

PART-I PHYSICS

SECTION-A

1. If $Z = \frac{A^2 B^3}{C^4}$, then the relative error in Z will be:

- (a) $\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$ (b) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} - \frac{4\Delta C}{C}$
(c) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}$ (d) $\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{\Delta C}{C}$

2. $\vec{A}$ is a vector quantity such that $|\vec{A}| = \text{nonzero constant}$. Which of the following expressions is true for $\vec{A}$?

- (a) $\vec{A} \cdot \vec{A} = 0$ (b) $\vec{A} \times \vec{A} < 0$
(c) $\vec{A} \times \vec{A} = 0$ (d) $\vec{A} \times \vec{A} > 0$

3. Which of the following relations is true for two- unit vectors $\hat{A}$ and $\hat{B}$ making an angle θ to each other?

- (a) $|\hat{A} + \hat{B}| = |\hat{A} - \hat{B}| \tan \frac{\theta}{2}$ (b) $|\hat{A} - \hat{B}| = |\hat{A} + \hat{B}| \tan \frac{\theta}{2}$
(c) $|\hat{A} + \hat{B}| = |\hat{A} - \hat{B}| \cos \frac{\theta}{2}$ (d) $|\hat{A} - \hat{B}| = |\hat{A} + \hat{B}| \cos \frac{\theta}{2}$

4. If force $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle having position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be:-

- (a) $3\hat{i} + 4\hat{j} - 2\hat{k}$ (b) $-10\hat{i} + 10\hat{j} + 5\hat{k}$
(c) $10\hat{i} + 5\hat{j} - 10\hat{k}$ (d) $10\hat{i} + \hat{j} - 5\hat{k}$

5. The height of any point P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at point P will be: (Given g = acceleration due to gravity at the surface of earth)

- (a) $g/2$ (b) $g/4$
(c) $g/3$ (d) $g/9$

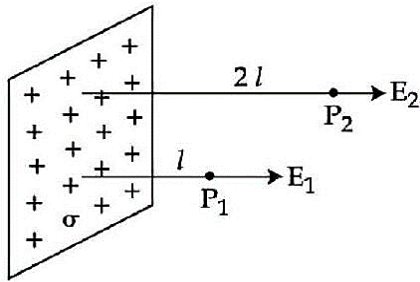
6. The terminal velocity (v_t) of the spherical rain drop depends on the radius (r) of the spherical rain drop as:

- (a) $r^{1/2}$ (b) r
(c) r^2 (d) r^3

7. The relation between root mean square speed (v_{rms}) and most probable speed (v_p) for the molar mass M of oxygen gas molecule at the temperature of 300 K will be :-

- (a) $v_{\text{rms}} = \sqrt{\frac{2}{3}} v_p$ (b) $v_{\text{rms}} = \sqrt{\frac{3}{2}} v_p$
(c) $v_{\text{rms}} = v_p$ (d) $v_{\text{rms}} = \sqrt{\frac{1}{3}} v_p$

8. In the figure, a very large plane sheet of positive charge is shown. P_1 and P_2 are two points at distance l and $2l$ from the charge distribution. If σ is the surface charge density, then the magnitude of electric fields E_1 and E_2 at P_1 and P_2 respectively are:



- (a) $E_1 = \sigma/\epsilon_0, E_2 = \sigma/2\epsilon_0$ (b) $E_1 = 2\sigma/\epsilon_0, E_2 = \sigma/\epsilon_0$
(c) $E_1 = E_2 = \sigma/2\epsilon_0$ (d) $E_1 = E_2 = \sigma/\epsilon_0$

9. Match List-I with List-II

List-I

List-II

- | | |
|--------------------|--|
| (A) AC generator | (I) Detects the presence of current in the circuit |
| (B) Galvanometer | (II) Converts mechanical energy into electrical energy |
| (C) Transformer | (III) Works on the principle of resonance in AC circuit |
| (D) Metal detector | (IV) Changes an alternating voltage for smaller or greater value |

Choose the correct answer from the options given below:-

- (a) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(b) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(c) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(d) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

10. A long straight wire with a circular cross-section having radius R , is carrying a steady current I . The current I is uniformly distributed across this cross-section. Then the variation of magnetic field due to current I with distance r ($r < R$) from its centre will be:

- (a) $B \propto r^2$ (b) $B \propto r$
(c) $B \propto \frac{1}{r^2}$ (d) $B \propto \frac{1}{r}$

