L-x})\mathrm{g}$

And also $\mathrm{fs_{max} = m_x g}$

$0.5\lambda(\mathrm{L-x})\mathrm{g} = \lambda \mathrm{xg}$

$$\frac{\mathrm{L-x}}{2} = \mathrm{x}$$

$$\frac{\mathrm{L}}{2} = \frac{3\mathrm{x}}{2} \Rightarrow \mathrm{x} = \frac{\mathrm{L}}{3} = \frac{6}{3} = 2\ \mathrm{m}$$

22. (600)

$\mathrm{U_i + K_i = U_f + K_f}$

$$\Rightarrow 0 + \frac{1}{2}\,\mathrm{m}(12)^2 = \frac{1}{2}\,\mathrm{K}(0.3)^2 + \frac{1}{2}\,\mathrm{m}(6)^2$$

$\Rightarrow 0.5(12^2 - 6^2) = \mathrm{K}(0.3)^2$

$\mathrm{K = 600\ N/m}$

23. (100) $F = \eta A \frac{\Delta v_x}{\Delta y}$

$$\frac{F}{A}=\eta\frac{\Delta v_x}{\Delta y}$$

$$\Rightarrow 10^{-3}=10^{-2}\times\frac{36\times1000}{h\times3600}$$

$$\Rightarrow h=10^{-2}\times\frac{36\times1000}{10^{-3}\times3600}=100\text{ m}$$

24. (31) Heat rejected $= mL_f + mS\Delta T$

$= (50\times540)+50(1)(100-20)$

$= 31000\text{Cal}$

$= 31\times10^3\text{Cal}$

25. (80) $f_1=\frac{2v}{2l_1}$

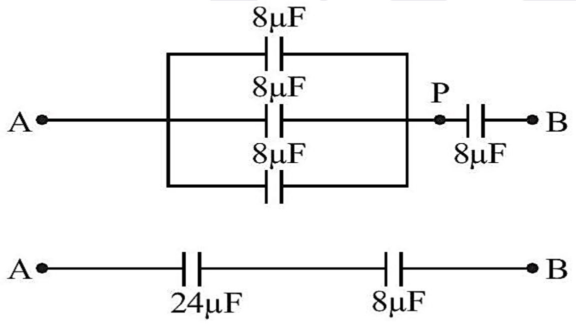
$f_2=\frac{v}{4l_2}$

$f_1=f_2$

$=\frac{2v}{2l_1}=\frac{v}{4l_2}$

$l_1=4l_2=80\text{ cm}$

26. (6) Two capacitors are short circuited



Finally equivalent capacitance

$=\frac{24\times8}{24+8}=\frac{24\times8}{32}=6\mu F$

27. (450) $H=i^2Rt$

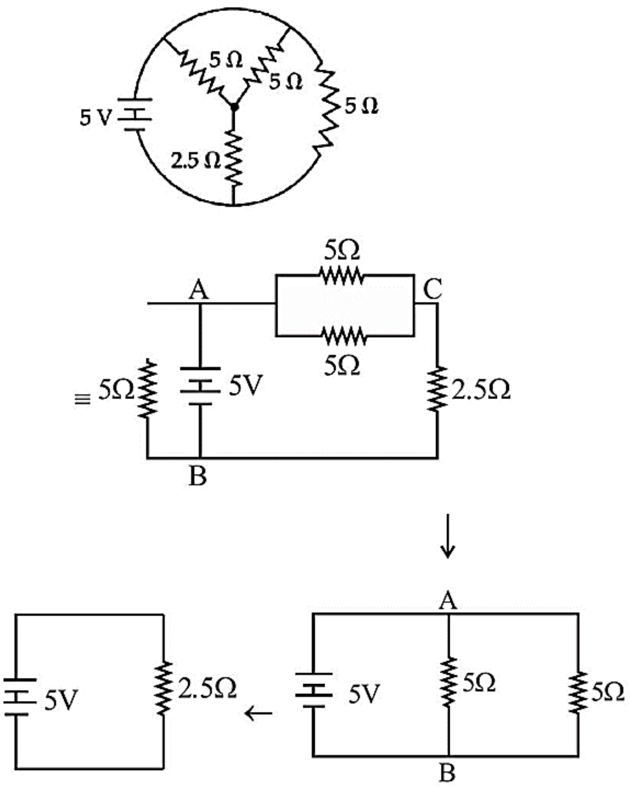
$300=2^2\times R\times15$

$\Rightarrow R=\frac{300}{60}=5\Omega$

Now, for $i=3\text{ A}, t=10\text{ s}, R=5\Omega$

$H=3^2\times5\times10=450\text{ J}$

28. (2)



Current supplied by 5 V battery

$$= \frac{5\text{ V}}{2.5\Omega} = 2\text{ A}$$

29. (4) $I = 2\sin(t^2) \Rightarrow dI = 4t\sin(t^2)dt$

If $\mathrm{I} = 0 \Rightarrow \mathrm{t} = 0$

and $\mathrm{I} = 2 \Rightarrow 2 = 2\sin \mathrm{t}^2$

$$\Rightarrow \mathrm{t} = \sqrt{\frac{\pi}{2}}$$

$$\mathrm{E} = \int \mathrm{LIdI}$$

$$= \int 2 \times 2\sin(\mathrm{t}^2) \times 4\mathrm{t}\cos(\mathrm{t}^2)\mathrm{dt}$$

$$= 8\int_0^{\sqrt{\pi/2}} \mathrm{t}\sin(2\mathrm{t}^2)\mathrm{dt}$$

$$= 2[-\cos(2\mathrm{t}^2)]_0^{\sqrt{\pi/2}}$$

$$= 2[-\cos\pi + \cos 0] = 4$$

30. (2) $\vec{\mathrm{F}} = 10\hat{\imath} + 5\hat{\jmath}$

$\mathrm{m} = 100\text{ g} = 0.1\text{ kg}$

$\vec{a}=\frac{\vec{F}}{m}=100\hat{i}+50\hat{j}$

$\vec{S}=\vec{u}t+\frac{1}{2}\vec{a}t^2=\frac{1}{2}\vec{a}t^2(\text{ as }\vec{u}=0)$

$=\frac{1}{2}(100\hat{i}+50\hat{j})2^2$

$=200\hat{i}+100\hat{j}$

$=a\hat{\imath}+b\hat{\jmath}$

$a=200, b=100$

$\therefore \frac{a}{b}=2$

31. **(c)** Bond strength ∝ Bond order removal of electron from antibonding MO increases B.O.