11. If wattless current flows in the AC circuit, then the circuit is

- (a) Purely Resistive circuit (b) Purely Inductive circuit
(c) LCR series circuit (d) RC series circuit only

12. The electric field in an electromagnetic wave is given by $E = 56.5 \sin \omega(t - x/c) \text{ NC}^{-1}$. Find the intensity of the wave if it is propagating along x-axis in the free space. (Given $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$)

- (a) 5.65 Wm^{-2} (b) 4.24 Wm^{-2}
(c) $1.9 \times 10^{-7} \text{ Wm}^{-2}$ (d) 56.5 Wm^{-2}

13. The two light beams having intensities I and $9I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point P and π at point Q. Then the difference between the resultant intensities at P and Q will be:

- (a) $2I$ (b) $6I$
(c) $5I$ (d) $7I$

14. A light wave travelling linearly in a medium of dielectric constant 4, incident on the horizontal interface separating medium with air. The angle of incidence for which the total intensity of incident wave will be reflected back into the same medium will be (Given : relative permeability of medium $\mu_r = 1$)

- (a) 10° (b) 20°
(c) 30° (d) 60°

15. Given below are two statements:-

Statement I: Davisson-Germer experiment establishes the wave nature of electrons.

Statement II: If electrons have wave nature, they can interfere and show diffraction.

In the light of the above statements choose the correct answer from the options given below: -

- (a) Both Statement I and Statement II are true
(b) Both Statement I and Statement II are false
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

16. The ratio for the speed of the electron in the 3rd orbit of He^+ to the speed of the electron in the 3rd orbit of hydrogen atom will be :-

- (a) 1:1 (b) 1:2
(c) 4:1 (d) 2:1

17. The photodiode is used to detect the optical signals. These diodes are preferably operated in reverse biased mode because.

- (a) fractional change in majority carriers produce higher forward bias current
(b) fractional change in majority carriers produce higher reverse bias current
(c) fractional change in minority carriers produce higher forward bias current
(d) fractional change in minority carriers produce higher reverse bias current

18. A signal of 100THz frequency can be transmitted with maximum efficiency by:

- (a) Coaxial cable (b) Optical fibre
(c) Twisted pair of copper wires (d) Water

19. The difference of speed of light in the two media A and B ($v_A - v_B$) is $2.6 \times 10^7 \text{ m/s}$. If the refractive index of medium B is 1.47, then the ratio of refractive index of medium B to medium A is : (Given : speed of light in vacuum $c = 3 \times 10^8 \text{ ms}^{-1}$)

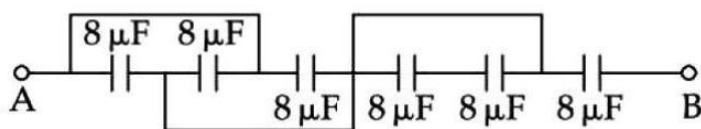
- (a) 1.303 (b) 1.318
(c) 1.13 (d) 0.12

20. A teacher in his physics laboratory allotted an experiment to determine the resistance (G) of a galvanometer. Students took the observations for $\frac{1}{3}$ deflection in the galvanometer. Which of the below is true for measuring value of G ?

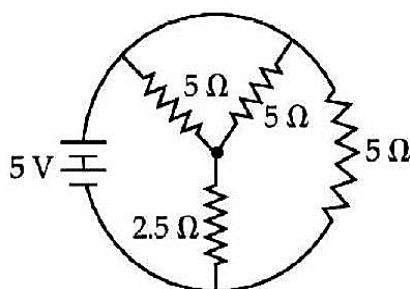
- (a) $\frac{1}{3}$ deflection method cannot be used for determining the resistance of the galvanometer.
- (b) $\frac{1}{3}$ deflection method can be used and, in this case, the G equals to twice the value of shunt resistance(s).
- (c) $\frac{1}{3}$ deflection method can be used and, in this case,, the G equals to three times the value of shunt resistance(s)
- (d) $\frac{1}{3}$ deflection method can be used and, in this case, the G value equals to the shunt resistance(s).

SECTION-B

- 21. A uniform chain of 6 m length is placed on a table such that a part of its length is hanging over the edge of the table. The system is at rest. The co-efficient of static friction between the chain and the surface of the table is 0.5, the maximum length of the chain hanging from the table is m.
- 22. A 0.5 kg block moving at a speed of 12 ms^{-1} compresses a spring through a distance 30 cm when its speed is halved. The spring constant of the spring will be Nm^{-1} .
- 23. The velocity of upper layer of water in a river is 36 kmh^{-1} . Shearing stress between horizontal layers of water is 10^{-3} Nm^{-2} . Depth of the river is m. (Co-efficiency of viscosity of water is 10^{-2} Pa. s)
- 24. A steam engine intake 50 g of steam at 100°C per minute and cools it down to 20°C . If latent heat of vaporization of steam is 540 calg^{-1} , then the heat rejected by the steam engine per minute is $\times 10^3 \text{ cal}$
- 25. The first overtone frequency of an open organ pipe is equal to the fundamental frequency of a closed organ pipe. If the length of the closed organ pipe is 20 cm. The length of the open organ pipe is cm.
- 26. The equivalent capacitance between points A and B in below shown figure will be μF .



- 27. A resistor develops 300 J of thermal energy in 15 s, when a current of 2 A is passed through it. If the current increases to 3 A, the energy developed in 10 s is J.
- 28. The total current supplied to the circuit as shown in figure by the 5 V battery is A



29. The current in a coil of self-inductance 2.0H is increasing according to $I = 2\sin(t^2)\text{A}$. The amount of energy spent during the period when current changes from 0 to 2A is J .
30. A force on an object of mass 100g is $(10\hat{i} + 5\hat{j})\text{N}$. The position of that object at $t = 2\text{s}$ is $(a\hat{i} + b\hat{j})\text{m}$ after starting from rest. The value of $\frac{a}{b}$ will be

PART-II CHEMISTRY

SECTION-A

31. Bonding in which of the following diatomic molecule(s) become(s) stronger, on the basis of MO Theory, by removal of an electron?

(A) NO (B) N_2 (C) O_2
(D) C_2 (E) B_2

Choose the most appropriate answer from the options given below: -

- (a) (A), (B), (C) only (b) (B), (C), (E) only
(c) (A), (C) only (d) (D) only

32. Incorrect statement for Tyndall effect is:-

- (a) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.
(b) The diameter of the dispersed particles is much smaller than the wavelength of the light used.
(c) During projection of movies in the cinemas hall, Tyndall effect is noticed.
(d) It is used to distinguish a true solution from a colloidal solution.

33. The pair, in which ions are isoelectronic with Al^{3+} is :-

(a) Br^- and Be^{2+} (b) Cl^- and Li^+
(c) S^{2-} and K^+ (d) O^{2-} and Mg^{2+}

34. Leaching of gold with dilute aqueous solution of NaCN in presence of oxygen gives complex [A], which on reaction with zinc forms the elemental gold and another complex [B]. [A] and [B], respectively are:-

- (a) $[\text{Au}(\text{CN})_4]^-$ and $[\text{Zn}(\text{CN})_2(\text{OH})_2]^{2-}$
(b) $[\text{Au}(\text{CN})_2]^-$ and $[\text{Zn}(\text{OH})_4]^{2-}$
(c) $[\text{Au}(\text{CN})_2]^-$ and $[\text{Zn}(\text{CN})_4]^{2-}$
(d) $[\text{Au}(\text{CN})_4]^{2-}$ and $[\text{Zn}(\text{CN})_6]^{4-}$

35. Number of electron deficient molecules among the following PH_3 , B_2H_6 , CCl_4 , NH_3 , LiH and BCl_3 is

(a) 0 (b) 1
(c) 2 (d) 3

36. Which one of the following alkaline earth metal ions has the highest ionic mobility in its aqueous solution?

(a) Be^{2+} (b) Mg^{2+}
(c) Ca^{2+} (d) Sr^{2+}

37. White precipitate of AgCl dissolves in aqueous ammonia solution due to formation of :

(a) $[\text{Ag}(\text{NH}_3)_4]\text{Cl}_2$ (b) $[\text{Ag}(\text{Cl})_2(\text{NH}_3)_2]$
(c) $[\text{Ag}(\text{NH}_3)_2]\text{Cl}$ (d) $[\text{Ag}(\text{NH}_3)\text{Cl}]\text{Cl}$

38. Cerium (IV) has a noble gas configuration. Which of the following is correct statement about it?

- (a) It will not prefer to undergo redox reactions.