$NO\&O_2$ has valence e − in π *orbital.

32. **(b)** The diameter of dispersed particle should be somewhat below or near the wavelength of light.

33. **(d)** Isoelectronic species have same no. of electrons Al^{+3}, O^{2-}, Mg^{+2} all have 10 electrons.

34. **(c)** $Au+NaCN \rightarrow Na[Au(CN)_2]$

$Zn+Na[Au(CN)_2] \rightarrow Na_2[Zn(CN)_4]+Au$

35. **(c)** Electron deficient species have less than 8 electrons (or two electrons for H) in their valence (incomplete octet) B_2H_6, BCl_3 have incomplete octet.

36. **(d)** Highest ionic mobility corresponds to lowest extent of hydration and highest size of gaseous ion. Hence Sr^{2+} has the highest ionic mobility in its aqueous solution

37. **(c)** $AgCl+2NH_3 \rightarrow \underset{\text{soluble}}{[Ag(NH_3)_2]^+Cl^-}$

38. **(b)** Cerium exists in two different oxidation state +3 , +4

$Ce^{+4}+e^- \rightarrow Ce^{3+}\ E^0=+1.61\ V$

$Ce^{+3}+3e^- \rightarrow Ce\ E^0=-2.336\ V$

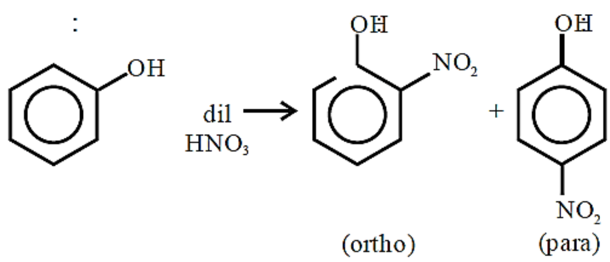
It shows Ce^{+4} acts as a strong oxidising agent & accepts electron.

39. **(a)** Strongest oxidising agent have highest reduction potential value

$E^0_{Mn^{+3}/Mn^{+2}}=1.51\ V$ (highest)

40. **(a)** Eutrophication of water body results in loss of Biodiversity.

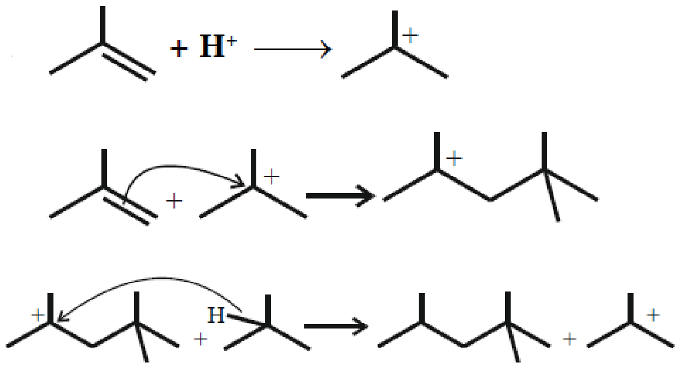
41. **(c)**



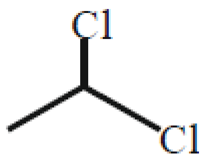
Para product has higher boiling point than ortho as intermolecular H-bond is possible in former, where as intramolecular H-bond is possible in ortho product. Steam distillation can separate them as ortho product is steam volatile.

42. (c) Dihedral angle: It's the angle b/w 2 specified groups ($-CH_3$ here) Staggered form is Given in option (c) & the angle is 180°

43. (b)

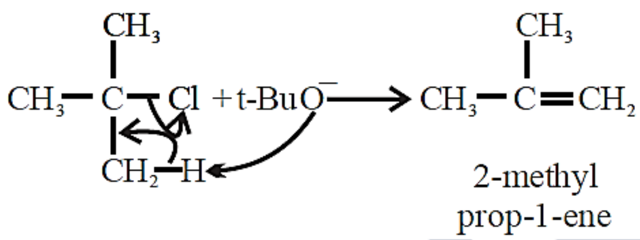


44. (d)

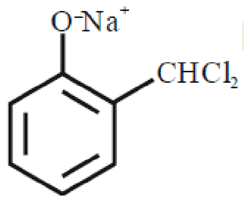


"1, 1-Dichloroethane is Ethylidene chloride"

45. (d) We have been given a bulky base, hence elimination will take place & not substitution.

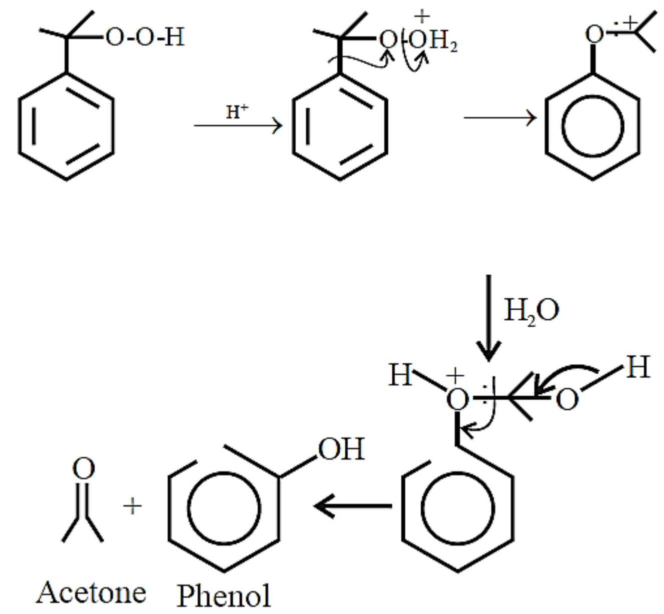


46. (c) It's a classic Reimer-Tiemann reaction.