- (b) It will prefer to gain electron and act as an oxidizing agent
- (c) It will prefer to give away an electron and behave as reducing agent
- (d) It acts as both, oxidizing and reducing agent.

39. Among the following, which is the strongest oxidizing agent?

- (a) Mn^{3+} (b) Fe^{3+}
(c) Ti^{3+} (d) Cr^{3+}

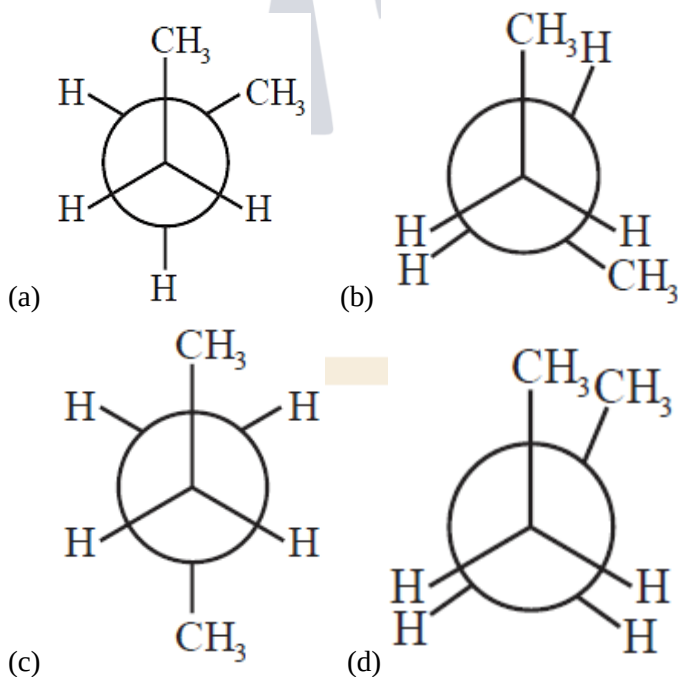
40. The eutrophication of water body results in:

- (a) loss of Biodiversity
- (b) breakdown of organic matter
- (c) increase in biodiversity
- (d) decrease in BOD.

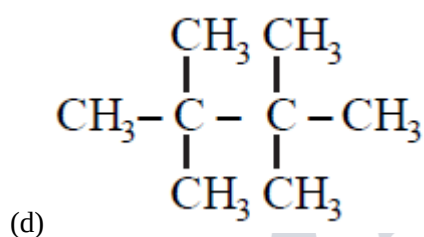
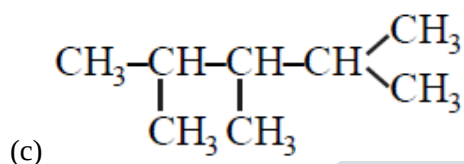
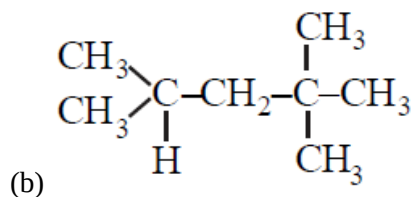
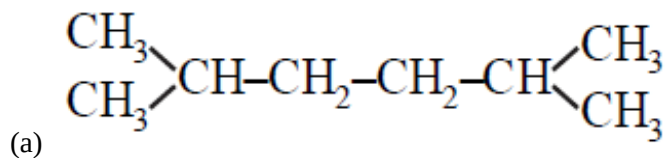
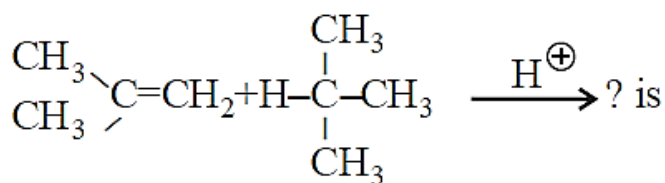
41. Phenol on reaction with dilute nitric acid, gives two products. Which method will be most effective for large scale separation?

- (a) Chromatographic separation
- (b) Fractional Crystallisation
- (c) Steam distillation
- (d) Sublimation

42. In the following structures, which one is having staggered conformation with maximum dihedral angle?



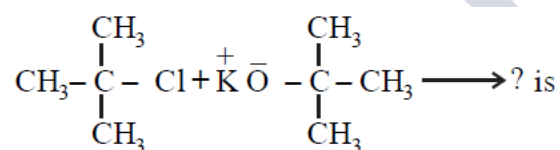
43. The products formed in the following reaction.



44. The IUPAC name of ethylidene chloride is:

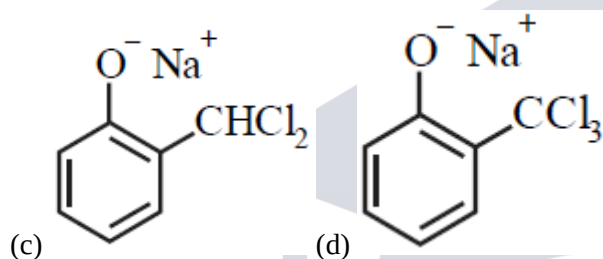
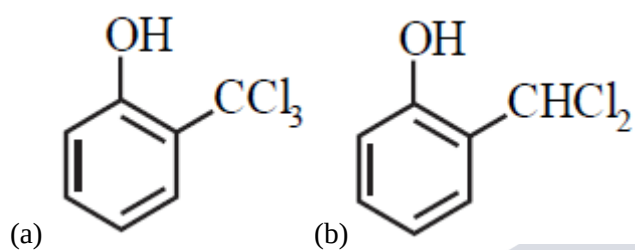
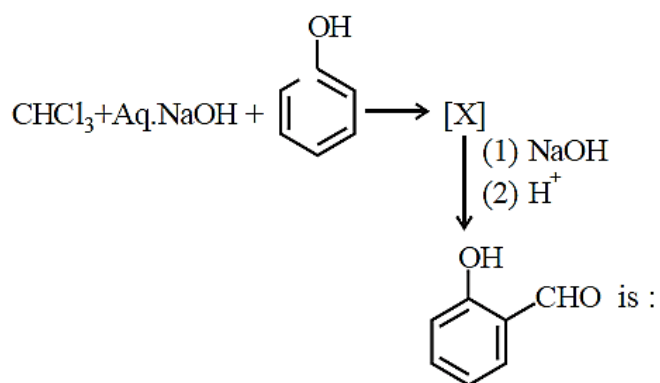
- (a) 1-Chloroethene (b) 1-Chloroethyne
(c) 1,2-Dichloroethane (d) 1,1-Dichloroethane

45. The major product in the reaction

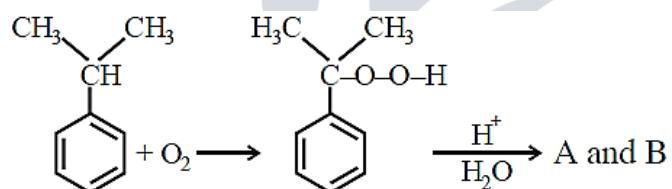


- (a) t-Butyl ethyl ether (b) 2,2-Dimethyl butane
(c) 2-Methyl pent-1-ene (d) 2-Methyl prop-1-ene

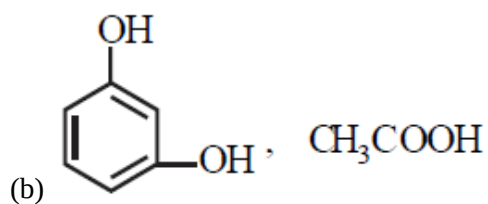
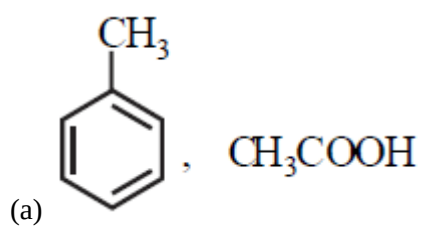
46. The intermediate X, in the reaction

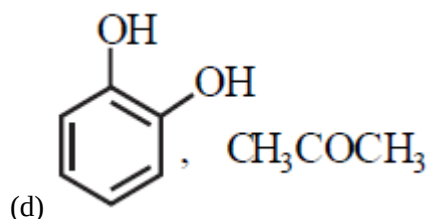
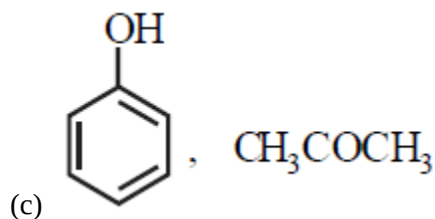


47. In the following reaction:



The compounds A and B respectively are:-





48. The reaction of $\text{R}-\text{C}(=\text{O})-\text{NH}_2$ with bromine and KOH gives RNH_2 as the end product. Which one of the following is the intermediate product formed in this reaction?