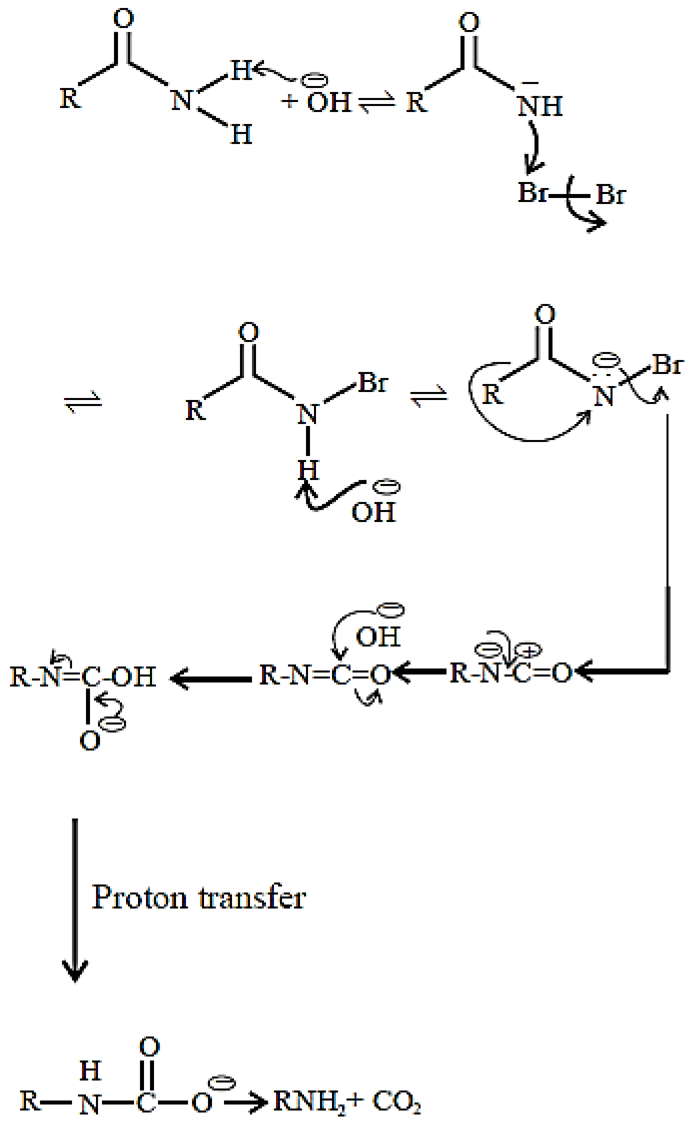


Will be the intermediate formed.

47. (c) Given reaction is cumene-Peroxide method for the preparation of phenol. In this reaction



48. (c) The given reaction is Hoffmann-Bromide degradation method.



49. (c) Micelle formation only takes place above CMC.

50. (b) Alitame is a second -generation dipeptide sweetner that is 200 times sweeter than sucrose.

51. (5418)

M.M. of $C_7H_5\ N_3O_6$ is $84 + 5 + 42 + 96 = 227$

$$n_{C_7H_5\ N_3O_6} = \frac{681}{227} = 3$$

$$n_N = \frac{681}{227} \times 3 = 9 \text{ mol}$$

no. of N atoms $= 9 \times 6.02 \times 10^{23}$

$= 5418 \times 10^{21}$

$\therefore$ The answer is 5418

52. (1) Unit cell formula $-Na_4Cl_4$

Mass per unit cell $= \frac{Z \times M.M.}{N_A} g$

$$= \frac{4 \times 58.5}{N_A} g$$

$$d_{\text{unit cell}} = \frac{m}{V} = \frac{m}{a^3}$$

$$\Rightarrow \frac{4 \times 58.5}{N_A \cdot a^3} = 43.1$$

$\Rightarrow a^3 = 9.02 \times 10^{-24}\ cm^3$

$\Rightarrow a = 2.08 \times 10^{-8}$ cm

$\Rightarrow a = 2.08 \times 10^{-10}$ m

Also $a = 2(r_{Na^+} + r_{Cl^-})$

$\Rightarrow r_{Na^+} + r_{Cl^-} = 1.04 \times 10^{-10}$ m

$\therefore$ 1

53. (4) We can -not calculate I.E. of lithium atom.

54. (300)

$\Delta G = \Delta H - T\Delta S = 0$ at equilibrium

$\Rightarrow -165 \times 10^3 - T \times (-505) = 0$

$\Rightarrow T = 300$ K

The answer is 300

55. (0)

$n_{H_2SO_4}$ in $Sol^n A = 50\%$ of original solution $= 0.01$ m mol.

$n_{H_2SO_4}$ in Final solution $= 0.01 + 0.01$

$= 0.02$mmol

$= 0.00002 \times 10^3$mmol

56. (710)

$N_2O_4 \rightleftharpoons 2NO_2$
$t = 0$ 1 mol
$t = t$ $(1 - 0.5)$mol 0.5×2 mol
$= 0.5$ mol 1 mol

$$k_P = \frac{\left(\frac{1}{1.5} \times 1\right)^2}{\left(\frac{0.5}{1.5} \times 1\right)} = \frac{1}{0.75} = \frac{100}{75}$$

$= 1.33$
$\Delta G^0 = -RT\ell nk_P$
$= -8.31 \times 300 \times \ell n(1.33) = -710.45$ J/mol
$= -710$ J/mol.