- (a) $\text{R}-\text{C}(=\text{O})-\text{NH}-\text{Br}$ (b) $\text{R}-\text{NH}-\text{Br}$
(c) $\text{R}-\text{N}=\text{C}=\text{O}$ (d) $\text{R}-\text{C}(=\text{O})-\text{NBr}_2$

49. Using very little soap while washing clothes, does not serve the purpose of cleaning of clothes because

- (a) soap particles remain floating in water as ions
(b) the hydrophobic part of soap is not able to take away grease
(c) the micelles are not formed due to concentration of soap, below its CMC value
(d) colloidal structure of soap in water is completely disturbed.

50. Which one of the following is an example of artificial sweetner?

- (a) Bithional (b) Alitame
(c) Salvarsan (d) Lactose

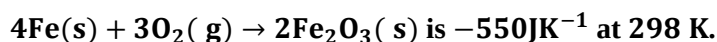
SECTION-B

51. The number of N atoms in 681 g of $\text{C}_7\text{H}_5\text{N}_3\text{O}_6$ is $x \times 10^{21}$. The value of x is ($N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$) (Nearest Integer)

52. The distance between Na^+ and Cl^- ions in solid NaCl of density 43.1 g cm^{-3} is $x \times 10^{-10} \text{ m}$. (Nearest Integer) (Given: $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$)

53. The longest wavelength of light that can be used for the ionisation of lithium atom (Li) in its ground state is $x \times 10^{-8} \text{ m}$. The value of x is (Nearest Integer) (Given: Energy of the electron in the first shell of the hydrogen atom is $-2.2 \times 10^{-18} \text{ J}$; $h = 6.63 \times 10^{-34} \text{ Js}$ and $c = 3 \times 10^8 \text{ ms}^{-1}$)

54. The standard entropy change for the reaction

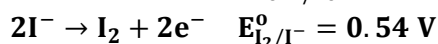


[Given: The standard enthalpy change for the reaction is -165 kJ mol^{-1}]. The temperature in K at which the reaction attains equilibrium is (Nearest Integer)

55. 1 L aqueous solution of H_2SO_4 contains 0.02 mmol H_2SO_4 . 50% of this solution is diluted with deionized water to give 1 L solution (a). In solution (a), 0.01 m mol of H_2SO_4 are added. Total m mols of H_2SO_4 in the final solution is $\times 10^3 \text{ m mols}$.

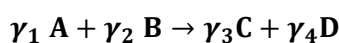
56. The standard free energy change (ΔG°) for 50% dissociation of N_2O_4 into NO_2 at 27°C and 1 atm pressure is $-x\text{Jmol}^{-1}$. The value of x is (Nearest Integer) [Given: $R = 8.31\text{ J K}^{-1}\text{ mol}^{-1}$, $\log 1.33 = 0.1239$, $\ln 10 = 2.3$]

57. In a cell, the following reactions take place



The standard electrode potential for the spontaneous reaction in the cell is $x \times 10^{-2}\text{ V}$ at 298 K. The value of x is (Nearest Integer)

58. For a given chemical reaction



Concentration of C changes from 10mmol dm^{-3} to 20mmol dm^{-3} in 10 seconds. Rate of appearance of D is 1.5 times the rate of disappearance of B which is twice the rate of disappearance A. The rate of appearance of D has been experimentally determined to be $9\text{mmol dm}^{-3}\text{ s}^{-1}$. Therefore the rate of reaction is $\text{mmol dm}^{-3}\text{ s}^{-1}$. (Nearest Integer)

59. If $[\text{Cu}(\text{H}_2\text{O})_4]^{2+}$ absorbs a light of wavelength 600 nm for d – d transition, then the value of octahedral crystal field splitting energy for $[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$ will be $\times 10^{-21}\text{ J}$. (Nearest Integer) (Given: $h = 6.63 \times 10^{-34}\text{ J s}$ and $c = 3.08 \times 10^8\text{ ms}^{-1}$)

60. Number of grams of bromine that will completely react with 5.0 g of pent-1-ene is $x \times 10^{-2}\text{ g}$. (Atomic mass of Br = 80 g/mol) (Nearest Integer)