57. (23)

$$\underset{\text{Cathode}}{Fe^{+3}} + \underset{\text{anode}}{I^-} \longrightarrow I_2 + Fe^{+2}$$

$E^0_{Cell} = E^0_{cathode} - E^0_{anode}$

$= 0.77 - 0.54$

$= 0.23$

$= 23 \times 10^{-2}$ V

58. (1) γ_1 A $+ \gamma_2$ B $\longrightarrow \gamma_3 C + \gamma_4 D$

Given: $+\frac{d[D]}{dt} = \frac{-3}{2}\frac{d[B]}{dt}$

$$\Rightarrow \frac{-1}{2}\frac{d[B]}{dt} = \frac{+1}{3}\frac{d[D]}{dt}$$

$$-\frac{d[B]}{dt} = -2\frac{d[A]}{dt} \Rightarrow -\frac{1}{2}\frac{d[B]}{dt} = \frac{-d(a)}{dt}$$

$$+\frac{d[B]}{dt} = 9\text{mmoldm}^{-3}\ s^{-1}$$

$$\frac{+d[C]}{dt} = \frac{20 - 10}{10} = 1\text{mmoldm}^{-3}\ s^{-1}$$

$$\frac{+d[C]}{dt} = \frac{1}{9} \times \frac{+d[D]}{dt}$$

$1\text{ A} + 2\text{ B} \longrightarrow \frac{1}{3}\text{C} + 3\text{D}$

$\Rightarrow 3\text{ A} + 6\text{ B} \longrightarrow \text{C} + 9\text{D}$

Rate of reaction $= \frac{+d[C]}{dt} = 1\text{mmoldm}^{-3}\text{ s}^{-1}$

59. (746) $\Delta_t = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3.08\times10^{8}}{600\times10^{-9}}$

$$= \frac{6.63 \times 3.08 \times 10^{-17}}{600}$$
$$= 0.034034 \times 10^{-17}$$

$= 340.34 \times 10^{-21}$ J

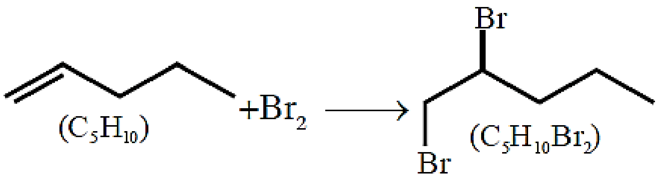
$\Delta_0 = \frac{9}{4}\Delta_t$

$= \frac{9}{4} \times 340.34 \times 10^{-21}$

$= 765.765 \times 10^{-21}$ J

$\approx 766 \times 10^{-21}$ J

60. (1143)



moles of Br_2 = moles of C_5H_{10}

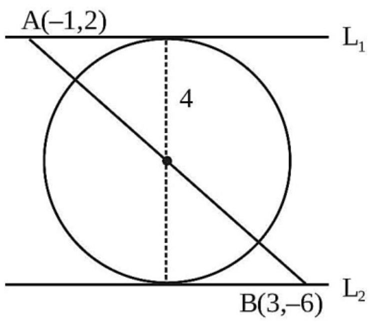
$\Rightarrow \frac{w}{160} = \frac{5}{70}$

$\Rightarrow w = \frac{5 \times 160}{70}$ g

$= 11.428$ g

$= 1142.8 \times 10^{-2}$ g $\approx 1143 \times 10^{-2}$ g

61. (c)



$L_1: 4x - 3y + K_1 = 0$

$L_2: 4x - 3y + K_2 = 0$

now

$-4 - 6 + K_1 = 0 \Rightarrow K_1 = 10$

$12 + 18 + K_2 = 0 \Rightarrow K_2 = -30$

$\Rightarrow$ Tangent to the circle are

$4x - 3y + 10 = 0$

$4x - 3y - 30 = 0$

Length of diameter $2r = \frac{|10+30|}{5} = 8$

$\Rightarrow r = 4$

Now centre is mid -point of A & B

$x = 1, y = -2$

Equation of circle

$(x-1)^2 + (y+2)^2 = 16$

62. (c)

$$\int_0^{\pi} \frac{e^{\cos x} \sin x}{(1+\cos^2 x)(e^{\cos x} + e^{-\cos x})} dx$$

Use King's property

$$I = \int_0^{\pi} \frac{e^{-\cos x} \sin x}{(1+\cos^2 x)(e^{-\cos x} + e^{\cos x})} dx$$

On adding equation (1) and (2), we get

$$2I = \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx = 2\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$$

On putting $\cos x = t$, we get

$$I = \int_0^{1} \frac{dt}{1+t^2} = (\tan^{-1} t)_0^1 = \frac{\pi}{4}$$

63. (a) $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9} = \lambda$

$a + b = 7\lambda, b + c = 8\lambda, a + c = 9\lambda$

$\Rightarrow a + b + c = 12\lambda$

Now $a = 4\lambda, b = 3\lambda, c = 5\lambda$

$\because c^2 = b^2 + a^2$

$\angle C = 90°$

$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}ab$

$\frac{R}{r} = \frac{c}{2\sin C} \times \frac{s}{\Delta} = \frac{c}{2} \times \frac{6\lambda}{\frac{1}{2}ab} = \frac{c}{ab} \times 6\lambda = \frac{5}{2}$

64. (c) $f: N \to R, f(x+y) = 2f(x)f(y)$

$f(1) = 2,$

$$\sum_{k=1}^{10} f(\alpha+k) = 2f(\alpha)\sum_{k=1}^{10} f(k)$$
$$= 2f(\alpha)(f(1) + f(2) + \cdots.. + f(10))$$

From (1)

$f(2) = 2f^2(1) = 2^3$

$f(3) = 2f(2)f(1) = 2^5$

$\vdots$

$f(10) = 2^9 f^{10}(1) = 2^{19}$

$f(\alpha) = 2^{2\alpha-1}; \alpha \in N$

from (2)

$$\sum_{k=1}^{10} f(\alpha+k) = 2(2^{2\alpha-1})(2 + 2^3 + 2^5 + \cdots. + 2^{19})$$

$$\frac{512}{3}(2^{20}-1) = 2^{2\alpha}\left(2\frac{(2^{20}-1)}{3}\right)$$

Hence $\alpha = 4$

65. (b) $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$$A\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$\Rightarrow c_1 = 1, c_2 = 1, c_3 = 2$

$$A\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} C_1 + a_1 \\ C_2 + a_2 \\ C_3 + a_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$\Rightarrow a_1 = -2, a_2 = -1, a_3 = -1$

$$A\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}a_1+b_1\\a_2+b_2\\a_3+b_3\end{bmatrix} = \begin{bmatrix}1\\1\\0\end{bmatrix}$$

$\Rightarrow b_1 = 3,\ b_2 = 2,\ b_3 = 1$

$$\Rightarrow A = \begin{bmatrix}-2 & 3 & 1\\-1 & 2 & 1\\-1 & 1 & 2\end{bmatrix}$$

$$\Rightarrow\ A - 2I = \begin{bmatrix}-4 & 3 & 1\\-1 & 0 & 1\\-1 & 1 & 0\end{bmatrix}$$

$|A - 2I| = 0$

Now, $\begin{bmatrix}-4 & 3 & 1\\-1 & 0 & 1\\-1 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}4\\1\\1\end{bmatrix}$

$-4x_1 + 3x_2 + x_3 = 4$

$-x_1 + x_3 = 1$

$-x_1 + x_2 = 1$

$(1) - [(2) + 3(3)]$

$0 = 0 \Rightarrow$ infinite solutions

66. (a) $f(x) = x^3 + x - 5$

$\Rightarrow f'(x) = 3x^2 + 1 \Rightarrow$ increasing function

$\Rightarrow$ invertible

$\Rightarrow g(x)$ is inverse of $f(x)$

$\Rightarrow g(f(x)) = x$

$\Rightarrow\ g'(f(x))f'(x) = 1$

$f(x) = 63$

$\Rightarrow x^3 + x - 5 = 63$

$\Rightarrow x = 4$

put $x = 4$

$$g'(f(4))f'(4) = 1$$
$$g'(63) \times 49 = 1\ \{f'(4) = 49\}$$
$$g'(63) = \frac{1}{49}$$

67. (c)

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \to (\sim p \vee q)$	$p \to \sim q$	$\sim (p \to \sim q)$	$p \wedge \sim q$	p_2
T	T	F	F	T	T	F	T	F	F
T	F	F	T	F	F	T	F	T	F
F	T	T	F	T	T	T	F	F	F
F	F	T	T	T	T	T	F	F	F

$p \to (\sim p \vee q)$ is F when p is true q is false

From table

P1 & P2 both are false

68. (d)

$$\frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \frac{1}{2^3 \cdot 3^8} + \cdots .. + \frac{1}{2^{10} \cdot 3} = \frac{K}{2^{10} \cdot 3^{10}}$$

$K = 2^9 + 2^8 \cdot 3 + 2^7 \cdot 3^2 + \cdots .. + 3^9$

$$= \frac{2^9\left(\left(\frac{3}{2}\right)^{10} - 1\right)}{\frac{3}{2} - 1} = 3^{10} - 2^{10}$$

Now, $3^{10} - 2^{10} = (3^5 - 2^5)(3^5 + 2^5)$

$$\begin{aligned} &= (211)(275) \\ &= (35 \times 6 + 1)(45 \times 6 + 5) \\ &= 6\lambda + 5 \end{aligned}$$

Remainder is 5 .

69. (a) $\text{Lt}\frac{f(x)}{x \to 1} = f'(1)(\text{ and } f(1) = 0)$

$f(x) + f'(x) + t''(x) = x^5 + 64$

$f'(x) + f''(x) + f'''(x) = 5x^4$

$f''(x) + f'''(x) + \text{fiv}^{iv}(x) = 20x^3$

$f'''(x) + f^{iv}(x) + f^{v}(x) = 60x^2$

$\therefore f^{v}(x) - f''(x) = 60x^2 - 20x^3$

$\Rightarrow 120 - f''(1) = 40 \Rightarrow f''(1) = 80$

Also $f(1) + f'(1) + f''(1) = 65 \Rightarrow f'(1) = -15.$

70. (c) (a) $P(E_1) \cdot P(E_2) = \frac{1}{6}\frac{1}{4} = \frac{1}{24} \neq P(E_1 \cap E_2)$

(b) $P(E_1' \cap E_2') = 1 - P(E_1 \cup E_2)$

$$\begin{aligned} &= 1 - \left(P(E_1) + P(E_2) - P(E_1 \cap E_2)\right) \\ &= 1 - \left(\frac{1}{6} + \frac{1}{4} - \frac{1}{8}\right) = \frac{17}{24} \end{aligned}$$

$P(E_1')P(E_2) = \frac{5}{6} \times \frac{1}{4} = \frac{5}{24}$

(c) $P(E_1 \cap E_2') = P(E_1) - P(E_1 \cap E_2) = \frac{1}{6} - \frac{1}{8} = \frac{1}{24}$

(d) $P(E_1 \cap E_2) = P(E_2) - P(E_1 \cap E_2) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$

71. (a) $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$$

$A^3 = -4\ A$

$A^4 = (-4I)(-4I) = (-4)^2 I$

$A^5 = (-4)^2\ A,\ \ A^6 = (-4)^3 I$

$$M = \sum_{k=1}^{10} A^{2k} = A^2 + A^4 + \cdots . + A^{20}$$

$= [-4 + (-4)^2 + (-4)^3 + \cdots . + (-4)^{20}]I$

$= -4\lambda I$

$\Rightarrow$ M is symmetric matrix

$$N = \sum_{k=1}^{10} A^{2k-1} = A + A^3 + \cdots .. + A^{19}$$

$= A[1 + (-4) + (-4)^2 + \cdots \ldots + (-4)^9]$

$= \lambda A \Rightarrow$ skew symmetric

$\Rightarrow N^2$ is symmetric matrix

$\Rightarrow MN^2$ is non identity symmetric matrix

72. (d) $\int \left(\frac{x(\cos x - \sin x)}{e^x+1} + \frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right) dx = \frac{xg(x)}{e^x+1} + c$