PART-III MATHEMATICS

SECTION-A

61. Let a circle C touch the lines $L_1: 4x - 3y + K_1 = 0$ and $L_2: 4x - 3y + K_2 = 0$, $K_1, K_2 \in \mathbb{R}$. If a line passing through the centre of the circle C intersects L_1 at $(-1, 2)$ and L_2 at $(3, -6)$, then the equation of the circle C is

(a) $(x - 1)^2 + (y - 2)^2 = 4$

(b) $(x + 1)^2 + (y - 2)^2 = 4$

(c) $(x - 1)^2 + (y + 2)^2 = 16$

(d) $(x - 1)^2 + (y - 2)^2 = 16$

62. The value of $\int_0^\pi \frac{e^{\cos x} \sin x}{(1 + \cos^2 x)(e^{\cos x} + e^{-\cos x})} dx$ is equal to

(a) $\frac{\pi^2}{4}$

(b) $\frac{\pi^2}{2}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

63. Let a, b and c be the length of sides of a triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of $\frac{R}{r}$ is equal to

- (a) $\frac{5}{2}$ (b) 2
(c) $\frac{3}{2}$ (d) 1

64. Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = 2f(x)f(y)$ for natural numbers x and y . If $f(1) = 2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}(2^{20} - 1) \text{ holds, is}$$

- (a) 2 (b) 3
(c) 4 (d) 6

65. Let A be a 3×3 real matrix such that

$$A \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}; A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \text{ and } A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix

of order 3, then the system $(A - 2I)X = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$ has

- (a) no solution
(b) infinitely many solutions
(c) unique solution
(d) exactly two solutions

66. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x^3 + x - 5$. If $g(x)$ is a function such that $f(g(x)) = x, \forall x \in \mathbb{R}$, then $g'(63)$ is equal to

- (a) $\frac{1}{49}$ (b) $\frac{3}{49}$
(c) $\frac{43}{49}$ (d) $\frac{91}{49}$

67. Consider the following two propositions:

$$P1: \sim (p \rightarrow \sim q)$$

$$P2: (p \wedge \sim q) \wedge ((\sim p) \vee q)$$

If the proposition $p \rightarrow ((\sim p) \vee q)$ is evaluated as FALSE, then:

- (a) P1 is TRUE and P2 is FALSE
(b) P1 is FALSE and P2 is TRUE
(c) Both P1 and P2 are FALSE
(d) Both P1 and P2 are TRUE

68. If $\frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \dots + \frac{1}{2^{10} \cdot 3} = \frac{K}{2^{10} \cdot 3^{10}}$, then the remainder when K is divided by 6 is

- (a) 1 (b) 2
(c) 3 (d) 5

69. Let $f(x)$ be a polynomial function such that $f(x) + f'(x) + f''(x) = x^5 + 64$. Then, the value of $\lim_{x \rightarrow 1} \frac{f(x)}{x-1}$

- (a) -15 (b) -60
(c) 60 (d) 15

70. Let E_1 and E_2 be two events such that the conditional probabilities $P(E_1 | E_2) = \frac{1}{2}$, $P(E_2 | E_1) = \frac{3}{4}$ and $P(E_1 \cap E_2) = \frac{1}{8}$. Then:

- (a) $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$
- (b) $P(E'_1 \cap E'_2) = P(E'_1) \cdot P(E'_2)$
- (c) $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$
- (d) $P(E'_1 \cap E_2) = P(E_1) \cdot P(E_2)$

71. Let $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is

- (a) a non-identity symmetric matrix
- (b) a skew-symmetric matrix
- (c) neither symmetric nor skew-symmetric matrix
- (d) an identity matrix

72. Let $g: (0, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that

$$\int \left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2} \right) dx = \frac{xg(x)}{e^x + 1} + c,$$

for all $x > 0$, where c is an arbitrary constant. Then.

- (a) g is decreasing in $(0, \frac{\pi}{4})$
- (b) g' is increasing in $(0, \frac{\pi}{4})$
- (c) $g + g'$ is increasing in $(0, \frac{\pi}{2})$
- (d) $g - g'$ is increasing in $(0, \frac{\pi}{2})$

73. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x) = \frac{1 - 2e^{2x}}{e^x}$.

Then, for which of the following range of α , the inequality $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$ holds?