On differentiating both sides w.r.t. x, we get

$$\left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2}\right) = \frac{(e^x + 1)(g(x) + xg'(x)) - e^x \cdot x \cdot g(x)}{(e^x + 1)^2}$$

$(e^x + 1)x(\cos x - \sin x) + g(x)(e^x + 1 - xe^x)$

$= (e^x + 1)(g(x) + xg'(x)) - e^x \cdot x \cdot g(x)$

$\Rightarrow g'(x) = \cos x - \sin x$

$\Rightarrow g(x) = \sin x + \cos x + C$

$g(x)$ is increasing in $(0, \pi/4)$

$g''(x) = -\sin x - \cos x < 0$

$\Rightarrow g'(x)$ is decreasing function

let $h(x) = g(x) + g'(x) = 2\cos x + C$

$\Rightarrow h'(x) = g'(x) + g''(x) = -2\sin x < 0$

$\Rightarrow$ h is decreasing

let $\phi(x) = g(x) - g'(x) = 2\sin x + C$

$\Rightarrow \phi'(x) = g'(x) - g''(x) = 2\cos x > 0$

$\Rightarrow \phi$ is increasing

73. **(a)** $f(x) = \log_e(x^2+1) - e^{-x} + 1$

$\Rightarrow f'(x) = \dfrac{2x}{x^2+1} + e^{-x} > 0\ \forall x \in R$

$\Rightarrow$ f is strictly increasing

$g(x) = \dfrac{1-2e^{2x}}{e^x} = e^{-x} - 2e^x$

$\Rightarrow g'(x) = -(2e^x + e^{-x}) < 0\ \forall x \in R$

$\Rightarrow$ g is decreasing

Now $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$

$\Rightarrow g\left(\dfrac{(\alpha-1)^2}{3}\right) > g\left(\alpha - \dfrac{5}{3}\right)$

$\Rightarrow \dfrac{(\alpha-1)^2}{3} < \alpha - \dfrac{5}{3}$

$\Rightarrow \alpha^2 - 5\alpha + 6 < 0$

$\Rightarrow (\alpha-2)(\alpha-3) < 0$

$\Rightarrow \alpha \in (2,3)$

74. **(b)** $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\vec{a} = \lambda\left(\dfrac{1}{\sqrt{3}}\hat{\imath} + \dfrac{1}{\sqrt{3}}\hat{\jmath} + \dfrac{1}{\sqrt{3}}\hat{k}\right) = \dfrac{\lambda}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$

Now projection of $\vec{a}$ on $\vec{b} = 7$

$\Rightarrow \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = 7$

$\dfrac{\lambda}{\sqrt{3}}\dfrac{(\hat{\imath} + \hat{\jmath} + \hat{k})\cdot(3\hat{\imath} + 4\hat{\jmath})}{5} = 7$

$\lambda = 5\sqrt{3}$

$\vec{a} = 5(\hat{\imath} + \hat{\jmath} + \hat{k})$

now $\vec{b} = 5\alpha(\hat{\imath} + \hat{\jmath} + \hat{k}) + \beta(\hat{\imath})$

$\vec{a} \cdot \vec{b} = 0$

$\Rightarrow 25\alpha(3) + 5\beta = 0$

$\Rightarrow 15\alpha + \beta = 0 \Rightarrow \beta = -15\alpha$

$\vec{b} = 5\alpha(-2\hat{\imath} + \hat{\jmath} + \hat{k})$

$|\vec{b}| = 5\sqrt{3}$

$\Rightarrow \alpha = \pm\frac{1}{\sqrt{2}}$

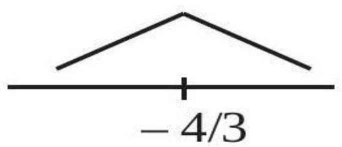
$\vec{b} = \pm\frac{5}{\sqrt{2}}(-2\hat{\imath} + \hat{\jmath} + \hat{k})$

Projection of $\vec{b}$ on $3\hat{\imath} + 4\hat{\jmath}$ is

$\frac{\vec{b} \cdot (3\hat{\imath} + 4\hat{\jmath})}{5} = \pm\frac{5}{\sqrt{2}}\left(\frac{-6+4}{5}\right) = \pm\sqrt{2}$

75. (b)

$$(x+1)dy - ydx = e^{3x}(x+1)^2$$
$$\frac{(x+1)dy - ydx}{(x+1)^2} = e^{3x}$$
$$d\left(\frac{y}{x+1}\right) = e^{3x} \Rightarrow \frac{y}{x+1} = \frac{e^{3x}}{3} + C$$
$$\left(0, \frac{1}{3}\right) \Rightarrow C = 0 \Rightarrow y = \frac{(x+1)e^{3x}}{3}$$
$$\frac{dy}{dx} = \frac{1}{3}((x+1)3e^{3x} + e^{3x}) = \frac{3^{3x}}{3}(3x+4)$$
$$\underbrace{(3/3}_{-4/3}$$



Clearly, $x = \frac{-4}{3}$ is point of local minima

76. (c) $C_1: x^2 + y^2 = 2$

$C_2: y^2 = x$

Let tangent to parabola be $y = mx + \frac{1}{4m}$.

It is also a tangent of circle so distance from centre of circle $(0,0)$ will be $\sqrt{2}$.

$$\left|\frac{\frac{1}{4m}}{\sqrt{1+m^2}}\right| = \sqrt{2} \Rightarrow 1 = 32\ m^2 + 32\ m^4$$

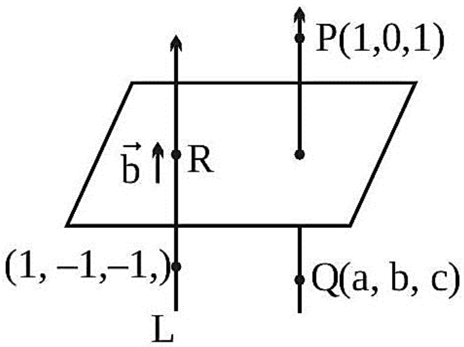
by solving

$m^2 = \frac{3\sqrt{2}-4}{8}$, $m^2 = \frac{-3\sqrt{2}-4}{8}$ (rejected)

$$m = \pm\sqrt{\frac{3\sqrt{2}-4}{8}}$$

so, $8|m_1 m_2| = 3\sqrt{2} - 4$

77. (b)



Let parallel vector of $L = \vec{b}$

mirror image of Q on given plane $x + y + z = 5$

$$\frac{a-1}{1} = \frac{b-0}{1} = \frac{c-1}{1} = \frac{-2(2-5)}{3}$$

$a = 3, b = 2, c = 3$

$Q \equiv (3,2,3)$

$\because \vec{b} \parallel \overrightarrow{PQ}$

so, $\vec{b} = (1,1,1)$

Equation of line

$$L: \frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{1}$$

Let point R, $(\lambda + 1, \lambda - 1, \lambda - 1)$

lying on plane $x + y + z = 5$,

so, $3\lambda - 1 = 5$

$\Rightarrow \lambda = 2$

Point R is (3,1,1)

$QR^2 = 5$

78. (c) $y^2dx - xydy = -(x^2 + y^2)dy$

$y(ydx - xdy) = -(x^2 + y^2)dy$

$-y(xdx - ydx) = -(x^2 + y^2)dy$

$$\frac{xdy - ydx}{x^2} = \left(1 + \frac{y^2}{x^2}\right)\frac{dy}{y}$$

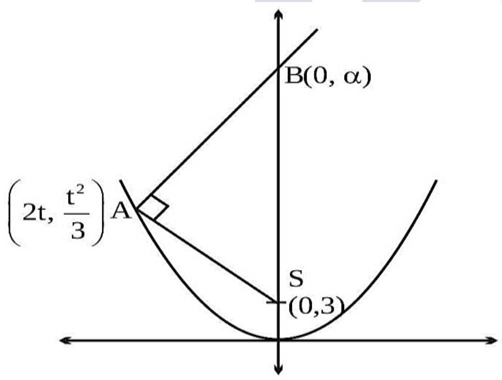
$$\Rightarrow \frac{d(y/x)}{1 + \frac{y^2}{x^2}} = \frac{dy}{y}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \ln y + C$$

$$(\alpha, \sqrt{3}\alpha) \Rightarrow \frac{\pi}{3} = \ln(\sqrt{3}\alpha) + \frac{\pi}{4}$$

$$\therefore \ln(\sqrt{3}\alpha) = \frac{\pi}{12}$$

79. (d)



parabola $x^2 = 12y$

$SA \perp SB$
so, $m_{AS} \cdot m_{AB} = -1$

$$\frac{\left(3 - \frac{t^2}{3}\right)}{(0 - 2t)} \cdot \frac{\left(\alpha - \frac{t^2}{3}\right)}{(0 - 2t)} = -1$$

by solving

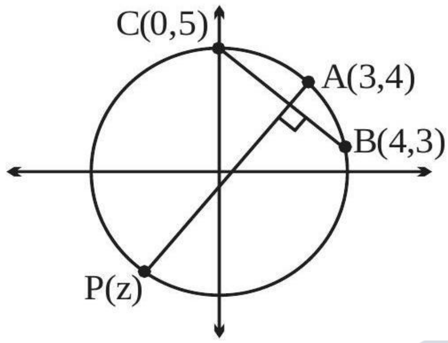
$$3\alpha = \frac{27t^2 + t^4}{t^2 - 9}$$

ordinate of centriod of $\triangle$ SAB $= K = \frac{\alpha+\frac{t^2}{3}+3}{3}$

$$k = \frac{9+3\alpha+t^2}{9}$$

$$\lim_{t\to1} k = \lim_{t\to1} \frac{1}{9}\left(9+t^2+\frac{27t^2+t^4}{(t^2-9)}\right) = \frac{13}{18}$$

80. (b)



Slope of BC $= \frac{3-5}{4-0} = -\frac{1}{2}$

Slope of AP $= 2$

equation of AP : $y-4 = 2(x-3)$

$\Rightarrow y = 2(x-1)$

P lies on circle $x^2+y^2 = 25$

$\Rightarrow x^2 + (2(x-1))^2 = 25$

$\Rightarrow x = -\frac{7}{5}$ and $y = -\frac{24}{5}$

$\Rightarrow \arg(z) = \tan^{-1}\left(\frac{24}{7}\right) - \pi$

81. (286)

82. (63) x y z $\leftarrow$ odd number

$z = 1,3,5,7,9$

$x+y+z = 7,14,21$ [sum of digit multiple of 7] $_{1}\,x_{\text{to9}}$ $+ y = 6,4,2,13,11,9,7,5,20,18,16,14,12$ $x+y = 6 \Rightarrow$ $(1,5), (2,4), (3,3), (4,2), (5,1), (6,0)$

$\rightarrow$ T.N. $= 6$

$x+y = 4 \Rightarrow (1,3), (2,2), (3,1), (4,0)$

$\rightarrow$ T.N $= 4$

$x+y = 2 \Rightarrow (1,1), (2,0)$

$\rightarrow$ T.N. $= 2$

$x + y = 13 \Rightarrow (4,9), (5,8), (6,7), (7,6), (8,5), (9,4)$

$\rightarrow$ T.N. $= 6$

$x + y = 11 \Rightarrow (2,9), (3,8), (4,7), (5,6), (6,5),$

$(6,5), (7,4), (8,3), (9,2)$

$\rightarrow$ T.N. $= 8$

$x + y = 9 \Rightarrow (1,8), (2,7), (3,8), (4,5), (5,4), \ldots. (8,1), (9,0)$

$\rightarrow$ T.N. $= 9$

$x + y = 7 \Rightarrow (1,8), (2,5), (3,4), \ldots. (8,1), (7,0)$

$\rightarrow$ T.N. $= 7$

$x + y = 5 \Rightarrow (1,4), (2,3), (3,2), (4,1), (5,0)$

$\rightarrow$ T.N. $= 5$

$x + y = 20 \Rightarrow$ Not possible

$x + y = 18 \Rightarrow (9,9) \rightarrow$ T.N. $= 1$

$x + y = 16 \Rightarrow (7,9), (8,8), (9,7)$

$\rightarrow$ T.N. $= 3$

$x + y = 14 \Rightarrow (5,9), (6,8), (7,7), (8,6), (9,5)$

$\rightarrow$ T.N. $= 5$

$x + y = 12 \Rightarrow (3,9), (4,8), (5,7), (6,6) \ldots (9,3)$

$\rightarrow$ T.N. $= 7$

83. (576)