- (a) $(2, 3)$
- (b) $(-2, -1)$
- (c) $(1, 2)$
- (d) $(-1, 1)$

74. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $a_i > 0$, $i = 1, 2, 3$ be a vector which makes equal angles with the coordinate axes OX , OY and OZ . Also, let the projection of $\vec{a}$ on the vector $3\hat{i} + 4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90° . If $\vec{a}$, $\vec{b}$ and x -axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is equal to

- (a) $\sqrt{7}$
- (b) $\sqrt{2}$
- (c) 2
- (d) 7

75. Let $y = y(x)$ be the solution of the differential equation $(x + 1)y' - y = e^{3x}(x + 1)^2$, with

$y(0) = \frac{1}{3}$. Then, the point $x = -\frac{4}{3}$ for the curve $y = y(x)$ is:

- (a) not a critical point
- (b) a point of local minima

(c) a point of local maxima

(d) a point of inflection

76. If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1 m_2|$ is equal to

- (a) $3 + 4\sqrt{2}$ (b) $-5 + 6\sqrt{2}$
(c) $-4 + 3\sqrt{2}$ (d) $7 + 6\sqrt{2}$

77. Let Q be the mirror image of the point P(1, 0, 1) with respect to the plane S: $x + y + z = 5$. If a line L passing through (1, -1, -1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to:

- (a) 2 (b) 5
(c) 7 (d) 11

78. If the solution curve $y = y(x)$ of the differential equation $y^2 dx + (x^2 - xy + y^2) dy = 0$, which passes through the point (1, 1) and intersects the line $y = \sqrt{3}x$ at the point $(\alpha, \sqrt{3}\alpha)$, then value of $\log_e(\sqrt{3}\alpha)$ is equal to

- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{12}$ (d) $\frac{\pi}{6}$

79. Let $x = 2t$, $y = \frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that $SA \perp BA$, where A is any point on the conic. If k is the ordinate of the centroid of ΔSAB , then $\lim_{t \rightarrow 1} k$ is equal to

- (a) $\frac{17}{18}$ (b) $\frac{19}{18}$
(c) $\frac{11}{18}$ (d) $\frac{13}{18}$

80. Let a circle C in complex plane pass through the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z (\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3 , then $\arg(z)$ is equal to :

- (a) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \pi$ (b) $\tan^{-1}\left(\frac{24}{7}\right) - \pi$
(c) $\tan^{-1}(3) - \pi$ (d) $\tan^{-1}\left(\frac{3}{4}\right) - \pi$

SECTION-B

81. Let C_r denote the binomial coefficient of x^r in the expansion of $(1+x)^{10}$. If $\alpha, \beta \in \mathbb{R}$. $C_1 + 3 \cdot 2C_2 + 5 \cdot 3C_3 + \dots$ upto 10 terms $= \frac{\alpha \times 2^{11}}{2^{\beta}-1} \left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots \right)$ upto 10 terms) then the value of $\alpha + \beta$ is equal to

82. The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is

83. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4$, $|\vec{b}| = 3$ $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$. Then $|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})|^2 + 4(\vec{a} \cdot \vec{b})^2$ is equal to

84. Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to

85. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14 \operatorname{cosec}^2 x - 2 \sin^2 x = 21 - 4 \cos^2 x$ holds, is

86. For a natural number n , let $a_n = 19^n - 12^n$. Then, the value of $\frac{31a_9 - a_{10}}{57a_8}$ is

87. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \left(2 \left(1 - \frac{x^{25}}{2}\right) (2 + x^{25})\right)^{\frac{1}{50}}$. If the function $g(x) = f(f(f(x))) + f(f(x))$, the the greatest integer less than or equal to $g(1)$ is

88. Let the lines

$$L_1: \vec{r} = \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in \mathbb{R}$$

$$L_2: \vec{r} = (\hat{i} + 3\hat{j} + \hat{k}) + \mu(\hat{i} + \hat{j} + 5\hat{k}); \mu \in \mathbb{R}$$

intersect at the point S . If a plane $ax + by - z + d = 0$ passes through S and is parallel to both the lines L_1 and L_2 , then the value of $a + b + d$ is equal to

89. Let A be a 3×3 matrix having entries from the set $\{-1, 0, 1\}$. The number of all such matrices A having sum of all the entries equal to 5, is

90. The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, \dots$ is equal to