$|\vec{a}| = 4, |\vec{b}| = 3\ \theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})|^2 + 4(\vec{a} \cdot \vec{b})^2$

$|\vec{a} \times \vec{b} - \vec{b} \times \vec{a}|^2 + 4a^2\, b^2 \cos^2\theta$

$2|\vec{a} \times \vec{b}|^2 + 4a^2b^2\cos^2\theta$

$4a^2b^2\sin^2\theta + 4a^2b^2\cos^2\theta$

$4a^2b^2 = 4 \times 16 \times 9 = 576$

84. (7) $2x^2 - rx + p = 0 < x_1$

$y^2 - sy - q = 0 < \begin{matrix} y_2 \\ y_1 \end{matrix}$

Equation of the circle with PQ as diameter is $2(x^2+y^2)-rx-2sy+p-2q=0$

on comparing with the given equation

$\mathrm{r}=11, \mathrm{s}=7$

$\mathrm{p}-2\mathrm{q}=-22$

$\therefore 2\mathrm{r}+\mathrm{s}-2\mathrm{q}+\mathrm{p}=22+7-22=7$

85. (4) $\mathrm{x}\in\left(\frac{\pi}{4},\frac{7\pi}{4}\right)$

$14\operatorname{cosec}^2 x-2\sin^2 x=21-4\cos^2 x$

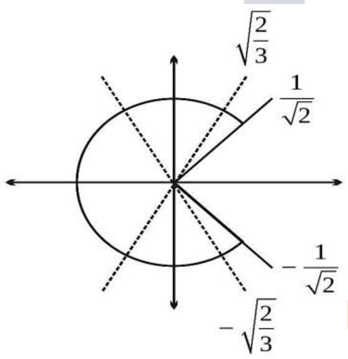
$=21-4(1-\sin^2 x)$

$=17+4\sin^2 x$

$14\operatorname{cosec}^2 x-6\sin^2 x=17$

let $\sin^2 \mathrm{x}=\mathrm{p}$

$$\frac{14}{p}-6p=17\Rightarrow 14-6p^2=17p$$
$$6p^2+17p-14=0$$
$$p=-3.5,\frac{2}{3}\Rightarrow \sin^2 x=\frac{2}{3}$$
$$\Rightarrow \sin x=\pm\sqrt{\frac{2}{3}}$$



86. (4) $\mathrm{a}_\mathrm{n}=19^\mathrm{n}-12^\mathrm{n}$

$$\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}=\frac{31(19^9-12^9)-(19^{10}-12^{10})}{57\alpha_8}$$
$$=\frac{19^9(31-19)-12^9(31-12)}{57\alpha_8}$$
$$=\frac{19^9\cdot 12-12^{19}\cdot 19}{57\alpha_8}$$
$$=\frac{12\cdot 19(19^8-12^8)}{57\alpha_8}=4$$

87. (2) $f(x) = \left[2\left(1 - \frac{x^{25}}{2}\right)(2 + x^{25})\right]^{\frac{1}{50}}$

$$f(x) = [(2 - x^{25})(2 + x^{25})]^{\frac{1}{50}}$$
$$= (4 - x^{50})^{1/50}$$

$$f(f(x)) = \left(4 - \left((4 - x^{50})^{1/50}\right)^{50}\right)^{1/50} = x$$

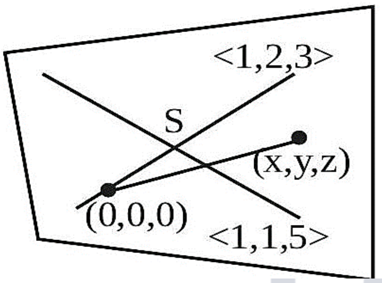
$g(x) = f(f(f(x))) + f(f(x))$

$= f(x) + x$

$g(1) = f(1) + 1 = 3^{1/50} + 1$

$[g(1)] = [3^{1/50} + 1] = 2$

88. (5) Both the lines lie in the same plane



$\therefore$ equation of the plane

$$\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix} = 0$$

$\Rightarrow 7x - 2y - z = 0$

$\therefore a + b + d = 5$

89. (414)

Case-I: $1 \rightarrow 7$ times and $-1 \rightarrow 2$ times

number of possible matrix $= \frac{9!}{7!2!} = 36$

Case-II: $1 \rightarrow 6$ times,

$-1 \rightarrow 1$ times

and $0 \rightarrow 2$ times

number of possible matrix $= \frac{9!}{6!2!} = 252$

Case-III: $1 \rightarrow 5$ times,

and $0 \rightarrow 4$ times

number of possible matrix $= \frac{9!}{5!4!} = 126$

Hence total number of all such matrix A $= 414$

90. (98) $\frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + \cdots.$

$\left(1 - \frac{2}{3}\right) + \left(1 - \frac{4}{9}\right) + \left(1 - \frac{8}{27}\right) + \left(1 - \frac{16}{81}\right)$ ... 100 terms

$$100 - \left[\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \cdots.\right]$$

$$100 - \frac{\frac{2}{3}\left(1 - \left(\frac{2}{3}\right)^{100}\right)}{1 - \frac{2}{3}}$$

$$100 - 2\left(1 - \left(\frac{2}{3}\right)^{100}\right)$$

$$S = 98 + 2\left(\frac{2}{3}\right)^{100}$$

$\Rightarrow [S] = 98$